Sub-string/Pattern Matching in Sub-Linear Time Using a Sparse Fourier Transform Approach

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ABSTRACT

We consider the problem of querying a string (or, a database) of length N bits to determine all the locations where a substring (query) of length M appears either exactly or is within a Hamming distance of K from the query. We assume that sketches of the original signal can be computed off line and stored. Using the sparse Fourier transform computation based approach introduced by Pawar and Ramchandran, we show that all such matches can be determined with high probability in sub-linear time. Specifically, if the query length $M = N^{\mu}$ and the number of matches is $L = N^{\lambda}$, as $N \to \infty$, we show that for $\lambda < 1 - \mu$, all the matching positions can be determined with a probability that approaches 1 as $N \to \infty$ for $K \leq \frac{1}{6}M$. More importantly our scheme has a worst-case computational complexity that is only $O\left(N^{\max(\alpha, \lambda+1-\alpha)} \log^2 N\right)$ where $\alpha = \max(\mu, 1-\mu)$ which means we can recover all the matching positions in *sub-linear* time for $\lambda < 1 - \mu$. Further, the number of Fourier transform coefficients that need to be computed, stored and accessed, i.e., the sketching complexity of this algorithm is only $O\left(N^{\max(\mu, 1-\mu)}\right)$. Several extensions of the main theme are also discussed.

CCS CONCEPTS

•**Computer systems organization** →**Embedded systems;** *Redundancy;* Robotics; •**Networks** →Network reliability;

KEYWORDS

Pattern Matching, String Matching, Big Data, Sub-linear time Algorithms, Sparse Signal Processing, Database, Information Retrieval

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*The full version of the author's guide is available as `acmart.pdf` document

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1 INTRODUCTION

The *proceedings* are the records of a conference[1]. ACM seeks to give these conference by-products a uniform, high-quality appearance. To do this, ACM has some rigid requirements for the format of the proceedings documents: there is a specified format (balanced double columns), a specified set of fonts (Arial or Helvetica and Times Roman) in certain specified sizes, a specified live area, centered on the page, specified size of margins, specified column width and gutter size.

2 THE BODY OF THE PAPER

Typically, the body of a paper is organized into a hierarchical structure, with numbered or unnumbered headings for sections, subsections, sub-subsections, and even smaller sections. The command `\section` that precedes this paragraph is part of such a hierarchy.[2] LaTeX handles the numbering and placement of these headings for you, when you use the appropriate heading commands around the titles of the headings. If you want a sub-subsection or smaller part to be unnumbered in your output, simply append an asterisk to the command name. Examples of both numbered and unnumbered headings will appear throughout the balance of this sample document.

Because the entire article is contained in the **document** environment, you can indicate the start of a new paragraph with a blank line in your input file; that is why this sentence forms a separate paragraph.

2.1 Type Changes and *Special* Characters

We have already seen several typeface changes in this sample. You can indicate italicized words or phrases in your text with the command `\textit`; emboldening with the command `\textbf` and typewriter-style (for instance, for computer code) with `\texttt`. But remember, you do not have to indicate typestyle changes when such changes are part of the *structural* elements of your article; for instance, the heading of this subsection will be in a sans serif[3] typeface, but that is handled by the document class file. Take care

[1] This is a footnote
[2] This is a footnote.
[3] Another footnote, here. Let's make this a rather short one to see how it looks.

with the use of[4] the curly braces in typeface changes; they mark the beginning and end of the text that is to be in the different typeface.

You can use whatever symbols, accented characters, or non-English characters you need anywhere in your document; you can find a complete list of what is available in the *LaTeX User's Guide* [7].

2.2 Math Equations

You may want to display math equations in three distinct styles: inline, numbered or non-numbered display. Each of the three are discussed in the next sections.

2.2.1 Inline (In-text) Equations. A formula that appears in the running text is called an inline or in-text formula. It is produced by the **math** environment, which can be invoked with the usual `\begin . . . \end` construction or with the short form `$ . . . $`. You can use any of the symbols and structures, from α to ω, available in LaTeX [7]; this section will simply show a few examples of in-text equations in context. Notice how this equation: $\lim_{n\to\infty} x = 0$, set here in in-line math style, looks slightly different when set in display style. (See next section).

