Propositions and Predicates

CSC165 Week 2 - Part 2

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Negating Logical Operators

NOT: $\neg (\neg p) \iff p$

It is not the case that x is not positive $\iff$ x is positive

OR: $\neg (p \vee q) \iff$

AND: $\neg (p \wedge q) \iff$

Demorgan's Laws

p	q	¬(p ∨ q)	
T	T		
T	F		
F	T		
F	F		

p	q	¬(p ∧ q)	

IF: $\neg(p \Longrightarrow q) \Longleftrightarrow$

IF AND ONLY IF: $\neg(p \Longleftrightarrow q) \Longleftrightarrow$

p	q	$p \implies q$	
T	T		
T	F		
F	T		
F	F		

p	q	$\neg(p \implies q)$	
T	T		
T	F		
F	T		
F	F		

p	q	$p \iff q$	
T	T		
T	F		
F	T		
F	F		

p	q	$\neg(p \iff q)$	
T	T		
T	F		
F	T		
F	F		

A useful domain: Factors

Definition 1.8. Let $n, d \in \mathbb{Z}$.[25] We say that d **divides** n, or n **is divisible by** d, when there exists a $k \in \mathbb{Z}$ such that $n = dk$. In this case, we use the notation $d \mid n$ to represent "d divides n."

630 is divisible by:

630

Using “|” or “=“ to describe divisibility

$2 \mid n$	“2 divides n” or “2 is a factor of n”
$\exists\, k \in \mathbb{Z},\ n = 2k$	“There exists an integer k such that n can be written as a product of 2 and k” or “n is a multiple of 2”

Both “$2 \mid n$” and “$n = 2k$” are predicates.

How do we symbolize “All multiples of 100 are divisible by 10”?

Precedence

1. $\neg$
2. $\vee$, $\wedge$
3. $\Rightarrow$, $\Leftrightarrow$
4. $\forall$, $\exists$

So for example the expression

$$(p \vee \neg q) \wedge r \Rightarrow \big((s \vee t) \wedge u\big) \vee (\neg v \wedge w)$$

$$\forall x, y \in \mathbb{N},\ \exists z \in \mathbb{N},\ x + y = z \land x \cdot y = z \Rightarrow x = y$$

represents

$$\forall x, y \in \mathbb{N},\ \left(\exists z \in \mathbb{N},\ \Big((x + y = z \land x \cdot y = z) \Rightarrow x = y\Big)\right).$$

Scope

✅ $\forall\, x \in \mathbb{N},\ \exists\, k \in \mathbb{Z},\ \exists\, m \in \mathbb{Z},\ \exists\, n \in \mathbb{Z},\ ((x = 2k) \wedge (x = 3m)) \implies (x = 6n)$

✅ $\forall\, x \in \mathbb{N},\ ((\exists\, k \in \mathbb{Z},\ x=2k) \wedge (\exists\, m \in \mathbb{Z},\ x = 3m)) \implies (\exists\, n \in \mathbb{Z},\ x = 6n)$

X $(\forall\, x \in \mathbb{N},\ (\exists\, k \in \mathbb{Z},\ x=2k) \wedge (\exists\, m \in \mathbb{Z},\ x = 3m)) \implies (\exists\, n \in \mathbb{Z},\ x = 6n)$

X
$$(\forall\, x \in \mathbb{N},\ (\exists\, k \in \mathbb{Z},\ x=2k) \wedge (\exists\, m \in \mathbb{Z},\ x = 3m))$$
$$\implies (\forall\, x \in \mathbb{N},\ \exists\, n \in \mathbb{Z},\ x = 6k)$$

Negating the universal quantifier

Let U = {students in this course},
Let $M(x)$ = "x is majoring in English Literature"

How do we symbolize:

"It is not the case that all students in this course are majoring in English Literature."

Negating the existential quantifier

Let U = {students in this course},
Let M(x) = “x is majoring in English Literature”

How do we symbolize:

“No one is majoring in English Literature.”
or
“It is not the case that someone in this course is majoring in English Literature.”

[redacted]

Double Quantifiers

Let U = {students in this course}, and $x \in U$, $y \in U$

Let $S(x,y)$ = “x studies with y”

$\forall x \in U, \forall y \in U, S(x,y) =$

$\forall x \in U, \exists y \in U, S(x,y) =$

$\exists x \in U, \forall y \in U, S(x,y) =$

$\exists x \in U, \exists y \in U, S(x,y) =$

Negating Double Quantifiers

$\neg (\forall x \in U, \forall y \in U, S(x,y)) =$

$\neg (\forall x \in U, \exists y \in U, S(x,y)) =$

$\neg (\exists x \in U, \forall y \in U, S(x,y)) =$

$\neg (\exists x \in U, \exists y \in U, S(x,y)) =$

Negating Double Quantifiers

$$\neg (\forall x \in U, \forall y \in U, S(x,y)) = \exists x \in U, \exists y \in U, \neg S(x,y)$$

$$\neg (\forall x \in U, \exists y \in U, S(x,y)) = \exists x \in U, \forall y \in U, \neg S(x,y)$$

$$\neg (\exists x \in U, \forall y \in U, S(x,y)) = \forall x \in U, \exists y \in U, \neg S(x,y)$$

$$\neg (\exists x \in U, \exists y \in U, S(x,y)) = \forall x \in U, \forall y \in U, S(x,y)$$