Today: - average-case runtime (quickly!)
- quick recap of algorithm analysis
- intro to graphs!

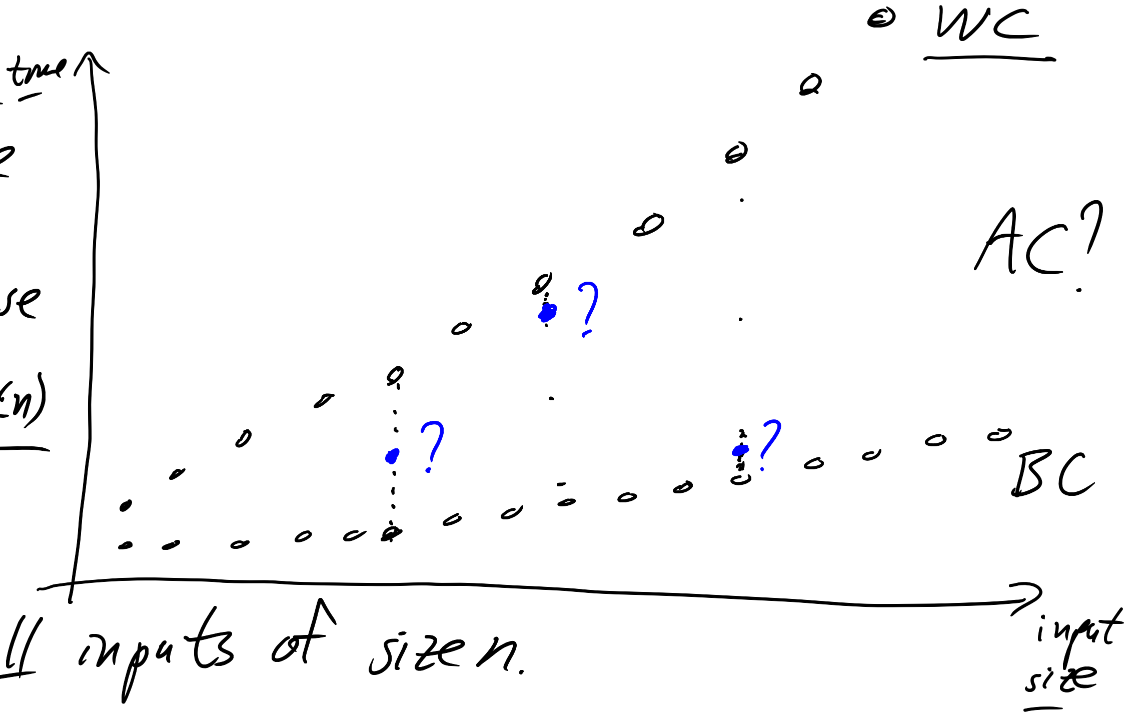
Average-case

WC(n): worst-case

BC(n): best-case

AC(n): average-case

$$AC(n) \neq \frac{WC(n) + BC(n)}{2}$$



AC(n) = average runtime over all inputs of size n.

Example:

```
def search(L: list, x: int) -> bool:
    for item in L:
        if item == x:
            return True
    return False
```

$WC(n) \in \Theta(n)$

$BC(n) \in \Theta(1)$

First step in average-case analysis is: define set of "all" inputs. — meaning each possible behaviour of the algorithm happens for <u>at least one</u> input.

$\mathcal{I}_n = \{\text{all inputs of size } n\}$

$$AC(n) = \frac{1}{|\mathcal{I}_n|} \sum_{x \in \mathcal{I}_n} RT(x)$$

runtime of algo. on x

add over all $x \in \mathcal{I}_n$

take arithmetic average

$Y_1 \cdots Y_m$: $\dfrac{Y_1 + \cdots + Y_m}{m} = \dfrac{1}{m} \sum_{i=1}^{m} Y_i$

$m = |\mathcal{I}_n| =$ size of $\mathcal{I}_n$

$Y_i = RT(x)$ for some $x \in \mathcal{I}_n$

[1,2,3] [2,3,1]
[1,3,2] [3,1,2]
[2,1,3] [3,2,1]

<u>Back to example:</u>

- <u>Idea 1:</u>
  define $\mathcal{I}_n = \{ (L, x) \mid L$ is <u>a permutation</u> of $[1,2,\ldots,n]$, $x = 1 \}$

  rearrangement

  <u>NOTE:</u> does <u>not</u> contain inputs with $x \notin L$...

- <u>Idea 2:</u> $\mathcal{I}_n' = \{ (L, x) \mid L = [1,2,\ldots,n],\ x \in \{0,1,\ldots,n\} \}$

$\mathcal{I}_n' = \{ ([1,2,\ldots,n], 0), ([1,2,\ldots,n], 1), \ldots, ([1,2,\ldots,n], n) \}$

$$AC(n) \doteq \frac{1}{|\mathcal{I}'_n|} \sum_{(L,x)\in \mathcal{I}'_n} RT(L,x)$$

\# iterations when looking for x

$$= \frac{1}{n+1}\left((n+1) + \sum_{x=1}^{n} x \right)$$

when $x=0$

$= \cdots$ simple arithmetic

$$\mathcal{I}''_n = \left\{ (L,x) \;\middle|\; L = [1,2,\dots,n],\ x \in \{1,2,\dots,\underline{n^2}\} \right\}$$

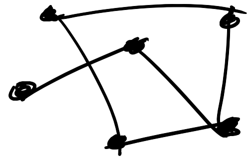
?

<u>Graphs:</u>

$G=(V,E)$

$V=$ vertices/nodes

$E=$ edges (pairs of vertices)



Statements about graphs

$\boxed{1}$ Prove $\underbrace{\forall G=(V,E),}$ $|E| \leq \frac{|V|(|V|-1)}{2}$

Notation: this introduces 3 related variables:

- graph G
- G's vertex set V
- G's edge set E

Proof: by definition, each elem. of E is a subset of V of size 2.

By worksheet #10, there are exactly $\frac{|V|(|V|-1)}{2}$ such subsets. $\square$