Sets, Functions, and Predicates

CSC165 Week 1

Lindsey Shorser, Winter 2021

Mathematical Sets

- A collection of elements
- unordered
- distinct

Examples:

$$\{1, 2, 3, 4\} = \{4, 1, 3, 2\}$$

$$\{1, 2, 5\} = \{1, 1, 5, 1, 2\}$$

{Monday, Tuesday, … Friday}

$\{2, 3, 5, 7, 11\}$ $\{2, 3, \ldots, 11\}$

$\varnothing$ = empty set = { }

An example of a set of subsets: (all combinations of a, b, and c)

S = {{a}, {b}, {c}, {ab}, {ac}, {bc}, {a,b,c}, $\varnothing$ }

singleton

2^3 elements because each subset will include a or not include a which is 2 choices. Do this for b and c and you get 2x2x2 combinations.

$x \in S$ "x is an element of S"

$A \subset S$ "A is a subset of S"

$A \subseteq S$

$\mathbb{R}$ = real numbers

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ including zero!

$\mathbb{Z} = \{ \ldots, -2, -1, 0, 1, 2, \}$

$\mathbb{Q} = \{ \quad \mid m, n \in \mathbb{Z}$ and $n \neq 0\}$

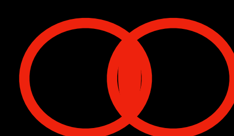
3 is an element of the rational numbers because

$$\frac{3}{1} \in \mathbb{Q}$$

Difference: $\mathbb{N} \setminus \{0\} = \{1, 2, 3, \ldots\} = \mathbb{Z}^+$ "positive integers"

$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$ "both"

A B



$A \cup B = \{ x \mid x \in A \text{ or } x \in B\}$ "either"

A B

Set Operations

$A \times B = \{ (a,b) \mid a \in A \text{ and } b \in B \}$

Let $A = \{ 1, x \}$

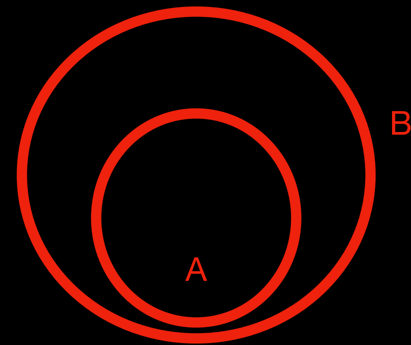
Power Set $P(A) = \{ \{1\}, \{x\}, \{1,x\}, \varnothing \}$

the empty set is an element

$\varnothing \notin A$

$\varnothing \in P(A)$

$A \subset B$ “all $a \in A$ are such that $a \in B$ and $A \neq B$”



strict subset

(there exists a $b \in B$ such that $b \notin A$)

$A \subseteq B$

$f: A \longrightarrow B$

$f(a) = b$

Domain Codomain

Functions

Summation Notation

Product Notation

Propositions and Logical Operators