TT4-Q2

Wednesday, April 14, 2021 4:51 PM

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Aids Allowed: ONLY your *own notes* taken during lectures and office hours, the lecture *slides and recordings* (for all sections), and the *Course Notes* (textbook).

Submission Instructions

- Submit your work directly on **MarkUs**—even if you are late!
- You may type your answers or hand-write them *legibly*, on paper or using a tablet and stylus.
- You may write your answers directly on the question paper, or on another piece of paper/document.
- You may submit your answers as a single file/document or as multiple files/documents. Each document may contain answers for only part of one question, an entire question, or multiple questions, but *please label each part of your answers* to make it clear what you are answering.
- There is no "required file", *but please give short names to your file(s)*, like "`Q2.png`" or "`TT4.pdf`".
- You **must** submit your answers in PDF or as photos (JPEG/JPG/GIF/PNG/HEIC/HEIF). **Other formats** (e.g., Word documents, LaTeX source files, ZIP files) **are NOT accepted**—you must **export** or **compile** documents to PDF, **convert** images into a supported format, and upload each file **individually**.

For all questions in this test, write your proofs *formally*, including a header and a proof body with justifications for each deduction. Remember that we are looking for evidence that you understand the conventions for writing correct proofs, so pay attention to the *structure* of your answers, in addition to their content!

2. [5 marks] Asymptotic Notation (Big-O/Omega/Theta)

Let $f_0 : \mathbb{N} \to \mathbb{R}^{\geq 0}$ and for each $k \in \mathbb{N}$, define $f_{k+1} : \mathbb{N} \to \mathbb{R}^{\geq 0}$ as follows: $f_{k+1}(n) = (f_k(n))^{k+1}$, for all $n \in \mathbb{N}$.

Suppose that $f_0 \in \Omega(n)$.

Use induction to prove that, for each $m \in \mathbb{N}$, $f_m \in \Omega(n^{m!})$.

You may use the fact that $(n^j)^k = n^{jk}$ for all $n, j, k \in \mathbb{N}$.

Base Case: $(m = 0)$

$f_0 \in \Omega(n^{0!})$ we know $f_0 \in \Omega(n)$.

Induction: Assume $f_m \in \Omega(n^{m!})$. WTS: $f_{m+1} \in \Omega(n^{(m+1)!})$

Let $n, m \in \mathbb{N}$.

we know, $f_m \in \Omega(n^{m!})$

$f_{m+1}(n) = (f_m(n))^{m+1}$

$= (n^{m!})^{m+1}$

$= (n^{(m+1)m!})$ By fact

$= (n^{(m+1)!})$

Therefore, $f_{m+1}(n) \in \Omega(n^{(m+1)!})$