# Sets, Functions, and Predicates

## CSC165 Week 1

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# Mathematical Sets

- A collection of elements
- unordered
- distinct

{1, 2, 3}

<u>Examples:</u>

{1, 2, 3, 4} = {4, 1, 3, 2}

{1, 2, 5} = {1, 1, 5, 1, 2}

{Monday, Tuesday, … Friday}

{2, 3, 5, 7, 11} ✓

{2, 3, …, 11} ✗

$\varnothing$ = empty set = { }

$\varnothing$

An example of a set of subsets: (all combinations of a, b, and c)

S = {{a}, {b}, {c}, {ab}, {ac}, {bc}, {a,b,c}, $\varnothing$ } ← P({a, b, c})

singleton

2 × 2 × 2

$2^3$ elements because each subset will include a or not include a which is 2 choices. Do this for b and c and you get 2x2x2 combinations.

$x \in S$ “x is an element of S”

$A \subset S$ “A is a subset of S”

$A \subseteq S$

$\mathbb{R}$ = real numbers

$\cancel{\mathbb{N}^+}$

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ including zero!

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

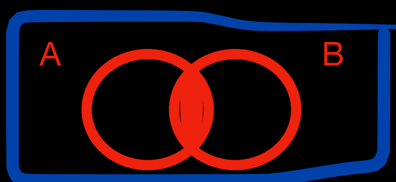
$\mathbb{Q} = \{\frac{m}{n} \mid m, n \in \mathbb{Z} \text{ and } n \neq 0\}$

$\longrightarrow$ 3 is an element of the rational numbers because

$$\frac{3}{1} \in \mathbb{Q}$$

Difference: $\mathbb{N} \setminus \{0\} = \{1, 2, 3, \ldots\} = \mathbb{Z}^+$ "positive integers"

$\longrightarrow$ $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$ "both"




$\longrightarrow$ $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$ "either"

# Set Operations

$\mathbb{R}^2 \rightarrow \mathbb{R} \times \mathbb{R} = \{(x,y) \mid x, y \in \mathbb{R}\}$

$\rightarrow$ A x B = { (a,b) | a ∈ A and b ∈ B }

Let A = { 1, x } $\leftarrow$ A

$q \in \mathbb{R}$

$\rightarrow$ Power Set P(A) = { {1}, {x}, {1,x}, ∅ }

all subsets of A

$\emptyset \subset A$

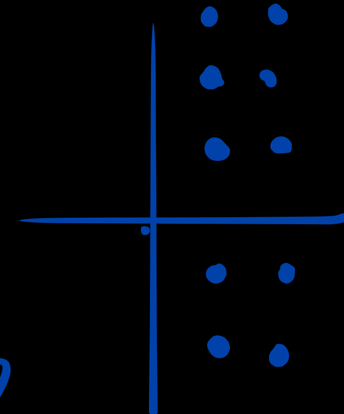
$f(x)$

$x(f)$

the empty set is an element

∅ ∉ A

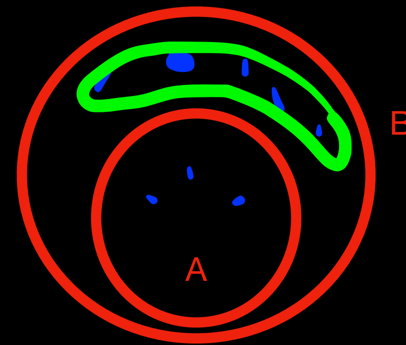
∅ ∈ P(A)



$\mathbb{Z} \times \mathbb{Z} = \{(m,n) \mid m,n \in \mathbb{Z}\}$

$A \subset B$ → "all $a \in A$ are such that $a \in B$ and $A \neq B$"

strict subset




→ (there exists a $b \in B$ such that $b \notin A$)

$A \subseteq B$ → $A \subset B$ or $A = B$

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$f: A \longrightarrow B$

Domain Codomain

$f(a) = b$

$B = \{\text{True}, \text{False}\}$

$A \subset B$ implies $\boxed{A \subseteq B}$

context

# ~~Functions~~

$P(\{1, x, \phi\}) = \{\{\}, \{1\}, \{x\}, \{\phi\}, \{1, \phi\}, \ldots, \phi\}$

Let $A = \phi$ ← elements?

$P(A) = \{\phi\}$ ← elements are sets

$= \{\{\}\}$

$B \times \phi = \phi$

~~Summation Notation~~

$\{1,2\} \times \{?, @, !\}$

$= \{(1,?), (1,@), (1,!), (2,!), (2,@), (2,?)\}$

Graph of $f: A \to B$

$\{(a, f(a)) \mid a \in A\}$

# Product Notation

# Propositions and Logical Operators