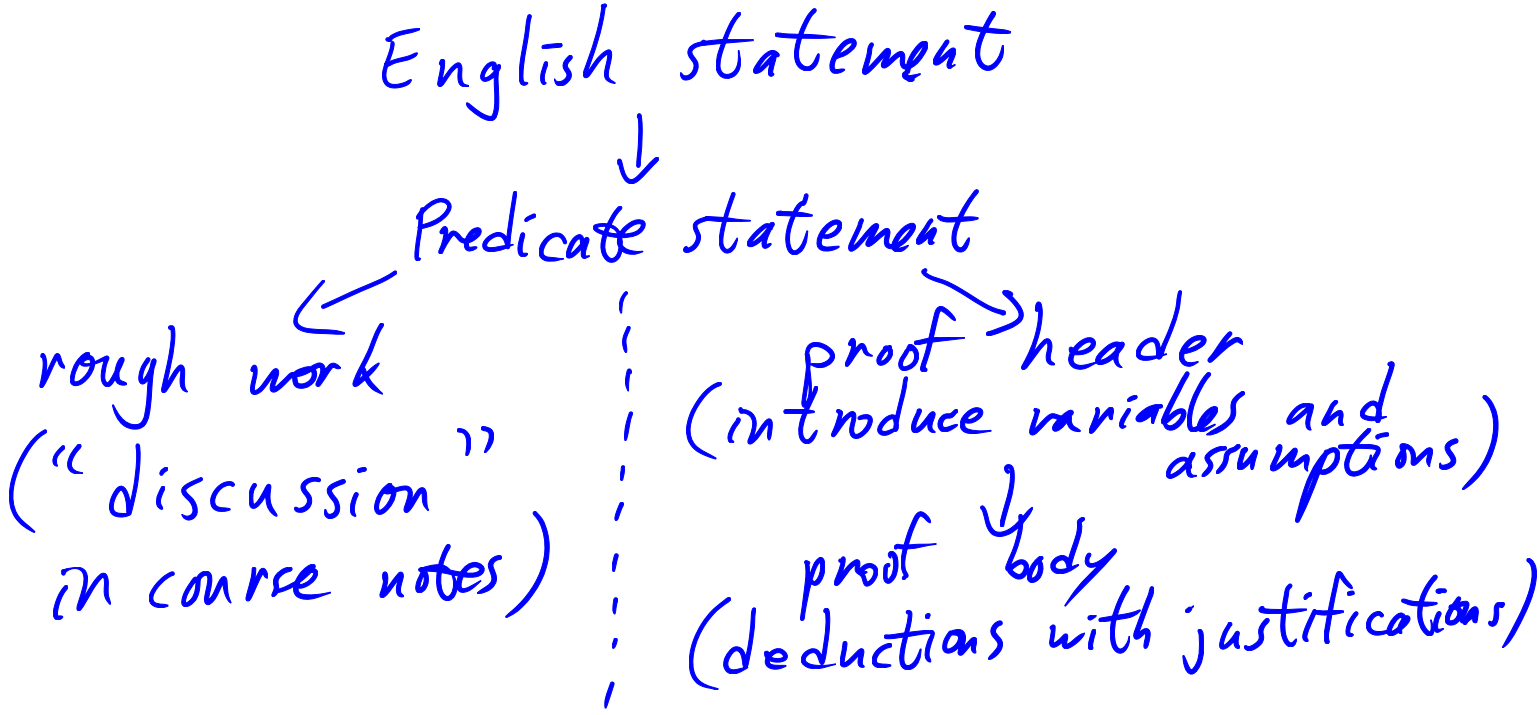
This week: PROOFS

- Proof = any convincing argument
- Generally,



General proof forms

statement	proof headers
$\forall x \in D,\ P(x)$	"Let $x \in D$" (let x denote a fixed but arbitrary element of D) now try to prove $P(x)$
$\exists x \in D,\ P(x)$	"Let $x =$ _____" (for some specific value) now prove $P(x)$
$P \Rightarrow Q$	"Assume P" (suppose P were True) now prove Q
$P \wedge Q$	no header necessary

Ex 1: Prove that every natural number n greater than 20 satisfies $1.5n - 4 \geq 3$.

Step 1: Predicate statement:

~~$\forall n \in \{x \in \mathbb{N} \mid x > 20\}, \; 1.5n - 4 \geq 3$~~ ✗

$\forall n \in \mathbb{N}, \; n > 20 \Rightarrow 1.5n - 4 \geq 3$

Step 2: Proof header

Let $n \in \mathbb{N}$.
Assume $n > 20$.

} Let $n \in \mathbb{N}$, and assume $n > 20$.

(Goal: prove $1.5n - 4 \geq 3$)

PAUSE THE PROOF — ROUGH WORK

KNOWN

$n \in \mathbb{N}$

$n > 20$

WANT

$1.5n - 4 \geq 3$

$1.5n - 4 \geq 3 \iff 1.5n \geq 7$

$\iff n \geq \frac{7}{1.5} = \frac{14}{3}$

$n > 20 \implies n \geq 14/3$

Back to proof...

Let $n \in \mathbb{N}$, and assume $n > 20$.

Then $n \geq 5$

optional

So $1.5n \geq 7.5$

So $1.5n - 4 \geq 3.5 \geq 3.$

□ ← QED
end of proof

Convention: justify every step — except for simple algebraic operations.