- TT2: not quite done...
- TT3 on Monday — see Quercus
- PS4 comes out today
- TT4 details & date are available (14 April)

Worst-case running time

Example:

```
def is_pal(s: str) -> bool:
    for i in range(len(s)):
        if s[i] != s[len(s)-1-i]:
            return False
    return True
```

NOTE:

- algo can stop early
- what is "the" runtime for input size n?
  - runtime does not depend on n only, but also on specific input s.

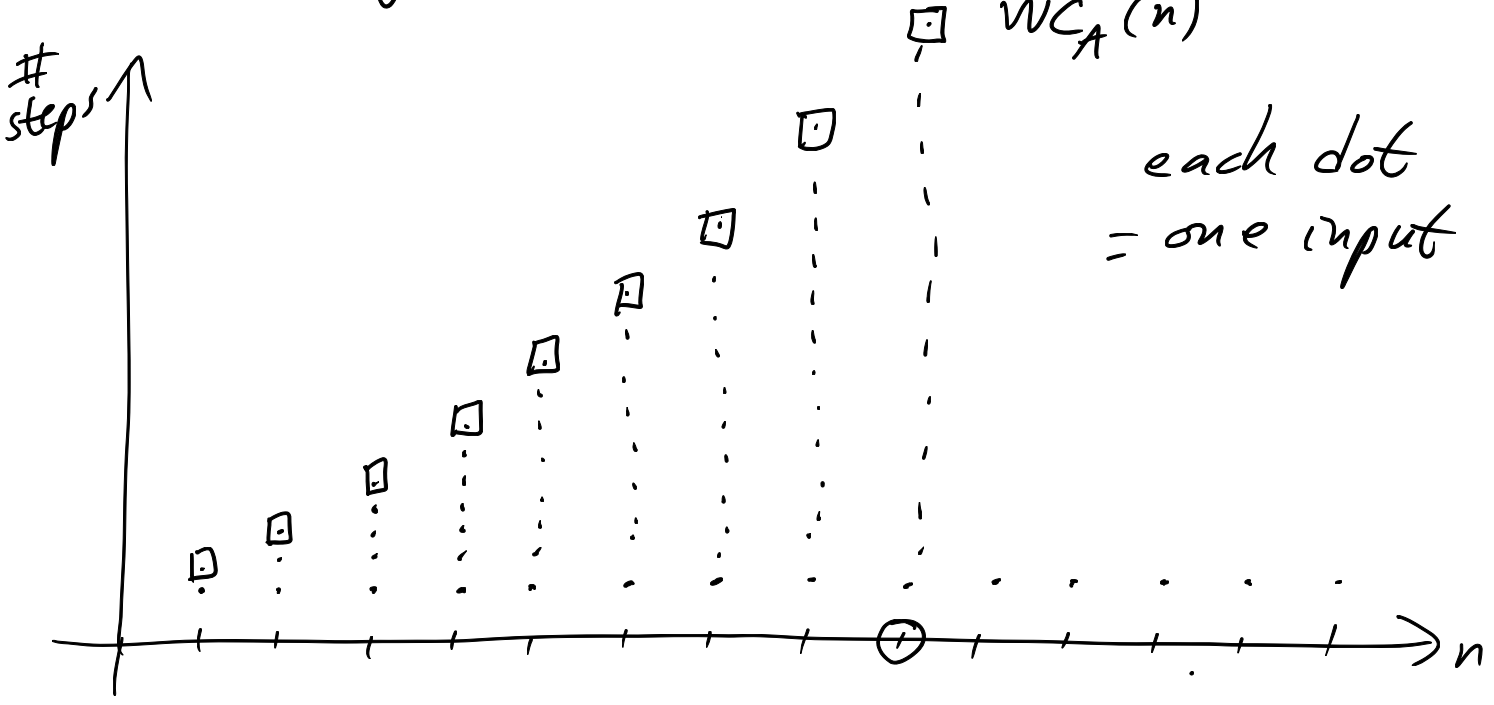
e.g., is_pal("abcdcba")
is_pal("aabbccd")

Definitions:

- Given algorithm A,
  $RT_A(x)$ = the runtime (# steps) of A on input x
- Define $\mathcal{I}_n = \{ x \mid x \text{ is an input of size } n \}$

- Define $WC_A(n) = \max\{ RT_A(x) \mid x \in \mathcal{I}_n \}$

worst-case runtime of A for input size n
= largest runtime of A for all inputs of size n.



- g is an <u>upper bound</u> on $WC_A(n)$?

