Applications of Spatial Coupling & Sparse Graph Codes for Sparse Recovery

Avinash Vem

Department of Electrical and Computer Engineering
Texas A&M University

Dec 02, 2016

Outline

1. Spatial Coupling(SC)
   - SC-LDPC Ensemble
   - Threshold Saturation Phenomenon

2. SC-LDPC Lattices
   - Introduction to Lattices
   - Proposed Lattice Construction
   - Poltyrev Goodness
   - Symmetric Interference Channel

3. Side-Information Problems
   - Gelfand-Pinsker & Wyner-Ziv
   - Compound LDGM/LDPC Codes
   - Spatial Coupling of Compound Codes

4. Write-Once Memory
   - Problem Statement
   - Coding Scheme

5. Research Summary

Outline

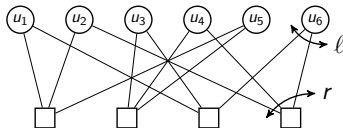# An (ℓ, r) LDPC Code

Tanner Graph

Parity-Check Matrix

$$H = \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$\ell = 2 \qquad r = 3$$

LDPC Code $\mathcal{C} = \{x : H \odot x = 0\}$

Compressed Representation

Belief Propagation Decoder & Threshold

Belief Propagation (BP)

- Popular choice
- Low-complexity
- Threshold: $\mathbf{h}^{\mathrm{BP}}$

Maximum a Posteriori (MAP)

- Optimal decoder
- *Not realizable*
- Threshold: $\mathbf{h}^{\mathrm{MAP}}$

$$\mathbf{h}^{\mathrm{BP}} < \mathbf{h}^{\mathrm{MAP}}$$

Threshold Comparison

LDPC	Shannon	AWGN		BSC	
(ℓ, r)	$\mathtt{h}^{\mathrm{Sh}}$	$\mathtt{h}^{\mathrm{BP}}$	$\mathtt{h}^{\mathrm{MAP}}$	$\mathtt{h}^{\mathrm{BP}}$	$\mathtt{h}^{\mathrm{MAP}}$
$(3, 6)$	0.5000	0.4293	0.4794	0.4160	0.4681
$(4, 6)$	0.6667	0.5211	0.6645	0.5203	0.6633
$(5, 6)$	0.8333	0.5731	0.8333	0.5773	0.8333

Threshold Comparison

LDPC	Shannon	AWGN		BSC	
(ℓ, r)	$\mathbf{h}^{\mathrm{Sh}}$	$\mathbf{h}^{\mathrm{BP}}$	$\mathbf{h}^{\mathrm{MAP}}$	$\mathbf{h}^{\mathrm{BP}}$	$\mathbf{h}^{\mathrm{MAP}}$
$(3, 6)$	0.5000	0.4293	0.4794	0.4160	0.4681
$(4, 6)$	0.6667	0.5211	0.6645	0.5203	0.6633
$(5, 6)$	0.8333	0.5731	0.8333	0.5773	0.8333

Spatial-Coupling aids in bridging this gap

(ℓ, r, N, w) Spatially-Coupled Ensemble

▶ An LDPC code of left-degree $\ell = 3$ and right-degree $r = 4$
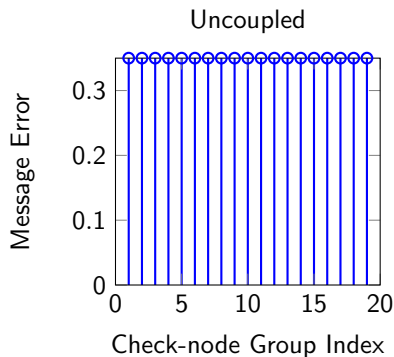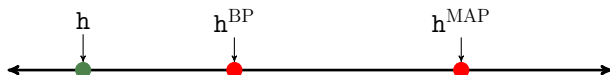
(ℓ, r, N, w) Spatially-Coupled Ensemble
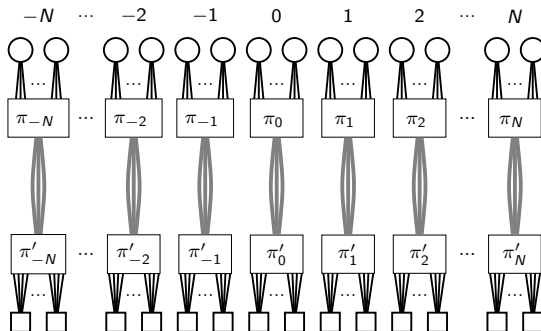
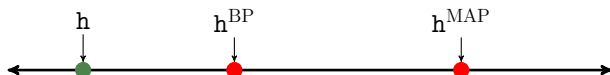# (ℓ, r, N, w) Spatially-Coupled Ensemble
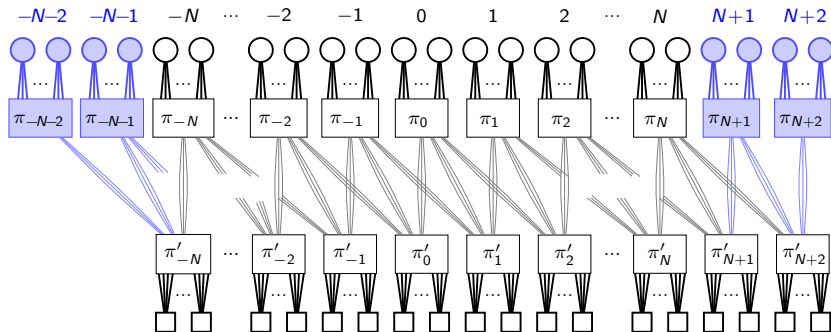
- Shown for $\ell = 3$, $r = 4$, and $w = 3$
- Check-nodes at Section $\{i\}$ are connected to variable-nodes in Sections $\{i - (w - 1), \ldots, i\}$
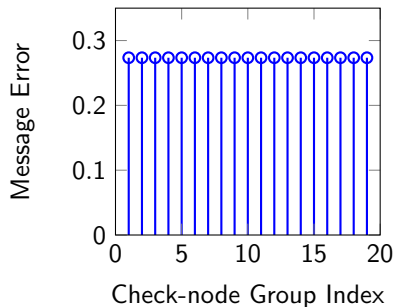- Shown to have near optimal BP thresholds
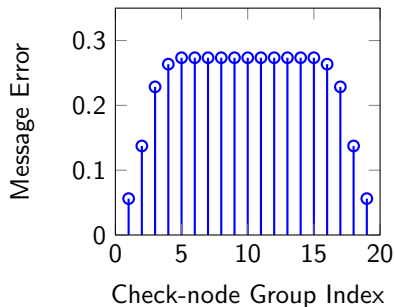
Threshold Saturation via Spatial Coupling (1)
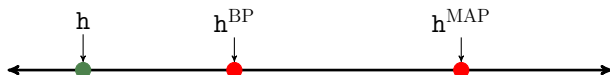
Threshold Saturation via Spatial Coupling (1)
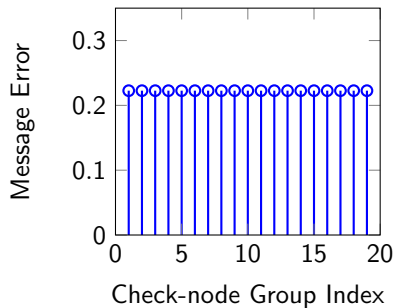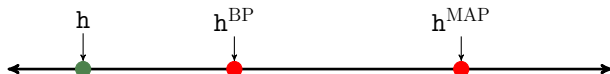
Threshold Saturation via Spatial Coupling (1)

Threshold Saturation via Spatial Coupling (1)

Threshold Saturation via Spatial Coupling (1)

Threshold Saturation via Spatial Coupling (1)

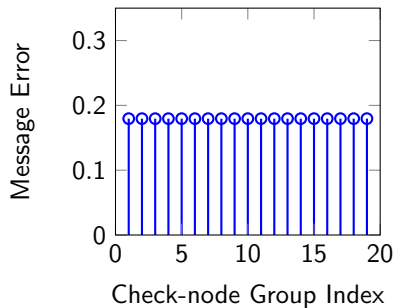# Threshold Saturation via Spatial Coupling (1)

Threshold Saturation via Spatial Coupling (1)
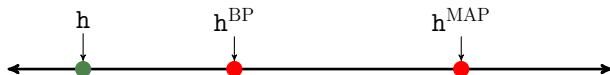
Threshold Saturation via Spatial Coupling (1)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (2)

Threshold Saturation via Spatial Coupling (3)

Threshold Saturation via Spatial Coupling (3)

Threshold Saturation via Spatial Coupling (3)

Threshold Saturation via Spatial Coupling (3)

Threshold Saturation via Spatial Coupling (3)

Threshold Saturation via Spatial Coupling (3)

Threshold Saturation via Spatial Coupling (3)

Threshold Saturation via Spatial Coupling (3)

Threshold Saturation Result

MAP Performance with a BP Decoder!

For large $N, w \quad \mathrm{h}_{\mathrm{c}}^{\mathrm{BP}} = \mathrm{h}^{\mathrm{MAP}}$

Threshold Saturation Result

MAP Performance with a BP Decoder!

For large N, w $\mathrm{h}_{\mathrm{c}}^{\mathrm{BP}} = \mathrm{h}^{\mathrm{MAP}}$

SC-LDPC (ℓ, r)	Shannon h^{Sh}	AWGN $\mathrm{h}_{\mathrm{c}}^{\mathrm{BP}}$	BSC $\mathrm{h}_{\mathrm{c}}^{\mathrm{BP}}$
$(3, 6)$	0.5000	0.4794	0.4681
$(4, 6)$	0.6667	0.6645	0.6633
$(5, 6)$	0.8333	0.8333	0.8333
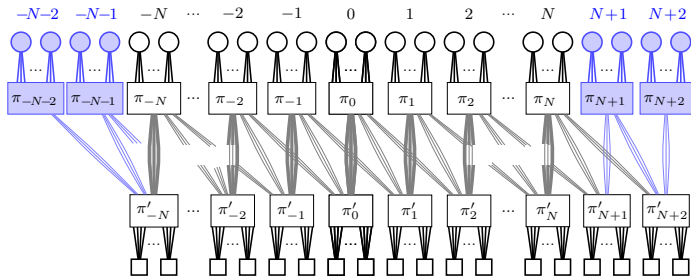
Rate loss for finite N and w

SC-LDPC (ℓ, r, N, w)	Shannon h^{Sh}	AWGN h_{c}^{BP}	BSC h_{c}^{BP}
$(3, 6, 10, 3)$	0.5434	0.4794	0.4681
$(3, 6, 20, 3)$	0.5222	0.4794	0.4681
$(3, 6, 30, 3)$	0.5149	0.4794	0.4681
$(4, 6, 10, 3)$	0.7245	0.6645	0.6633
$(4, 6, 20, 3)$	0.6963	0.6645	0.6633
$(4, 6, 30, 3)$	0.6866	0.6645	0.6633
$(5, 6, 10, 3)$	0.9056	0.8333	0.8333
$(5, 6, 20, 3)$	0.8704	0.8333	0.8333
$(5, 6, 30, 3)$	0.8582	0.8333	0.8333

Pros & Cons

Pros

- ▶ Significant improvment in thresholds

- ▶ Achieves capacity under simple BP decoding

- ▶ Universality - works for all channels models!

