INTRODUCTION TO FINANCIAL ENGINEERING

OPTION PRICING

Outline

- Used AAPL stock.
- Used YAHOO finance for stock history
- Use past one-year data to estimate the annual volatility.
- Uses 10 years' US treasury rate as the rate of interest.
- Evaluated Binomial for different strike prices and time of maturity to evaluate the call/ put option.
- Evaluated Black Scholes for different strike prices and time of maturity to evaluate the call/ put option.
- Showed convergence of Black Scholes and Binomial for large n
- Comparison with the actual market data
- Created a delta neutral portfolio
- Used numerical methods to get implied volatility.

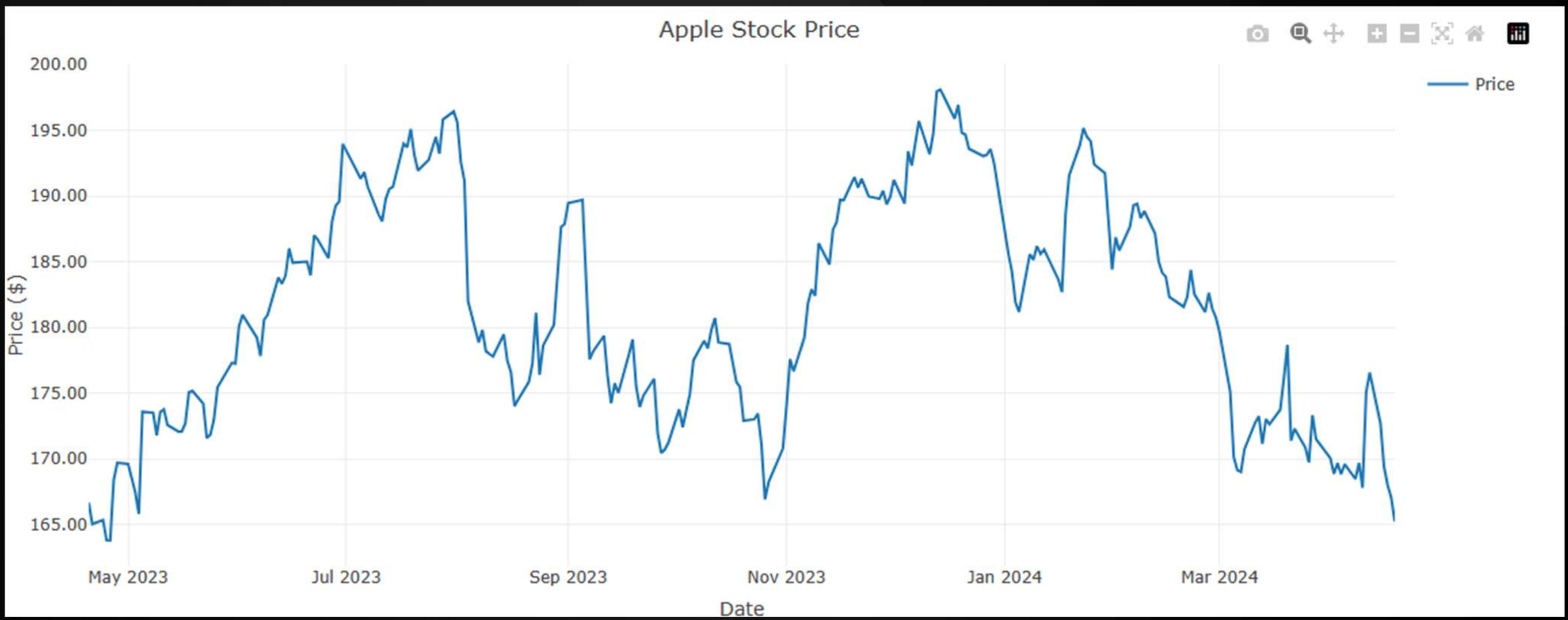
Procedure

Data Sources and Overview

Used AAPL stock:
We chose Apple Inc. (AAPL) as our underlying stock for this project.

Used YAHOO finance for stock history:
For collecting historical stock data, we utilized Yahoo Finance's API.
We downloaded data from **March 28, 2023**, to **April 1, 2024**, for AAPL stock.

Stock - AAPL



Apple Stock Price
Price
200.00
195.00
190.00
185.00
180.00
175.00
170.00
165.00
Price ($)
May 2023
Jul 2023
Sep 2023
Nov 2023
Jan 2024
Mar 2024
Date

Interest Rate

Used 10 years' US treasury rate as the rate of interest

r = 4.518%

Volatility

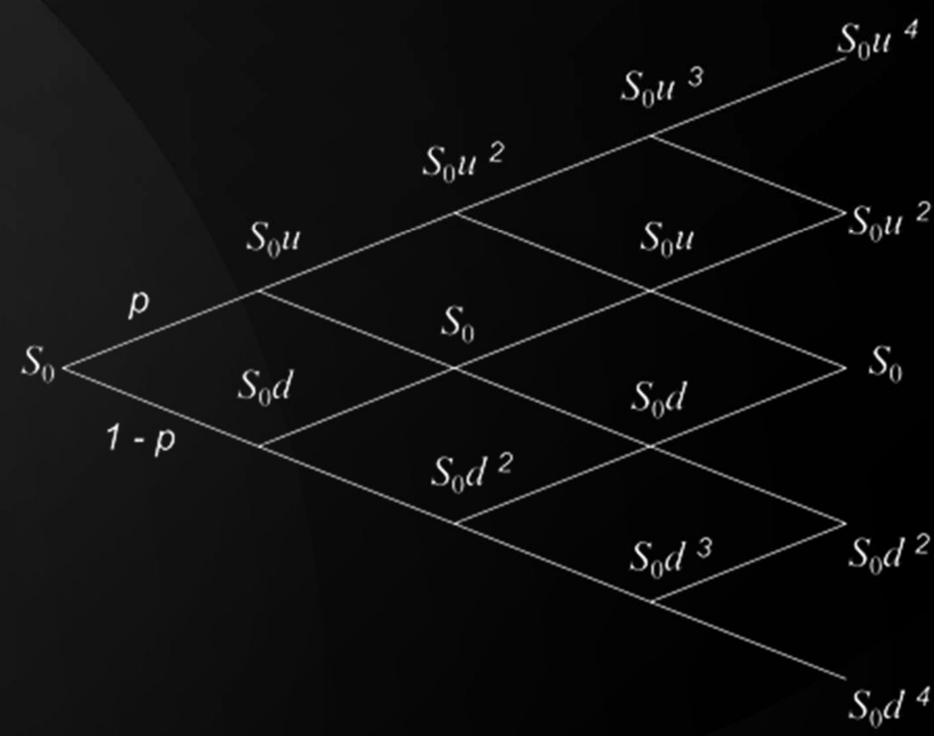
$$\sigma_{daily} = 1.22\%$$

$$\sigma_{annual} = \sigma_{daily} * \sqrt{252} = 19.36\%$$

Binomial Model

- The binomial option pricing model assesses options by employing an iterative method that involves multiple periods to determine their value.

