TT3-Q1

Monday, March 22, 2021 7:28 PM

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Aids Allowed: Your *own notes* taken during lectures and office hours, the lecture *slides and recordings* (for all sections), and the *Course Notes* (textbook).

Submission Instructions

- Submit your work directly on **MarkUs**—even if you are late!
- You may type your answers or hand-write them *legibly*, on paper or using a tablet and stylus.
- You may write your answers directly on the question paper, or on another piece of paper/document.
- You may submit your answers as a single file/document or as multiple files/documents. Each document may contain answers for only part of one question, an entire question, or multiple questions, but *please label each part of your answers* to make it clear what you are answering.
- You may name your file(s) any way you want (there is no "required file").
- You **must** submit your answers in PDF or as photos (JPEG/JPG/GIF/PNG/HEIC/HEIF). **Other formats** (e.g., Word documents, LaTeX source files, ZIP files) **are NOT accepted**—you must **export** or **compile** documents to PDF, **convert** images into a supported format, and upload each file **individually**.

For all questions in this test, "proof" means a *formal* proof that includes a header, and a proof body with justifications for each deduction. Each question can be answered correctly in less than one (1) page. You will NOT be penalized directly if you use more space for your answer, but longer answers increase the chance of errors... Remember that we are looking for evidence that you understand the conventions for writing correct proofs, so pay attention to the *structure* of your answers in addition to their content!

1. [5 marks] Asymptotic Notation I.

You may use https://www.desmos.com/calculator to look at the graph of the function in this question, *but NO other online resource is allowed* for any question on this test. Also, you still need to provide rigorous arguments for each proof: remember that *a graph is NOT a rigorous argument.*

Consider the function $f : \mathbb{N} \to \mathbb{R}^{\geq 0}$ defined by the formula

$$f(n) = n^2(\cos(n\pi) + 1) + 1.$$

Prove that:

(a) [2 marks] $f \in \Omega(1)$

(b) [3 marks] $f \notin \Theta(1)$

a) WTS: $\exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow n^2(\cos(n\pi)+1)+1 \geq c$

Pf: Let $c = n$, $n_0 = \sqrt{\frac{n-1-n^2}{\cos(n\pi)}}$

$n \geq n_0$

$n \geq \sqrt{\frac{n-1-n^2}{\cos(n\pi)}}$

$n^2(\cos(n\pi)+1)+1 \geq n$

$n^2(\cos(n\pi)+1)+1 \geq c$ $(n=c)$

$\square$

b) $\forall c_1, c_2, n_0 \in \mathbb{R}^+, \exists n \in \mathbb{N}, n \geq n_0 \wedge c_1 > n^2(\cos(n\pi)+1)+1) > c_2$

Part 1 $n =$

By definition $n \geq \max(n_0, \ldots) \geq n_0$

*Don't forget: this test contains **four** separate questions (plus the Academic Integrity statement)!*

Part 2

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