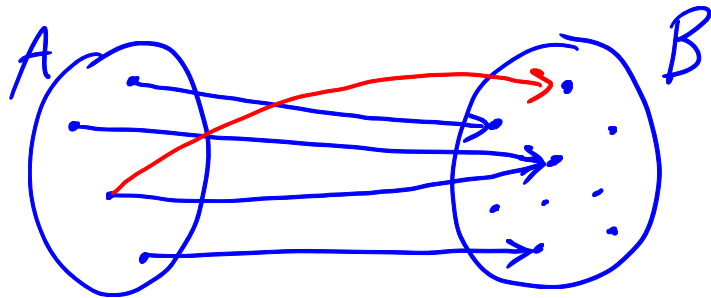


Functions, Summation & Product notation, propositional logic

Function: $f : A \rightarrow B$

name → "is a function" → domain → codomain



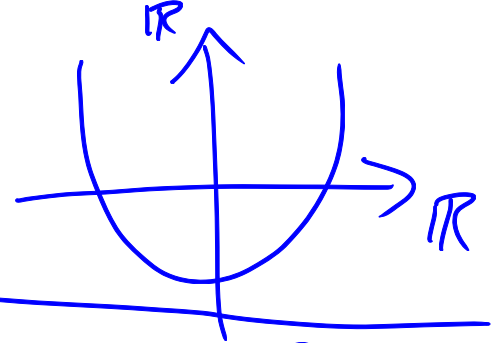
mapping elements of A to elements of B

$f(x)$ for some $x \in A$
 $\in B$

$$f(x) = \frac{x^2}{x-3}$$
$$\text{domain} = \mathbb{R} \setminus \{3\}$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = x^2 - 4$$

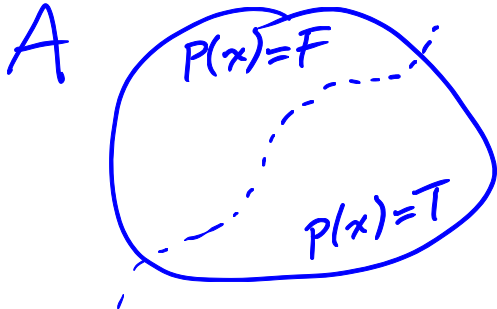


Predicates: $P: A \rightarrow \{\text{True, False}\}$

$P(x)$

$\{T, F\}$
 $\{0, 1\}$

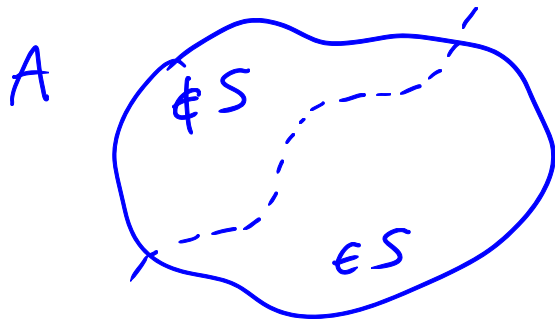
Predicates and sets



$$P: A \rightarrow \{T, F\}$$

e.g.: $P_1: \mathbb{R} \rightarrow \{T, F\}$

$$P(x) = \underline{x > 165}$$



$$S \subseteq A$$

- given any predicate $P: A \rightarrow \{T, F\}$
construct specific subset $S_P \subseteq A$ as follows:
$$S_P = \{x \in A \mid P(x) = T\}$$
- given any subset $S \subseteq A$
construct specific predicate $P_S: A \rightarrow \{T, F\}$
as follows:
$$P_S(x) = \begin{cases} T & \text{if } x \in S \\ F & \text{if } x \notin S \end{cases}$$

$$P_S(x): x \in S$$

Summation & Product notation

Ex: $\frac{2^2}{2-1} + \frac{3^2}{3-1} + \frac{4^2}{4-1} + \dots + \frac{165^2}{165-1}$

end value \leftarrow 165

general term: $\frac{j^2}{j-1}$

summation variable \leftarrow $j=2$ \rightarrow start value for j

fixed step of +1 for j

$$\sum_{j=2}^2 \frac{j^2}{j-1} = \frac{2^2}{2-1}$$

$$\sum_{j=2}^1 \frac{j^2}{j-1}$$

$\frac{2^2}{2-1} + \frac{1^2}{1-1}$ ~~X~~ fixed +1 step
 undefined ~~X~~ leads to more special cases

0 — identity for +
 $(x+0=x)$

Product:

$$\prod_{i=1}^{165} \frac{i^2}{i-3} = \underbrace{\left(\frac{1^2}{1-3}\right) \left(\frac{2^2}{2-3}\right) \left(\frac{3^2}{3-3}\right) \dots \left(\frac{165^2}{165-3}\right)}$$

$$\prod_{i=3}^1 i^3 = 1$$

→ see notes for properties of Σ, Π

Propositional Logic

1. ~~Def:~~ proposition = any statement that is True or False

2. ✓ it is sunny outside

3. ✓ there is intelligent life on Jupiter

4. ~~How are you?~~

5. ✓ $x > 165$ also a predicate

(def: a glarble is a pink umbrella)

no meaning
outside the definition

Worksheet

- Join a breakout room with $< 4-6$ people

• Breakout room 1 reserved:
The Silent Room!

- Main Room (here):
 - quiet work (or not-so-quiet when I discuss logistics with TAs & other students)
 - general questions — not about worksheet

You can ask questions in here too!
Don't hesitate to raise your hand.