2.2.2 Display Equations. A numbered display equation—one set off by vertical space from the text and centered horizontally—is produced by the **equation** environment. An unnumbered display equation is produced by the **displaymath** environment.

Again, in either environment, you can use any of the symbols and structures available in LaTeX; this section will just give a couple of examples of display equations in context. First, consider the equation, shown as an inline equation above:

$$\lim_{n\to\infty} x = 0 \tag{1}$$

Notice how it is formatted somewhat differently in the **displaymath** environment. Now, we'll enter an unnumbered equation:

$$\sum_{i=0}^{\infty} x + 1$$

and follow it with another numbered equation:

$$\sum_{i=0}^{\infty} x_i = \int_0^{\pi+2} f \tag{2}$$

just to demonstrate LaTeX's able handling of numbering.

2.3 Citations

Citations to articles [2–4, 6], conference proceedings [4] or maybe books [7, 8] listed in the Bibliography section of your article will occur throughout the text of your article. You should use BibTeX to automatically produce this bibliography; you simply need to insert one of several citation commands with a key of the item cited in the proper location in the `.tex` file [7]. The key is a short reference you invent to uniquely identify each work; in this sample document, the key is the first author's surname and a word from the title. This identifying key is included with each item in the `.bib` file for your article.

The details of the construction of the `.bib` file are beyond the scope of this sample document, but more information can be found in the *Author's Guide*, and exhaustive details in the *LaTeX User's Guide* by Lamport [7].

This article shows only the plainest form of the citation command, using `\cite`.

[4] A third, and last, footnote.

Table 1: Frequency of Special Characters

Non-English or Math	Frequency	Comments
Ø	1 in 1,000	For Swedish names
π	1 in 5	Common in math
\$	4 in 5	Used in business
Ψ_1^2	1 in 40,000	Unexplained usage

2.4 Tables

Because tables cannot be split across pages, the best placement for them is typically the top of the page nearest their initial cite. To ensure this proper "floating" placement of tables, use the environment **table** to enclose the table's contents and the table caption. The contents of the table itself must go in the **tabular** environment, to be aligned properly in rows and columns, with the desired horizontal and vertical rules. Again, detailed instructions on **tabular** material are found in the *LaTeX User's Guide*.

Immediately following this sentence is the point at which Table 1 is included in the input file; compare the placement of the table here with the table in the printed output of this document.

To set a wider table, which takes up the whole width of the page's live area, use the environment **table*** to enclose the table's contents and the table caption. As with a single-column table, this wide table will "float" to a location deemed more desirable. Immediately following this sentence is the point at which Table 2 is included in the input file; again, it is instructive to compare the placement of the table here with the table in the printed output of this document.

It is strongly recommended to use the package booktabs [5] and follow its main principles of typography with respect to tables:

(1) Never, ever use vertical rules.
(2) Never use double rules.

It is also a good idea not to overuse horizontal rules.

2.5 Figures

Like tables, figures cannot be split across pages; the best placement for them is typically the top or the bottom of the page nearest their initial cite. To ensure this proper "floating" placement of figures, use the environment **figure** to enclose the figure and its caption.

This sample document contains examples of `.eps` files to be displayable with LaTeX. If you work with pdfLaTeX, use files in the `.pdf` format. Note that most modern TeX systems will convert `.eps` to `.pdf` for you on the fly. More details on each of these are found in the *Author's Guide*.

As was the case with tables, you may want a figure that spans two columns. To do this, and still to ensure proper "floating" placement of tables, use the environment **figure*** to enclose the figure and its caption. And don't forget to end the environment with **figure***, not **figure**!

Table 2: Some Typical Commands

Command	A Number	Comments
`\author`	100	Author
`\table`	300	For tables
`\table*`	400	For wider tables

Figure 1: A sample black and white graphic.

