Propositions and Predicates

CSC165 Week 2 - Part 1

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Propositions are statements that have a true or false value.
The value of a proposition is fixed.

Predicates are functions that have codomain {True, False}
The value of a predicate depends on one or more input values.

$(2 < 1) \vee (2 \in \mathbb{Z})$ vs. $(x < 1) \vee (x \in \mathbb{Z})$

This is a proposition because we know that it is ____________

This is a predicate because its truth value depends on the value of x.

$(x < 1) \implies (x \geq 2)$	Proposition ?	Predicate ?
18 January 2021 is a Monday.	Proposition ?	Predicate ?
Today is Monday.	Proposition ?	Predicate ?
5 is an even number	Proposition ?	Predicate ?
All dogs are animals.	Proposition ?	Predicate ?
If x is a dog, then x is an animal.	Proposition ?	Predicate ?

Quantifiers

A quantifier tells you the quantity of something.

$\forall$ is called the "universal quantifier".

It means "for all", "each", "every" or "all".

Example: $\forall\ x \in \mathbb{N}, x \geq 0$

means "Every natural number x is greater than or equal to zero."

$\exists$ is called the "existential quantifier".

It means "there exists", "at least one", "for some element".

Example: $\exists\ x \in \mathbb{N}, x \geq 5$

means "There exists a natural number x that is greater than or equal to five."

Reading quantifiers in precise and colloquial language

Statement 1: All dogs are animals.

Statement 2: If x is a dog, then x is an animal.

Statement 3: Let U = the set of living things on Earth.

$\forall x \in U$ If x is a dog, then x is an animal.

Which quantifiers are hidden in these sentences?

Example 1: All rational numbers are real numbers.

Example 2: At least one prime number is even.

Example 3: Someone in this class is wearing a purple t-shirt.

Example 4: It is always possible to add 1 to any integer to get another integer.

How to symbolize "There are infinitely many primes"

Let $U = \mathbb{N}$ and $P(x)$ = "x is prime"

Double Quantifiers

Let U = {students in this course}, and $x \in U$, $y \in U$

Let $S(x,y)$ = "x studies with y"

$\forall x \in U, \forall y \in U, S(x,y) =$

$\forall x \in U, \exists y \in U, S(x,y) =$

$\exists x \in U, \forall y \in U, S(x,y) =$

$\exists x \in U, \exists y \in U, S(x,y) =$