Cons

- ▶ Need large blocklengths to leverage the gains

Outline

Lattices and Lattice Codes

Lattice

A lattice of dimension n is a discrete subgroup of $\mathbb{R}^n$ isomorphic to $\mathbb{Z}^n$

$$\Lambda = \{\mathbf{G}\mathbf{z}, \mathbf{z} \in \mathbb{Z}^n\}$$

Lattices and Lattice Codes

Lattice

A lattice of dimension n is a discrete subgroup of $\mathbb{R}^n$ isomorphic to $\mathbb{Z}^n$

$$\Lambda = \{\mathbf{G}\mathbf{z}, \mathbf{z} \in \mathbb{Z}^n\}$$

- Efficient structures for
  - Mathematics: sphere packing and sphere covering problems
  - Information Theory: channel coding & quantization
- Single user Gaussian channel - Erez and Zamir
- Coding with side information - Wyner-Ziv and Costa, Zamir, Erez and Shamai
- Secrecy - He and Yener
- Dirty multiple access channel - Philosof, Khisti, Erez and Zamir

"Lattices are everywhere" by Ram Zamir

Prior Work

New perspectives for dealing with interference:

- Interference alignment - Sridharan, Jafarian, Vishwanath and Jafar
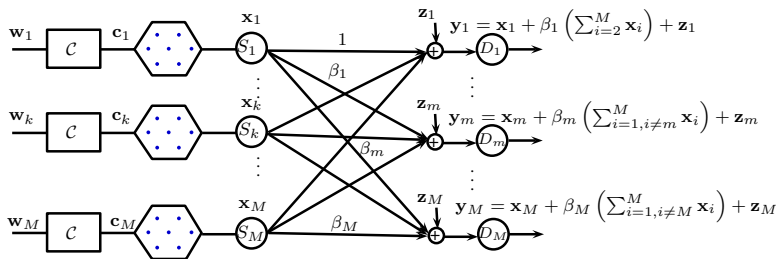
Prior Work

New perspectives for dealing with interference:

- ▶ Interference alignment - Sridharan, Jafarian, Vishwanath and Jafar
- ▶ Compute-and-forward - Nazer & Gastpar
- ▶ Physical layer network coding - Wilson et al, Nam et al
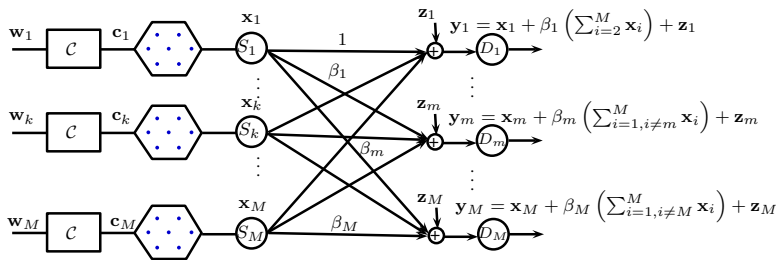
Prior Work

New perspectives for dealing with interference:

- ▶ Interference alignment - Sridharan, Jafarian, Vishwanath and Jafar
- ▶ Compute-and-forward - Nazer & Gastpar
- ▶ Physical layer network coding - Wilson et al, Nam et al
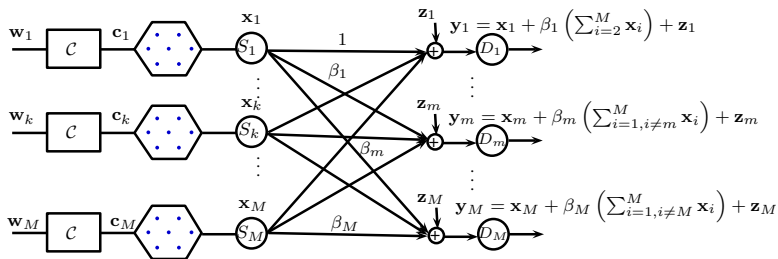
- ▶ Above schemes are all based on lattices good for channel coding

Goodness of Lattices for Channel Coding

- Voronoi region $\mathcal{V}$ of a lattice Λ, $\mathcal{V} := \{\mathbf{x} : \|\mathbf{x}\| \leq \|\mathbf{x} - \mathbf{c}\| \quad \forall \mathbf{c} \in \Lambda\}$
- Fundamental volume of Λ, $V(\Lambda)$: $\text{Vol}(\mathcal{V})$
- Let a lattice point $\lambda \in \Lambda$ is trasnmitted via AWGN channel of variance σ^2
- Volume-to-noise ratio(VNR) of Λ:

$$\text{VNR} = \frac{V(\Lambda)^{2/n}}{2\pi e \sigma^2}$$

- $P(\Lambda, \sigma^2) := \Pr(d(\lambda, \lambda + \mathbf{z}) \geq d(\lambda', \lambda' + \mathbf{z}))$ for some $\lambda' \in \Lambda$

Goodness of Lattices for Channel Coding

- Voronoi region $\mathcal{V}$ of a lattice Λ, $\mathcal{V} := \{\mathbf{x} : \|\mathbf{x}\| \leq \|\mathbf{x} - \mathbf{c}\| \quad \forall \mathbf{c} \in \Lambda\}$
- Fundamental volume of Λ, $V(\Lambda)$: $\text{Vol}(\mathcal{V})$
- Let a lattice point $\lambda \in \Lambda$ is trasnmitted via AWGN channel of variance σ^2
- Volume-to-noise ratio(VNR) of Λ:

$$\text{VNR} = \frac{V(\Lambda)^{2/n}}{2\pi e \sigma^2}$$

- $P(\Lambda, \sigma^2) := \Pr(d(\lambda, \lambda + \mathbf{z}) \geq d(\lambda', \lambda' + \mathbf{z}))$ for some $\lambda' \in \Lambda$

Poltyrev Goodness for Channel Coding

For any VNR> 1 $\exists \{\Lambda_n\}$ such that $P(\Lambda_n, \sigma^2) \to 0$ as $n \to \infty$.

- *Poltyrev*-good lattices are at the core of such lattice coding schemes

Objective

Motivating questions

- All the existing results were based on Construction-A.
  - Linear codes over *increasing field sizes* and their ML decoding

Objective

Motivating questions

- All the existing results were based on Construction-A.
    - Linear codes over *increasing field sizes* and their ML decoding
- Is this construction fundamental to good lattices?
- Can we work with just binary codes under practical decoding schemes?

Main Result

Codes over $\mathbb{F}_2$ and BP decoding suffice

- Recall Forney et al's result - based on nested random binary linear codes
- Propose capacity-achieving nested SC LDPC ensemble
- Construct lattices using Construction-D, based on the above ensemble
- Show existence of sequence of lattices that are *Poltyrev*-good under BP

Main Result

Codes over $\mathbb{F}_2$ and BP decoding suffice

- ▶ Recall Forney et al's result - based on nested random binary linear codes
- ▶ Propose capacity-achieving nested SC LDPC ensemble
- ▶ Construct lattices using Construction-D, based on the above ensemble
- ▶ Show existence of sequence of lattices that are *Poltyrev*-good under BP

Applications

- ▶ Apply proposed lattices to Symmetric Interference Channel
- ▶ Can be applied to other problems which adopt Construction A lattices

Construction D with L levels

- Barnes and Sloane '83, Forney, Chung and Trott '00, Yan, Ling, Wu ' 13
- Choose $G_1 \subseteq \ldots \subseteq G_L$ where G_l is a gen matrix of code $\mathcal{C}_l$ over $\mathbb{F}_2$.

Construction D with L levels

- Barnes and Sloane '83, Forney, Chung and Trott '00, Yan, Ling, Wu ' 13
- Choose $G_1 \subseteq \ldots \subseteq G_L$ where G_l is a gen matrix of code $\mathcal{C}_l$ over $\mathbb{F}_2$.
- $\underline{\lambda} = \underline{w}_1 \mathbf{G}_1 + 2\underline{w}_2 \mathbf{G}_2 \ldots + 2^{L-1}\underline{w}_{L-1}\mathbf{G}_{L-1} + 2^L \mathbb{Z}^N \in \Lambda$

Multi-Level Decoding(Successive Cancellation)

- $\underline{y} = \boxed{\underline{w}_1 \mathbf{G}_1 + 2\underline{w}_2 \mathbf{G}_2 \ldots + 2^{L-1}\underline{w}_{L-1}\mathbf{G}_{L-1} + 2^L \mathbb{Z}^N} + \underline{n}$

Multi-Level Decoding(Successive Cancellation)

- $\underline{y} = \boxed{\underline{w}_1 \mathbf{G}_1 + 2\underline{w}_2 \mathbf{G}_2 \ldots + 2^{L-1}\underline{w}_{L-1}\mathbf{G}_{L-1} + 2^L\mathbb{Z}^N} + \underline{n}$

- $\underline{y} \bmod 2 = [\underline{w}_1 \mathbf{G}_1 + \underline{n}] \bmod 2 = \underline{w}_1 \odot \mathbf{G}_1 + \boxed{\underline{n} \bmod 2}$

- Decode $\underline{w}_1$, reconstruct $\underline{w}_1 \mathbf{G}_1$ and subtract from $\underline{y}$
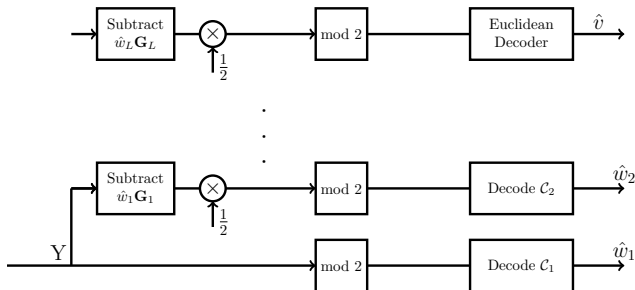
Theorem (Forney, Trott & Chung)

There exists a sequence of Construction D lattices based on $\mathcal{C}_1 \subseteq \mathcal{C}_2 \ldots \subseteq \mathcal{C}_L$ such that the VNR $\to 1$ and the $Pr(\lambda, \sigma^2) \to 0$.

- ▶ Take L large enough.
- ▶ It's sufficient that $\mathcal{C}_i$ at each level is capacity achieving for the mod-2 AWGN channel.

Theorem (Forney, Trott & Chung)

There exists a sequence of Construction D lattices based on $\mathcal{C}_1 \subseteq \mathcal{C}_2 \ldots \subseteq \mathcal{C}_L$ such that the VNR $\to 1$ and the $Pr(\lambda, \sigma^2) \to 0$.

- Take L large enough.
- It's sufficient that $\mathcal{C}_i$ at each level is capacity achieving for the mod-2 AWGN channel.

Objective:

- Capacity achieving nested code constructions, preferably under BP decoding.

