Multi-Place Predicate Logic Symbolization

Part I – Basic Symbolization

Alex Koo

F^1: *a* is fast.

L^2: *a* likes *b*.

A^2: {1} sees {2}.

B^3: {1} borrows {2} from {3}.

n-place predicates

1. Use brackets for multi-place predicates

IMPORTANT

Fx. Gy. A(xy). B(ywza). ✓

F(x). G(y). Axy. Bywza. ✗

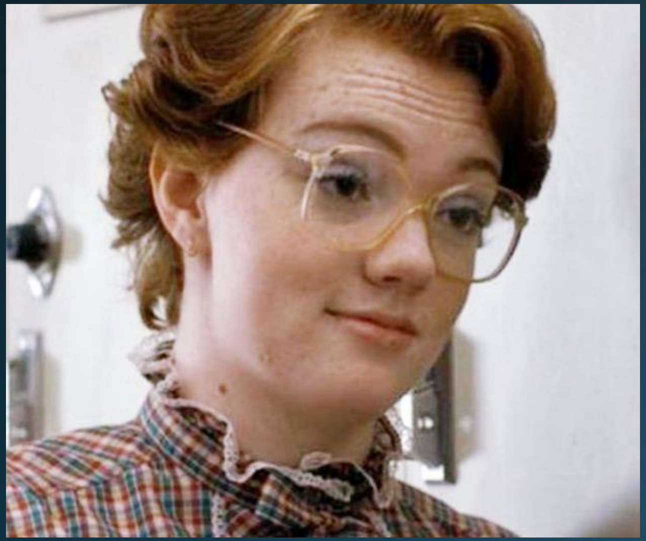
2. Order matters!

$L(ab) \neq L(ba)$

F^1: *a* is a person. L^2: *a* likes *b*. a^0: Barb.

Someone likes Barb.

$$\exists x(Fx \land L(xa))$$



Barb likes someone.

$$\exists x(Fx \land L(ax))$$

F^1: *a* is a person. L^2: *a* likes *b*. a^0: Barb.

Not everyone likes Barb.

$\exists x(Fx \wedge {\sim}L(xa))$ $\qquad$ ${\sim}\forall x(Fx \rightarrow L(xa))$

Barb doesn't like anyone.

${\sim}\exists x(Fx \wedge L(ax))$ $\qquad$ $\forall x(Fx \rightarrow {\sim}L(ax))$

Everyone loves Someone

F^1: {1} is a person. L^2: {1} loves {2}.

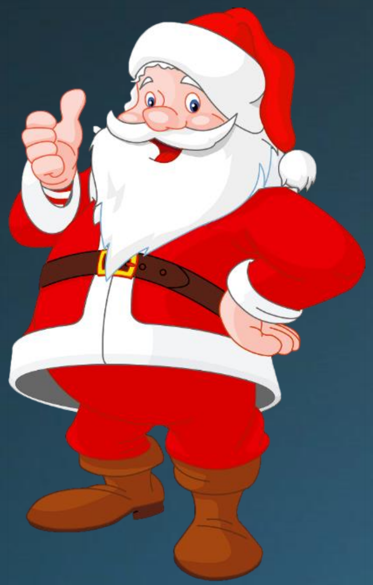
$\forall x(Fx \rightarrow \exists x(Fx \land L(xx)))$

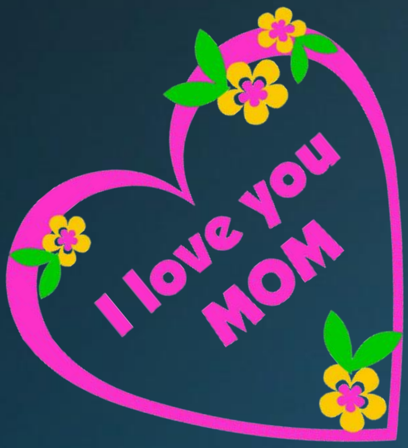
$\forall x(Fx \rightarrow \exists y(Fy \land L(xy)))$

Every person, x, loves some person, y.

Everyone loves Someone

Every person, x, loves some person, y.

