INDIAN INSTITUTE OF TECHNOLOGY, GUWAHATI

DEPARTMENT OF MECHANICAL ENGINEERING

OPTIMIZATION METHODS IN ENGINEERING – ME 609

PROJECT PHASE – 3

CONSTRAINTS OPTIMIZATION WITH PENALTY METHOD

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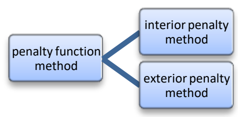
SPECIALIZATION: COMPUTATIONAL MECHANICS

Penalty function method ⇨

- The penalty method is one of the constrained optimization method . In this method if the constraint violated then objective functions get penalized .
- Now the constraint transformed into the unconstraint problem by adding the penalty term of each constraint violation.
- It works in the series of sequence and each time penalty added with new point obtained and form a sequence of series .
- Penalty method are two types:



Interior & exterior penalty method

- This method works for the feasible points and penalize points that are close to constraint boundary.
- This methods penalize the infeasible points but not the feasible solutions.

Powell's conjugate direction method ⇨

- This method uses previous points to create new points.
- First of all we set a n linearly independent search direction and perform a series of unidirectional search along the each of search direction from previous best point.
- This procedure help in the finding of minimum by one pass in the n universal directions.

Parallel subspace property ⇨

- It is method to reach the optimum solution by using the property of conjugate direction.
- It is method which gives exact solution of quadratic problem by traversing all the direction once.

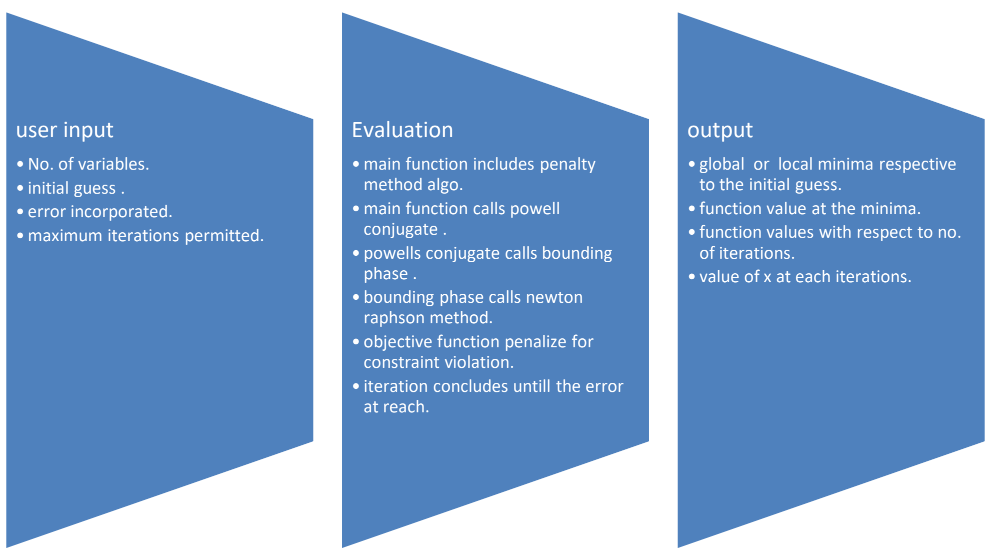
Bounding phase method ⇨

- Bounding phase method is a single variable optimization direct search bracketing method.

- This method begins with the initial guess.
- Then finds a search direction based on the two function evaluations at the initial guess.
- After that a exponential search strategy is used to bracket the minimum point .
- Large value of delta respond to fast bracketing but the accuracy is poor.
- Small value of delta respond to better accuracy but it requires more number of iterations .

Newton Raphson method⇨

- Convergence plot depends on the initial points and nature of objective function.
- At the every iterations the first and second derivatives of the function values are calculated .
- If the initial guess is near to the minima then it converges and give exact value.



