- Sets
- Functions

set = collection of elements — anything

- unordered
- elems. are distinct

$$\underbrace{\{1,2,3\}} = \underbrace{\{3, \cancel{3}, 1, 2, \cancel{1}, \cancel{2}, \cancel{1}\}}$$

$\{3, \square, \{2, hi\}\}$

$\emptyset = \{\}$ Is $\emptyset$ = nothing?

$\{3, 1, \emptyset, hi\} \neq \{hi, 1, 3\}$

$x \in A$ (for object x, set A)

↳ "x is an elem. of A", "x belongs to A"

$\emptyset \in \{3, 1, \emptyset, hi\}$ $\emptyset \notin \{hi, 1, 3\}$

Standard sets

$\mathbb{R}$ = real numbers

$\mathbb{Q}$ = rational numbers

$\mathbb{Z}$ = integers : $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$\mathbb{Z}^+$ = positive integers : $\{1, 2, 3, \ldots\}$

$\mathbb{N}$ = natural numbers

$= \{0, 1, 2, 3, \ldots\} = \mathbb{Z}^+ \cup \{0\}$

Operations:

$x \in A$

$A \subseteq B$: every $x \in A$ also belongs to B

$\emptyset \in A$: not true for every A

$\emptyset \subseteq A$: true for every set A

$|A|$ = size of A = # elems.

$|\emptyset| = 0$ $\qquad$ $|A|$ undefined if A is infinite

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$ $\qquad$ union

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$ $\qquad$ intersection

$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$ $\qquad$ difference

$$\underbrace{\{1,2,3\}}_{A} \cup \underbrace{\{2,4,6\}}_{B} = \{1,3,4,6,2\}$$

$$A \times B = \{\underbrace{(x,y)}_{\text{general form of elements}} \mid \underbrace{x \in A \text{ and } y \in B}_{\text{conditions on variables}}\}$$

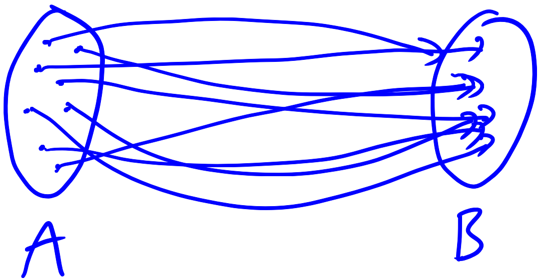
"where":

eg: $\{1,2\} \times \{a,b,c\} =$
$\{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c)\}$

"power set"
$$\mathcal{P}(A) = \{ s \mid s \subseteq A \}$$
$$\mathcal{P}(\{1,a\}) = \{\emptyset, \{1\}, \{a\}, \{1,a\}\}$$

Functions: $\underset{\text{name}}{f} : \underset{\text{domain}}{A} \to \underset{\text{co-domain}}{B}$

mapping of elements in A to element in B



formally,
$f \subseteq A \times B$
where each element of A appears in exactly one pair

Actually... we will allow $\underbrace{f(x)}$ to be undefined for some $x \in A$

the elem. in B associated with x

$\underline{(x, f(x))}$

- functions with more than one argument?

say f takes 2 real numbers as arguments

$f: \underbrace{\mathbb{R} \cup \mathbb{R}}_{\mathbb{R}} \to \mathbb{R}$ ✗ $\qquad f: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

(x, y)

<u>Special case:</u>

codomain = $\{\text{True}, \text{False}\}$

$f: A \to \{\text{True}, \text{False}\}$ —— f is a "predicate"

- For all predicates $P: A \to \{T, F\}$

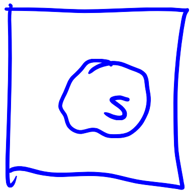
$S_P = \{x \in A \mid P(x) = \text{True}\}$

- For every subset $S \subseteq A$

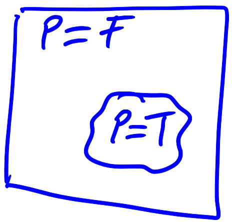
$P_S(x)$: $x \in S$

→ $P_S(x)$ is defined so that
$P_S(x) = \text{True}$ when $x \in S$
$= \text{False}$ when $x \notin S$

A :



A :



$f(x) = \frac{3}{x-5}$

$f: \mathbb{R}\setminus\{5\} \to \mathbb{R}$

$\boxed{f: \mathbb{R} \to \mathbb{R}}$

$f: \mathbb{R} \to \mathbb{R} \cup \{\text{undef.}\}$

$P \rightsquigarrow S_P \rightsquigarrow P_{S_P} = P$

$$\underbrace{S \rightsquigarrow P_S}_{} \rightsquigarrow S_{P_S} = S$$

starting from subset S
to create a matching predicate P_S