- TT2 issues? Please be patient, we <u>will</u> respond to all!
- PS3 is out! You already know what you need for Q1, Q2, and will be all set for Q3 by the middle of next week.

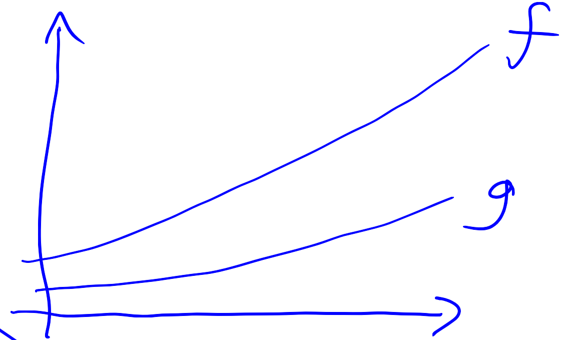
Algorithm Analysis

1. Correctness (why does my program work?) → CSC236
2. Complexity (how <u>efficient</u> is my program?)
   - ↳ How much <u>time</u> does it take to execute?
   - ↳ Measure how many <u>"steps"</u> the program executes?
     - Expressed as a <u>function</u> of the <u>input size</u>.
     - ✗ Want <u>approximate</u> representation of that function — capture the "rate of growth".

First, develop math. tools (O, Ω, Θ) to compare functions.

- Working with functions $f, g, h, \ldots : \underline{\mathbb{N}} \to \mathbb{R}^{\geq 0}$

$$\mathbb{R}^{\geq 0} = [0, \infty) = \{ x \in \mathbb{R} \mid x \geq 0 \} = \mathbb{R}^{+} \cup \{0\}$$



g is absolutely dominated by f

$\Downarrow$

g is dominated by f up to a constant factor

$\Uparrow$ X

$f(n) = \frac{n}{2}$

$g(n) = n$

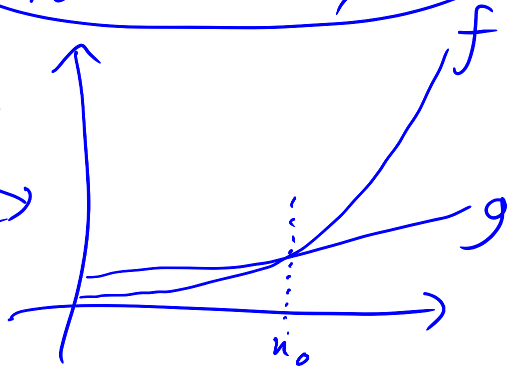
$\Rightarrow$

g is eventually dominated by f

not abs.

$g(n) = n + 4$

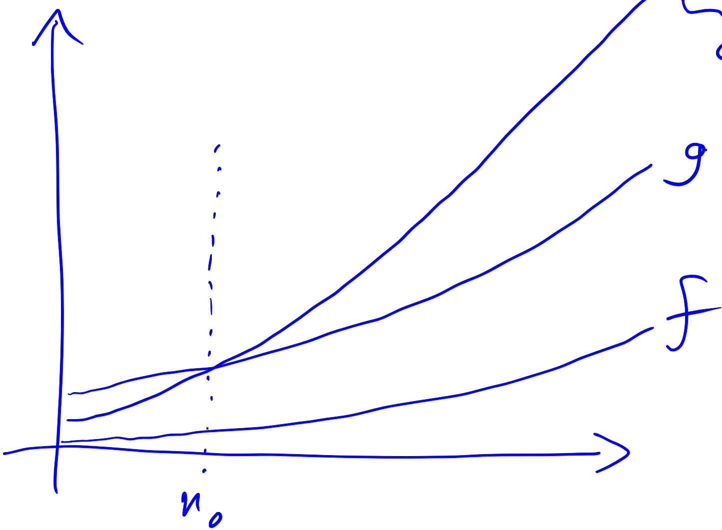
$f(n) = n^2$ $\Rightarrow$



$$g \in O(f)$$

$$\exists c, n_0 \in \mathbb{R}^+,\ \forall n \in \mathbb{N},\ n \geq n_0 \Rightarrow g(n) \leq c \cdot f(n)$$

$\underbrace{5.5}_{c} \cdot f$



g is eventually dominated by f, up to const. factor

Ex: Prove that $\forall a,b \in \mathbb{R}^+$, $an+b \in O(n^2)$

NOTE: "$an+b \in O(n^2)$" means

$g \in O(f)$, where $g(n) = an+b$, $f(n) = n^2$ $\forall n \in \mathbb{N}$.

$g: \mathbb{N} \to \mathbb{R}^{\geq 0}$

Proof: Let $a,b \in \mathbb{R}^+$.

Let $c =$ ______

$n_0 =$ ______

Let $n \in \mathbb{N}$. Assume $n \geq n_0$

WTS: $an+b \leq c n^2$

ROUGH WORK

WANT:

$an + b \le cn^2$

KNOW

$a, b \in \mathbb{R}^+$

$c = \boxed{?} \in \mathbb{R}^+$

$n_0 = \boxed{?} \in \mathbb{R}^+$

$n \in \mathbb{N}$. $n \ge n_0$

Idea 1: focus on c

know $an \le an^2$ a long as $n \ge 1$

$b \le bn^2$

$an + b \le (a+b)n^2$ $\longrightarrow$ pick $n_0 = 1$ $\in \mathbb{R}^+$ ✓

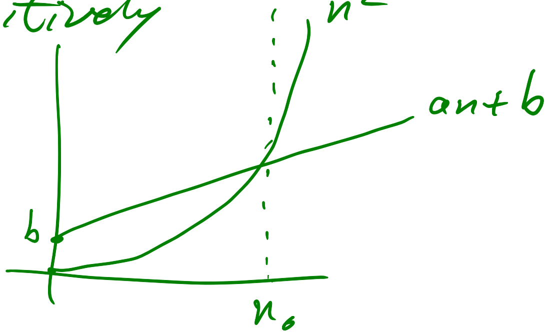
$c = a + b$

$c \in \mathbb{R}^+$ ✓

Idea 2: focus on n_0

important: do NOT need exact value where $n^2 = an + b$ (might not be in $\mathbb{N}$)

intuitively



want $an + b \le n^2$

$\Leftarrow$ $an + b \le \frac{n^2}{2} + \frac{n^2}{2}$

$an \le \frac{n^2}{2} \Leftarrow \underline{n \ge 2a}$ $\qquad$ $b \le \frac{n^2}{2} \Leftarrow \underline{n \ge \sqrt{2b}}$

pick $n_0 = \max(2a, \sqrt{2b})$

$c = 1$

EXERCISE: turn these ideas into complete proofs!