TT1-Q3

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Aids Allowed: Your *own notes* taken during lectures and office hours,
the lecture *slides and recordings* (for all sections), and the *Course Notes* (textbook).

Submission Instructions

- Submit your work directly on **MarkUs**—even if you are late!
- You may type your answers or hand-write them *legibly*, on paper or using a tablet and stylus.
- You may write your answers directly on the question paper, or on another piece of paper/document.
- You may submit your answers as a single document or as multiple documents.
- You may name your file(s) any way you want (there is no "required file").
- You must submit your answers in PDF or as photos (JPEG/JPG/GIF/PNG/HEIC/HEIF).
  Other formats (e.g., Word documents, LaTeX source files) **are NOT accepted**—you must **export** or **compile** documents to PDF, and **convert** images into a supported format.

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3. **[6 marks] Disproofs.**

   Consider the following statement:
   $$\forall x, y \in \mathbb{Z}, \ x \mid y \wedge y \mid x \Rightarrow x = y$$

   (a) **[1 mark]** Write the *negation* of this statement in predicate logic.

   $$\exists x, y \in \mathbb{Z}, (x \mid y \wedge y \mid x) \wedge x \neq y$$

   (b) **[5 marks]** *Disprove* the original statement by proving its negation. Take the time to write your proof carefully: it will be marked on its *structure* as well as its *content*,

   Pf: Let $x = -10$

   Let $y = 10$

   We will prove that $x \mid y$ and $y \mid x$ $(\exists k_1 \in \mathbb{Z}, x = y k_1,$ and $\exists k_2 \in \mathbb{Z}, y = x k_2)$

   and that $x \neq y$.

   Let $k_1 = -1$ and $k_2 = -1$

   Proof by arithmetic: $\quad x = y(-1) \qquad y = x(-1)$

   $\qquad\qquad\qquad\qquad -10 = 10(-1) \qquad 10 = (-10)(-1)$

   $\qquad\qquad\qquad\qquad -10 = -10 \qquad\qquad 10 = 10$

   Thus, $x \mid y$ and $y \mid x$ and

   By the original statement $x \neq y$ as $10 \neq -10$.

$\square$

*Don't forget: there are **two more questions** in this test, in separate documents!*