Proposed Nested Spatially-Coupled LDPC Ensemble

1. Begin with a (d_v^1, d_c) SC LDPC code. For ex, $(d_v^1 = 3, d_c = 6, L = 3, w = 2)$.
2. Group check nodes into type $\mathcal{T}_k$, $k \in \{1, \ldots, d_v^1\}$
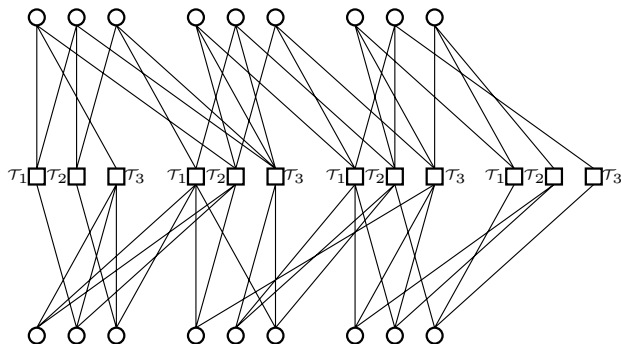
Proposed Nested Spatially-Coupled LDPC Ensemble

1. Begin with a (d_v^1, d_c) SC LDPC code. For ex, $(d_v^1 = 3, d_c = 6, L = 3, w = 2)$.
2. Group check nodes into type $\mathcal{T}_k$, $k \in \{1, \ldots, d_v^1\}$
3. Remove all check nodes of type $\mathcal{T}_1, \ldots, \mathcal{T}_{d_v^1 - d_v^2}$. Ex: $(d_v^2 = 2, 6)$ sup-code.
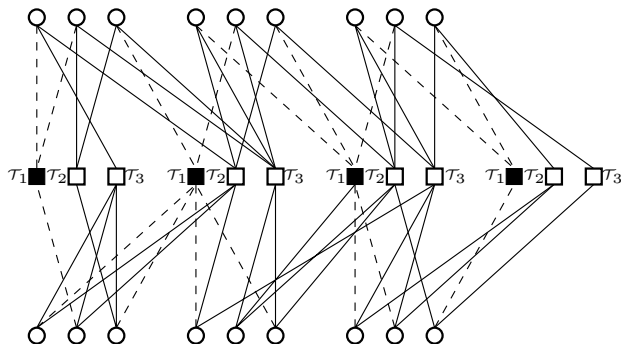
Proposed Nested Spatially-Coupled LDPC Ensemble

1. Begin with a (d_v^1, d_c) SC LDPC code. For ex, $(d_v^1 = 3, d_c = 6, L = 3, w = 2)$.
2. Group check nodes into type $\mathcal{T}_k$, $k \in \{1, \ldots, d_v^1\}$
3. Remove all check nodes of type $\mathcal{T}_1, \ldots, \mathcal{T}_{d_v^1 - d_v^2}$. Ex: $(d_v^2 = 2, 6)$ sup-code.
4. Results in a super-code that is a (d_v^2, d_c) SC LDPC code.
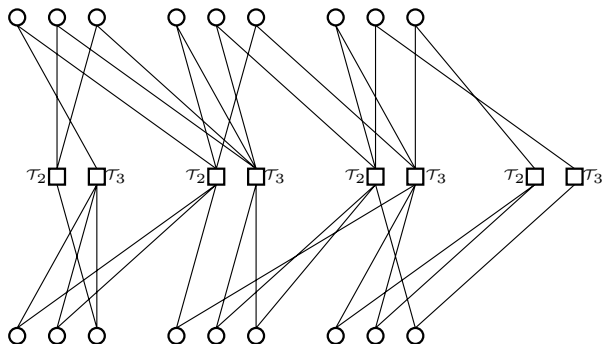
Lattice Design based on the proposed Nested SC LDPC ensemble

1. For a given σ, compute the capacity of the mod-2 AWGN channel at each level:

$$\underline{y_i} = \underline{w_i}\mathbf{G}_i + \frac{1}{2^{i-1}}\underline{n} \mod 2 = \underline{w_i} \odot \mathbf{G}_i + \boxed{\frac{1}{2^{i-1}}\underline{n} \mod 2}$$

2. Fix check node degree d_c. Choose $d_v^1, \ldots, d_v^r$ such that the rate of the code at each level is arbitrarily close to the capacity at the respective level.

Lattice Design based on the proposed Nested SC LDPC ensemble

1. For a given σ, compute the capacity of the mod-2 AWGN channel at each level:

$$\underline{y_i} = \underline{w_i}\mathbf{G}_i + \frac{1}{2^{i-1}}\underline{n} \mod 2 = \underline{w_i} \odot \mathbf{G}_i + \boxed{\frac{1}{2^{i-1}}\underline{n} \mod 2}$$

2. Fix check node degree d_c. Choose $d_v^1, \ldots, d_v^r$ such that the rate of the code at each level is arbitrarily close to the capacity at the respective level.

Lemma

Given nested binary linear codes $\mathcal{C}_1 \subseteq \mathcal{C}_2 \subseteq \ldots \subseteq \mathcal{C}_r$ there exists nested generator matrices for these codes.

Proposed Ensemble is Capacity achieving

Theorem

Each code ensemble in the proposed nested Spatially-Coupled LDPC ensemble is capacity achieving.

Proof.

- ▶ Show that the mod 2 AWGN channel is BMS.
- ▶ Each derived protograph has the same spatially coupled structure.
- ▶ The proof follows from Kudekar & Urbanke, Kumar & Pfister's results.

□

Proposed Lattices are Poltyrev-Good

Theorem

There exists a sequence of SC LDPC lattices with $VNR(\Lambda, \sigma^2) \to 1$ for which, under multistage BP decoding, $\mathbb{E}\left[P(\lambda, \sigma^2)\right] \to 0$ as $w, L, M \to \infty$.

Proof.

- ▶ The proposed nested ensemble achieve capacity.
- ▶ Follows from Forney's result.

□

Proposed Lattices are Poltyrev-Good

Theorem

There exists a sequence of SC LDPC lattices with VNR$(\Lambda, \sigma^2) \to 1$ for which, under multistage BP decoding, $\mathbb{E}\left[P(\lambda, \sigma^2)\right] \to 0$ as $w, L, M \to \infty$.

Proof.

- ▶ The proposed nested ensemble achieve capacity.
- ▶ Follows from Forney's result.

□

- ▶ Binary codes and more importantly practical BP decoding suffices.
- ▶ Practically we observe that two levels of coding gets you lattices very close to Poltyrev limit.

Design Example of Poltyrev-Good Lattice

Target error probability $P(2^L\mathbb{Z}^n, \sigma_L^2) = 10^{-4}$ in the uncoded level $\implies \sigma_L = 0.08$

Design Example of Poltyrev-Good Lattice

Target error probability $P(2^L\mathbb{Z}^n, \sigma_L^2) = 10^{-4}$ in the uncoded level $\implies \sigma_L = 0.08$

1. Capacities for the mod 2 AWGN channel for respective levels:

	Level L-1	Level L-2	Level L-3
σ_{eff}	0.16	0.32	0.64
Cap	0.99	0.57	0.02
(14,30) (3,30)	0.9	0.533	0

Design Example of Poltyrev-Good Lattice

Target error probability $P(2^L\mathbb{Z}^n, \sigma_L^2) = 10^{-4}$ in the uncoded level $\implies \sigma_L = 0.08$

1. Capacities for the mod 2 AWGN channel for respective levels:

	Level L-1	Level L-2	Level L-3
σ_{eff}	0.16	0.32	0.64
Cap	0.99	0.57	0.02
(14,30) (3,30)	0.9	0.533	0

2. Fix L=3 and use $(3, 30)$, $(14, 30)$ nested SC-LDPC codes.
   - Note $P(4\mathbb{Z}^n, \sigma^2) \approx nP(4\mathbb{Z}, \sigma^2)$
   - We fix $n = 2 \times 10^5$

(d_c, d_v^1, d_v^2)	(L,w)	$P(4\mathbb{Z}, \sigma^2)$	σ_{max}	VNR	VNR$_{\text{rate-loss}}$
(30,14,3)	(32,4)	5×10^{-10}	0.3184	1.02dB	1.347dB

Design Example of Poltyrev-Good Lattice

Target error probability $P(2^L \mathbb{Z}^n, \sigma_L^2) = 10^{-4}$ in the uncoded level $\implies \sigma_L = 0.08$

1. Capacities for the mod 2 AWGN channel for respective levels:

	Level L-1	Level L-2	Level L-3
σ_{eff}	0.16	0.32	0.64
Cap	0.99	0.57	0.02
(14,30) (3,30)	0.9	0.533	0

2. Fix L=3 and use $(3, 30)$, $(14, 30)$ nested SC-LDPC codes.
   - Note $P(4\mathbb{Z}^n, \sigma^2) \approx n P(4\mathbb{Z}, \sigma^2)$
   - We fix $n = 2 \times 10^5$

(d_c, d_v^1, d_v^2)	(L,w)	$P(4\mathbb{Z}, \sigma^2)$	$\sigma_{\max}$	VNR	VNR$_{\text{rate-loss}}$
(30,14,3)	(32,4)	5×10^{-10}	0.3184	1.02dB	1.347dB
(60, 26, 3)	(72, 12)	5×10^{-10}	0.3200	0.482dB	0.927dB
(60, 27, 3)	(64, 9)	5×10^{-10}	0.3203	0.57dB	0.951dB

Alternate Nested SC LDPC ensemble

- Derive a lower rate code by "splitting the checks"
- Consider a $(3, 8)$ code
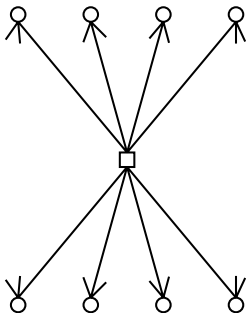
Alternate Nested SC LDPC ensemble

