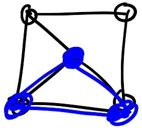
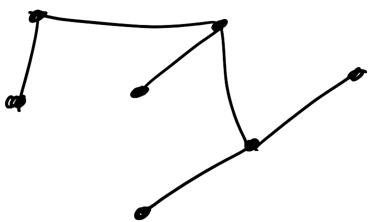
While we wait... what shows are you hoping to catch up on after the term? ... ¿

Def: A *tree* $T=(V,E)$ is a graph that is *connected* ($\forall u,v \in V$, $\exists$ path between u,v) and *acyclic* (no path of length ≥ 1 starts & ends on same vertex).



EX:



Q: how does $|E|$ relate to $|V|$ for trees?

Claim: $\forall$ non-empty trees $T=(V,E)$, $|E|=|V|-1$.

Proof? use induction

$$\underbrace{\forall n \in \mathbb{Z}^+, \ \forall T=(V,E), \ (T \text{ is a tree}) \wedge |V|=n \implies |E| = |V|-1.}_{P(n)}$$

- <u>Base Case</u>: EXERCISE ... :) (for $n=1$)
- <u>Ind. Hyp.</u>: Let $n \in \mathbb{Z}^+$ and assume $P(n)$:

  $\forall T=(V,E)$, T is a tree $\wedge$ $|V|=n \implies |E|=|V|-1$.
- <u>Ind. Step</u>: WTP: $P(n+1)$:

  $\forall T_1=(V_1,E_1)$, T_1 is a tree $\wedge$ $|V_1|=n+1 \implies |E_1|=|V_1|-1$

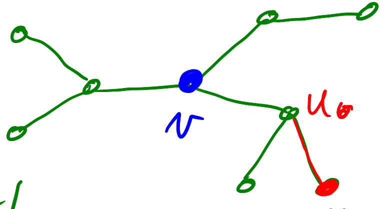
  (NOTE: set up proof headers based on WTP)

  Let $T_1=(V_1,E_1)$ and assume T_1 is a tree and $|V_1|=n+1$.

  WTP: $|E_1|=|V_1|-1$.

ROUGH WORK:

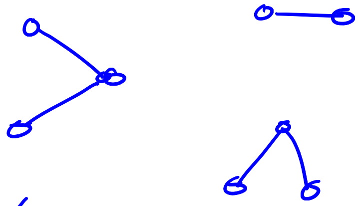
T_1:



$|V_1| = n+1$

Idea: remove <u>some</u> vertex v

"$T_1 - v$":



n vertices but <u>not</u> a tree...

Insight: remove a <u>leaf</u> from T_1

(<u>Def:</u> a leaf is a vertex with degree 1, i.e., with exactly 1 neighbour)

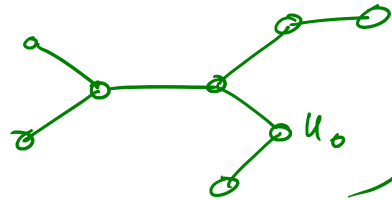
<u>Q:</u> How do we know T_1 contains a leaf?

For now, treat this as an unproved assumption...

Assume T_1 contains at least one leaf v_0.

[NEEDS PROOF!] <u>LEMMA</u>

Then, $T' = (V', E')$ where $V' = V_1 - \{v_0\}$
and $E' = E_1 - \{(v_0, u_0) \mid u_0 \text{ is } v_0\text{'s neighbour in } T_1\}$
is a tree.

(T': )

- T' is still connected because v_0 was a leaf in T_1
- T' is still acyclic

Also, $\underline{|V'|} = |V_1| - 1 = (n+1) - 1 = \underline{n}$

(1) $|E'| = |E_1| - 1$.

$\underline{\text{By I.H., } |E'| = |V'| - 1 = n - 1.}$ (2)

so $|E_1| - 1 = n - 1 \iff |E_1| = n = (n+1) - 1$
$= |V_1| - 1.$ $\square$

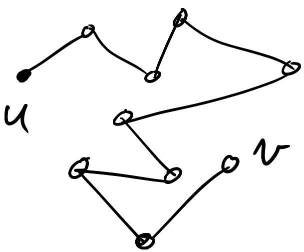
WAIT: what about assumption?

LEMMA = fact needed for main proof, that requires its own proof.

Proof that every tree with $n \geq 2$ vertices contains at least one leaf.

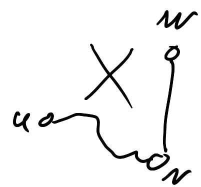
Let $T = (V, E)$ be a tree with $|V| \geq 2$.

Let $u \in V$. Find a longest path in T, starting from u, consider the endpoint v of this path.

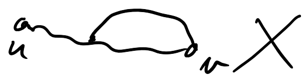


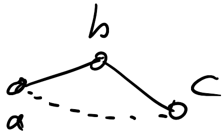
Claim: degree$(v) = 1$.

Otherwise, if degree$(v) > 1$, either there would be a longer path from u



or there would be a cycle in T





length (a—b—c) = 2 (2 edges)

length (a—b—c—a) = 3 (3 edges)

REVIEW — CORE TOPICS & SKILLS

- prop. & pred. logic (translations)
- proof techniques (read & write proofs)
  (direct, indirect, cases, contradiction, induction; $\exists$, $\forall$, $\Rightarrow$, ...)
- domains: - number theory (divisibility, primes)
  - number representations
- $O/\Omega/\Theta$ (proofs, disproofs)
- * algorithm analysis: (apply analysis)
  RT, WC/BC, AC,
  upper bound, lower bounds
- graphs — domain

TT4: Emphasis on algorithm analysis

- Content: - more than half on algo. analysis
  - rest on other topics covering entire course
- Difficulty: - range of difficulties

Advice

1. Read the questions!
2. Show what you know!
3. Manage your time!
4. Explain what you're doing!