Introduction to Proofs

CSC165 Week 3 - Part 1

Lindsey Shorser, Winter 2021

How do I write a proof?

There is a difference between:

Am I expressing the correct idea?

and

Am I expressing the idea in a way that will be understood?

To avoid confusion in CS, write logical expressions so that:

- There are commas after each “$x \in S$” statement.
- You do not use “such that” or “s.t.”
- Each logical operator has the correct number of inputs without overlap. For example: $(x > 0) \wedge (x < 1)$ instead of $0 < x < 1$.
- You can use as much English as you’d like in your proof. However do NOT use words in your translation of the problem into logical symbols.

What goes into a proof:

- The header
  - what we want to show,
  - “let” statements,
  - any assumptions we can make

- The body of the proof is an argument that relies on:
  - definitions
  - assumptions that were made in the proof header
  - previous deductions from something that already appeared in the proof body
  - external true statements that we already know

Prove or disprove:

Note: I will use W.T.S for "want to show" in these slides to save space. Please do not do that in your solutions.

$$\forall\, n \in \mathbb{N},\ n > 20 \Rightarrow 1.5n - 4 \geq 3$$

$$\forall\, n \in \mathbb{N},\ n > 20 \wedge 1.5n - 4 \geq 3$$

$$\exists\, n \in \mathbb{N},\ n > 20 \wedge 1.5n - 4 \geq 3$$

$$\exists\, n \in \mathbb{N},\ n > 20 \Rightarrow 1.5n - 4 \geq 3$$

p	**q**	**$p \wedge q$**
T	**T**	T
T	**F**	F
F	**T**	F
F	**F**	F

p	**q**	**$p \Longrightarrow q$**
T	**T**	T
T	**F**	F
F	**T**	T
F	**F**	T

A typical proof of an existential.

Given statement to prove: $\exists x \in S,\ P(x)$.

Proof. Let $x =$ ________.

[Proof that $P($________$)$ is True.]

$\blacksquare$

A typical proof of a universal.

Given statement to prove: $\forall x \in S, P(x)$.

Proof. Let $x \in S$. (That is, let x be an arbitrary element of S.)

[Proof that $P(x)$ is True].

$\blacksquare$

A typical proof of an implication (direct).

Given statement to prove: $p \Rightarrow q$.

Proof. Assume p.

[Proof that q is True.]

$\blacksquare$

Prove or disprove:

Note: I will use W.T.S for "want to show" in these slides to save space. Please do not do that in your solutions.

$$\forall\, n \in \mathbb{N},\ n > 20 \Rightarrow 1.5n - 4 \geq 3$$

$$\forall\, n \in \mathbb{N},\ n > 20 \wedge 1.5n - 4 \geq 3$$

$$\exists\, n \in \mathbb{N},\ n > 20 \wedge 1.5n - 4 \geq 3$$

$$\exists\, n \in \mathbb{N},\ n > 20 \Rightarrow 1.5n - 4 \geq 3$$

p	**q**	**p∧q**
T	**T**	T
T	**F**	F
F	**T**	F
F	**F**	F

p	**q**	**p⟹q**
T	**T**	T
T	**F**	F
F	**T**	T
F	**F**	T

Proof:

We want to show that: $\forall\ n \in \mathbb{N},\ n > 20 \Rightarrow 1.5n - 4 \geq 3$

Rough work:

Proof:

We want to show that: $\forall\, n \in \mathbb{N},\, n > 20 \Rightarrow 1.5n - 4 \geq 3$

Let $n \in \mathbb{N}$. Assume that $n > 20$.

Example: $\forall\, x \in \mathbb{Z},\ x \mid (x+5) \Rightarrow x \mid 5.$

Rough Work:

Proof: We want to show that

$$\forall\, x \in \mathbb{Z},\ \ x \mid (x+5) \Rightarrow x \mid 5$$

Let $x \in \mathbb{Z}$.

Assume $x \mid (x+5)$.

[redacted]

$$\forall\, x \in \mathbb{Z},\ \ x \mid (x+5) \Rightarrow x \mid 5$$

Could 5 be replaced by a different prime number in the proof?

$$\forall\, x \in \mathbb{Z},\ \ x \mid (x+5) \Rightarrow x \mid 5$$

Could 5 be replaced by a non-prime number?

$$\forall\, x \in \mathbb{Z},\ \ x \mid (x+5) \Rightarrow x \mid 5$$

Which definitions did we use?

$$\forall\, x \in \mathbb{Z},\ \ x \mid (x+5) \Rightarrow x \mid 5$$

What was the overall proof structure?

Example: $\exists\, x \in \mathbb{Z},\ \forall\, y \in \mathbb{Z},\ x = y+1$

We want to show that $\forall\ x \in \mathbb{Z},\ \exists\ y \in \mathbb{Z},\ x \neq y+1$