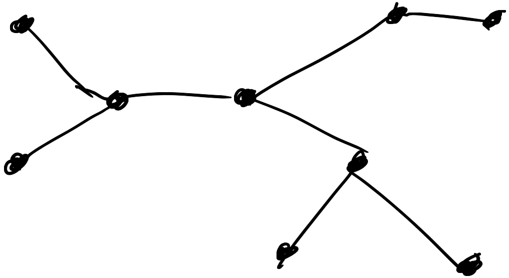
While we're waiting...
what shows will you watch once the term is over? :)

Def: A tree is a connected, acyclic graph

(connected: $\forall u, v \in V$, $\exists$ path between u and v;
acyclic: no path with length >1 starts & ends at same vertex)

EX:



Proof: $\forall$ trees $T=(V,E)$, $|E| = |V| - 1$

By induction

$\forall n \in \mathbb{Z}^+, \boxed{\forall T=(V,E), (T \text{ is a tree}) \wedge |V| = n \Rightarrow |E| = |V|-1}$ $P(n)$

- **Base Case:** EXERCISE... :)
- **Ind. Hyp.:** Let $n \in \mathbb{Z}^+$, assume $P(n)$:

  $\forall T=(V,E), (T \text{ is a tree}) \wedge |V| = n \Rightarrow |E| = |V| - 1$
- **Ind. Step:** WTP

  ▷ $\forall T_1 = (V_1, E_1), (T_1 \text{ is a tree}) \wedge |V_1| = n+1 \Rightarrow |E_1| = |V_1| - 1$

  Let $T_1 = (V_1, E_1)$.

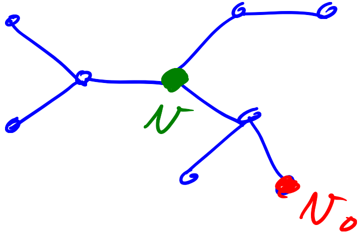
  Assume T_1 is a tree and $|V_1| = n+1$

  WTP: $|E_1| = |V_1| - 1$.

ROUGH WORK ...

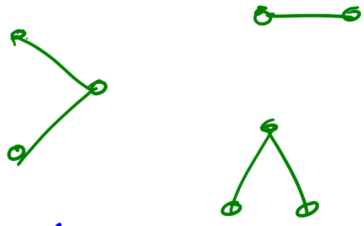
idea: remove <u>some</u> vertex v

T_1 :



$|V_1| = n+1$

"$T_1 - v$" :



Insight: remove v with degree 1
(just <u>one</u> edge adjacent to v).

<u>WAIT!</u> How do we know there is such a vertex?

For now: assume there is — we'll prove later.

- Assume T_1 contains at least one vertex v_0 with degree 1 — <u>to be proved later...</u>

Then $T' = (V', E')$ where $V' = V_1 - \{v_0\}$
$E' = E_1 - \{(u, v_0) \mid (u, v_0) \in E_1\}$

$E' = E_1 - \{$single edge in E_1 adjacent to $v_0\}$

$|V'| = n$, T' is a tree

- T' contains no cycle (because T_1 is acyclic)
- T' is connected (because T_1 is connected and we removed only v_0 and its one edge)

So by IH, $|E'| = |V'| - 1 = n-1$

and $|E_1| = |E'| + 1 = (n-1)+1 = n = (n+1) - 1$
$= |V_1| - 1.$

$\square$

Proof of Lemma

a fact needed for the main proof, that requires its own proof

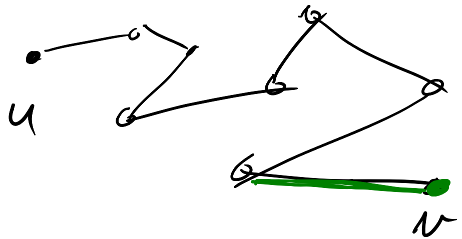
Every tree with $n \geq 2$ vertices contains at least one <u>leaf</u> (vertex of degree 1)

<u>Proof</u>: Let $T=(V,E)$ be a tree with $|V|=n\geq 2$.

Let u be any vertex from V.

- If $\deg(u)=1 \Rightarrow$ done.
- If $\deg(u)>1 \Rightarrow$ find a <u>longest</u> path in T, starting from u.

(NOTE: this case is not really necessary...)

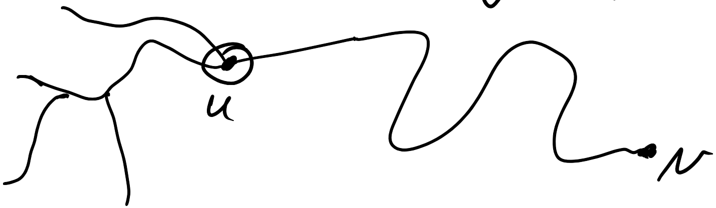


Let v be the other endpoint of this path.

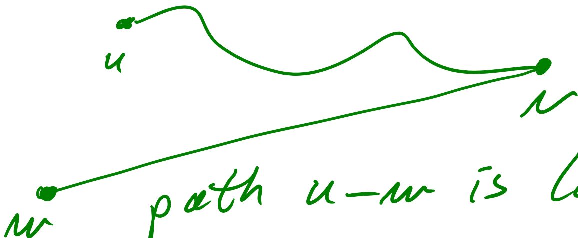
<u>Claim</u>: $\deg(v)=1$.

Otherwise, either T contains a <u>longer</u> path starting at u (contradiction), or T <u>contains a cycle</u> (also a contradiction). □

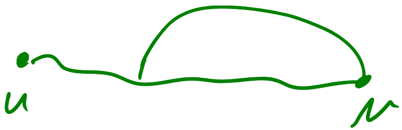
– what about longer paths starting at v?



– intuition



path u–w is longer ✗



cycle ✗

REVIEW

CORE TOPICS

- prop. & pred. logic
- proof techniques
- number theory (divisibility, primes)
- number representation
- $O/\Omega/\Theta$
- algorithm analysis
  - RT, WC, BC, AC
  - upper/lower bounds
- intro. to graphs

SKILLS

translation

read & write proofs

conversion

proofs/disproofs

apply analysis to algorithms

TT4:

1-hour term test on algo. analysis

1-hour test on all the material