QUESTION NO. 1

$$\boldsymbol{min\ f(x) = (x_1 - 10)^3 + (x_2 - 20)^3}$$

$$\boldsymbol{g_1(x) = (x_1 - 5)^2 + (x_2 - 5)^2 - 100 \geq 0}$$

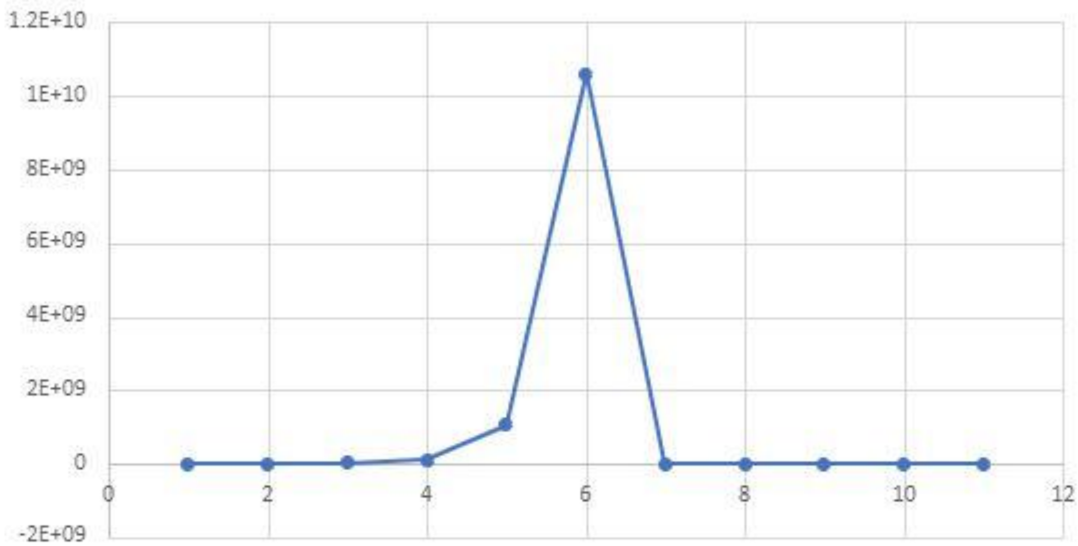
$$\boldsymbol{g_2(x) = (x_1 - 6)^2 + (x_2 - 5)^2 - 82.81 \leq 0}$$

$$\boldsymbol{13 \leq x_1 \leq 20, \quad 0 \leq x_2 \leq 4}$$

At initial guess X1 = 18, X2 = 4;

X1*	X2*	F(X*)
5.708198	-18.582888	-35101.670536
28.168565	-18.582569	879349.448069
29.445654	-18.582533	10415710.220063
29.568885	-18.582530	105796712.707921
29.581165	-18.582529	1059608324.811890
29.582393	-18.582529	10597724626.985151
14.089197	0.830944	-6968.586754
14.094418	0.841755	-6962.492578
14.094942	0.842840	-6961.881760
14.094994	0.842949	-6961.820664
14.094994	0.842949	-6961.759581

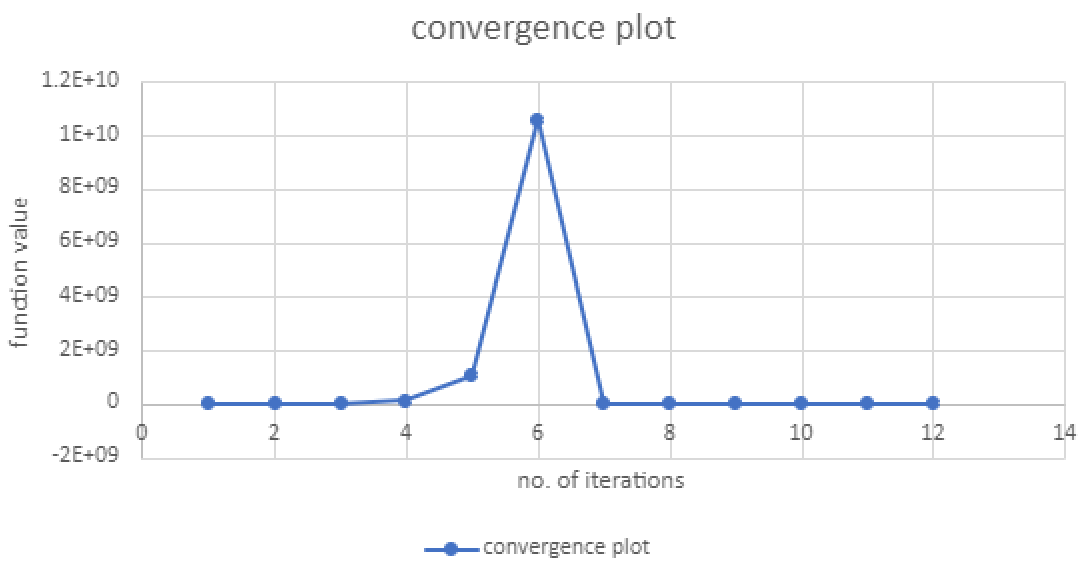
Optimum function value V/S No. of iterations ⇨