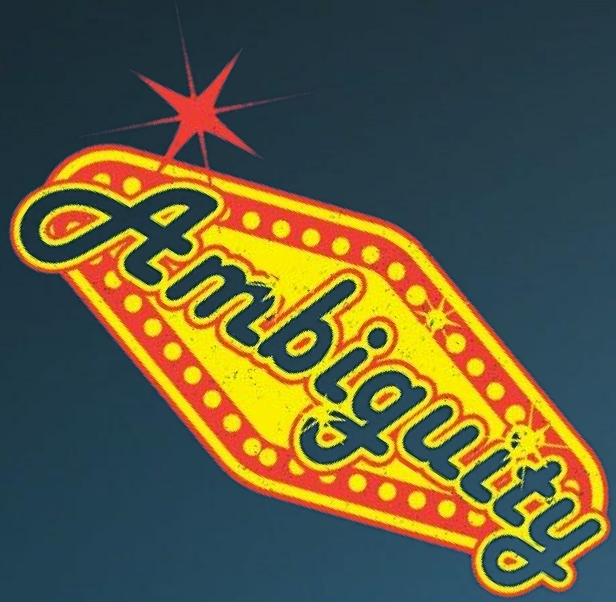




Who is y?

Is y a specific or generic person?

Everyone loves Someone



F^1: {1} is a person. L^2: {1} loves {2}.

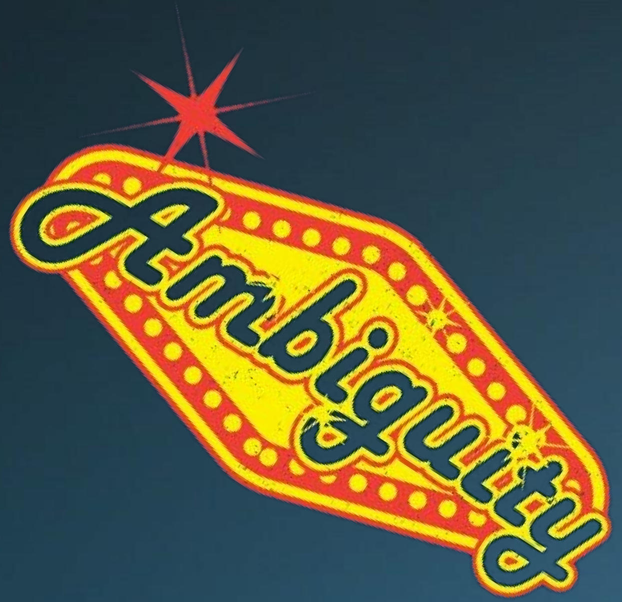
$\forall x(Fx \rightarrow \exists y(Fy \wedge L(xy)))$

Everyone loves some generic person

$\exists y(Fy \wedge \forall x(Fx \rightarrow L(xy)))$

Everyone loves some specific person

$\forall x \exists y$ vs $\exists y \forall x$

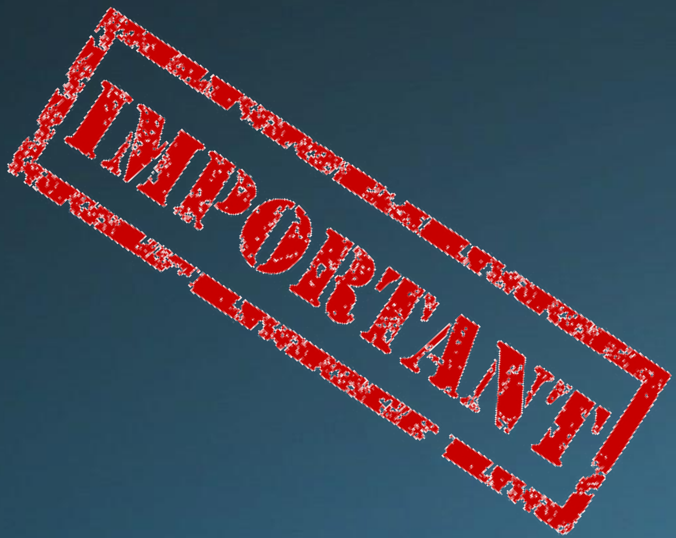


$\forall x \exists y$: for each thing x, there is some generic thing y

$\exists y \forall x$: for some specific thing y, there are all things x

Identical quantifiers are not ambiguous

1. Use brackets for multi-place predicates

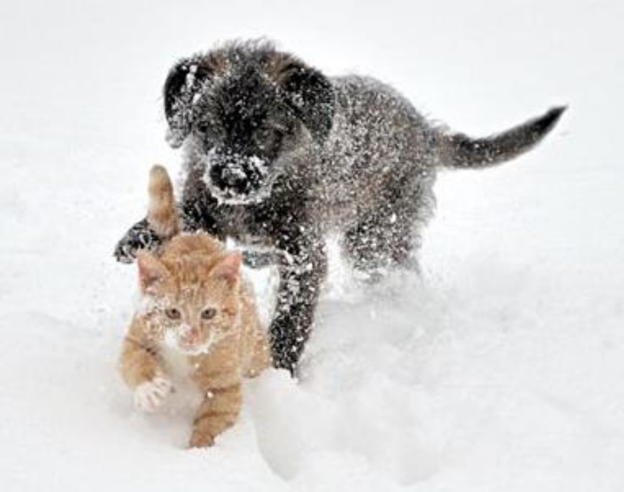


2. Order matters

3. Use new variables for nested quantifiers

4. Order of quantifier matters

Some dogs chase cats.