- Within this model, each iteration results in two potential outcomes – an **upward movement** or a **downward movement** following a binomial tree.



Call Option Pricing Formula

$$C = \sum_{n=0}^{N} \binom{N}{n} p^n (1-p)^{N-n} \quad (S \quad u^n \quad d^{N-n} - K)^+ \; e^{-rT}$$

Put Option Pricing Formula

$$P = \sum_{n=0}^{N} \binom{N}{n} p^n (1-p)^{N-n} \quad (K - S \cdot u^n \quad d^{N-n})^+ \; e^{-rT}$$

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

$$\boldsymbol{\Delta t} = T / n$$

$$\boldsymbol{u} = e^{\sqrt{(\sigma \Delta t)}}$$

$$\boldsymbol{d} = e^{-\sqrt{(\sigma \Delta t)}}$$

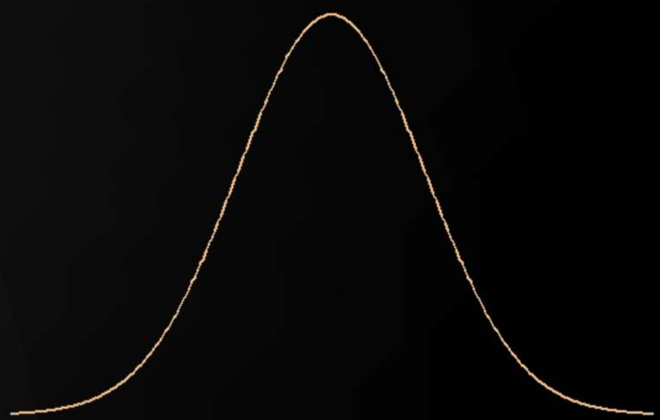
Or

Use Put-Call Parity

	Strike Price	Time to Maturity (Years)	Call Option Price	Put Option Price
0	120	0.5	43.5182	0.0677969
1	120	1	46.5035	0.432497
2	120	1.5	49.5183	0.885316
3	120	2	52.4738	1.33607
4	120	5	68.033	2.99877
5	140	0.5	25.0843	1.1872
6	140	1	29.5731	2.61865
7	140	1.5	33.5711	3.62767

Black Scholes Model

- The model considers **five** key components:
  - The **price** of the underlying asset.
  - The **strike price** of the option.
  - The **time to expiration.**
  - The **risk-free interest rate**, and
  - The **volatility** of the underlying asset's returns.



- The Black-Scholes model provides a theoretical framework for pricing options.

$$C = SN(d_1) - Ke^{-rt}N(d_2)$$

where:

$$d_1 = \frac{ln\frac{S}{K} + (r + \frac{\sigma_v^2}{2})t}{\sigma_s\sqrt{t}}$$

and

$$d_2 = d_1 - \sigma_s\sqrt{t}$$

and where:

