

False

$$\exists f, g, h, [(g: \mathbb{R} \rightarrow \mathbb{R}) \wedge (h: \mathbb{R} \rightarrow \mathbb{R}) \wedge (\exists x_1 \in \mathbb{R}, g(-x_1) = -g(x_1)) \wedge (\exists x_2 \in \mathbb{R}, h(-x_2) = -h(x_2)) \wedge f(x) = g(x) - h(x)] \implies (\forall x_3 \in \mathbb{R}, f(-x_3) = f(x_3)) \wedge (\exists k \in \mathbb{R}, f(x_4) \neq k)$$

In order to disprove the statement above, we will prove its negation:

$$\forall f, g, h, [(g: \mathbb{R} \rightarrow \mathbb{R}) \wedge (h: \mathbb{R} \rightarrow \mathbb{R}) \wedge (\exists x_1 \in \mathbb{R}, g(-x_1) = -g(x_1)) \wedge (\exists x_2 \in \mathbb{R}, h(-x_2) = -h(x_2)) \wedge f(x) = g(x) - h(x)] \wedge (\exists x_3 \in \mathbb{R}, f(-x_3) \neq f(x_3)) \vee (\forall k \in \mathbb{R}, f(x_4) = k)$$

Let $g(-x) = -g(x)$ and let $h(-x) = -h(x)$. We will prove that $f(x)$ is not an even function (e.g. is odd function):

$$\begin{aligned} f(x) &= g(x) - h(x) \\ f(-x) &= g(-x) - h(-x) \\ &= -g(x) + h(x) \\ &= -f(x) \end{aligned}$$

By definition of an odd function, $f(x) = -f(-x)$. Therefore, the negation of the statement is true and the original statement is false.