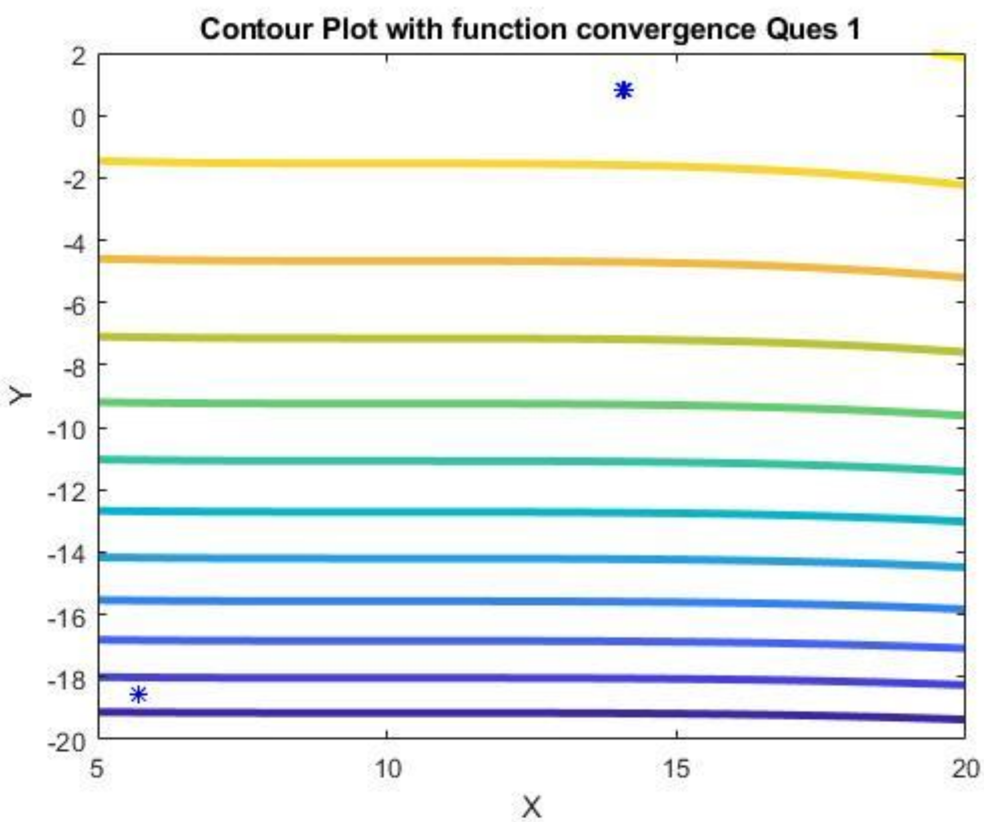


At initial guess X1 = 13, X2 = 0;

X1*	X2*	F(X*)
5.708200	-18.582889	-35101.670536
28.168566	-18.582570	879349.650930
29.445655	-18.582534	10415712.423899
29.568886	-18.582531	105796734.922819
29.581166	-18.582530	1059608547.142989
29.582394	-18.582530	10597726850.559156
14.089197	0.830944	-6968.586754
14.094418	0.841755	-6962.492578
14.094942	0.842840	-6961.881760
14.133667	0.919113	-6873.298637
14.094999	0.842960	-6961.814554
14.094999	0.842960	-6961.808447