C = Call option price

S = Current stock (or other underlying) price

K = Strike price

r = Risk-free interest rate

t = Time to maturity

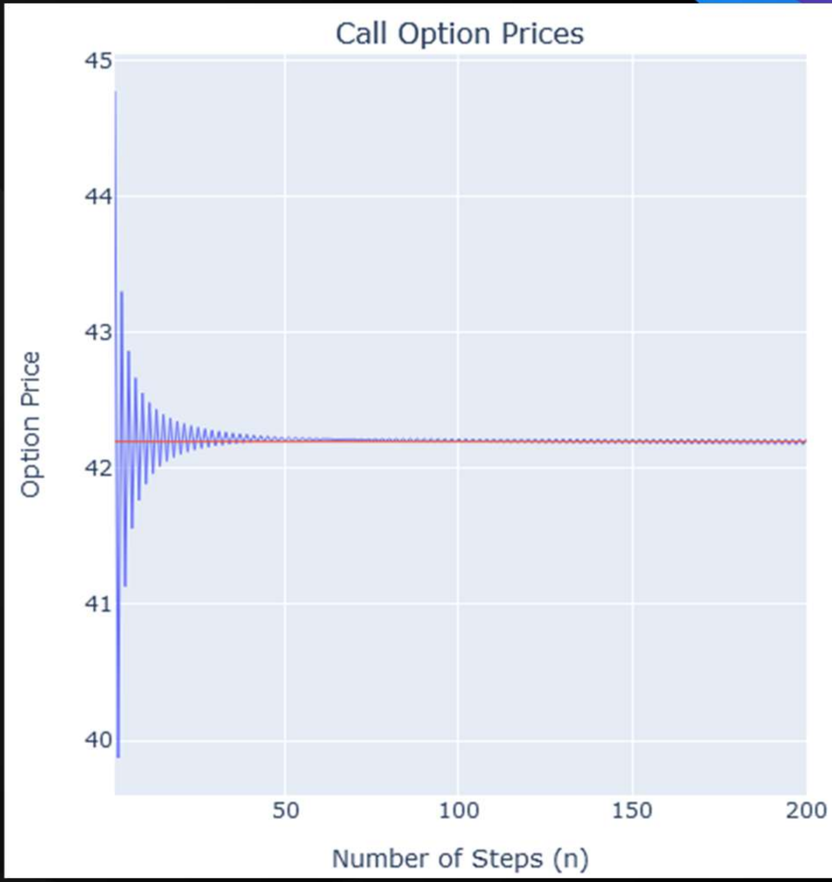
N = A normal distribution

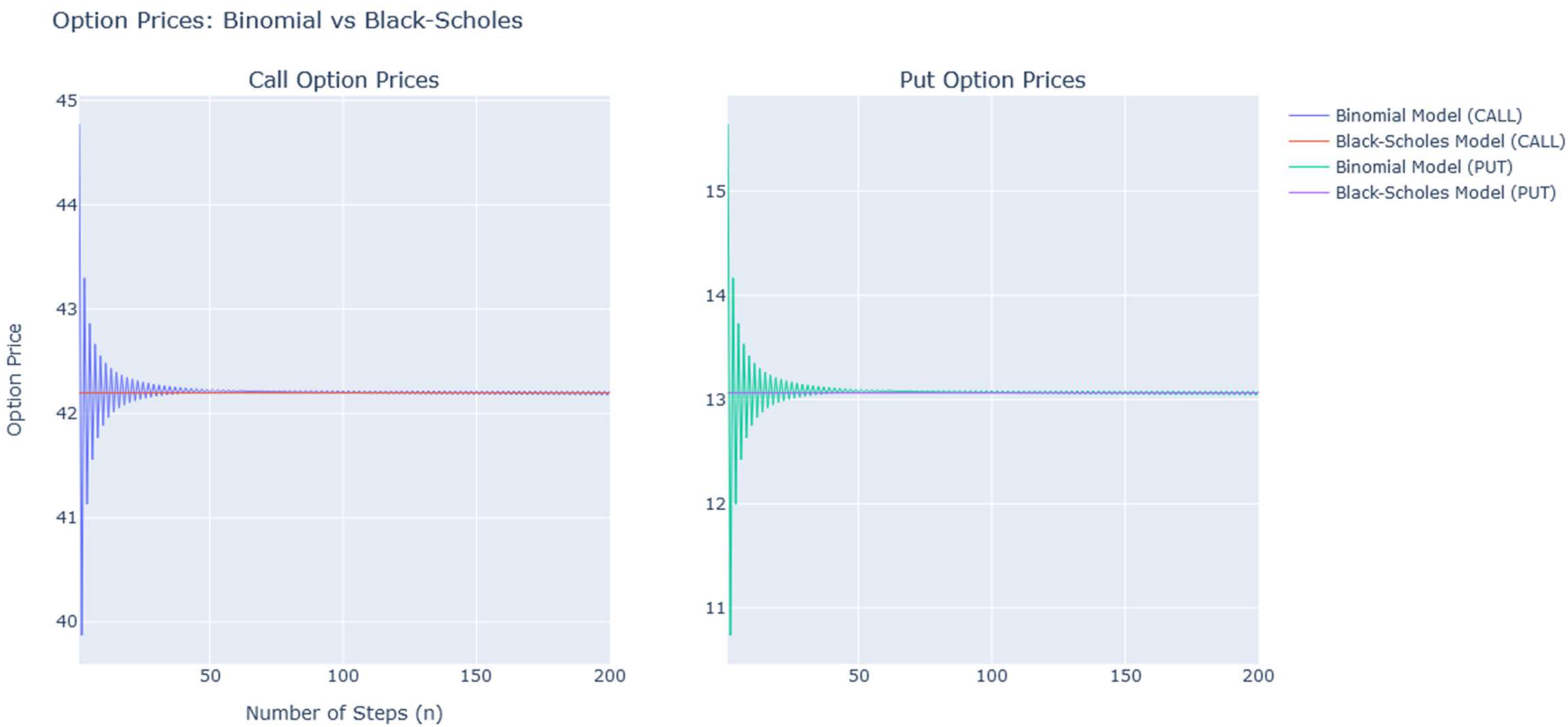
To find P
Use Put-Call Parity

	Strike Price	Time to Maturity (Years)	Call Option Price	Put Option Price
0	120	0.5	43.5189	0.068449
1	120	1	46.5011	0.430124
2	120	1.5	49.5214	0.888491
3	120	2	52.4673	1.32953
4	120	5	68.0167	2.98246
5	140	0.5	25.0934	1.19623
6	140	1	29.5576	2.60316
7	140	1.5	33.5816	3.63812

Convergence

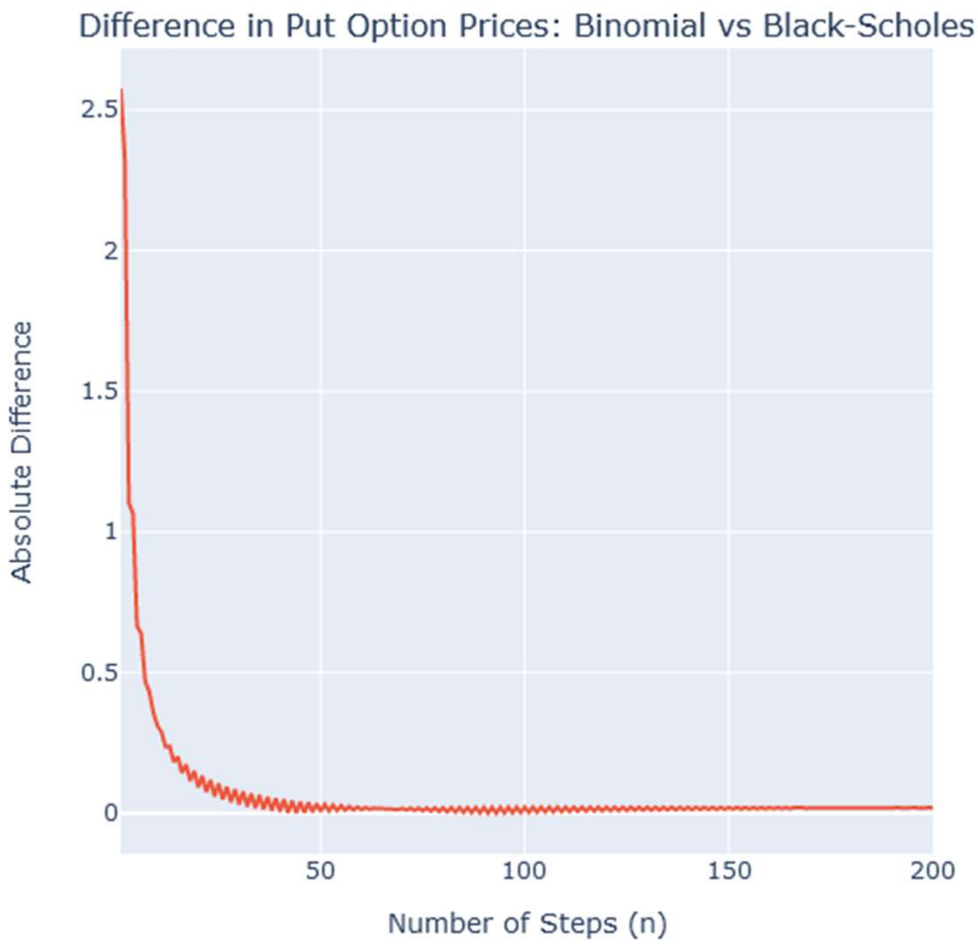
- Increasing the number of time steps (nn) in the binomial model leads to finer granularity and smaller time intervals between steps.
- This finer granularity results in the binomial model approaching the continuous-time dynamics assumed by the Black-Scholes model, indicating convergence between the two models.