D^1: *a* is a dog. C^1: *a* is a cat. C^2: *a* chases *b*.

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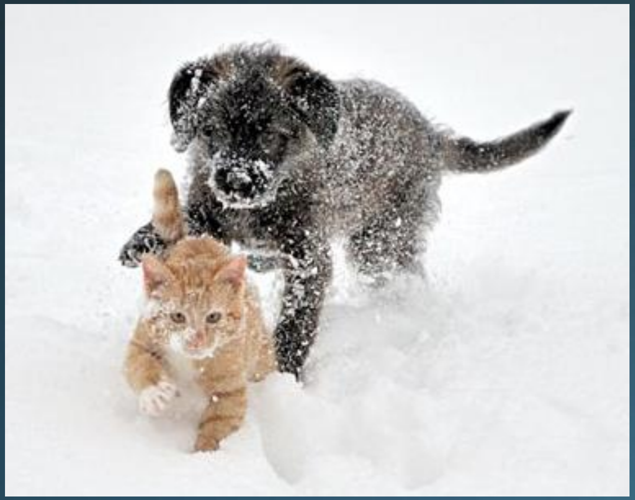

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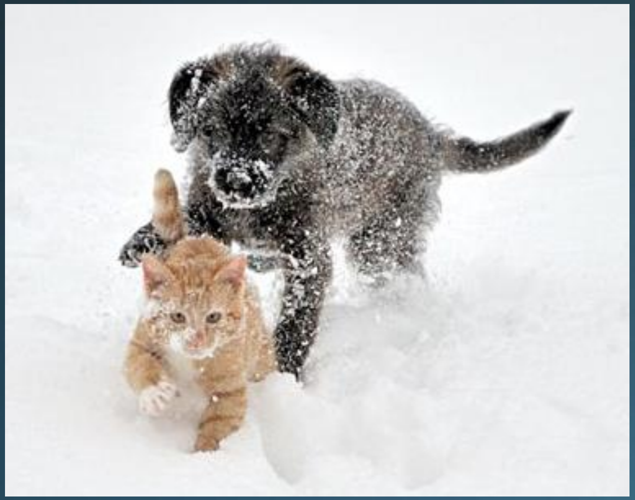

$\exists x(Dx \wedge$

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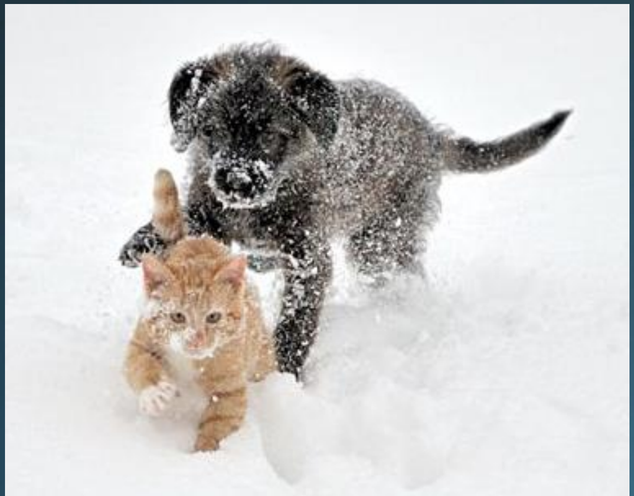

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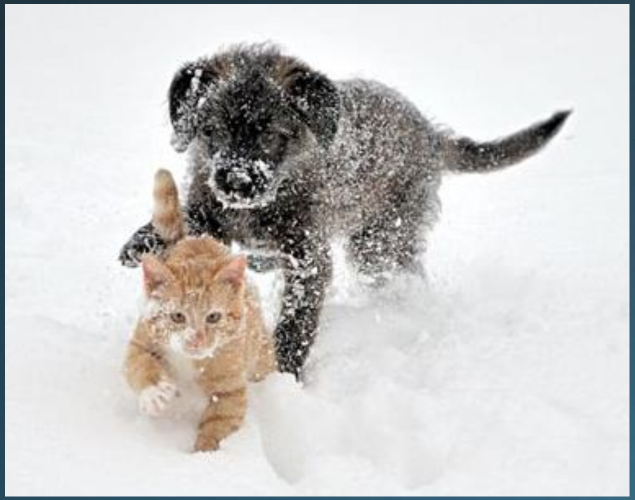