- Derive a lower rate code by "splitting the checks"
- Consider a $(3, 8)$ code
- Split each check into "two" checks to derive a $(3, 4)$ sub-code
- Easy to prove that resulting code is from the $(3, 4)$ SC LDPC ensemble
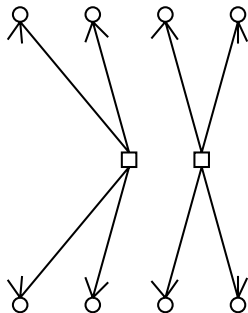
Simulation Results
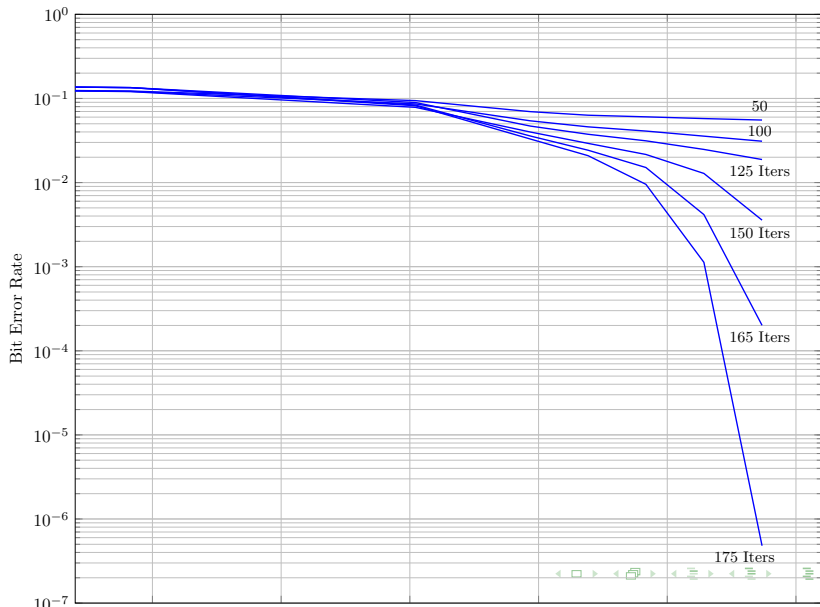
Simulation Results
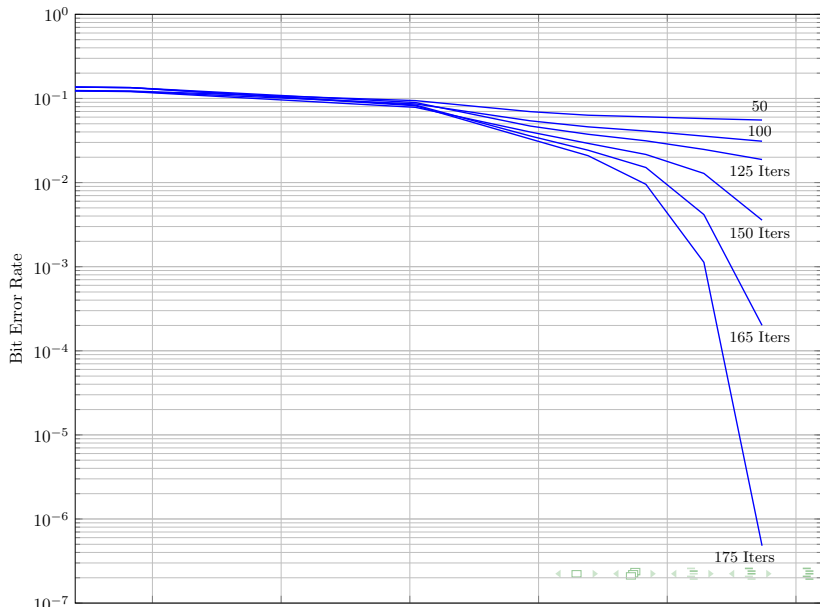
3-User Symmetric Interference Channel
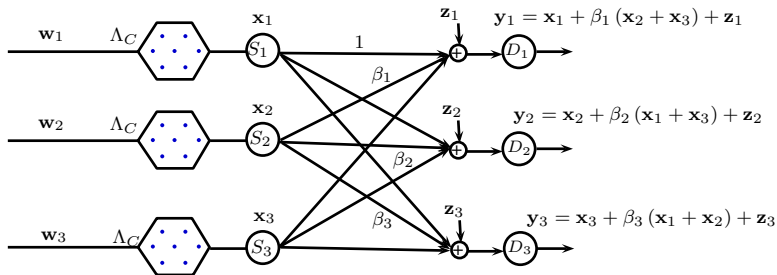
3-User Symmetric Interference Channel
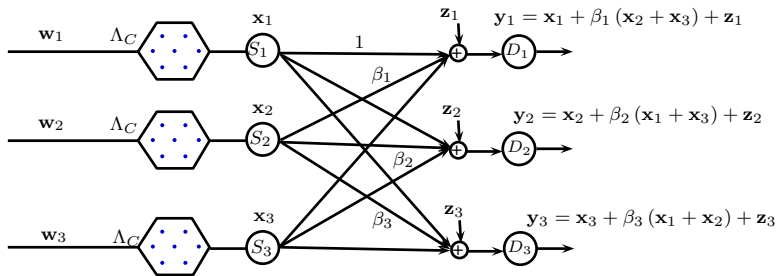
- $\mathbf{x}_i \in \Lambda_C \triangleq \Lambda \cap \mathbb{Z}_4^N$ is transmitted.

Symmetric Interference Channel - Decoding Sums

Interference at Destination 1:

$$\mathbf{x}_2 + \mathbf{x}_3 = (\underline{w}_2^1 + \underline{w}_3^1)\mathbf{G}_1 + 2(\underline{w}_2^2 + \underline{w}_3^2)\mathbf{G}_2 + 4\mathbf{k}_{23}$$
$$= (\underline{w}_2^1 \oplus \underline{w}_3^1)\mathbf{G}_1 + 2(\underline{c}_{23}^1 \oplus \underline{w}_2^2 \oplus \underline{w}_3^2)\mathbf{G}_2 + 4(\underline{c}_{23}^2 + \mathbf{k}_{23})\mathbf{Z}$$

where the carry overs are

$$\underline{c}_{23}^1 = 0.5\left(\underline{w}_1^1 + \underline{w}_1^2 - \underline{w}_1^1 \oplus \underline{w}_1^2\right),$$
$$\underline{c}_{23}^2 = 0.5\left(\underline{c}_{23}^1 + \underline{w}_2^1 + \underline{w}_2^2 - \underline{c}_{23}^1 \oplus \underline{w}_2^1 \oplus \underline{w}_2^2\right)$$
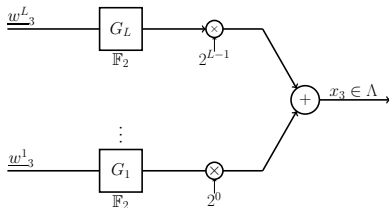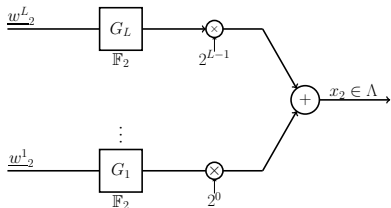
Achievable Information Rates
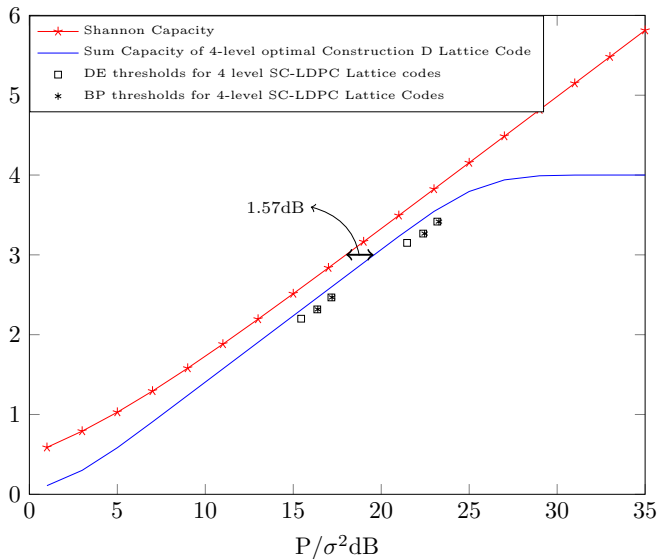
Concluding Remarks

- Multilevel constructions - efficient ways to decode integer combinations

- Need capacity achieving nested codes

- Multilevel construction is provably good under message passing decoding

Concluding Remarks

- Multilevel constructions - efficient ways to decode integer combinations

- Need capacity achieving nested codes

- Multilevel construction is provably good under message passing decoding

- Coding schemes based on *binary codes and iterative decoding* suffice

Outline

Lossy Source Coding Problem

$$X^n = (X_1, \cdots, X_n), \ X_i \sim \text{Bernoulli}(\tfrac{1}{2})$$

Binary code $\mathcal{C} = (n, k)$, rate $R = k/n$

Lossy Source Coding Problem

$$X^n = (X_1, \cdots, X_n), \ X_i \sim \text{Bernoulli}(\tfrac{1}{2})$$

Binary code $\mathcal{C} = (n, k)$, rate $R = k/n$

Lossy Source Coding

- Compress X^n to $\hat{X}^n \in \mathcal{C}$
- Min. Hamming distortion

$$D = \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}|X_i - \hat{X}_i|$$

Lossy Source Coding Problem

$$X^n = (X_1, \cdots, X_n),\ X_i \sim \text{Bernoulli}(\tfrac{1}{2})$$

Binary code $\mathcal{C} = (n, k)$, rate $R = k/n$

Lossy Source Coding

▶ Compress X^n to $\hat{X}^n \in \mathcal{C}$

▶ Min. Hamming distortion

$$D = \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}|X_i - \hat{X}_i|$$

▶ Rate-Distortion theory:

$$R > 1 - h(D)$$

▶ $h(\cdot)$ is binary entropy function

$$h(D) = -D \log_2 D - (1-D) \log_2(1-D)$$
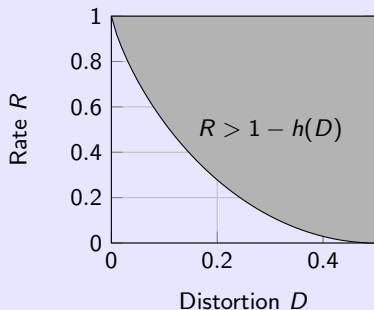
Side-Information Problems: Wyner-Ziv
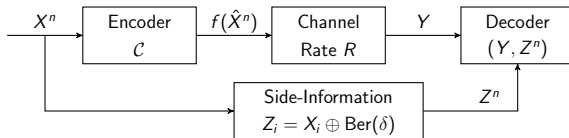
Wyner-Ziv Formulation

- Side-information Z^n about X^n
- Decoder additionally has Z^n
- Say $Z_i = X_i \oplus \text{Ber}(\delta)$

Side-Information Problems: Wyner-Ziv
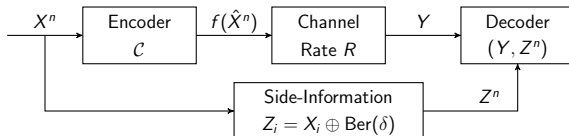
Wyner-Ziv Formulation

- ▶ Side-information Z^n about X^n
- ▶ Decoder additionally has Z^n
- ▶ Say $Z_i = X_i \oplus \text{Ber}(\delta)$
- ▶ Wyner-Ziv theory:

    $$R > l.c.e\{h(D * \delta) - h(D), (\delta, 0)\}$$

- ▶ $D * \delta = D(1 - \delta) + \delta(1 - D)$
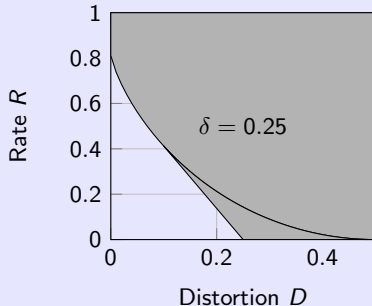
Side-Information Problems: Gelfand-Pinsker
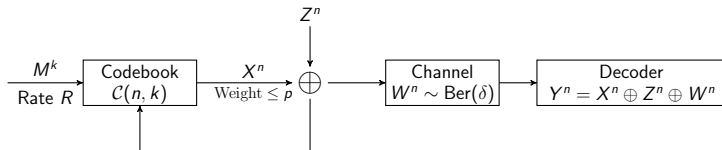
Side-Information Problems: Gelfand-Pinsker
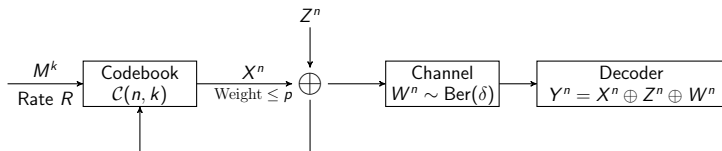
Gelfand-Pinsker Formulation

- Message M^k encoded to $X^n \in \mathcal{C}$ with $\frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] \leq p \leq \frac{1}{2}$
- Side-information Z^n is available *only at the encoder*

