TT2-Q1

Monday, March 1, 2021 7:13 PM

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Aids Allowed: Your *own notes* taken during lectures and office hours, the lecture *slides and recordings* (for all sections), and the *Course Notes* (textbook).

Submission Instructions

- Submit your work directly on **MarkUs**—even if you are late!
- You may type your answers or hand-write them *legibly*, on paper or using a tablet and stylus.
- You may write your answers directly on the question paper, or on another piece of paper/document.
- You may submit your answers as a single file/document or as multiple files/documents. Each document may contain answers for only part of one question, an entire question, or multiple questions, but *please label each part of your answers* to make it clear what you are answering.
- You may name your file(s) any way you want (there is no "required file").
- You **must** submit your answers in PDF or as photos (JPEG/JPG/GIF/PNG/HEIC/HEIF). **Other formats** (e.g., Word documents, LaTeX source files, ZIP files) **are NOT accepted**—you must **export** or **compile** documents to PDF, **convert** images into a supported format, and upload each file **individually**.

For all questions in this test, "proof" means a *formal* proof that includes a header, and a proof body with justifications for each deduction. Each question can be answered correctly in less than one (1) page, and some questions have *very* short correct answers. You will NOT be penalized directly if you use more space for your answer, but longer answers increase the chance of errors... Remember that we are looking for evidence that you understand the conventions for writing correct proofs—NOT just how to answer these specific questions—so pay close attention to the *structure* of your answers!

1. **[6 marks]** For both statements below:

(i) Write the negation of the original statement without using the $\neg$ symbol.

(ii) Write whether the original statement is true or false.

(iii) If the original statement is true, prove it. If the original statement is false, disprove it.

(NOTE: The notation $\mathbb{R}^{\geq 0}$ represents the set $[0, \infty) = \{x \in \mathbb{R} \mid x \geq 0\}$.)

(a) $\forall x \in \mathbb{R}, \forall n \in \mathbb{Z}, (x > n) \Rightarrow (3x > 5n + 2)$

(b) $\forall x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{Z}, (x \geq n) \wedge (4x + 6 > 5 - 2n^2)$

a) $\exists x \in \mathbb{R}, \exists n \in \mathbb{Z}, x > n \wedge 3x \leq 5n + 2$

False (Disproof). We will show that $x > n$ and $3x \leq 5n+2$

Pf: Let $x = 2$ $(x \in \mathbb{R})$.

Let $n = 1$ $(n \in \mathbb{Z})$. So $x > n$.

$3x \leq 5n + 2$

$3(2) \leq 5(1) + 2$

$6 \leq 7$ $\square$

Don't forget: this test contains ***four*** *separate questions (plus the Academic Integrity statement)!*

b) $\exists x \in \mathbb{R}^{\geq 0}, \forall n \in \mathbb{Z}, (x < n) \vee (4x+6 \leq 5-2n^2)$

Pf: True (Proof).

We will show that $x \geq n$ and $4x+6 > 5-2n^2$

Let $x \in \mathbb{R}^{\geq 0}$

Let $n = x$

$$4x+6 > 5-2n^2$$

$$4x+6 > 5-2x^2 \qquad (n=x)$$

$$4x+2x^2 > 5-6$$

$$2x(2+x) > -1 \qquad (\text{Since } x \in \mathbb{R}^{\geq 0}) \quad \square$$