Option Prices: Binomial vs Black-Scholes
Call Option Prices
Put Option Prices
Option Price
Number of Steps (n)
45
44
43
42
41
40
15
14
13
12
11
50
100
150
200
Binomial Model (CALL)
Black-Scholes Model (CALL)
Binomial Model (PUT)
Black-Scholes Model (PUT)


Difference in Call Option Prices: Binomial vs Black-Scholes
Absolute Difference
2.5
2
1.5
1
0.5
0
50
100
150
200
Number of Steps (n)


Difference in Put Option Prices: Binomial vs Black-Scholes
Absolute Difference
2.5
2
1.5
1
0.5
0
50
100
150
200
Number of Steps (n)

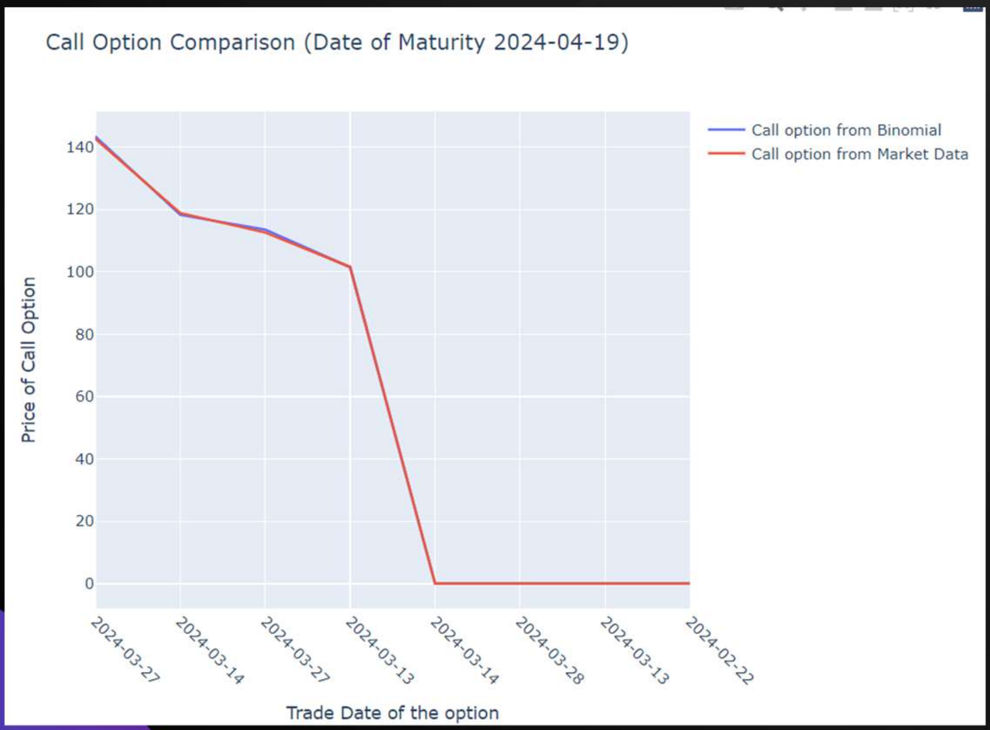
Comparison with Actual Market

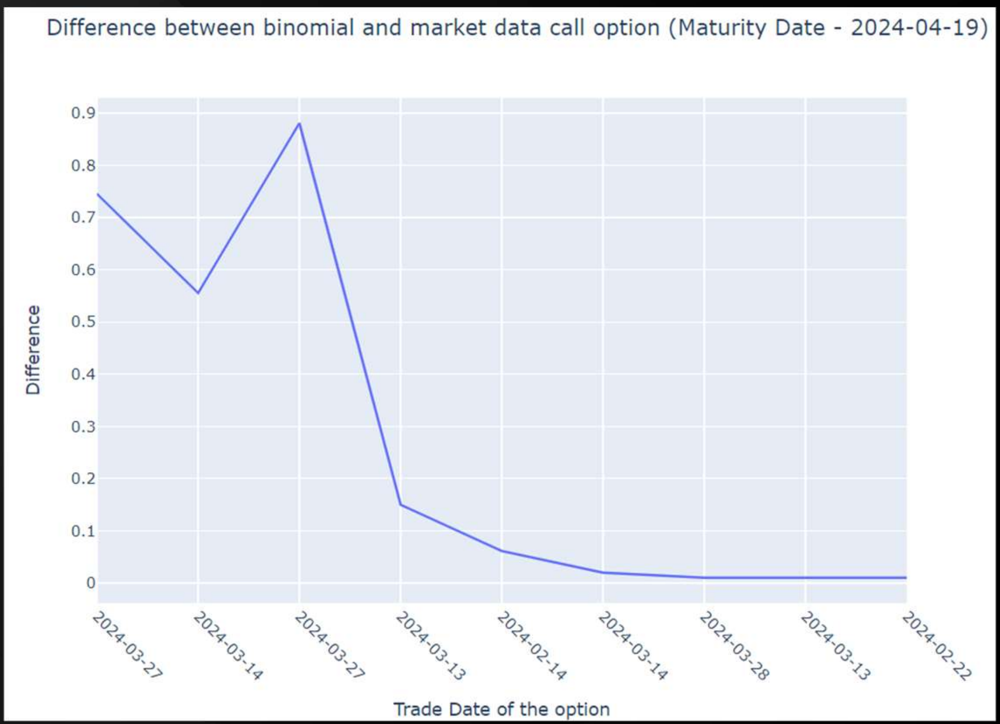
- Applying Black scholes and Binomial model on the dataset of options (collected from YahooFinance)
- Period of maturity = ExpireDate - LastTradeDate (In days)
- Steps of Binomial Model = 1 day

