$\forall n \in \mathbb{N}, \quad \sqrt{n+1} - \sqrt{n} \geq \dfrac{1}{2\sqrt{n+1}}$

$\forall n \in \mathbb{N}, \; n \geq 5 \Rightarrow \displaystyle\sum_{i=1}^{n} \sqrt{i} \leq \frac{3}{4} n\sqrt{n}$

$k\sqrt{k} = (k+1)\sqrt{k+1} - \sqrt{k+1} - k(\sqrt{k+1} - \sqrt{k})$

$\underline{n=5}$

$$\sum_{i=1}^{5} \sqrt{i} = \sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5}$$

$1 + \cancel{2} + \cancel{\sqrt{2} + \sqrt{3} + \sqrt{5}}$

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$\frac{3}{4} 5 \cdot \sqrt{5} = \frac{15}{4} \simeq 3.5\sqrt{5}$

$\cancel{\sqrt{8} + \sqrt{5}} + \sqrt{5} + \frac{\sqrt{5}}{2}$

$\cancel{2+1}$

$n = k$

$$\sum_{i=1}^{k} \sqrt{i} \leq \frac{3}{4} k\sqrt{k}$$

$n = k+1$

$$\cancel{A} = \sum_{i=1}^{k} \sqrt{i} + \sqrt{k+1} = A$$

$$\sum_{i=1}^{k+1} \sqrt{i} \le \frac{3}{4} k\sqrt{k} + \sqrt{k+1}$$

$$\le \frac{3}{4}\left(k\sqrt{k} + \sqrt{k+1}\right) + \frac{\sqrt{k+1}}{4}$$

$$\le \frac{3}{4}\left((k+1)\sqrt{k+1}\right) - \frac{3}{4}\left(k\sqrt{k+1}\right) - \frac{3}{4}\sqrt{k}$$

$$+ \frac{\sqrt{k+1}}{4}$$

$$\le \frac{3}{4} k+1\left(\sqrt{k+1}\right) - \frac{3}{4}\sqrt{k} + \frac{\sqrt{k+1}}{4}\left(1-3k\right)$$

$k \ge 5$ $\therefore$ $\frac{3}{4}\sqrt{k} > 0$ $\therefore 1-3k<0$

$$\le \frac{3}{4} k+1\left(\sqrt{k+1}\right)$$

$\square$