Side-Information Problems: Gelfand-Pinsker
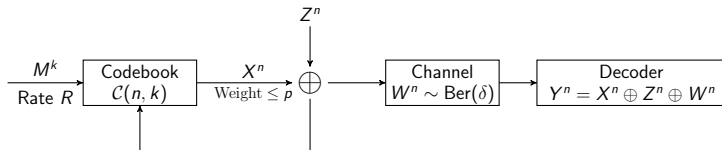
Gelfand-Pinsker Formulation

- Message M^k encoded to $X^n \in \mathcal{C}$ with $\frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] \leq p \leq \frac{1}{2}$
- Side-information Z^n is available *only at the encoder*
- The output at the decoder is

$$Y^n = X^n \oplus Z^n \oplus W^n, \quad \{W_i\} \sim \text{Ber}(\delta)$$

Side-Information Problems: Gelfand-Pinsker
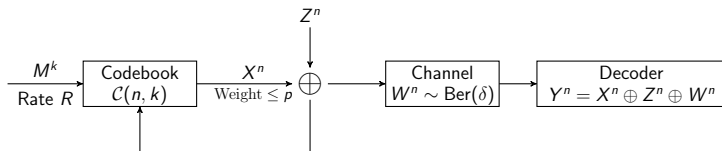
Gelfand-Pinsker Formulation

- Message M^k encoded to $X^n \in \mathcal{C}$ with $\frac{1}{n}\sum_{i=1}^{n}\mathbb{E}[X_i] \leq p \leq \frac{1}{2}$
- Side-information Z^n is available *only at the encoder*
- The output at the decoder is

$$Y^n = X^n \oplus Z^n \oplus W^n, \quad \{W_i\} \sim \text{Ber}(\delta)$$

- Capacity region by Gelfand-Pinsker:

$$R < h(p) - h(\delta)$$

Main Result

Objective

▶ Construct *low-complexity* coding schemes that achieve the complete rate regions of Wyner-Ziv and Gelfand-Pinsker

  • Low-complexity encoding and decoding

Main Result

Objective

- Construct *low-complexity* coding schemes that achieve the complete rate regions of Wyner-Ziv and Gelfand-Pinsker
  - Low-complexity encoding and decoding

Idea

- Wainwright et al. used compound LDGM/LDPC codes with *optimal encoding/decoding*

- Message-passing algorithms have non-negligible gap

Main Result

Objective

▶ Construct *low-complexity* coding schemes that achieve the complete rate regions of Wyner-Ziv and Gelfand-Pinsker
- Low-complexity encoding and decoding

Idea

▶ Wainwright et al. used compound LDGM/LDPC codes with *optimal encoding/decoding*

▶ Message-passing algorithms have non-negligible gap

▶ Remedy via *Spatial-Coupling*
- Channel coding in coupled compound codes (Kasai et al.)
- Lossy source coding with spatially-coupled LDGM (Aref et al.)
- Encoding with *compound codes has additional challenges*

Compound LDGM/LDPC Codes
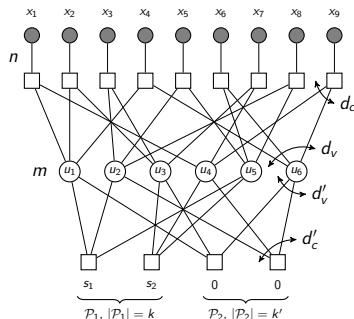
- Codebook $\mathcal{C}(n, m - k - k')$

- Message constraints

$$u_1 \oplus u_2 \oplus u_5 = s_1, \quad u_1 \oplus u_3 \oplus u_6 = 0$$

- Codeword $(x_1, \cdots, x_9)$:

$$x_1 = u_1 \oplus u_4, \qquad x_2 = \cdots$$

Compound LDGM/LDPC Codes
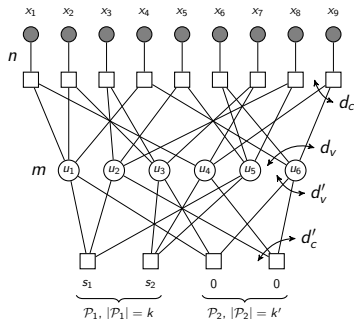
- ▶ Codebook $\mathcal{C}(n, m - k - k')$
- ▶ Message constraints

$$u_1 \oplus u_2 \oplus u_5 = s_1, \quad u_1 \oplus u_3 \oplus u_6 = 0$$

- ▶ Codeword $(x_1, \cdots, x_9)$:

$$x_1 = u_1 \oplus u_4, \qquad x_2 = \cdots$$

Key Properties

- ▶ Compound code is
  - a *good source code* under optimal encoding
  - a *good channel code* under optimal decoding
- ▶ LDGM code is
  - a *good source code* under optimal encoding
  - (side note) LDGM code is *not* a good channel code

Good Code

"Good" source code

- ▶ Rate of the code is $R = 1 - h(D) + \varepsilon$
- ▶ When this code is used to *optimally encode* $\text{Ber}(\frac{1}{2})$
- ▶ The average Hamming distortion is at most D

"Good" channel code

- ▶ Rate of the code is $R = 1 - h(\delta) - \varepsilon$
- ▶ When this code is used for channel coding on $\text{BSC}(\delta)$
- ▶ Message est. under *optimal decoding* with error at most ε

Coding Scheme: Wyner-Ziv
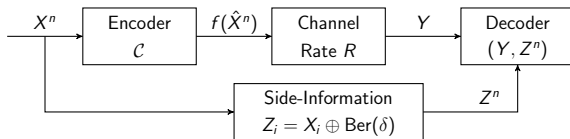
Coding Scheme: Wyner-Ziv
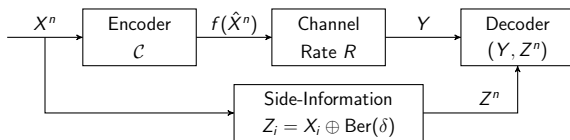
- Encode X^n to $\hat{X}^n$ using *LDGM* w/ Distortion $\approx D$
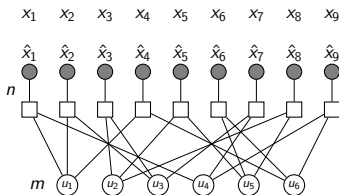
$$\frac{m}{n} \approx 1 - h(D) + \varepsilon$$

Coding Scheme: Wyner-Ziv
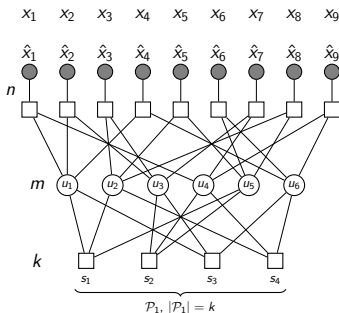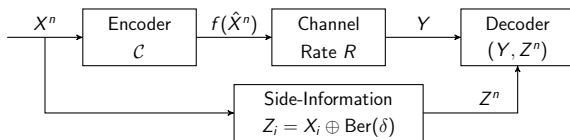
- Encode X^n to $\hat{X}^n$ using *LDGM* w/ Distortion $\approx D$

- Compute & transmit s_i's
  $R = \frac{k}{n} \approx h(D * \delta) - h(D)$

- Decoder has Z^n:
  $$Z_i = X_i \oplus \text{Ber}(\delta)$$
  $$\approx \hat{X}_i \oplus \text{Ber}(D) \oplus \text{Ber}(\delta)$$
  $$= \hat{X}_i \oplus \text{Ber}(D * \delta)$$

- Decode $\hat{X}^n$ from Z^n & s_i

$\frac{m}{n} \approx 1 - h(D) + \varepsilon \qquad \frac{m-k}{n} \approx 1 - h(D * \delta) + \varepsilon$

Coding Scheme: Gelfand-Pinsker
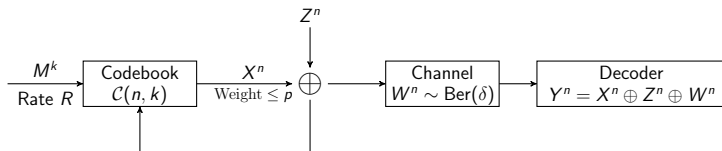
Coding Scheme: Gelfand-Pinsker
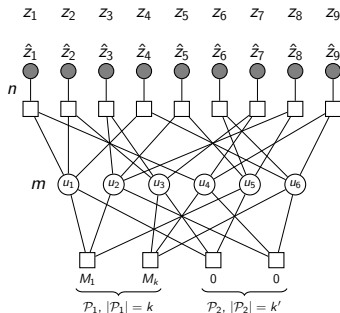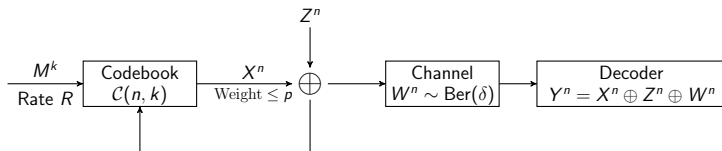
- With message M^k, encode Z^n to $\hat{Z}^n$ (Distortion $\approx p$)
- Transmit $X^n = Z^n \oplus \hat{Z}^n$

$$\frac{m-k-k'}{n} \approx 1 - h(p) + \varepsilon \qquad \frac{m-k'}{n} \approx 1 - h(\delta) + \varepsilon$$

Coding Scheme: Gelfand-Pinsker
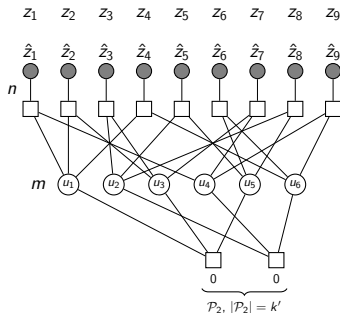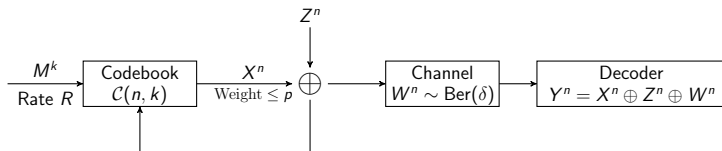
- With message M^k, encode Z^n to $\hat{Z}^n$ (Distortion $\approx p$)

- Transmit $X^n = Z^n \oplus \hat{Z}^n$

- Decoder has
$$Y^n = X^n \oplus Z^n \oplus W^n$$
$$= \hat{Z}^n \oplus W^n$$

- Decode $\hat{Z}^n$ and compute M^k

Coding Scheme: Gelfand-Pinsker
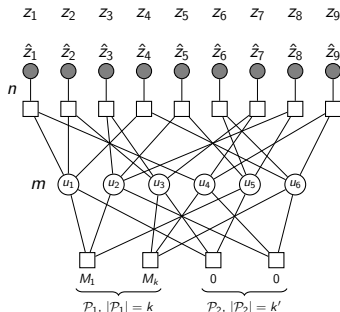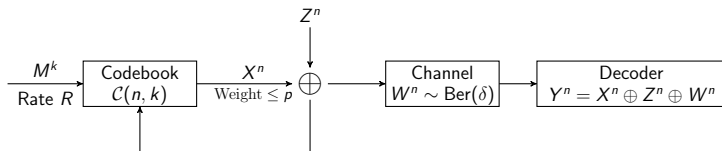
- With message M^k, encode Z^n to $\hat{Z}^n$ (Distortion $\approx p$)
- Transmit $X^n = Z^n \oplus \hat{Z}^n$
- Decoder has
$$Y^n = X^n \oplus Z^n \oplus W^n$$
$$= \hat{Z}^n \oplus W^n$$
- Decode $\hat{Z}^n$ and compute M^k
- $R = \frac{k}{n} \approx h(p) - h(\delta)$