Optimum function value V/S No. of iterations ⇨


convergence plot
1.2E+10
1E+10
8E+09
6E+09
4E+09
2E+09
0
-2E+09
function value
0
2
4
6
8
10
12
14
no. of iterations
convergence plot


Contour Plot with function convergence Ques 1
2
0
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
Y
5
10
15
20
X

Values at different initials points:

N	X1	X2	X1*	X2*	F(X*)
1	18	4	14.094994	0.842949	-6961.759581
2	14	1	14.094994	0.842949	-6961.759580
3	15	3	14.094994	0.842949	-6961.759584
4	16	1	14.107546	0.869491	-6932.012634
5	17	2	14.094994	0.842949	-6961.759566
6	19	4	14.094999	0.842960	-6961.808447
7	20	1	14.094994	0.842949	-6961.759583
8	20	4	14.094994	0.842949	-6961.759546
9	18	1	14.094994	0.842949	-6961.759698
10	16	3	14.094994	0.842949	-6961.759568

	Function values
Best	-6961.808447
Worst	-6932.012634
Mean	- 6958.78978
Median	-6961.7595
Standard deviation	8.9257268

QUESTION NO. 2

$$\boldsymbol{max\, f(x) = \frac{sin^3(2\pi x_1) sin(2\pi x_2)}{x_1^3(x_1 + x_2)}}$$

$$\boldsymbol{g_1(x) = x_1{}^2 - x_2 - 1 \leq 0}$$

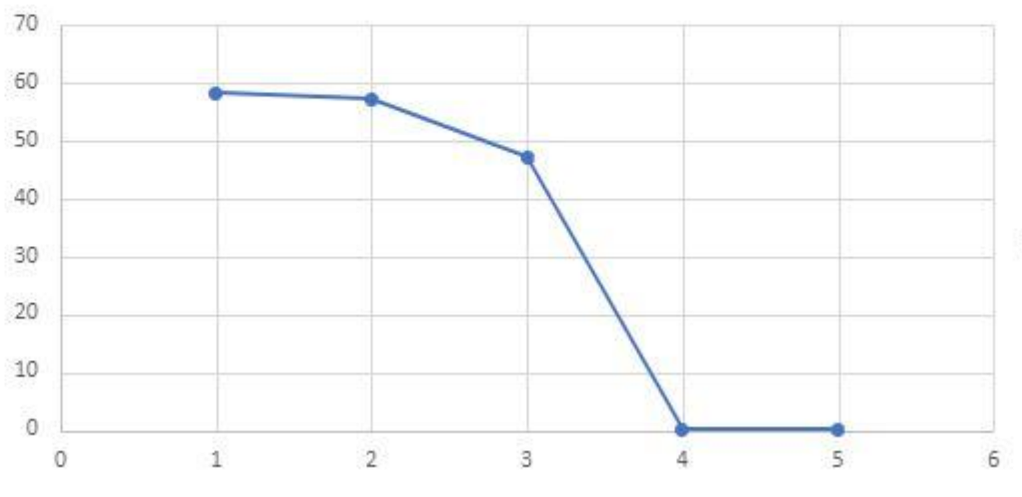
$$\boldsymbol{g_2(x) = 1 - x_1 + (x_2 - 4)^2 \leq 0}$$

$$\boldsymbol{0 \leq x_1} \leq 10,\ \ \boldsymbol{0 \leq x_2} \leq 10$$

At initial guess X1= 6 , X2=2;

X1*	X2*	F(X*)
-0.005884	4.243985	58.333162
-0.005092	4.243991	57.312358
-0.000820	4.243981	47.172704
1.227972	4.245377	0.095825
1.227972	4.245377	0.095825