  $WC_A(n) \in O(g(n))$: $\exists c, n_0 \in \mathbb{R}^+$,

  $\forall n \in \mathbb{N}, n \geq n_0 \Rightarrow \forall x \in \mathcal{I}_n, RT_A(x) \leq c \cdot g(n)$

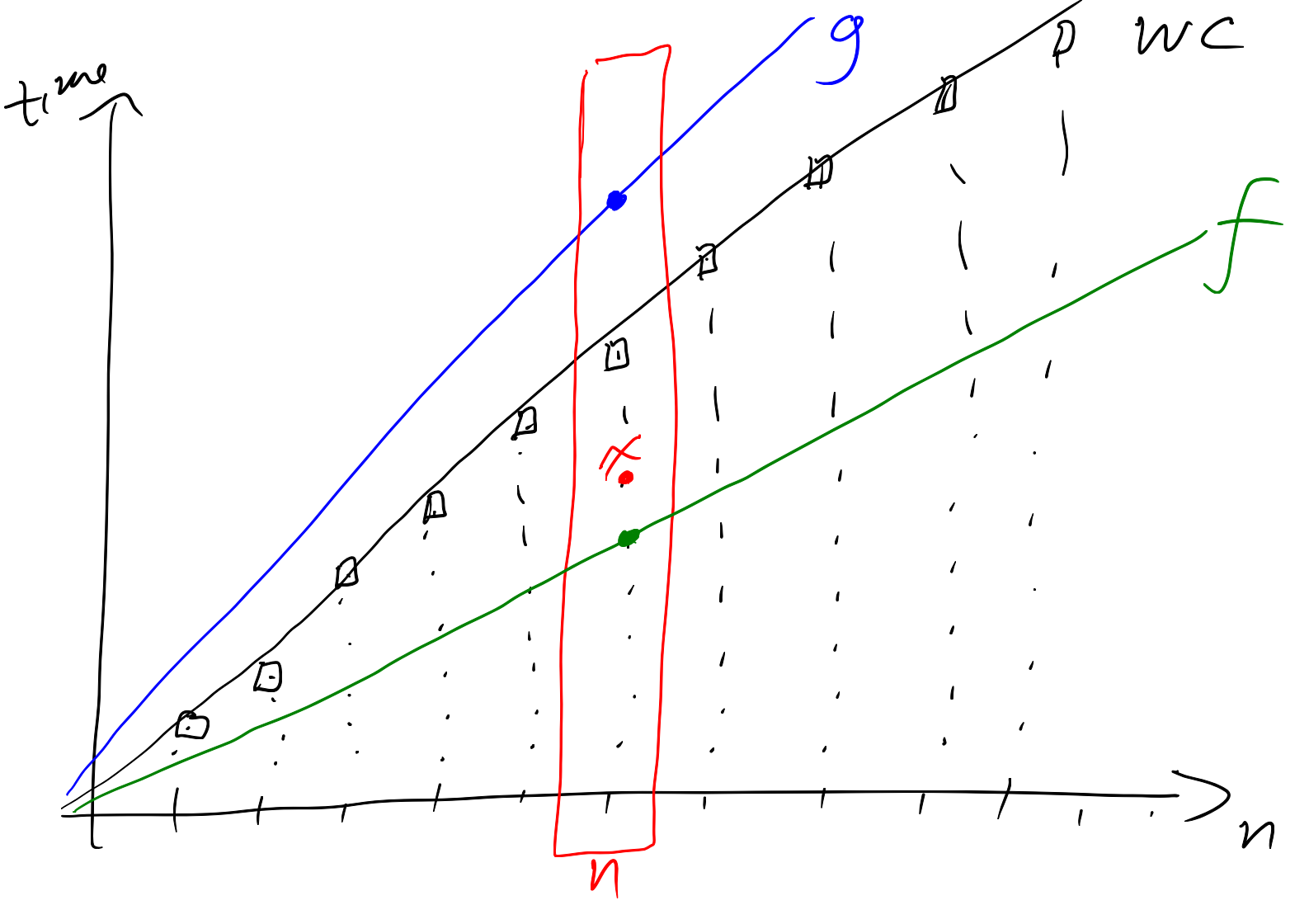
- f is a <u>lower bound</u> on $WC_A(n)$?

  $WC_A(n) \in \Omega(f(n))$: $\exists c, n_0 \in \mathbb{R}^+$,

  $\forall n \in \mathbb{N}, n \geq n_0 \Rightarrow \exists x \in \mathcal{I}_n, RT_A(x) \geq c\, f(n)$

NOTE:

<u>lower bound on worst-case $\neq$ best-case</u>


time
g
p
wc
f
x
n
n

Back to is_pal example...

- **Upper bound**: Let $n \in \mathbb{N}$. Let s be any string of length n.
  Then if the loop does not stop early, it performs n iterations, each one taking time 1.
  So total is $O(n)$.

NOTE: - general argument about all inputs
- we do not try to find "the worst input"

- **Lower bound**: Let $n \in \mathbb{N}$.
  Let $s = \underbrace{"aaa\ldots a"}_{n \text{ times}}$

$Xs = "\underset{q}{a}bcde\ldots"$ ↑

Then, if condition inside the loop is never true, so loop executes all n iterations, each taking time 1 $\Rightarrow$ total $\in \Omega(n)$

NOTE: – need a concrete input "family" (arbitrary size n)

– don't need "the" worst-case, just some input that is "bad enough"

Conclusion: $WC_{is\_pal}(n) \in \Theta(n)$