- PS0 or TTO concerns? Please be patient — we are working through them.
- PS1 TA office hours: check Quercus Announcement

"Every natural number n greater than 20 satisfies $1.5n - 4 \geq 3$."

$\forall n \in \mathbb{N}, \; n > 20 \Rightarrow 1.5n - 4 \geq 3$

Proof: Let $n \in \mathbb{N}$, and assume $n > 20$.

Then, $n \geq 4\frac{2}{3}$

So $1.5n \geq 7$

So $\boxed{1.5n - 4 \geq 3}$. $\square$

- "linking words": then, so, therefore, thus, ...
  optional but useful — so use them!
- <u>justification:</u> always justify steps in your deduction — <u>except</u> for simple algebraic manipulations

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<u>Interlude...</u>

$\forall n \cancel{>} 20, P(n)$

S_1: $\forall n \in \mathbb{N}, \; n > 20 \Rightarrow P(n)$

S_2: $\forall n \in \mathbb{N}, \; n > 20 \wedge P(n)$

CSC165 standard

$\boxed{\begin{array}{l} S_1 \text{ True} \\ S_2 \text{ False} \end{array} \text{ for } P(n): 1.5n - 4 \geq 3}$

~~S_3~~: $\forall n \in \mathbb{N}, \; n > 20 \, / \, P(n)$ syntax error

~~S_4~~: $\forall n, \; \underbrace{n \in \mathbb{N} \wedge n > 20 \Rightarrow}_{\text{alternate style — NOT IN CSC165}} P(n)$ syntex error

$\forall$ (variable) $\in$ (domain) — set given in question
($\exists$) — standard number set $(\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R})$

S_1: $\exists n \in \mathbb{N},\ n < 2 \wedge 1.5n - 4 \geq 3$

S_2: $\exists n \in \mathbb{N},\ n < 2 \Rightarrow 1.5n - 4 \geq 3$

S_1 is False

S_2 is True

pick $n = 3$. $3 < 2$ is False so

$\underbrace{3 < 2 \Rightarrow}$ $1.5 \cdot 3 - 4 \geq 3$ is True (vacuously)

$S_2 \equiv \exists n \in \mathbb{N},\ n \geq 2 \lor 1.5n - 4 \geq 3$

$n \not< 2$ $\qquad (p \Rightarrow q \equiv \neg p \lor q)$

"logigically equivalent to"

$A \equiv B$ "A is logically equiv. to B"

$A \Leftrightarrow B$ is always true

Ex 2: Prove that for all integers x, if $x \mid x+5$, then $x \mid 5$

1. $\forall x \in \mathbb{Z}, \underbrace{(x \mid x+5)} \Rightarrow (x \mid 5)$

$\cancel{(x \mid x)+5}$ T/F + 5 ?

$x \mid (x+5)$

? $x \mid x + \cancel{(5 \Rightarrow x)} \mid 5$

2. Proof header:

Let $x \in \mathbb{Z}$, and assume $x \mid x+5$

(WTP: $x \mid 5$) → note to self NOT part of proof

<u>what next?</u>

1. expand definition of "$|$"

$$\forall x \in \mathbb{Z}, \left(\exists k_1 \in \mathbb{Z}, x+5 = k_1 \cdot x\right) \Rightarrow \left(\exists k_2 \in \mathbb{Z}, 5 = k_2 \cdot x\right)$$

2. Let $x \in \mathbb{Z}$.

$\boxed{\text{Assume } \exists}$ $k_1 \in \mathbb{Z}$, $x+5 = k_1 \cdot x$.

Note: from this point on in the proof, k_1 is a variable I can use to refer to a value that satisfies

Let $k_2 =$ ____________ $\leftarrow$ concrete value allowed to use earlier variables (x, k_1), with constants, etc.