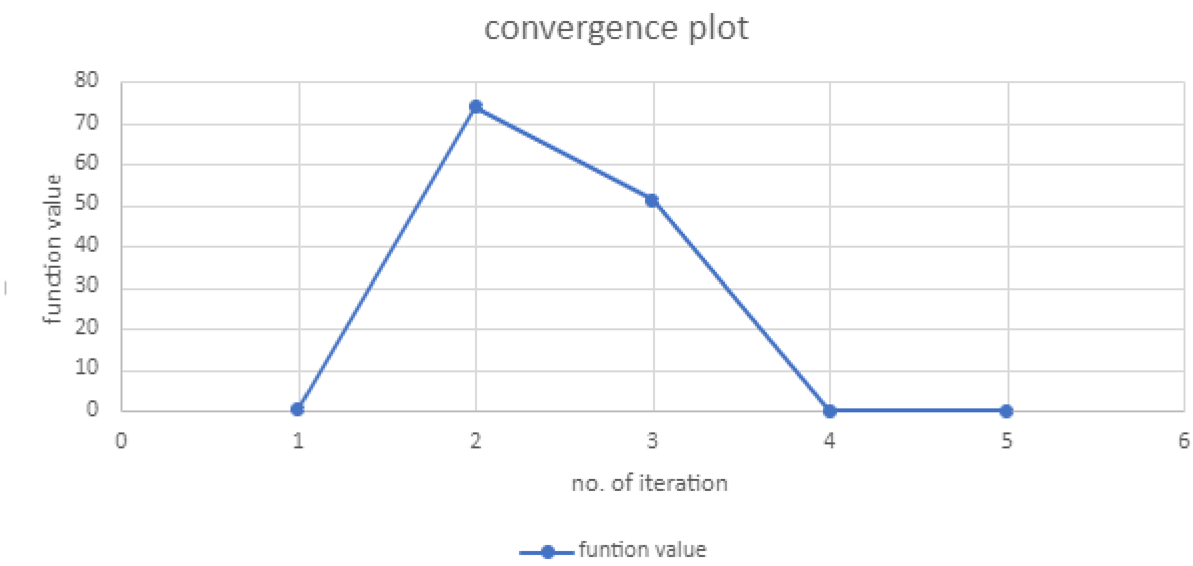
Optimum function value V/S No. of iterations ⇨



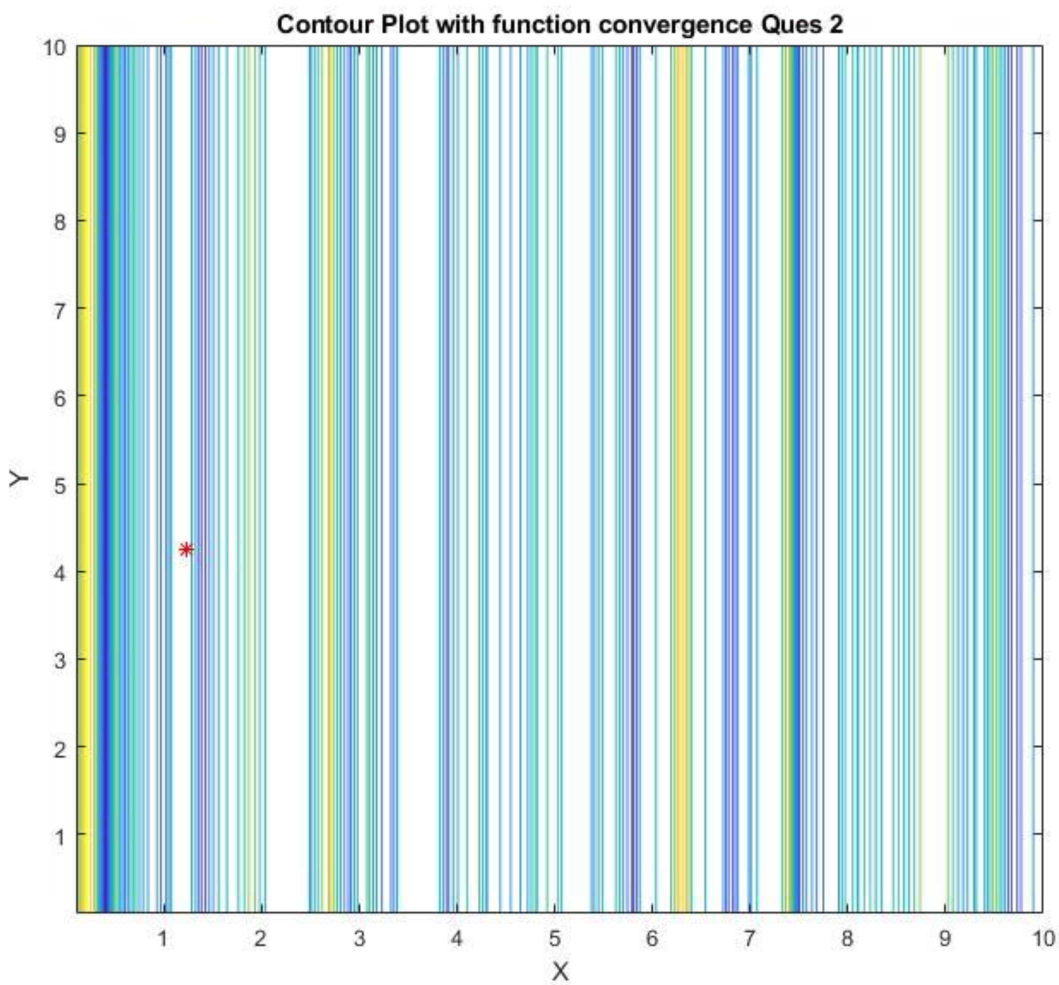
At initial guess X1= 4 , X2=9;