Contract Name	Last Trade Date (EDT)	Expire Date	Last Price	Bid	Ask	Change	% Change	Volume	Open Interest	Implied Volatility
AAPL240419C00005000	4/16/2024 4:14 PM	**2024-04-19**	164.30	159.40	161.50	0.00	0.00%	1	519	3,559.38%
AAPL240517C00005000	4/18/2024 6:35 PM	**2024-05-17**	161.69	159.35	160.45	0.00	0.00%	1	3	660.94%
AAPL240621C00005000	3/21/2024 1:30 PM	**2024-06-21**	171.95	159.20	160.80	0.00	0.00%	6	7	50.00%
AAPL240719C00005000	4/2/2024 1:48 PM	**2024-07-19**	164.00	158.75	160.75	0.00	0.00%	1	3	412.11%
AAPL240816C00005000	3/21/2024 3:09 PM	**2024-08-16**	167.00	158.85	160.55	0.00	0.00%	-	12	337.89%
AAPL240920C00005000	4/18/2024 7:11 PM	**2024-09-20**	162.05	158.60	161.10	0.00	0.00%	1	12	348.63%
AAPL241018C00005000	3/18/2024 2:56 PM	**2024-10-18**	172.06	162.45	164.00	0.00	0.00%	1	0	480.66%
AAPL241115C00005000	2/29/2024 2:32 PM	**2024-11-15**	175.80	164.50	168.45	0.00	0.00%	-	1	0.00%
AAPL250117C00005000	4/15/2024 1:35 PM	**2025-01-17**	169.40	158.65	161.90	0.00	0.00%	2	41	197.07%

