Problem Set 1

1

(a)

$$\forall a, b \in T, \forall f \in F, \neg BelongsTo(a, f) \land \neg BelongsTo(b, f) \implies a = b$$

(b)

$$\exists f \in F, \forall t \in T, BelongsTo(t, f) \implies Oak(t)$$

(c)

$$\exists f \in F, \forall t \in T, Pine(t) \implies BelongsTo(t, f)$$

(d)

$$\forall f \in F, \forall a, b \in T, Pine(a) \land Oak(b) \implies \neg BelongsTo(a, f) \lor \neg BelongsTo(b, f)$$

(e)

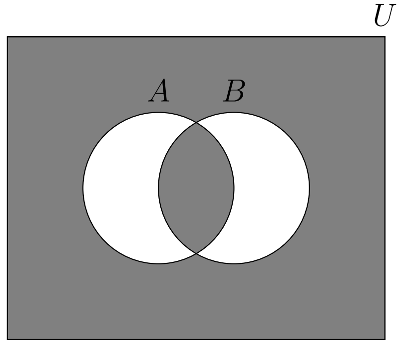
$$\forall g \in P(T), g \in F$$

2

(a)

$$A \setminus B$$

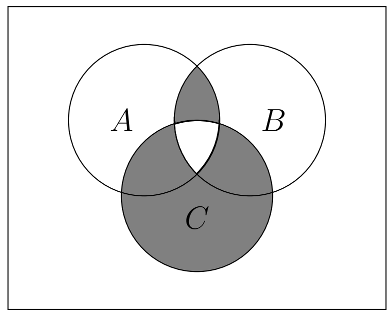
(b)



(c)

$$(A \setminus B \cup C) \cup (B \setminus A \cup C) \cup (C \setminus A \cup B) \cup (A \cap B \cap C)$$

(d)



3

(a)
Let $S = \{-1, 1, 2\}$.
Let $T = \{-1, 1\}$.
Let $P(x) = x$ is odd.
Let $Q(x, y) = x \cdot y > 0$.

$$\forall x \in S, \exists y \in T, (P(x) \vee Q(x, y)) \wedge (\neg P(x) \vee \neg Q(x, y))$$

If we pick -1 from S, pick 1 from T.
If we pick 1 from S, pick -1 from T.
If we pick 2 from S, pick 1 from T.
Therefore, our definitions satisfy first statement.

$$\exists y \in T, \forall x \in S, (P(x) \vee Q(x, y)) \wedge (\neg P(x) \vee \neg Q(x, y))$$

The negation of it is

$$\forall y \in T, \exists x \in S, (\neg P(x) \wedge \neg Q(x, y)) \vee (P(x) \wedge Q(x, y))$$

If we pick -1 from T, pick -1 from S.
If we pick 1 from T, pick 1 from S.
Therefore, our definitions are true for the first statement and fail for the second statement.

(b)
It is impossible because the first statement implies the second statement.
Assume the second statement is true.
Let $y \in T$ that satisfies $\forall x \in S, (P(x) \vee Q(x, y)) \wedge (\neg P(x) \vee \neg Q(x, y))$
Since the second statement is true, it is valid.
Let $x \in S$.

Then, $(P(x) \vee Q(x,y)) \wedge (\neg P(x) \vee \neg Q(x,y))$ is true.
Therefore, it is impossible to provide one definiton of sets S and T, and predicates P and Q, that makes the first statement false and the second statement true.

4

(a)

$$\exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n \leq n_0 \vee (2 \nmid n+1)$$

(b)

$$2|0 \wedge (\forall n \in \mathbb{N}, 2|n \implies 2|n+2)$$

(c)

$$\forall n_0 \in \mathbb{N}, \exists n \in \mathbb{N}, n > n_0 \wedge n = \sum_{i=1}^{n-1} i \cdot \mathbb{I}(i|n) \wedge 2|n$$

(d)

$$\exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, (n = \sum_{i=1}^{n-1} i \cdot \mathbb{I}(i|n)) \wedge (n > n_0) \implies (2|n+1)$$