X1*	X2*	F(X*)
-0.714352	3.747087	0.522052
-0.006780	3.243763	74.009320
0.017003	3.243832	51.405839
1.227972	4.245377	0.095825
1.227972	4.245377	0.095825

Optimum function value V/S No. of iterations ⇨



convergence plot
function value
no. of iteration
funtion value
0
10
20
30
40
50
60
70
80
1
2
3
4
5
6



Contour Plot with function convergence Ques 2
Y
X

Values at different initials points:

N	X1	X2	X1*	X2*	F(X*)
1	2	8	1.227972	4.245377	0.095825
2	3	10	1.227972	4.245377	0.095825
3	4	7	1.734144	4.746096	0.029144
4	5	9	1.227972	4.245377	0.095825
5	6	6	1.734144	4.746096	0.029144
6	7	1	1.324399	3.430439	0.027264
7	8	1	1.680372	3.795697	0.019621
8	8	10	1.227972	4.245377	0.095825
9	9	4	1.227972	4.245377	0.095825
10	10	6	1.227972	4.245377	0.095825

	Function values
Best	0.095825
Worst	0.019621
Mean	0.0680123
Median	0.095825
Standard deviation	0.0341539

QUESTION NO. 3

$$\boldsymbol{min\ f(x) = x_1 + x_2 + x_3}$$

$$\boldsymbol{g_1(x) = -1 + 0.0025(x_4 + x_6) \leq 0}$$

$$\boldsymbol{g_2(x) = -1 + 0.0025(-x_4 + x_5 + x_7) \leq 0}$$

$$g_3(x) = -1 + 0.01(-x_6 + x_8) \leq 0$$

$$g_4(x) = 100x_1 - x_1x_6 + 833.33252x_4 - 83333.333 \leq 0$$

$$g_5(x) = x_2x_4 - x_2x_7 - 1250x_4 + 1250x_5 \leq 0$$

$$g_6(x) = x_3x_5 - x_3x_8 - 2500x_5 + 1250000 \leq 0$$

$100 \leq x_1 \leq 1000$,

$10 \leq x_i \leq 1000$, **i=4,5,6,7,8**

$1000 \leq x_i \leq 10000$ **,i=2,3**

Due to the more no. of constraints it might be the penalty increases too much and the function value coming very large.

Observation ⟹

- Then minima of the function depends upon the initial guess .
- At some initial guesses it converges into local minima and at some initial guesses it converges into global minima due to movement in various direction.
- When function converges to minima then penalty become zero.
- First of all it minimize the function after that when R increases then it minimize

  Sequence of constraint which added with function by adding penalty.
- In the some function at different initial guess first the function value increases after that it converges to minima after some iteration.
- In some cases the function works very well and minimize after certain iteration.

Conclusion ⟹

- The penalty method is the best method used in constrained optimization which includes penalty for constraint violation .
- Any constraint convex , concave , linear and non-linear can be handled.

- At every iteration the   penalized function get distorted as compare to objective function.
- Due to penalized function convergence result in slower convergence.
- It requires more no. of function evaluations.
- Local minima can exist due to distorted penalized function.

Thank You