Good morning/afternoon/evening, welcome back!
How was your reading week?

- PS0, TT0 remarks are done
- PS1 marks are out — remark deadline is <u>Mar. 5</u>
- TT1 marks out this week (almost done)
- PS2: - lots of information on Piazza
  - TA office hours today
  - remember <u>academic integrity</u>: submit only the work & ideas of your group; cite any additional resources used

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<u>Recap.:</u>
- propositional & predicate notation, for precise expression.
- proof techniques — connection to logical structure of statements

Starting now: CS domains

Binary representation

Def: A binary representation of $x \in \mathbb{N}$ consists of: $k \in \mathbb{N}$ $b_0, b_1, \ldots, b_{k-1} \in \{0,1\}$

such that $x = \sum_{i=0}^{k-1} b_i \cdot 2^i = b_{k-1} \cdot 2^{k-1} + b_{k-2} \cdot 2^{k-2} + \cdots + b_1 2 + b_0$

notation: $x = (b_{k-1} b_{k-2} \cdots b_1 b_0)_2$

e.g. $(\underset{b_2}{1}\underset{b_1}{0}\underset{b_0}{1})_2 = \sum_{i=0}^{2} b_i \cdot 2^i = b_2 \cdot 2^2 + b_1 \cdot 2^1 + b_0 \cdot 2^0$

$= 1 \cdot 4 + 0 \cdot 2 + 1 \cdot 1$

$= 5$

Note: same idea applies to other bases

$b > 1$: $x = \sum_{i=0}^{k-1} d_i \cdot b^i$, where $d_i \in \{0, 1, \ldots, b-1\}$

Note: base 1 — "unary" — is different

$$x = \underbrace{111 \cdots 1}_{x \text{ times}}$$

Def: $B(n, x)$: "$\exists\, b_0, b_1, \ldots, b_{n-1} \in \{0,1\}$", $x = (b_{n-1} \cdots b_0)_2 = \sum_{i=0}^{n-1} b_i 2^i$

where $n, x \in \mathbb{N}$

$B(n,x)$ is true iff x can be represented in binary using exactly n bits e.g.: $B(3,5)$ = "5 can be represented using 3 bits"

True because $5 = (101)_2$

00	0
01	1
10	2
11	3

$B(4,5)$ = True because $5 = (0101)_2$

$B(2,5)$ = False

Q: in general, what is the relationship between n and x whenever $B(n,x)$ = True?

Discussion:

How many values can we represent with n bits?

Guess: 2^n different values

$0, 1, \ldots\ldots, 2^n - 1$

Proof that $\underline{\forall n \in \mathbb{N},}\ \forall x \in \mathbb{N},\ x \leq 2^n - 1 \Rightarrow B(n, x)$

<u>Idea 1</u>: Let $n \in \mathbb{N}$, $x \in \mathbb{N}$. Assume $x \leq 2^n - 1$

→ Try to prove $B(n, x)$.

... not obvious ...

<u>Idea 2</u>: try induction... but on which variable? No clear reason to choose one over the other — try n.

- First, define $P(n)$: $\forall z \in \mathbb{N},\ z \leq 2^n - 1 \Rightarrow B(n, z)$
- <u>Base case</u>: WTS $P(0)$: $\underline{\forall x \in \mathbb{N},}\ x \leq 2^0 - 1 \Rightarrow B(0, x)$

Let $x \in \mathbb{N}$. Assume $x \leq 2^0 - 1$.

Then, $x \le 1-1 = 0$, so $x = 0$.

WTS: $B(0, x)$

$\hookrightarrow$ x can be represented in binary using 0 bits...

$$0 = ()_2 = \sum_{i=0}^{-1} b_i \cdot 2^i$$

empty sum $= 0$