Figure 2: A sample black and white graphic that has been resized with the `includegraphics` command.

2.6 Theorem-like Constructs

Other common constructs that may occur in your article are the forms for logical constructs like theorems, axioms, corollaries and proofs. ACM uses two types of these constructs: theorem-like and definition-like.

Here is a theorem:

Theorem 2.1. *Let f be continuous on $[a, b]$. If G is an antiderivative for f on $[a, b]$, then*

$$\int_a^b f(t)\,dt = G(b) - G(a).$$

Here is a definition:

Definition 2.2. If z is irrational, then by e^z we mean the unique number that has logarithm z:

$$\log e^z = z.$$

The pre-defined theorem-like constructs are **theorem**, **conjecture**, **proposition**, **lemma** and **corollary**. The pre-defined definition-like constructs are **example** and **definition**. You can add your own constructs using the *amsthm* interface [1]. The styles used in the `\theoremstyle` command are **acmplain** and **acmdefinition**.

Another construct is **proof**, for example,

Proof. Suppose on the contrary there exists a real number L such that

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = L.$$

Then

$$l = \lim_{x\to c} f(x) = \lim_{x\to c} \left[gx \cdot \frac{f(x)}{g(x)}\right] = \lim_{x\to c} g(x) \cdot \lim_{x\to c} \frac{f(x)}{g(x)} = 0 \cdot L = 0,$$

which contradicts our assumption that $l \neq 0$. □

3 CONCLUSIONS

This paragraph will end the body of this sample document. Remember that you might still have Acknowledgments or Appendices; brief samples of these follow. There is still the Bibliography to deal with; and we will make a disclaimer about that here: with the exception of the reference to the LaTeX book, the citations in this paper are to articles which have nothing to do with the present subject and are used as examples only.

A HEADINGS IN APPENDICES

The rules about hierarchical headings discussed above for the body of the article are different in the appendices. In the **appendix** environment, the command **section** is used to indicate the start of each Appendix, with alphabetic order designation (i.e., the first is A, the second B, etc.) and a title (if you include one). So, if you need hierarchical structure *within* an Appendix, start with **subsection** as the highest level. Here is an outline of the body of this document in Appendix-appropriate form:

A.1 Introduction

A.2 The Body of the Paper

A.2.1 Type Changes and Special Characters.

A.2.2 Math Equations.

Inline (In-text) Equations.

Display Equations.

A.2.3 Citations.

A.2.4 Tables.

A.2.5 Figures.

A.2.6 Theorem-like Constructs.

A Caveat for the TeX Expert.

A.3 Conclusions

A.4 References

Generated by bibtex from your `.bib` file. Run latex, then bibtex, then latex twice (to resolve references) to create the `.bbl` file. Insert that `.bbl` file into the `.tex` source file and comment out the command `\thebibliography`.

B MORE HELP FOR THE HARDY

Of course, reading the source code is always useful. The file `acmart.pdf` contains both the user guide and the commented code.

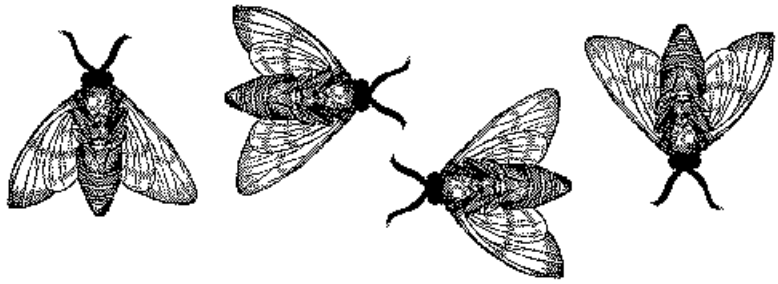

Figure 3: A sample black and white graphic that needs to span two columns of text.

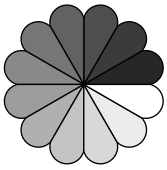

Figure 4: A sample black and white graphic that has been resized with the `includegraphics` command.

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