- TT2 issues? Thank you for your patience — we will answer everyone!
- PS3 is out! You already know what you need for Q1 & Q2, and will be ready for Q3 after Monday.

Algorithm Analysis

1. Correctness (why does my program work?) → CSC236
2. Complexity (how efficient is my program?)
   - How much time does my program take?
     - Abstractly, time is measured by counting "steps".
     - Express running time as a function of the input size.
     - Want to capture rate of growth of functions.

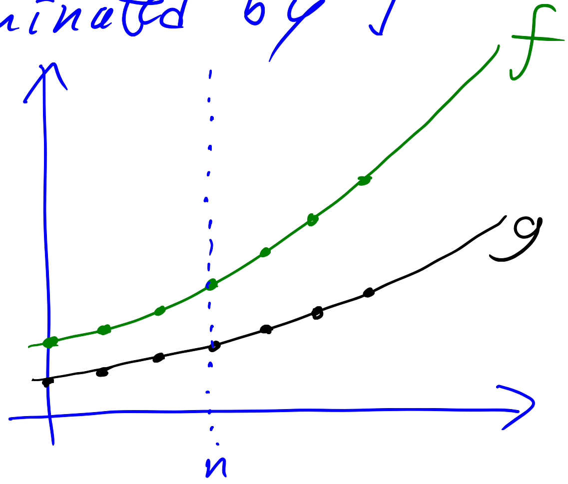
First, develop mathematical tools to compare functions ↪ (O, Ω, Θ)

↳ formalize the idea that one function is "bigger" than another.

Idea 1: g is absolutely dominated by f

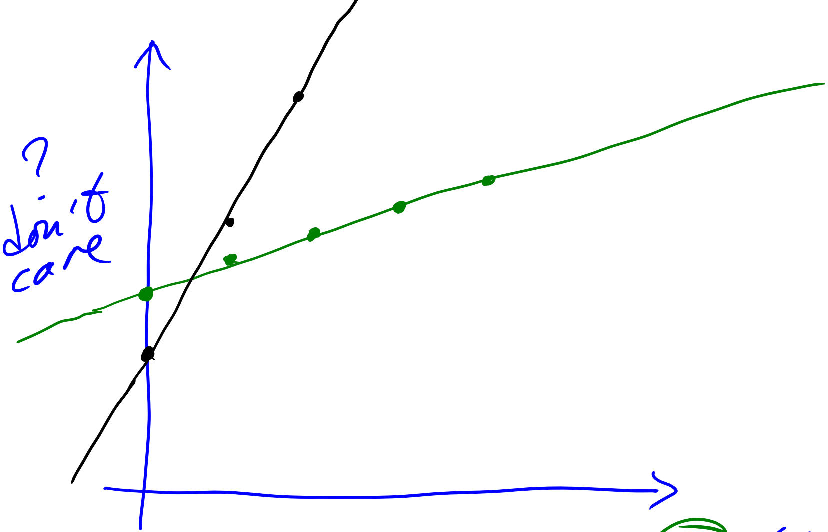
(where $g, f : \mathbb{N} \to \mathbb{R}^{\geq 0}$)

$\forall n \in \mathbb{N},\ g(n) \leq f(n)$



Q: which is bigger: $2n+3$?

$\frac{n}{2}+5$



* g is dominated by f up to a constant factor:

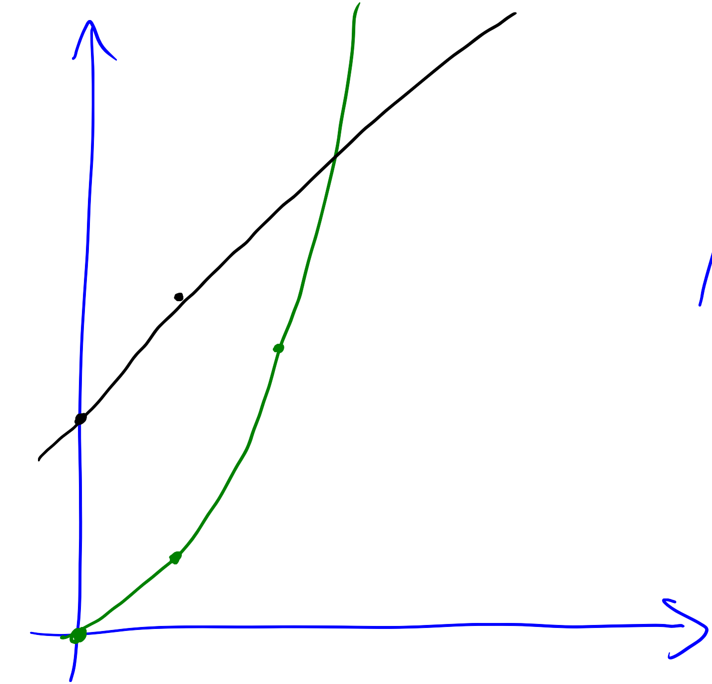
$\exists c \in \mathbb{R}^+, \forall n \in \mathbb{N}, g(n) \le c \cdot f(n)$

Here, $2n+3 \le 4\left(\frac{n}{2}+5\right) = 2n+20$ ✓

$\frac{n}{2}+5 \le 2(2n+3) = 4n+6$

- less restrictive than "absolutely dominates"

Q: $2n+3$ vs. n^2 ?