Comparison with Actual Market

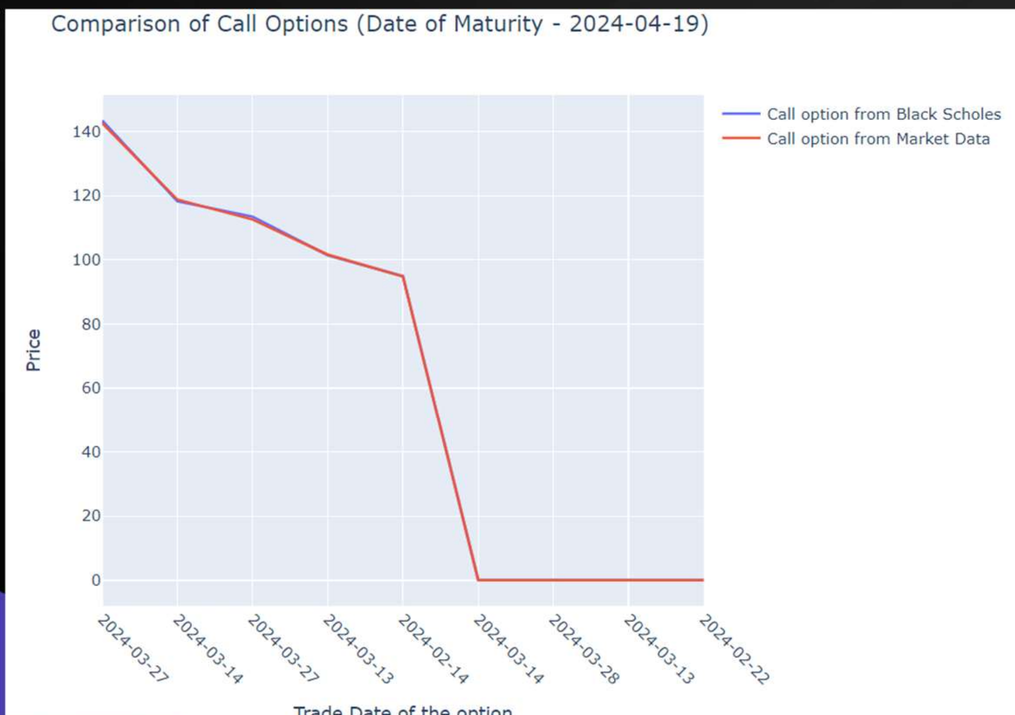
Call Option using Binomial Model

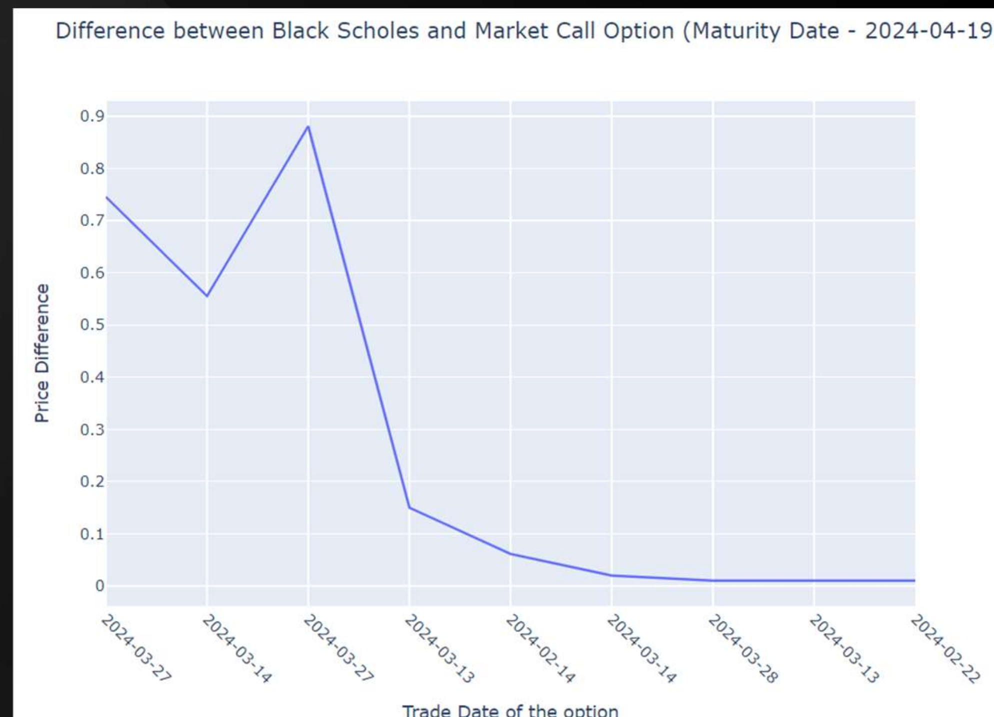




Comparison with Actual Market

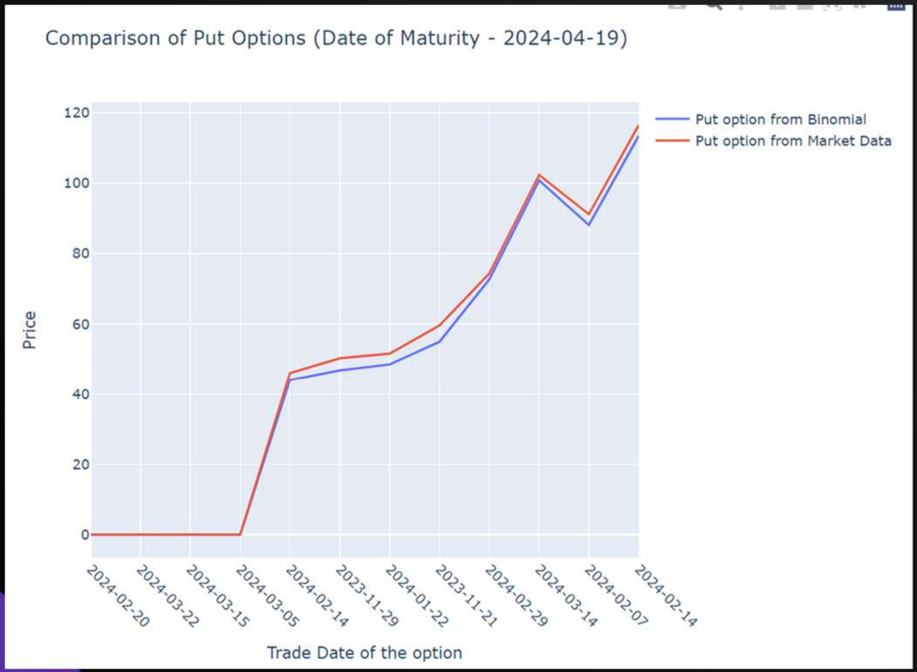
Call Option using Black Scholes Model





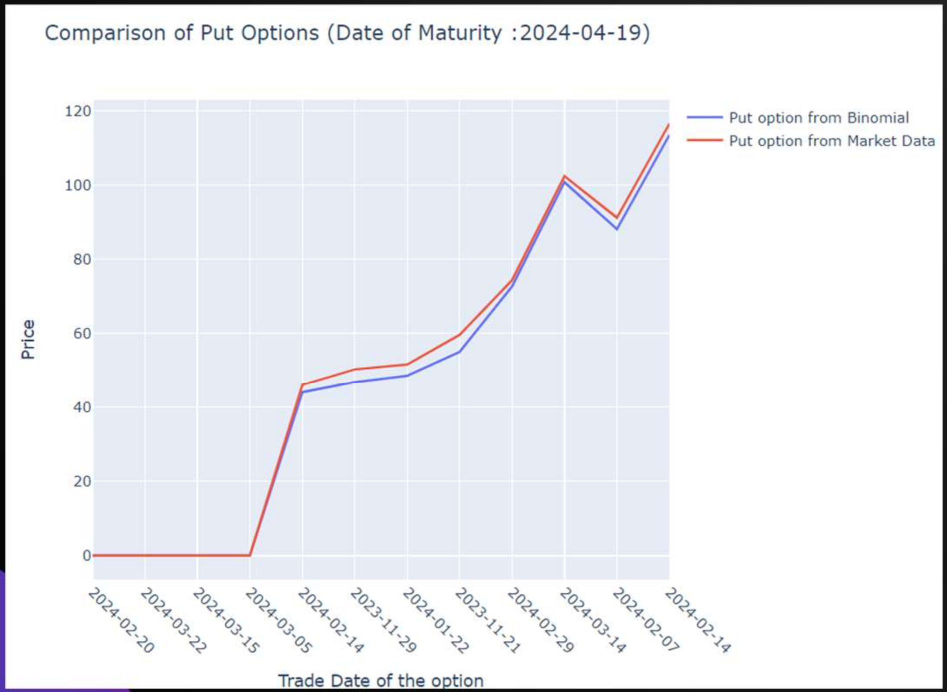
Comparison with Actual Market

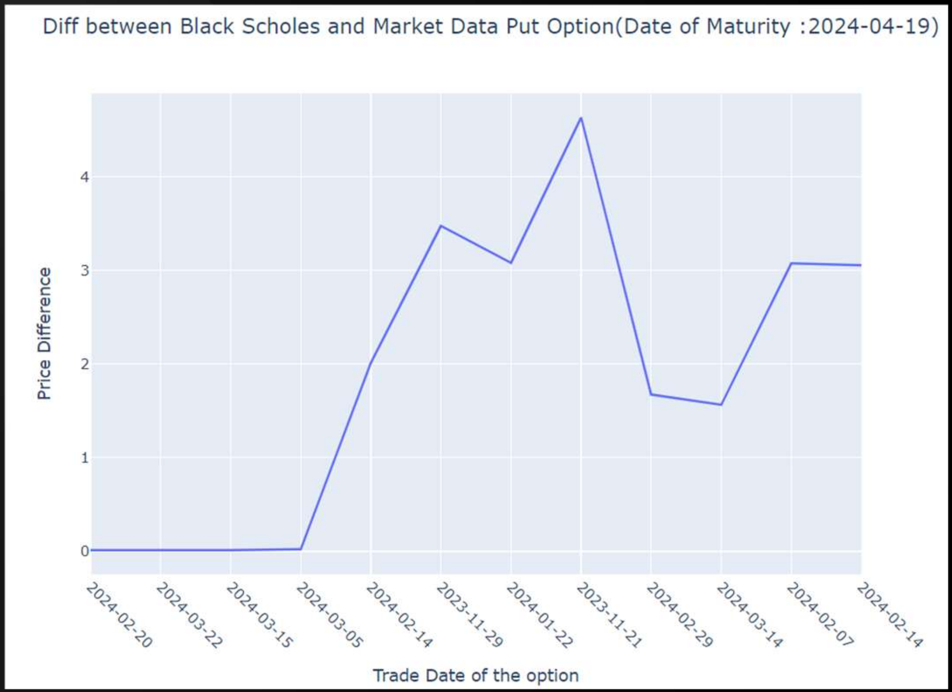
Put Option using Binomial Model



Comparison with Actual Market

Put Option using Black Scholes Model





Comparison with Actual Market

Binomial vs Black Scholes vs Actual Market

Comparison with Actual Market

Conclusion

- With a combination of shorter time to expiration and a sufficiently large number of steps in the binomial model, the practical difference between the two models becomes negligible.
- Black-Scholes and Binomial models are really good at predicting prices, which makes them valuable tools for making decisions in finance today.
- This shows that Binomial and Black-Scholes are applicable in actual market conditions and real world trading.