$\frac{m-k-k'}{n} \approx 1 - h(p) + \varepsilon$ $\frac{m-k'}{n} \approx 1 - h(\delta) + \varepsilon$

Remarks

- Need codes that are *simultaneously good* for channel and source coding

- Use message-passing algorithms instead of *optimal*

- Use spatial-coupling for *goodness* of codes under message-passing
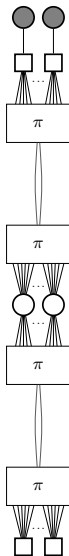
Spatially-Coupled Compound LDGM/LDPC Codes
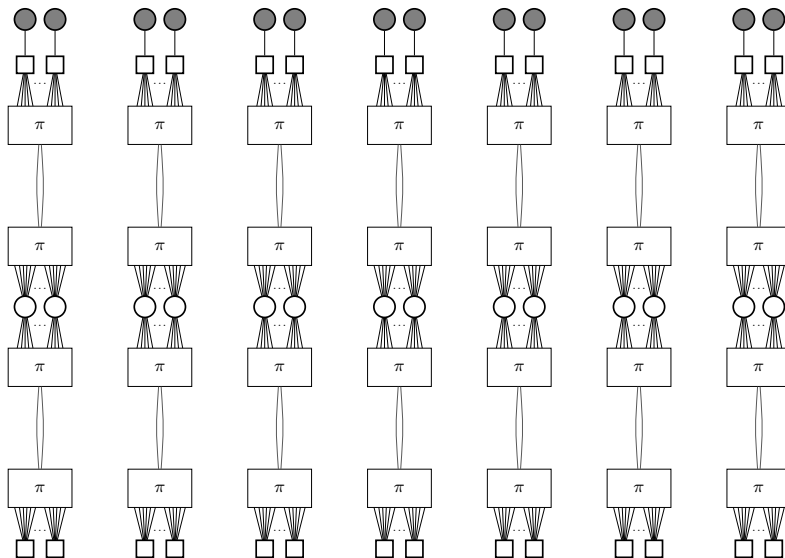
Spatially-Coupled Compound LDGM/LDPC Codes
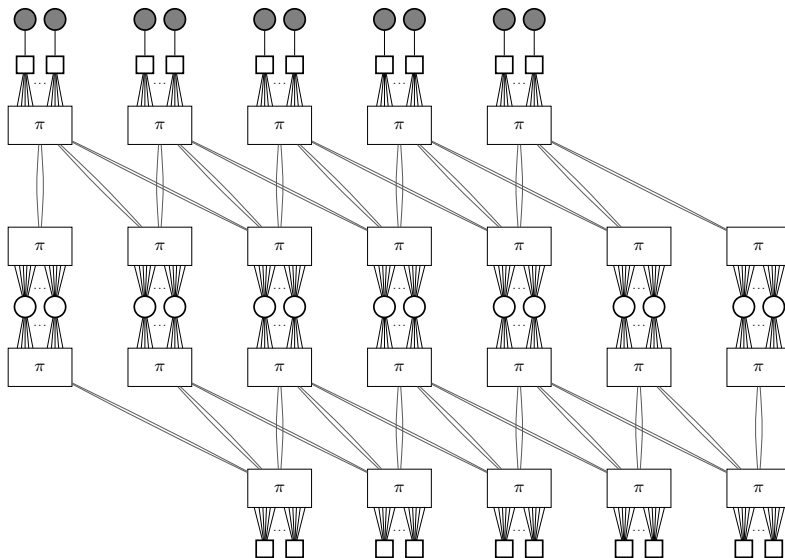
Spatially-Coupled Compound LDGM/LDPC Codes
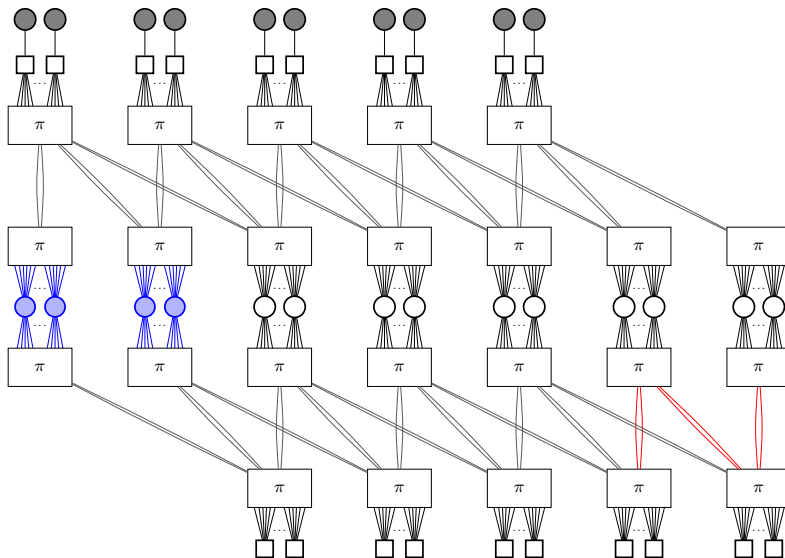
Spatially-Coupled Compound LDGM/LDPC Codes
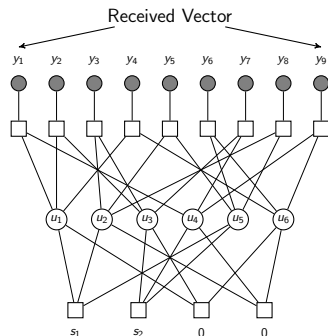
Decoding in Spatially-Coupled Compound Codes

Remarks

- Standard message-passing algorithm
- Threshold saturation proven for SC compound codes on BEC
- Empirically observed for BMS channels

Encoding in Spatially-Coupled Compound Codes
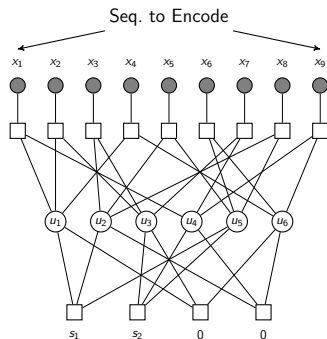
Remarks

- Inverse temperature parameter β
- Message-passing rules are the same
- However, a *crucial decimation step is needed*

Encoding in SC Compound Codes: BPGD Algorithm

Encoding in SC Compound Codes: Remarks

- **Randomization** in setting u_{i*} is crucial

- BPGD applied to uncoupled code *always failed*

- *Spatially-coupled structure* is crucial for successful encoding
  - In addition, distortion is close to optimal thresholds
  - *Does not encode* if decimated from both left and right
  - *Does not encode* if both left and right boundary is set to 0

Encoding in SC Compound Codes: Numerical Example

Block length (n)	4-cycles	Attempts $1/2/3/4/ \geq 5$
9000	yes	5/3/5/2/35
9000	no	21/12/5/3/9
27000	no	35/15/0/0/0
45000	no	40/9/0/0/1
63000	no	44/6/0/0/0
81000	no	50/0/0/0/0

Remarks

- ▶ # Attempts to encode 50 seq. in $(6, 3)$ LDGM / $(3, 6)$ LDPC
- ▶ $L = 20$, $w = 4$, $\beta = 0.65$, $T = 10$
- ▶ Removing *4-cycles* dramatically improves success
- ▶ How much do 6-cycles matter?

Numerical Results: Wyner-Ziv

LDGM (d_v, d_c)	LDPC (d_v', d_c')	(L, w)	(D_*, δ_*)	(D, δ)
$(6, 3)$	$(3,6)$	$(20,4)$	$(0.111, 0.134)$	$(0.1174, 0.122)$
$(8, 4)$	$(3,6)$	$(20,4)$	$(0.111, 0.134)$	$(0.1149, 0.120)$
$(10, 5)$	$(3,6)$	$(20,4)$	$(0.111, 0.134)$	$(0.1139, 0.122)$

Remarks

- D_* and δ_* are calculated based on the rate of the respective code:

$$D_* = h^{-1}(1 - R1) \qquad\qquad \delta_* = h^{-1}(1 - R2)$$

- $n \approx 140000$, $\beta = 1.04$, $T = 10$

Numerical Results: Gelfand-Pinsker

LDGM (d_v, d_c)	LDPC (d'_v, d'_c)	(L, w)	(p_*, δ_*)	(p, δ)
$(6, 3)$	$(3,6)$	$(20,4)$	$(0.215, 0.157)$	$(0.2200, 0.152)$
$(8, 4)$	$(3,6)$	$(20,4)$	$(0.215, 0.157)$	$(0.2230, 0.151)$
$(10, 5)$	$(3,6)$	$(20,4)$	$(0.215, 0.157)$	$(0.2200, 0.151)$

Remarks

- p_* and δ_* are calculated based on the rate of the respective code:

$$p_* = h^{-1}(1 - R1) \qquad\qquad \delta_* = h^{-1}(1 - R2)$$

- $n \approx 140000$, $\beta = 0.65$, $T = 10$

Concluding Remarks

Conclusion

- Spatially-coupled codes achieve the rate regions of Wyner-Ziv and Gelfand-Pinsker problems

- Coupling structure is also crucial
  - to achieve optimum thresholds
  - for encoding to succeed with decimation

Open Questions

- Effect of degree profiles, short-cycles on encoding success

- Precise trade-offs with polar codes

Outline

Write-Once Memories

Flash Memory

- In typical flash memory, changing from 0 to 1 is easy
- Resetting 1 to 0 requires rewriting whole block
- Write-once memories model such storage systems

Write-Once Memories
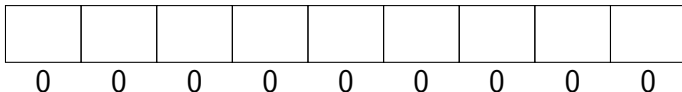
0	0	0	0	0	0	0	0	0

Flash Memory

- In typical flash memory, changing from 0 to 1 is easy
- Resetting 1 to 0 requires rewriting whole block
- Write-once memories model such storage systems

Binary Write-Once Memories

- $0 \longrightarrow 1$ is allowed

Write-Once Memories
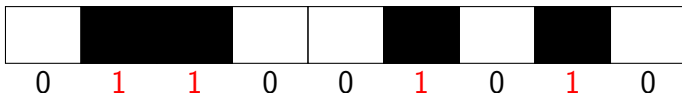
0	1	1	0	0	1	0	1	0

Flash Memory

▶ In typical flash memory, changing from 0 to 1 is easy

▶ Resetting 1 to 0 requires rewriting whole block

▶ Write-once memories model such storage systems

Binary Write-Once Memories

▶ 0 $\longrightarrow$ 1 is allowed

▶ 1 $\longrightarrow$ 0 *is forbidden*

Capacity Region (I) - Noiseless

Message

| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |

Write-Once Memory without Noise

- ▶ In 1982, Rivest and Shamir gave first WOM codes
  - • 2 bits in 2 writes with only 3 cells
- ▶ Only about $nt/\log(t)$ cells required to store n bits for t writes

