- Propositional logic ($\neg \ \wedge \ \vee \ \Rightarrow \ \Leftrightarrow$)
- Predicate logic ($\forall \ \exists$)

Implication $p \Rightarrow q$ "p implies q"

"if p, then q" — but beware

p	q	$p \Rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

"vacuous truth"

$$\underset{\text{hypothesis}}{p} \Rightarrow \underset{\text{conclusion}}{q}$$

$p \Rightarrow q$

$q \Rightarrow p$ converse of $p \Rightarrow q$

$\neg q \Rightarrow \neg p$ contrapositive of $p \Rightarrow q$

predicates $P : A \to \{T, F\}$

$\underbrace{\forall}_{\text{quant.}} \underset{\text{new variable}}{x} \underbrace{\in A}_{\text{domain of } x} \, / \, \boxed{P(x)}$: universal quantifier

"<u>for all</u> $x \in A$, $P(x)$ is true/holds"

$\underline{\exists}\, x \in A,\ P(x)$: existential

"<u>for some</u> $x \in A$, $P(x)$ holds"

at least one

"there exists $x \in A$ such that $P(x)$"

<u>Example</u>: finite

$D = \{$ all <u>strings</u> over <u>alphabet $\{a, b, c\}$</u> $\}$

$= \{\varepsilon, a, b, c, aa, ab, ac, ba, bb, bc, ca, cb, cc, aaa, \ldots\}$

ε represent empty sequence (empty string)

Define: $P(x,y)$: "x and y start with same character"] def. (meaning)

name & arguments

for $x,y \in D$

domain for arguments

$\exists x \in D, P(x,y)$

"some string starts with the same character as y"

value? unknown — depends on y

$\exists x \in D, P(x, bacb)$

value? True: pick $x = bb$

$\forall x \in D, P(x, bacb)$ value? False

pick $x = abc$

$\exists x \in D, \exists y \in D, P(x,y)$

value? meaning?

"there are two strings with same first character"

$\exists y \in D, \exists x \in D, P(y,x)$

$\exists u \in D, \exists t \in D, P(u,t)$

$\exists y \in D, \exists x \in D, P(x,y)$

Generally $\exists x \in D, \exists y \in D, \ldots$

is equivalent to $\exists y \in D, \exists x \in D, \ldots$

Same for $\forall x \in D, \forall y \in D$

$\forall y \in D, \forall x \in D$

① $\forall x \in D, \exists y \in D, P(x,y)$
② $\exists y \in D, \forall x \in D, P(x,y)$
] using same example D, P as earlier

① means: "every string starts with same first char. as some ~~(other)~~ string"

② means: "some string has same first char. as every string"

② is false: no single string has same first char. as all other strings

① is true: for each $x \in D$, we can pick a different $y \in D$ with same first char.

Last observation

$D = \{x_0, x_1, x_2, \ldots\}$

Generally, $\forall x \in D, P(x)$

like $P(x_0) \wedge P(x_1) \wedge \ldots$

$\exists x \in D, P(x)$ like $P(x_0) \vee P(x_1) \vee \ldots$