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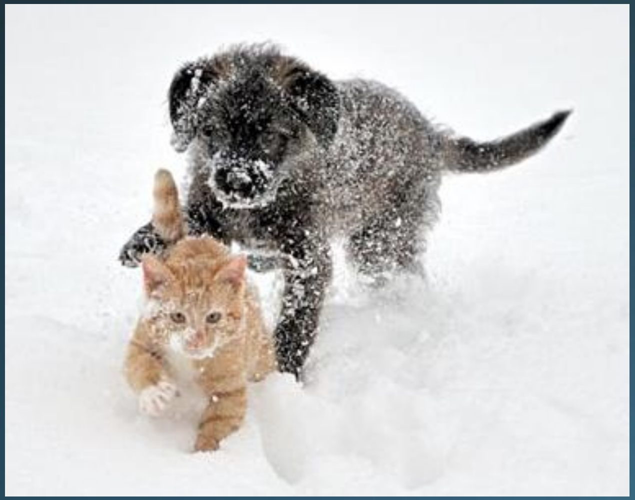

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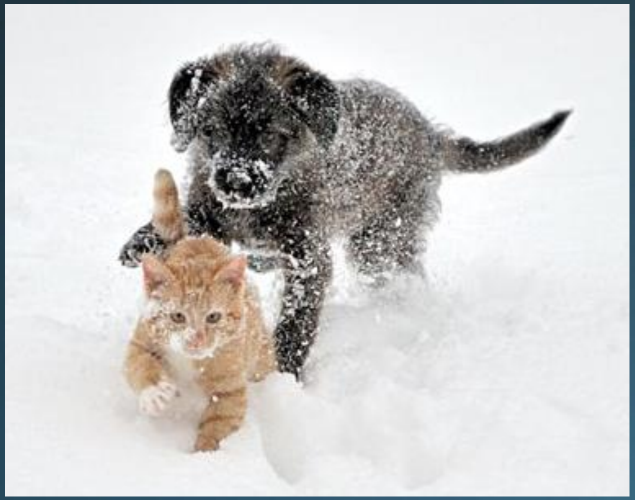

D^1: *a* is a dog. C^1: *a* is a cat. C^2: *a* chases *b*.

➡ $\exists x(Dx \wedge \exists y(Cy \wedge C(xy)))$ ✓

➡ $\exists x(Cx \wedge \exists y(Dy \wedge C(yx)))$ ✓

$\exists x \exists y(Dx \wedge Cy \wedge C(xy))$ ✓

Some dogs chase cats.

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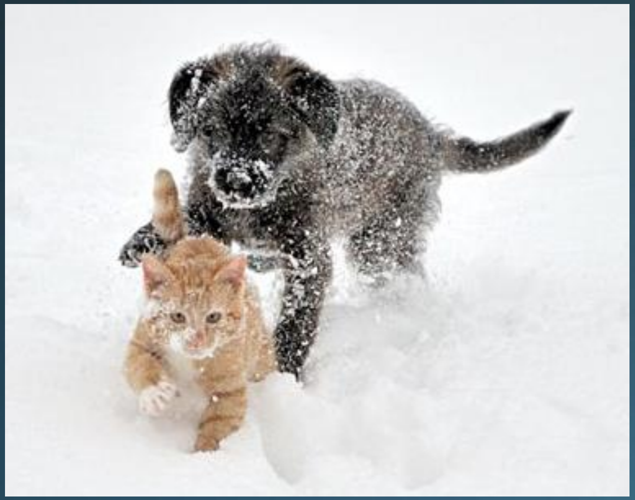

