This document shows **all versions** of each question (or part of a question) on the test, along with their sample solution. Each individual test paper contained only one version of each question (or each part).

1. [6 marks] For both statements below:

(i) Write the negation of the original statement without using the $\neg$ symbol.

(ii) Write whether the original statement is true or false.

(iii) If the original statement is true, prove it. If the original statement is false, disprove it.

(NOTE: The notation $\mathbb{R}^{\geq 0}$ represents the set $[0, \infty) = \{x \in \mathbb{R} \mid x \geq 0\}$.)

(a) $\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (x > n) \Rightarrow (6x < 2n + 3)$

(a) $\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (x > n) \Rightarrow (3x > 5n + 2)$

(a) $\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (x > n) \Rightarrow (5x < 2n + 1)$

(a) $\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (x > n) \Rightarrow (4x > 3n + 6)$

(a) $\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (x > n) \Rightarrow (7x < 4n - 3)$

(a) $\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (x < n) \Rightarrow (2x > 3n + 3)$

(a) $\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (x < n) \Rightarrow (6x < 2n + 1)$

(a) $\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (x < n) \Rightarrow (3x > 7n - 4)$

(a) $\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (x < n) \Rightarrow (5x < 3n - 1)$

(b) $\forall x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{Z}, (x \geq n) \wedge (3x + 1 > 3n^2)$

(b) $\forall x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{Z}, (x \geq n) \wedge (4x + 6 > 5 - 2n^2)$

(b) $\forall x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{Z}, (x \geq n) \wedge (2 - 3x < 3n^2 + 2)$

(b) $\forall x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{Z}, (x \geq n) \wedge (8x + 1 > 4n^2 + 5)$

(b) $\forall x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{Z}, (x \geq n) \wedge (7 - 3x < 2n^2 + 1)$

(b) $\forall x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{Z}, (x \leq n) \wedge (6 + 10x > 3n^2 + 3)$

(b) $\forall x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{Z}, (x \leq n) \wedge (5x + 2 < 5 + 3n^2)$

(b) $\forall x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{Z}, (x \leq n) \wedge (3x + 2 > n^2 - 4)$

(b) $\forall x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{Z}, (x \leq n) \wedge (3x + 4 > 5n^2 - 1)$