Prep 6 Quiz

Due Feb 22 at 9pm **Points** 10 **Questions** 6 **Available** until Apr 12 at 9pm **Time Limit** None **Allowed Attempts** Unlimited

Instructions

Readings

Please read the following parts of the **Course Notes**. This includes material we covered in Week 5 and some new material for Week 6. This quiz may not ask questions about every topic covered in the readings for this week.

- Pages 73–83 (but you can skip the proofs of Theorem 4.1 and Lemma 4.3).

General instructions

You can review the general instructions for all prep quizzes on the **Course Syllabus**. Remember that you can submit multiple times! You might consider printing this quiz out so that you can work on paper first.

Take the Quiz Again

Attempt History

	Attempt	Time	Score
KEPT	Attempt 2	2 minutes	10 out of 10
LATEST	Attempt 3	less than 1 minute	0 out of 10
	Attempt 2	2 minutes	10 out of 10
	Attempt 1	1,543 minutes	6.43 out of 10

Score for this attempt: **0** out of 10
Submitted Feb 22 at 7:48pm
This attempt took less than 1 minute.

Question 1

0 / 1 pts

Consider this "proof" of the statement in Example 3.11 (p. 75) of the Course Notes: "the sum of the first n odd numbers is a perfect square."

Proof. We first define the following predicate:

$$P(n): \ \exists x \in \mathbb{N}, \ \sum_{i=0}^{n-1}(2i+1) = x^2, \text{ where } n \in \mathbb{N}$$

We'll prove by induction that P(n) holds for all natural numbers n.

Base Case: Let $n = 0$. We'll prove that P(0) holds.

Let $x = 0$. We want to prove that $\sum_{i=0}^{n-1}(2i+1) = x^2$, which we can do by calculating both sides.

For the left-hand side, we substitute $n = 0$ to obtain $\sum_{i=0}^{-1}(2i+1) = 0$ (since this is an empty sum). For the right-hand side, we substitute $x = 0$ to obtain $0^2 = 0$.

Induction Step: Let $k \in \mathbb{N}$, and assume that P(k) holds. We'll prove that P(k+1) also holds.

By the induction hypothesis, we know that $\exists x_1 \in \mathbb{N}, \ \sum_{i=0}^{k-1}(2i+1) = x_1^2$. Let $x_1 = k$. We want to prove that $\exists x_2 \in \mathbb{N}, \ \sum_{i=0}^{k}(2i+1) = x_2^2$. Let $x_2 = k+1$. We will prove that $\sum_{i=0}^{k}(2i+1) = x_2^2$.

Then we can calculate starting with the left-hand side of the target equation:

$\sum_{i=0}^{k}(2i+1) = \sum_{i=0}^{k-1}(2i+1) + (2k+1)$ (pulling out the last term in the sum)

$\sum_{i=0}^{k}(2i+1) = k^2 + (2k+1)$ (by the I.H.)

$\sum_{i=0}^{k}(2i+1) = (k+1)^2$

$\sum_{i=0}^{k}(2i+1) = x_2^2$ (by our definition of x_2)

This proof has an error in it. What is the problem?

Correct Answer

- Because x_1 is introduced by the assumption of an existential, we cannot choose a value for it in the proof.
- The value we choose for x_2 is not allowed to depend on the value of x_1.
- It defines the wrong predicate for using induction.
- The inductive proof structure is not correct.

You Answered

- One of the calculation steps is invalid.

Unanswered

Question 2

0 / 4 pts

We saw in lecture the basic inductive proof structure for proving a statement of the form $\forall n \in \mathbb{N},\ P(n)$:

- Prove P(0) and for all natural numbers k, P(k) implies P(k+1).

We also saw one variation of this structure, for proving a statement of the form $\forall n \in \mathbb{N},\ n \geq M \Rightarrow P(n)$:

- Prove P(M) and for all natural numbers k that are greater than or equal to M, P(k) implies P(k+1).

There are many more variations of the inductive proof structure that can be used to prove predicates for different subsets of not just the natural numbers, but the integers as well!

Your task is to match each inductive proof structure below to the statement it proves. (Assume that P is a predicate defined for all integers.) The same answer may be used more than once.

You Answered

P(0) and for all natural numbers k, P(k) implies P(k+2)

Correct Answer P(n) is true for all even natural numbers n

You Answered

P(0) and P(1) and for all natural numbers k, P(k) implies P(k+2)

Correct Answer P(n) is true for all natural numbers n

You Answered

P(0) and for all integers k, P(k) implies P(k-1)

Correct Answer P(n) is true for all integers n <= 0

You Answered

P(0) and for all positive integers k, P(k-1) implies P(k)

Correct Answer P(n) is true for all natural numbers n

You Answered

P(0) and for all natural numbers k, P(k) implies P(k+5)

Correct Answer P(n) is true for all natural numbers n that are divisible by 5

You Answered

P(0) and for all integers k, P(k) implies both P(k-1) and P(k+1)

Correct Answer P(n) is true for all integers n

You Answered

P(0) and for all natural numbers k, (P(k+1) is False) implies (P(k) is False)

Correct Answer P(n) is true for all natural numbers n

Other Incorrect Match Options:

- P(n) is true for all odd integers n
- P(n) is true for NO integer n

Unanswered

Question 3

0 / 1 pts

What is the decimal representation of the binary number 101101?

(Just write the number, with no spaces or other characters.)

You Answered

Correct Answers 45 (with margin: 0)

Unanswered

Question 4

0 / 1 pts

What is the binary representation of the decimal number 26?

(Just write the number, with no spaces or other characters.)

You Answered

Correct Answers 11,010 (with margin: 0)

Unanswered

Question 5

0 / 2 pts

If $b_{k-1}b_{k-2}\cdots b_1b_0$ is a binary representation of $n \in \mathbb{Z}^+$, then which of the following statements are true?

Select all that apply.

Correct Answer

- [ ] At least one of $b_{k-1}, b_{k-2}, \ldots, b_1, b_0$ is equal to 1
- [ ] $b_{k-1}b_{k-2}\cdots b_1b_0 1$ is a binary representation for $n+1$
- [ ] $b_{k-1} = 1$

Correct Answer

- [ ] $n = \sum_{i=0}^{k-1} b_i 2^i$

Unanswered

Question 6

0 / 1 pts

Suppose we have a binary representation $b_{k-1}b_{k-2}\ldots b_1b_0$ of some positive integer n. Which of the following is a correct binary representation of the number $4n+1$?

- ○ $b_{k-1}b_{k-2}\ldots b_1b_0 00$

Correct Answer

- ○ $b_{k-1}b_{k-2}\ldots b_1b_0 01$
- ○ $b_{k-1}b_{k-2}\ldots b_1b_0 1$
- ○ $4b_{k-1}b_{k-2}\ldots b_1b_0 1$

Quiz Score: **0** out of 10