

Aids Allowed: ONLY your *own notes* taken during lectures and office hours, the lecture *slides and recordings* (for all sections), and the *Course Notes* (textbook).

Submission Instructions

- Submit your work directly on [MarkUs](#)—even if you are late!
 - You may type your answers or hand-write them *legibly*, on paper or using a tablet and stylus.
 - You may write your answers directly on the question paper, or on another piece of paper/document.
 - You may submit your answers as a single file/document or as multiple files/documents. Each document may contain answers for only part of one question, an entire question, or multiple questions, but *please label each part of your answers* to make it clear what you are answering.
 - There is no “required file”, *but please give short names to your file(s)*, like “Q2.png” or “TT4.pdf”.
 - You **must** submit your answers in PDF or as photos (JPEG/JPG/GIF/PNG/HEIC/HEIF). **Other formats** (e.g., Word documents, L^AT_EX source files, ZIP files) **are NOT accepted**—you must **export** or **compile** documents to PDF, **convert** images into a supported format, and upload each file **individually**.
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For all questions in this test, write your proofs *formally*, including a header and a proof body with justifications for each deduction. Remember that we are looking for evidence that you understand the conventions for writing correct proofs, so pay attention to the *structure* of your answers, in addition to their content!

2. [5 marks] Asymptotic Notation (Big-O/Omega/Theta)

Let $f_0 : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$ and for each $k \in \mathbb{N}$, define $f_{k+1} : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$ as follows: $f_{k+1}(n) = (f_k(n))^{k+1}$, for all $n \in \mathbb{N}$.

Suppose that $f_0 \in \Omega(n)$.

Use *induction* to prove that, for each $m \in \mathbb{N}$, $f_m \in \Omega(n^{m!})$.

You may use the fact that $(n^j)^k = n^{jk}$ for all $n, j, k \in \mathbb{N}$.