$\exists x(Dx \wedge \exists y Cy) \wedge C(xy)$ ✗

$\exists x(Dx \wedge C(xy) \wedge \exists y Dy)$ ✗

$C(DxCy)$ ✗

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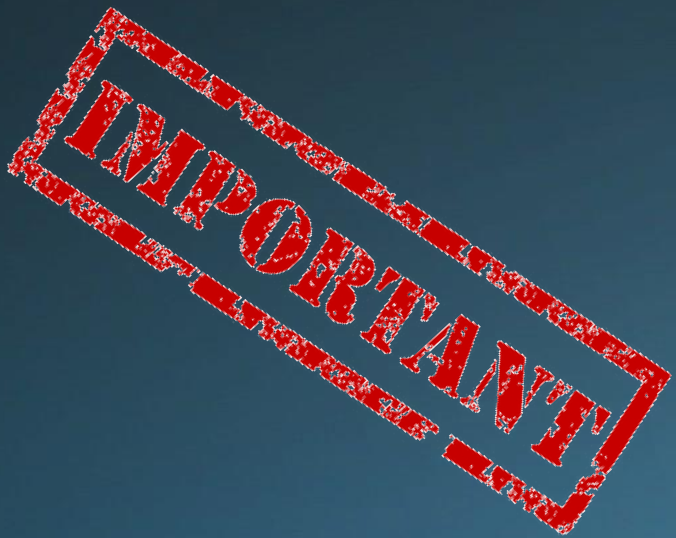

2. Order matters!

3. Use new variables for nested quantifiers

4. Order of quantifier matters

5. Paraphrase and use the canonical forms

6. Introduce everything and watch scope

No one watches every summer blockbuster movie.

A^1: a is a person. B^1: a is a blockbuster.

D^1: a is a movie. G^1: a comes out in the summer.
D^2: a watches b.

Tip: Work backwards from the main predicate

No one watches every
summer blockbuster movie.

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D(xy)

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D(xy)

No one Summer blockbuster movie

$\sim\exists x(Ax\wedge$ $\forall y(Gy\wedge By\wedge Dy\rightarrow$

No one watches every
summer blockbuster movie.

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D^1: *a* is a movie. G^1: *a* comes out in the summer.
D^2: *a* watches *b*.

$$\sim\exists x(Ax \wedge \forall y(Gy \wedge By \wedge Dy \rightarrow D(xy)))$$

It’s not the case that there is someone who
watches every summer blockbuster movie

No one watches every summer blockbuster movie.

If you're a person, then it's not the case that you watch every summer blockbuster movie.

$$\forall x(Ax \rightarrow \sim\forall y(Gy \wedge By \wedge Dy \rightarrow D(xy)))$$

If you're a person, then there is a summer blockbuster move that you do not watch.

$$\forall x(Ax \rightarrow \exists y(Gy \wedge By \wedge Dy \wedge \sim D(xy)))$$

Translate from the main predicate out

$$\forall x(Fx \wedge Gx \rightarrow \exists y(Hy \wedge A(yx)))$$

y stands in the A-relation to x

Translate from the main predicate out

$$\forall x(Fx \land Gx \to \exists y(Hy \land A(yx)))$$

y stands in the A-relation to x

Translate from the main predicate out

$$\forall x(Fx \land Gx \rightarrow \exists y(Hy \land A(yx)))$$

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All F’s and G’s

Translate from the main predicate out

$$\forall x(Fx \wedge Gx \rightarrow \exists y(Hy \wedge A(yx)))$$

y stands in the A-relation to x

All F's and G's

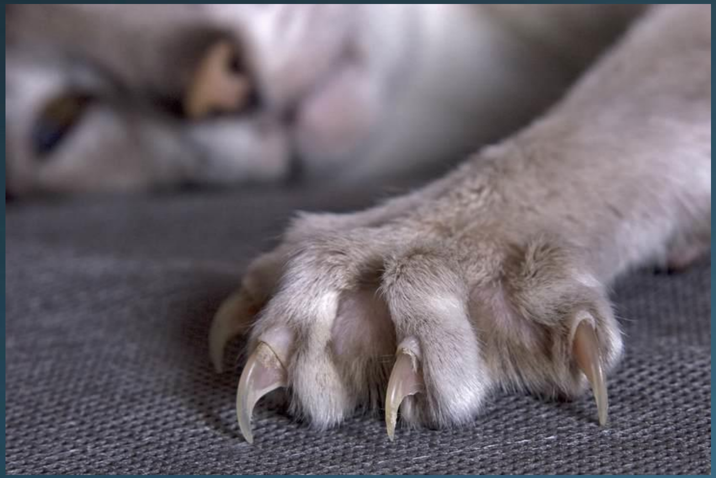
Some generic H

Translate from the main predicate out

$$\forall x(Fx \land Gx \rightarrow \exists y(Hy \land A(yx)))$$

Some generic H stands in the
A-relation to all F's and G's

F^1: {1} is a cat. G^1: {1} is scary.
H^1: {1} is a person. A^2: {1} is clawed by {2}.



$$\forall x(Fx \land Gx \rightarrow \exists y(Hy \land A(yx)))$$

Some generic H stands in the A-relation to all F's and G's

Some generic person is clawed by every scary cat.