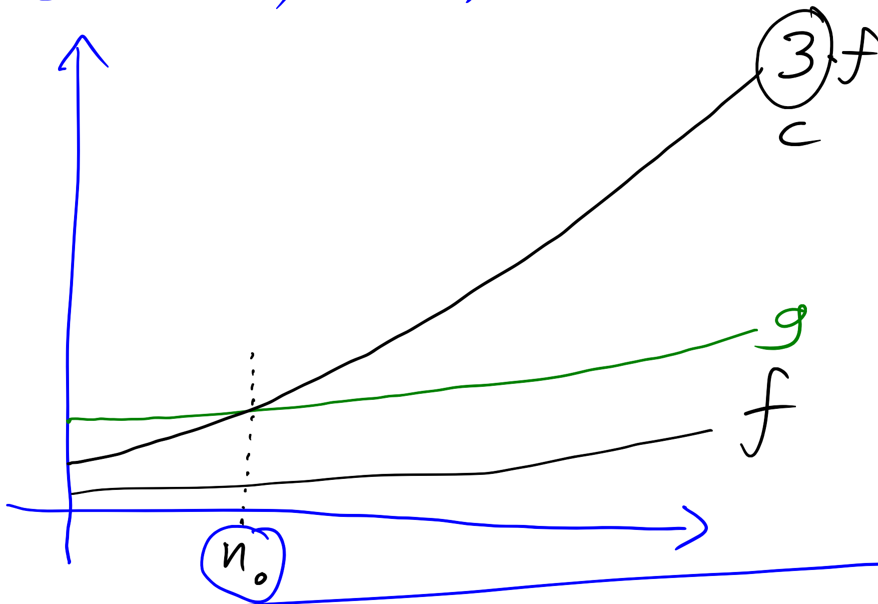
Is there a constant c s.t. $2n+3 \le c \cdot n^2 \; \forall n$?

No because $2(0)+3 > c \cdot 0^2$ no matter what c is...

* g is eventually dominated by f:

$\exists n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow g(n) \leq f(n)$

$\cdot\ g \in O(f):\ \exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow g(n) \leq c \cdot f(n)$



$g \in O(f)$ intuitively means "$g \leq f$"

f absolutely dominates g $\Rightarrow$ f dominates g up to const. $\Rightarrow$ $g \in O(f)$

f absolutely dominates g $\Rightarrow$ f eventually dominates g $\Rightarrow$ $g \in O(f)$

EX: Prove that $\forall a, b \in \mathbb{R}^+$, $an + b \in O(n^2)$

NOTE: "$an+b \in O(n^2)$" means

$g \in O(f)$ where $g(n) = an+b$

$f(n) = n^2$

Proof: Let $a, b \in \mathbb{R}^+$. $(*)$

Let $c =$ ____ and $n_0 =$ ____

Let $n \in \mathbb{N}$, and assume $n \geq n_0$

WTS: $an+b \leq c \cdot n^2$

ROUGH WORK:

Idea 1: focus on c

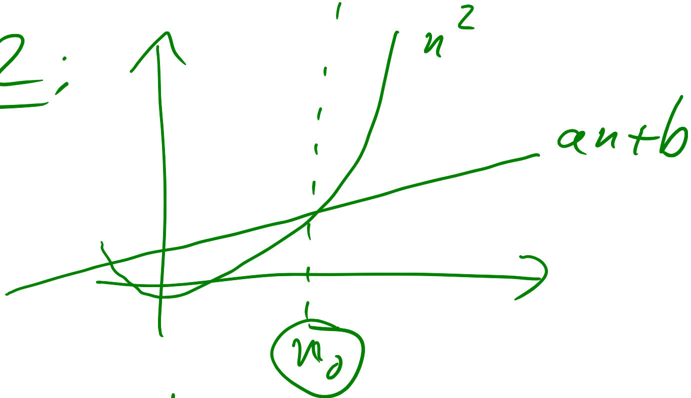
$$\left.\begin{array}{rcl} an &\le& an^2 \\ b &\le& bn^2 \end{array}\right\}$$ as long as $n \ge 1$

$$an+b \le (a+b)\,n^2$$

pick $n_0 = 1$

$c = \underbrace{a+b}_{\in \mathbb{R}^+}$

<u>Idea 2:</u>



<u>want:</u>

$an+b \le n^2$

solve for n:

$n^2 - an - b \ge 0$

want $an+b \le n^2$

$\Longleftarrow \quad an+b \le \frac{n^2}{2} + \frac{n^2}{2}$

split this: $\quad an \le \frac{n^2}{2} \quad \wedge \quad b \le \frac{n^2}{2}$

$n \geq 2a$ $\qquad\qquad$ $n \geq \sqrt{2b}$

$$\left.\begin{array}{l} \text{if } \underline{n \geq 2a} \wedge \underline{n \geq \sqrt{2b}} \\ \text{then } n^2 \geq an+b \end{array}\right\}$$

pick
$n_0 = \max(2a, \sqrt{2b})$
$c = 1$

EXERCISE: write the proofs!