Aids Allowed: ONLY your *own notes* taken during lectures and office hours,
the lecture *slides and recordings* (for all sections), and the *Course Notes* (textbook).

Submission Instructions

- Submit your work directly on **MarkUs**—even if you are late!

- You may type your answers or hand-write them *legibly*, on paper or using a tablet and stylus.

- You may write your answers directly on the question paper, or on another piece of paper/document.

- You may submit your answers as a single file/document or as multiple files/documents. Each document may contain answers for only part of one question, an entire question, or multiple questions, but *please label each part of your answers* to make it clear what you are answering.

- There is no "required file", *but please give short names to your file(s)*, like "Q2.png" or "TT4.pdf".

- You **must** submit your answers in PDF or as photos (JPEG/JPG/GIF/PNG/HEIC/HEIF). **Other formats** (e.g., Word documents, LATEX source files, ZIP files) **are NOT accepted**—you must **export** or **compile** documents to PDF, **convert** images into a supported format, and upload each file **individually**.

For all questions in this test, write your proofs *formally*, including a header and a proof body with justifications for each deduction. Remember that we are looking for evidence that you understand the conventions for writing correct proofs, so pay attention to the *structure* of your answers, in addition to their content!

4. **[8 marks] Algorithm Analysis: Worst-Case or Best-Case**

Consider the following algorithm.

```python
def big_short(lst: list, t: int) -> int:
    ''' Return the length of a shortest slice of lst whose sum is at least t
        (return len(lst) + 1 if sum(lst) < t).  Preconditions: t >= 0;
        lst is non-empty; every element of lst is non-negative. '''
    n = len(lst)
    m = n + 1  # min length found so far
    for i in range(n):          # Loop 1
        j = i
        s = 0  # sum of lst[i:j]
        while s < t and j < n:   # Loop 2
            s = s + lst[j]
            j = j + 1
        if s >= t and j-i < m:
            m = j - i
    return m
```

(a) **[3 marks]** Find, with proof, an **upper bound** on the **worst-case** running time of `big_short`. Show your work. For full marks, your upper bound must match the lower bound from the next part.

(b) **[3 marks]** Find, with proof, a **lower bound** on the **worst-case** running time of `big_short`. Show your work. For full marks, your lower bound must match the upper bound from the previous part.

(c) **[2 marks]** Find, with proof, an input family for which the running time of `big_short` is $\Theta(n \log n)$. Show your work. *(Warning! This part is tricky and worth few marks; you should keep it for last.)*

*Reminder: this test contains **five (5)** separate questions, plus the Academic Integrity statement!*