Delta Neutral Portfolio

- Delta neutral is a portfolio strategy that uses multiple positions to balance positive and negative deltas so the overall delta of the assets totals zero.

- A delta-neutral portfolio evens out the response to market movements for a certain range to bring the net change of the position to zero.

Neutralizing using options only

For Black Scholes model,

$$\boxed{\frac{d(C^E(S))}{dS} = N(d_1)}$$

$$\boxed{\frac{d(P^E(S))}{dS} = N(d_1) - 1}$$

$$x \cdot \Delta_C + y \cdot \Delta_P = 0$$

Call options

Strike (K)	Time to Maturity (T in days)	Call Delta	Portfolio (number of put options)
30	23	0.999742	38731.2
55	36	0.998498	6648.62
60	23	0.996792	3107.03
70	37	0.997174	3528.12
90	65	0.999097	11065.1

Put options

Strike (K)	Time to Maturity (T in days)	Put Delta	Portfolio (number of call options)
65	59	-0.000626	0.0062663
70	28	-0.00432	0.0433755
75	35	-0.00367	0.0368403
85	45	-0.00299	0.0299823
230	65	-0.00588	0.059131

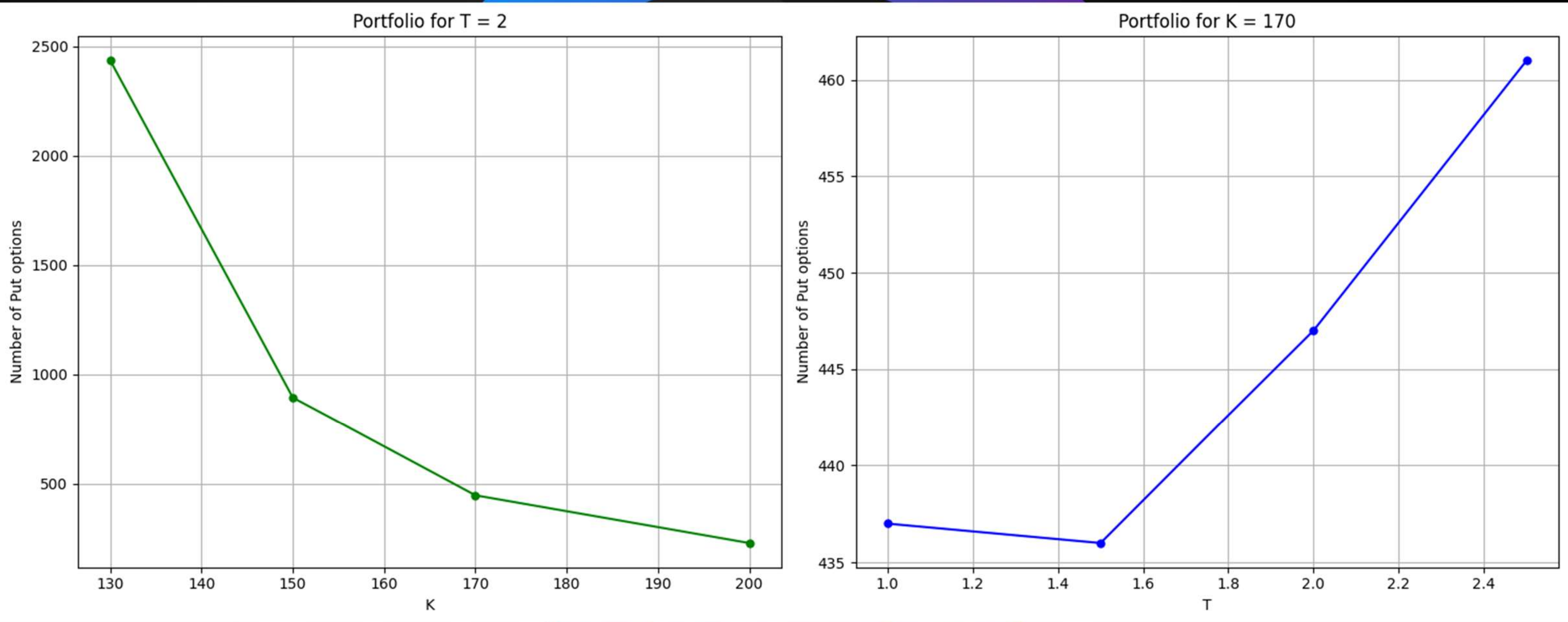
Neutralizing using options and number of shares

