

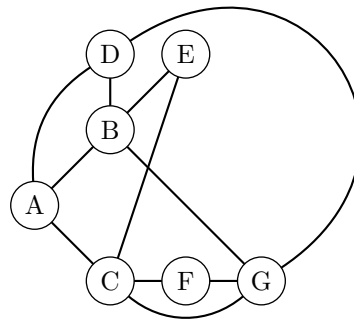
Learning Objectives

By the end of this worksheet, you will:

- Apply basic graph definitions to answer questions about properties of graphs.

1. **Terminology review.** One of the tricky things about learning graphs is that there's a lot of terminology to understand. This exercise will give you the opportunity to practice reading and using this terminology.

Consider the graph below.



- (a) How many vertices does this graph have?
- (b) How many edges does this graph have?
- (c) List all the vertices that are adjacent to vertex G.
- (d) What is the distance between A and G? Is there more than one shortest path between A and G?
- (e) Find a path that goes through all vertices of the graph. (Remember that a path cannot have any duplicate vertices.)

2. **Vertex degree.** Consider the following definition.

Definition 1 (degree). Let $G = (V, E)$ be a graph, and v be a vertex in V . The **degree** of v , denoted $d(v)$, is the number of neighbours of v .

Answer the following questions about this definition.

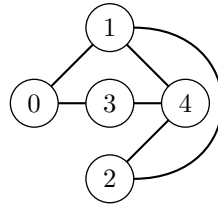
- (a) In the graph on the previous page, what is the degree of vertex D?
- (b) In the graph on the previous page, which vertex/vertices have the largest degree?
- (c) Let $G = (V, E)$ be a graph, and assume that for all $v \in V$, $d(v) \leq 5$. Find and prove a good upper bound (exact, not asymptotic) on the total number of edges, $|E|$, in terms of the number of vertices, $|V|$.
Formally, you can think of this as proving the following statement (after filling in the blank):

$$\forall G = (V, E), (\forall v \in V, d(v) \leq 5) \Rightarrow |E| \leq \underline{\hspace{2cm}}$$

[Note: once you have the right number in mind, the proof isn't computationally complex. Think about trying to count the number of edges a particular vertex can be an endpoint for.]

3. **Adjacency matrix.** Suppose we have a graph $G = (V, E)$, where the vertex set is $\{0, 1, \dots, n-1\}$ for some $n \in \mathbb{N}$. One of the most common ways to represent such a graph in a computer program is by using a two-dimensional array A , where $A[i][j]$ is set to 1 if vertices i and j are adjacent, and 0 if they aren't. This graph representation is known as the *adjacency matrix*.

(a) Consider the graph below.



Write down the adjacency matrix of this graph. The first row (which shows the adjacency relationships of vertex 0) has been done for you:

0 1 0 1 0

(b) Write an algorithm which does the following:

- Given as input an adjacency matrix A representing a graph, and a number i representing a vertex in the graph,
- the algorithm returns the degree of the given vertex in the given graph.