TT4-Q3

Wednesday, April 14, 2021 4:51 PM

TT4-Q3

Aids Allowed: ONLY your *own notes* taken during lectures and office hours, the lecture *slides and recordings* (for all sections), and the *Course Notes* (textbook).

Submission Instructions

- Submit your work directly on **MarkUs**—even if you are late!
- You may type your answers or hand-write them *legibly*, on paper or using a tablet and stylus.
- You may write your answers directly on the question paper, or on another piece of paper/document.
- You may submit your answers as a single file/document or as multiple files/documents. Each document may contain answers for only part of one question, an entire question, or multiple questions, but *please label each part of your answers* to make it clear what you are answering.
- There is no "required file", *but please give short names to your file(s)*, like "`Q2.png`" or "`TT4.pdf`".
- You **must** submit your answers in PDF or as photos (JPEG/JPG/GIF/PNG/HEIC/HEIF). **Other formats** (e.g., Word documents, LaTeX source files, ZIP files) **are NOT accepted**—you must **export** or **compile** documents to PDF, **convert** images into a supported format, and upload each file **individually**.

For all questions in this test, write your proofs *formally*, including a header and a proof body with justifications for each deduction. Remember that we are looking for evidence that you understand the conventions for writing correct proofs, so pay attention to the *structure* of your answers, in addition to their content!

3. [7 marks] Algorithm Analysis: Counting Iterations

Consider the following algorithm.

```
1  def woohoo(n: int) -> int:
2      ''' Precondition: n > 0. '''
3      i = 0
4      while i < n:              # Loop 1
5  ①       for j in range(i):    # Loop 2
6              print(j)
7          if i % 5 == 0:
8  ②           if i % 3 == 0:
9                  i = i + 1
10             else:
11                 i = i + 2
12     ③   else:
13             i = i + 4
```

① body takes constant time 1. iterates $i \le n$ times

②+③ body takes constant time.

a) if $i \% 5 \neq 0$, loop 1 takes time $\lceil \frac{n}{4} \rceil + \sum_{i=1}^{4} i \geq n$ total time

b) $\frac{n(n-1)}{2} + \lfloor \frac{n}{4} \rfloor + \lfloor \frac{n}{2} \rfloor + \lfloor \frac{n}{3} \rfloor \leq n^2$ total time

c) $\Theta(n^2)$, lower bound can be $\lceil \frac{n}{4} \rceil + \frac{n(n-1)}{2}$.

(a) **[2 marks]** Find a *lower bound* on **the number of iterations** of Loop 1, as a function of the input n, *without using Omega or Theta notation*. Show your work (in other words, explain how you obtained your answer and show your calculations). *Hint*: Don't try to make your lower bound tight.

(b) **[2 marks]** Find an *upper bound* on **the number of iterations** of Loop 1, as a function of the input n, *without using Big-O or Theta notation*. Show your work (in other words, explain how you obtained your answer and show your calculations). *Hint*: Don't try to make your upper bound tight.

(c) **[3 marks]** Give a Theta bound on **the running time** of function `woohoo`, as a function of its input n. Show your work (in other words, explain how you obtained your answer and show your calculations).

*Reminder: this test contains **five (5)** separate questions, plus the Academic Integrity statement!*