Capacity Region (I) - Noiseless

Message

0	0	1	0	0	1	0	1	0

Write-Once Memory without Noise

- In 1982, Rivest and Shamir gave first WOM codes
  - 2 bits in 2 writes with only 3 cells
- Only about $nt/\log(t)$ cells required to store n bits for t writes
- In 1985, Heegard gave the *capacity* for t-write system
- For a 2-write system, it is

$$\left\{ (R_1, R_2) \mid 0 \leq R_1 < h(\delta),\ 0 \leq R_2 < 1 - \delta \right\}$$

Capacity Region (II) - Read Errors

Message

| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 |

$\mathrm{Ber}(p)$

0   0   0   0   0   1   1   1   0
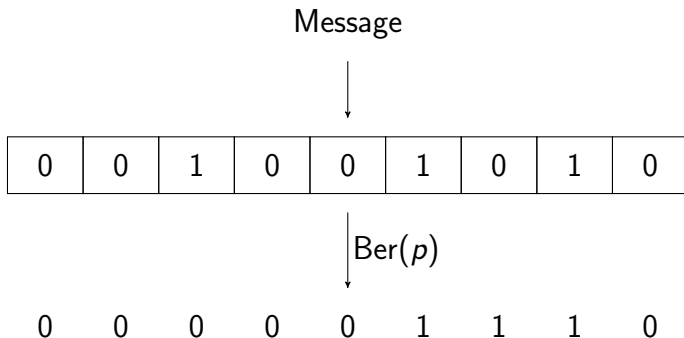
Write-Once Memory with Read Errors

- ▶ Different from write errors
- ▶ $Y = X \oplus \mathrm{Ber}(p)$, where $\mathrm{Ber}(p)$ denotes the Bernoulli noise
- ▶ Capacity region is *unknown*

Main Result

Objective

▶ Construct *low-complexity* coding schemes that achieve the capacity region of the WOM system

- Low-complexity encoding and decoding

Main Result

Objective

▶ Construct *low-complexity* coding schemes that achieve the capacity region of the WOM system

  • Low-complexity encoding and decoding

▶ Focus on the 2-write WOM system

  • Achieves the capacity region of the noiseless system
  • For read errors, achieves

$$R_1 < h(\delta) - h(p), \qquad\qquad R_2 < 1 - \delta - h(p).$$

Main Result

Objective

- Construct *low-complexity* coding schemes that achieve the capacity region of the WOM system
  - Low-complexity encoding and decoding
- Focus on the 2-write WOM system
  - Achieves the capacity region of the noiseless system
  - For read errors, achieves

$$R_1 < h(\delta) - h(p), \qquad\qquad R_2 < 1 - \delta - h(p).$$

- Extension to multi-write systems *seems possible with BPGD*

Main Result

Objective

► Construct *low-complexity* coding schemes that achieve the capacity region of the WOM system

- Low-complexity encoding and decoding

► Focus on the 2-write WOM system
- Achieves the capacity region of the noiseless system
- For read errors, achieves

$$R_1 < h(\delta) - h(p), \qquad\qquad R_2 < 1 - \delta - h(p).$$

► Extension to multi-write systems *seems possible with BPGD*

Idea

► Use compound LDGM/LDPC codes
► Encoding for second write is *erasure quantization*
► Use spatial coupling with message-passing

Compound LDGM/LDPC Codes
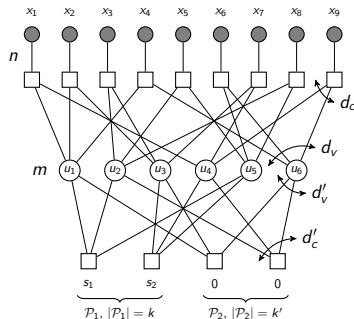
- Codebook $(n, m - k - k')$
- Message constraints

$$u_1 \oplus u_2 \oplus u_5 = s_1, \quad u_1 \oplus u_3 \oplus u_6 = 0$$

- Codeword $(x_1, \cdots, x_9)$:

$$x_1 = u_1 \oplus u_4, \qquad x_2 = \cdots$$

- Parametrized by s^k: $\mathcal{C}(s^k)$

Compound LDGM/LDPC Codes
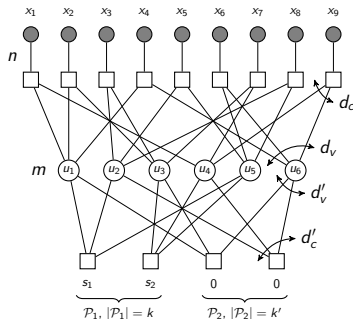
- ▶ Codebook $(n, m - k - k')$
- ▶ Message constraints

$$u_1 \oplus u_2 \oplus u_5 = s_1, \quad u_1 \oplus u_3 \oplus u_6 = 0$$

- ▶ Codeword $(x_1, \cdots, x_9)$:

$$x_1 = u_1 \oplus u_4, \qquad x_2 = \cdots$$

- ▶ Parametrized by s^k: $\mathcal{C}(s^k)$

Key Properties of Compound Codes

- ▶ a natural coset decomposition: $\mathcal{C} = \bigcup_{s^k \in \{0,1\}^k} \mathcal{C}(s^k)$
- ▶ achieves capacity over eras. chan. under MAP (when $m = n$)
- ▶ a good source code under optimal encoding
- ▶ a good channel code under optimal decoding

Good Code

"Good" source code

- Rate of the code is $R = 1 - h(\delta) + \varepsilon$
- When this code is used to *optimally encode* $\text{Ber}(\frac{1}{2})$
- The average Hamming distortion is at most δ

"Good" channel code

- Rate of the code is $R = 1 - h(p) - \varepsilon$
- When this code is used for channel coding on $\text{BSC}(p)$
- Message est. under *optimal decoding* with error at most ε

Coding Scheme for 2-write WOM: First Write

$$R_1 < h(\delta) - h(p)$$

- With message s^k, encode 0^n to x^n (Distortion $\approx \delta$)
- Store x^n

$$\frac{m-k-k'}{n} \approx 1 - h(\delta) \qquad \frac{m-k'}{n} \approx 1 - h(p)$$
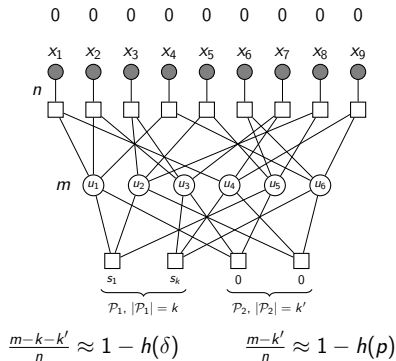
Coding Scheme for 2-write WOM: First Write
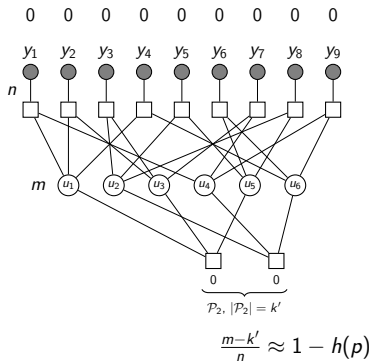
$$R_1 < h(\delta) - h(p)$$

- With message s^k, encode 0^n to x^n (Distortion $\approx \delta$)
- Store x^n
- Decoder has

$$y_i = x_i \oplus \mathsf{Ber}(p)$$

Coding Scheme for 2-write WOM: First Write

$$R_1 < h(\delta) - h(p)$$
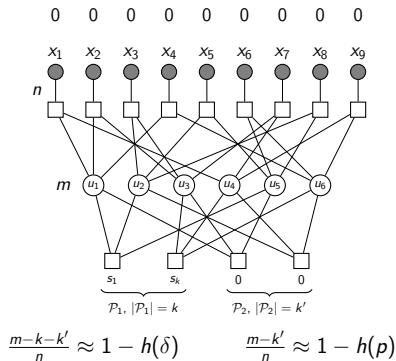
- With message s^k, encode 0^n to x^n (Distortion $\approx \delta$)
- Store x^n
- Decoder has

$$y_i = x_i \oplus \mathrm{Ber}(p)$$

- Dec. x^n and compute s^k
- $R_1 = \frac{k}{n} \approx h(\delta) - h(p)$

Coding Scheme for 2-write WOM: Second Write
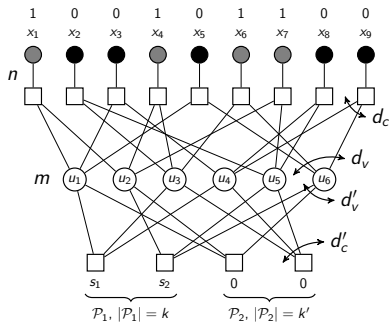
- Need to find a *consistent* codeword in $\mathcal{C}(s^k)$

Coding Scheme for 2-write WOM: Second Write
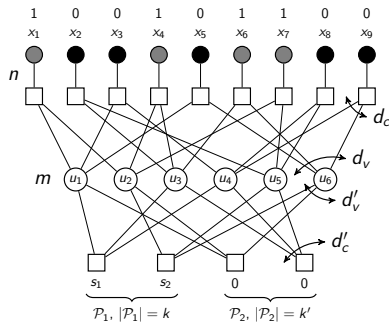
- Need to find a *consistent* codeword in $\mathcal{C}(s^k)$
- Closely related to Binary Erasure Quantization (BEQ)
- En Gad, Huang, Li and Bruck (ISIT 2015)

Binary Erasure Quantization

- Quantize a sequence in $\{0, 1, *\}^n$ to $x^n \in \mathcal{C} \subset \{0, 1\}^n$
  - 0's and 1's should *match exactly*
  - $*$'s can take either 0 or 1

- Can map the second write of 2-write WOM to BEQ
  - Map 0's to $*$'s and keep 1's
  - Quantize to codeword in $\mathcal{C}(s^k)$

- BEQ is the dual of decoding on binary erasure channel
  - Martinian and Yedidia (Allerton 2003)
  - Can quan. all seq. with erasure pattern $e^n \in \{0, 1\}^n$ to $\mathcal{C}$
    $$\Updownarrow$$
    Chan. dec. for $\mathcal{C}^\perp$ can correct all vectors with eras. $1^n \oplus e^n$

- Choose a good (dual) code $\mathcal{C}(s^k)$

Coding Scheme for 2-write WOM: Second Write
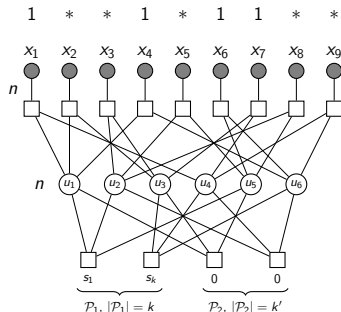
$$R_2 < 1 - \delta - h(p)$$

- ► *Change 0's to *'s*
- ► With message s^k, encode seq. to $\mathcal{C}(s^k)$

Coding Scheme for 2-write WOM: Second Write
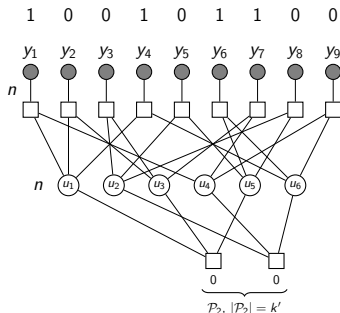
$$R_2 < 1 - \delta - h(p)$$

