

# Propositions and Predicates

CSC165 Week 2 - Part 1

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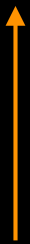
Propositions are statements that have a true or false value.  
The value of a proposition is fixed.

Predicates are functions that have codomain {True, False}  
The value of a predicate depends on one or more input values.

$$(2 < 1) \vee (2 \in \mathbb{Z})$$

vs.

$$(x < 1) \vee (x \in \mathbb{Z})$$



This is a proposition because  
we know that it is \_\_\_\_\_



This is a predicate because its truth  
value depends on the value of x.

$(x < 1) \implies (x \geq 2)$

Proposition ?

Predicate ?

18 January 2021 is a Monday.

Proposition ?

Predicate ?

Today is Monday.

Proposition ?

Predicate ?

5 is an even number

Proposition ?

Predicate ?

All dogs are animals.

Proposition ?

Predicate ?

If  $x$  is a dog, then  $x$  is an animal.

Proposition ?

Predicate ?

# Quantifiers

A quantifier tells you the quantity of something.

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$\forall$  is called the “universal quantifier”.

It means “for all”, “each”, “every” or “all”.

Example:  $\forall x \in \mathbb{N}, x \geq 0$

means “Every natural number  $x$  is greater than or equal to zero.”

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$\exists$  is called the “existential quantifier”.

It means “there exists”, “at least one”, “for some element”.

Example:  $\exists x \in \mathbb{N}, x \geq 5$

means “There exists a natural number  $x$  that is greater than or equal to five.”

# Reading quantifiers in precise and colloquial language

Statement 1: All dogs are animals.

Statement 2: If  $x$  is a dog, then  $x$  is an animal.

Statement 3: Let  $U$  = the set of living things on Earth.

$\forall x \in U$  If  $x$  is a dog, then  $x$  is an animal.

# Which quantifiers are hidden in these sentences?

Example 1: All rational numbers are real numbers.

Example 2: At least one prime number is even.

Example 3: Someone in this class is wearing a purple t-shirt.

Example 4: It is always possible to add 1 to any integer to get another integer.

# How to symbolize “There are infinitely many primes”

Let  $U = \mathbb{N}$  and  $P(x) = \text{“}x \text{ is prime”}$

# Double Quantifiers

Let  $U = \{\text{students in this course}\}$ , and  $x \in U, y \in U$

Let  $S(x,y) = \text{“}x \text{ studies with } y\text{”}$

$$\forall x \in U, \forall y \in U, S(x,y) =$$

$$\forall x \in U, \exists y \in U, S(x,y) =$$

$$\exists x \in U, \forall y \in U, S(x,y) =$$

$$\exists x \in U, \exists y \in U, S(x,y) =$$



