More Proofs

CSC165 Week 4 - Part 2

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Last time, we proved:

- $\forall\, x \in \mathbb{Z},\ \exists\, y \in \mathbb{Z},\ x \neq y+1$
- $\forall\, d \in \mathbb{N},\ \forall\, x \in \mathbb{N},\ x \mid (x+d) \Rightarrow x \mid d$ (Fact 1)
- $\forall\, x, \in \mathbb{N},\ \forall\, p \in \mathbb{N},\ \text{Prime}(p) \wedge x \mid (x+p) \Rightarrow x=1 \vee x=p$

Notice that the "$\mathbb{Z}$"s were replaced by "$\mathbb{N}$"
to make the last statement true and provable.

Example: $\forall\ a,b \in \mathbb{Z},\ 2\nmid a \wedge 2\nmid b \implies 2\nmid ab$

Generalization? $\forall\ d \in \mathbb{Z},\ \forall\ a,b \in \mathbb{Z},\ d \nmid a \wedge d \nmid b \implies d \nmid ab$

Example: $\forall\, d \in \mathbb{Z}\ (\forall\, a,b \in \mathbb{Z},\ d \nmid a \wedge d \nmid b \Longrightarrow d \nmid ab)$

$\Rightarrow \mathrm{Prime}(d) \vee d \leq 1$

Example: $\forall\ d \in \mathbb{Z}\ (\forall\ a,b \in \mathbb{Z},\ d \nmid a \wedge d \nmid b \Longrightarrow d \nmid ab)$

$\Rightarrow \text{Prime}(d) \vee d \leq 1$

Recall: Prime(p) = "$p>1 \wedge \forall d \in \mathbb{N},\ d | p \Longrightarrow d=1 \vee d=p$"