- *Change 0's to *'s*
- With message s^k, encode seq. to $\mathcal{C}(s^k)$
- Decoder has

$$y_i = x_i \oplus \text{Ber}(p)$$

Coding Scheme for 2-write WOM: Second Write

$$R_2 < 1 - \delta - h(p)$$
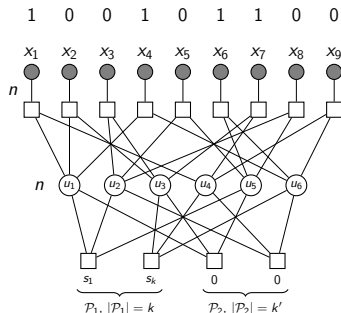
- ▶ *Change 0's to ∗'s*
- ▶ With message s^k, encode seq. to $\mathcal{C}(s^k)$
- ▶ Decoder has

$$y_i = x_i \oplus \mathsf{Ber}(p)$$

- ▶ Dec. x^n and compute s^k
- ▶ $R_2 = \frac{k}{n} \approx 1 - \delta - h(p)$

Iterative Erasure Quantization Algorithm
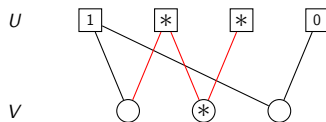
- *Peeling type encoder*

Remarks

- Need codes that are *simultaneously good* for channel/source coding and erasure quantization

- Use message-passing algorithms instead of *optimal*

- Use spatial-coupling for *goodness* of codes under message-passing
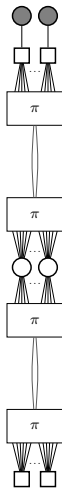
Spatially-Coupled Compound LDGM/LDPC Codes
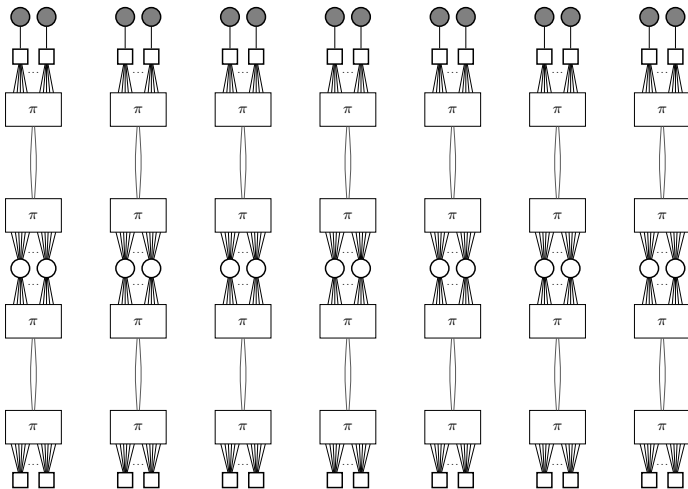
Spatially-Coupled Compound LDGM/LDPC Codes
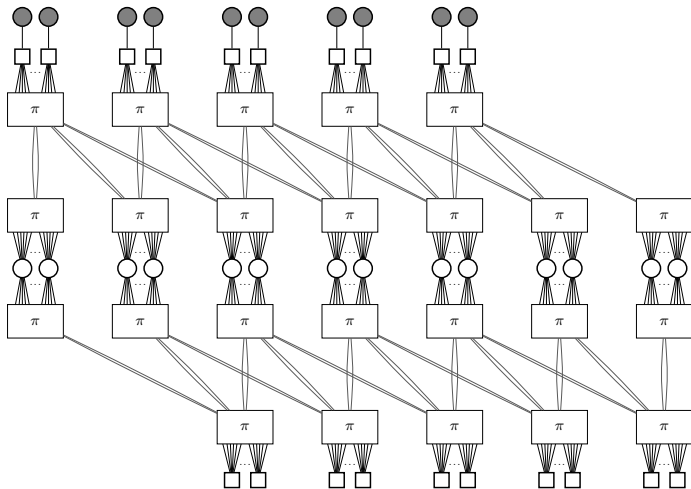
Spatially-Coupled Compound LDGM/LDPC Codes
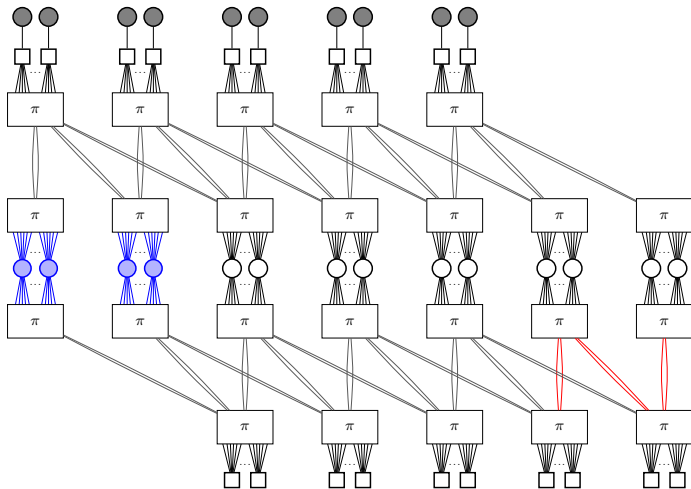
Spatially-Coupled Compound LDGM/LDPC Codes

Decoding in Spatially-Coupled Compound Codes
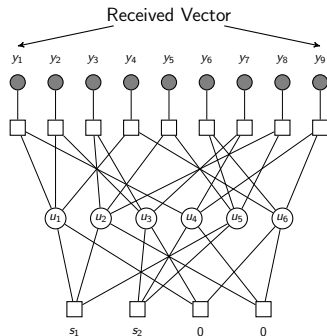
Remarks

- Standard message-passing algorithm
- Threshold saturation proven for SC compound codes on BEC
- Empirically observed for BMS channels

Numerical Results: Noiseless WOM

LDGM/LDPC (d_v, d_c, d'_v, d'_c)	δ^*	δ $w = 2$	δ $w = 3$	δ $w = 4$
$(3, 3, 3, 6)$	0.500	0.477	0.492	0.494
$(3, 3, 4, 6)$	0.333	0.294	0.324	0.326
$(3, 3, 5, 6)$	0.167	0.095	0.156	0.158
$(4, 4, 3, 6)$	0.500	0.461	0.491	0.492
$(4, 4, 4, 6)$	0.333	0.278	0.323	0.325
$(4, 4, 5, 6)$	0.167	0.086	0.155	0.159
$(5, 5, 3, 6)$	0.500	0.436	0.488	0.491
$(5, 5, 4, 6)$	0.333	0.260	0.320	0.324
$(5, 5, 5, 6)$	0.167	0.079	0.154	0.159

Remarks

- δ^* is the Shannon threshold
- $L = 30$, Single system length ≈ 24000

Numerical Results: WOM with Read Errors

LDGM/LDPC (d_v, d_c, d'_v, d'_c)	w	(δ^*, p^*)	(δ, p)
$(3, 3, 4, 6)$	3	$(0.333, 0.0615)$	$(0.321, 0.0585)$
$(3, 3, 4, 8)$	3	$(0.500, 0.0417)$	$(0.490, 0.0387)$
$(3, 3, 6, 8)$	4	$(0.250, 0.0724)$	$(0.239, 0.0684)$
$(4, 4, 4, 6)$	4	$(0.333, 0.0615)$	$(0.324, 0.0585)$
$(4, 4, 4, 8)$	4	$(0.500, 0.0417)$	$(0.492, 0.0387)$
$(4, 4, 6, 8)$	4	$(0.250, 0.0724)$	$(0.241, 0.0694)$

Remarks

- δ^* and p^* are the Shannon thresholds
- $L = 30$, Single system length ≈ 30000

Numerical Results: Small Blocklength
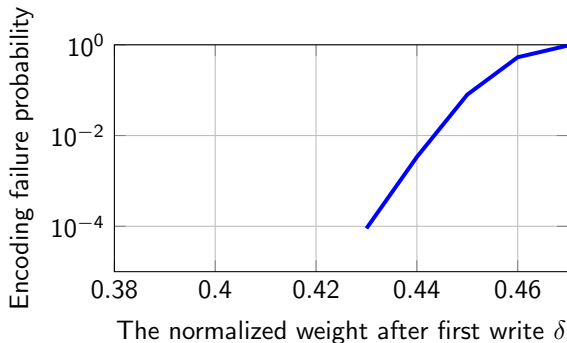
Remarks

- $(L, w) = (30, 3)$, Single system length 1200, Shannon threshold of 0.5
- A total of 10^5 were attempted to encode
- No failures for $\delta < 0.43$

Concluding Remarks

Conclusion

- Spatially-coupled compound codes achieve the capacity of 2-write systems

- Coupling structure is also crucial
  - to achieve optimum thresholds
  - for encoding to succeed

Multi-Write Systems

- Will BPGD work for multi-write systems?

Outline

1. Spatial Coupling(SC)
   - SC-LDPC Ensemble
   - Threshold Saturation Phenomenon

2. SC-LDPC Lattices
   - Introduction to Lattices
   - Proposed Lattice Construction
   - Poltyrev Goodness
   - Symmetric Interference Channel

3. Side-Information Problems
   - Gelfand-Pinsker & Wyner-Ziv
   - Compound LDGM/LDPC Codes
   - Spatial Coupling of Compound Codes

4. Write-Once Memory
   - Problem Statement
   - Coding Scheme

5. Research Summary

- ▶ SC-LDPC Lattices [C1]
- ▶ SC-Compound Codes
  - • Side-Information Problems [C2]
  - • Coding for WOM [C3]
- ▶ Tools from Sparse graph coding for sparse recovery problems
  - • Bipartite sparse graph for compressed sensing [C5]
  - • Group testing*
  - • Pattern matching*
- ▶ Uncoordinated multiple access
  - • Universal schemes for massive uncoordinated multiple access [C4]
  - • Optimal distributions for finite user multiple access*
- ▶ Coding for low latency requirements*

C1. A. Vem, Y. C. Huang, K. R. Narayanan and H. D. Pfister, "Multilevel lattices based on spatially-coupled LDPC codes with applications", in *Proc. IEEE. ISIT*, pp. 2336–2340, 2014.
C2. S. Kumar, A. Vem, K. R. Narayanan and H. D. Pfister,"Spatially-coupled codes for side-information problems", in *Proc. IEEE. ISIT*, pp. 516–520, 2014.
C3. S. Kumar, A. Vem, K. R. Narayanan and H. D. Pfister,"Spatially-coupled codes for write-once memories", in *Proc. Allerton. Conf*, pp. 125–131, 2015.
C4. A. Taghavi, A. Vem, J.-F. Chamberland and K. R. Narayanan "On the design of universal schemes for massive uncoordinated multiple access", in *Proc. IEEE. ISIT*, pp. 345–349, 2016.
C5. A. Vem, N. T. Janakiraman, and K. R. Narayanan,"Sub-linear time compressed sensing for support recovery using left and right regular sparse-graph codes", in *Proc. IEEE. ITW*, pp. 429–433, 2016.

*-To be submitted