Putnam preparation

Induction, pigeonholing

Induction: Let a be an integer and $P(n)$ a proposition (statement) about n for each integer $n \geq a$. The principle of mathematical induction states that:

If
(i) $P(a)$ is true, and
(ii) for each integer $k \geq a$, $P(k)$ true implies $P(k+1)$ true,
then $P(n)$ is true for all integers $n \geq a$.

The following variant, sometimes called **strong induction**, is occasionally useful:

If
(i) $P(a)$ is true, and
(ii) for each integer $k \geq a$, $P(a), P(a+1), \ldots, P(k)$ true implies $P(k+1)$ true,
then $P(n)$ is true for all integers $n \geq a$.

Pigeonhole principle: If $kn+1$ objects ($k \geq 1$) are distributed among n boxes, one of the boxes will contain at least $k+1$ objects.