

# Propositions and Predicates

CSC165 Week 2 - Part 1

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Propositions are statements that have a true or false value.

The value of a proposition is fixed.

$$f : A \longrightarrow \{T, F\}$$

Predicates are functions that have codomain {True, False}

The value of a predicate depends on one or more input values.

$$\begin{array}{cc} F & T \\ (2 < 1) \vee (2 \in \mathbb{Z}) \end{array}$$

vs.

$$(x < 1) \vee (x \in \mathbb{Z})$$

This is a proposition because we know that it is T

This is a predicate because its truth value depends on the value of x.

$(x < 1) \implies (x \geq 2)$

18 January 2021 is a Monday.

Today is Monday.

x

5 is an even number

All dogs are animals.

✓ If x is a dog, then x is an animal.

Proposition ?

Predicate ?

Proposition ?

Predicate ?

Proposition ?

Predicate ?

Proposition ?

Predicate ?

Proposition ?

Predicate ?

Proposition ?

Predicate ?

# Quantifiers

A quantifier tells you the quantity of something.

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$\forall$  is called the “universal quantifier”.

It means “for all”, “each”, “every” or “all”.

Example:  $\forall x \in \mathbb{N}, x \geq 0$

means “Every natural number  $x$  is greater than or equal to zero.”

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$\exists$  is called the “existential quantifier”.

It means “there exists”, “at least one”, “for some element”.

Example:  $\exists x \in \mathbb{N}, x \geq 5$

means “There exists a natural number  $x$  that is greater than or equal to five.”

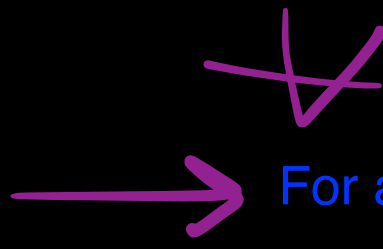
# Reading quantifiers in precise and colloquial language

Statement 1: All dogs are animals.

Statement 2: If  $x$  is a dog, then  $x$  is an animal.

Statement 3: Let  $U =$  the set of living things on Earth.

$\forall x \in U$  If  $x$  is a dog, then  $x$  is an animal.

  $\forall x \in U, D(x) \implies A(x)$

where  $D(x) =$  "x is a dog" and  $A(x) =$  "x is an animal"

# Which quantifiers are hidden in these sentences?

Example 1: All rational numbers are real numbers.

$$\forall x \in \mathbb{Q}, x \in \mathbb{R}$$

Example 2: At least one prime number is even.

$$\exists x \in \mathbb{N}, \text{prime}(x) \wedge (\exists k \in \mathbb{Z}, x = 2k)$$

Example 3: Someone in this class is wearing a purple t-shirt.

$$\exists x \in \{ \text{people in this class} \}, p(x) \quad \text{where } p(x) = \text{"x is wearing a purple t-shirt"}$$

Example 4: It is always possible to add 1 to any integer to get another integer.

$$\forall n \in \mathbb{Z}, n+1 \in \mathbb{Z} \quad \boxed{\{x \in \mathbb{R} \mid \dots\}}$$

$s(x,y)$

# How to symbolize “There are infinitely many primes”

→ Let  $U = \mathbb{N}$  and  $P(x) = \text{“}x \text{ is prime”}$

There are prime numbers that get infinitely large.

$$\forall n \in \mathbb{N}, \exists p \in \mathbb{N}, (P(p) \wedge (n < p))$$

“we can find” =  $\exists$   $f(n, p)$

# Double Quantifiers

Let  $U = \{\text{students in this course}\}$ , and  $x \in U$ ,  $y \in U$

Let  $S(x,y) = \text{"x studies with y"}$

- $\forall x \in U, \forall y \in U, S(x,y) =$  Every student in the class studies with every student in the class.  
Everyone studies with everyone.
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- $\rightarrow \forall x \in U, \exists y \in U, S(x,y) =$  For each student, they have someone they study with.  
"Everyone has a study buddy"  
"Everyone studies with someone"
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- $\exists x \in U, \forall y \in U, S(x,y) =$  Some student in the class studies with all students.  
Someone studies with everyone.  
(super studier)
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- $\rightarrow \exists x \in U, \exists y \in U, S(x,y) =$  A student studies with a student.  
Someone studies with someone.



