

Def: Prime(x): " $x > 1 \wedge$
where $x \in \mathbb{N}$ $\forall d \in \mathbb{Z}^+, d \mid x \Rightarrow d = 1 \vee d = x$ "

- $\forall d \in \mathbb{Z}^+, d \neq 1 \wedge d \neq x \Rightarrow d \nmid x$ $\xrightarrow{\text{contrapositive}}$
 - also equivalent: $\neg \exists d \in \mathbb{Z}^+, d \mid x \wedge d \neq 1 \wedge d \neq x$
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This Week: Proofs

- Proof = any convincing argument
- Generally ...

English statement



Predicate statement

rough work
("discussion" in
course notes)



Proof headers
(introduce variables
and assumptions)



Proof body
(deductions with
justifications)

General forms for proof headers

...

statement

$\forall x \in D, P(x)$

want to prove \uparrow

proof header

Let $x \in D$.

(let x be arbitrary, but fixed)

now prove $P(x)$
for that x

$\exists x \in D, P(x)$

Let $x = \underline{\hspace{2cm}}$ \uparrow

(pick a specific value)
make sure $x \in D$ \leftarrow

now prove $P(x)$
for that x

$P \Rightarrow Q$

Assume P

now prove Q

Ex 1: Prove that every natural number n greater than 20 satisfies $1.5n - 4 \geq 3$.

Step 1: Predicate statement

$$\forall n \in \mathbb{N}, \underline{n > 20} \Rightarrow \boxed{1.5n - 4 \geq 3}$$

Step 2:

~~$p(n)$ $p: \mathbb{N} \rightarrow \mathbb{R}$~~
not necessary

Let $n \in \mathbb{N}$.

Assume $n > 20$.

WTP: $1.5n - 4 \geq 3$

→ "What To Prove"

ROUGH WORK — NOT PART OF PROOF!

KNOW

$n \in \mathbb{N}$

$n > 20$

$\leftarrow \dots \rightarrow$

WANT

$1.5n - 4 \geq 3$

$\hookrightarrow \underline{1.5n - 4 \geq 3} \Leftrightarrow 1.5n \geq 7$

$\Leftrightarrow n \geq 7/1.5$
"
 $14/3$
"
 $4.66\dots$

Back to proof:

Let $n \in \mathbb{N}$, and assume $n > 20$.

Then, $n \geq 4.666\dots$

$$\text{So } 1.5n \geq 7$$

$$\text{So } \underline{1.5n - 4 \geq 3.}$$



(QED
end of proof)