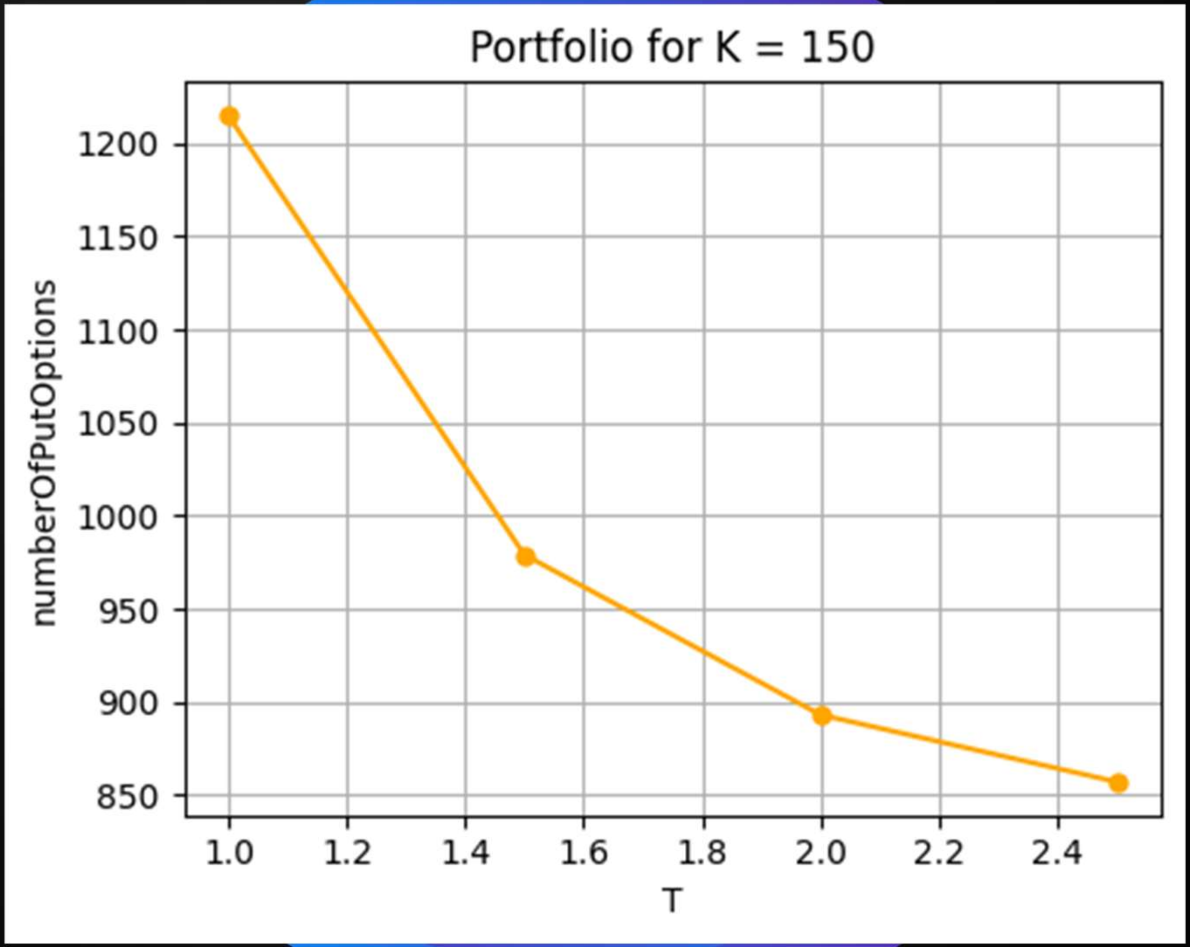
Let's suppose we own 100 shares of AAPL stocks, now to neutralize the delta we need to buy Put options.

$$\boxed{\Delta_s = +1}$$

$$\boxed{\frac{d(P^E(S))}{dS} = N(d_1) - 1}$$

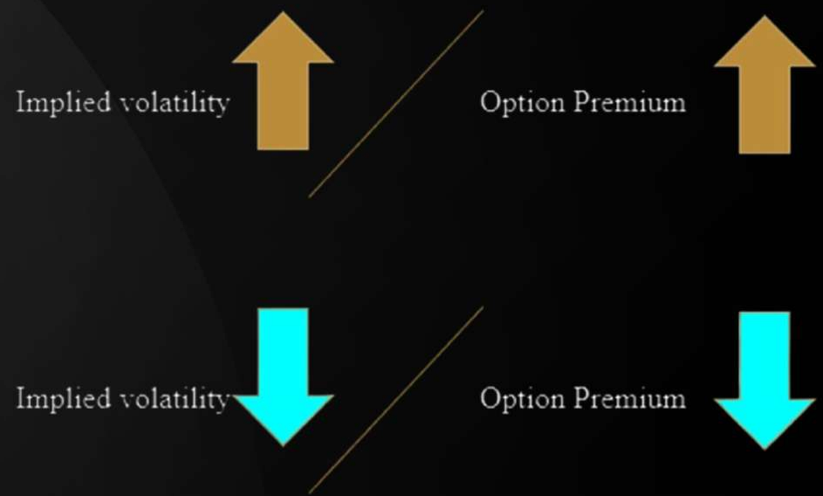
$$100 \cdot \Delta_s + y \cdot \Delta_p = 0$$


Portfolio for T = 2
Number of Put options
K
Portfolio for K = 170
Number of Put options
T


Portfolio for K = 150
numberOfPutOptions
1200
1150
1100
1050
1000
950
900
850
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
T

Implied Volatility

- IV is often used to price options contracts where high implied volatility results in options with higher premiums and vice versa.
- Implied volatility usually increases in bearish markets and decreases when the market is bullish.



Numerical Method

We need to solve for σ

$$(C(S_0, K, T, r, \sigma) - C_0)^2 = 0$$

For this we will use Newton-Raphson method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Call Options:

	K	C	T	S0	r	Implied Volatility(%)
0	30	142.65	23	173.31	0.04518	0
1	55	118.8	36	173	0.04518	0
2	60	112.6	23	173.31	0.04518	0
3	70	101.6	37	171.13	0.04518	122.397
4	90	94.81	65	184.15	0.04518	0
5	255	0.02	36	173	0.04518	43.4183
6	270	0.01	22	171.48	0.04518	60.461
7	285	0.01	37	171.13	0.04518	51.7409
8	290	0.01	57	184.37	0.04518	37.0416

Put Options:

	K	P	T	S0	r	Implied Volatility(%)
1	70	0.01	28	172.28	0.04518	105.88
2	75	0.01	35	172.62	0.04518	89.2486
3	85	0.02	45	170.12	0.04518	70.5573
4	230	46.04	65	184.15	0.04518	42.2296
5	240	50.27	142	189.37	0.04518	35.089
6	245	51.6	88	193.89	0.04518	41.897
7	250	59.6	150	190.64	0.04518	40.3173
8	255	74.35	50	180.75	0.04518	63.3845
9	275	102.34	36	173	0.04518	92.9561
10	280	91.18	72	189.41	0.04518	66.3324
11	300	116.5	65	184.15	0.04518	82.2096

THANK YOU