# TT4-Q1

Wednesday, April 14, 2021 4:51 PM

TT4-Q1

**Aids Allowed**: ONLY your *own notes* taken during lectures and office hours, the lecture *slides and recordings* (for all sections), and the *Course Notes* (textbook).

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**Submission Instructions**

- Submit your work directly on **MarkUs**—even if you are late!
- You may type your answers or hand-write them *legibly*, on paper or using a tablet and stylus.
- You may write your answers directly on the question paper, or on another piece of paper/document.
- You may submit your answers as a single file/document or as multiple files/documents. Each document may contain answers for only part of one question, an entire question, or multiple questions, but *please label each part of your answers* to make it clear what you are answering.
- There is no "required file", *but please give short names to your file(s)*, like "`Q2.png`" or "`TT4.pdf`".
- You **must** submit your answers in PDF or as photos (JPEG/JPG/GIF/PNG/HEIC/HEIF). **Other formats** (e.g., Word documents, LaTeX source files, ZIP files) **are NOT accepted**—you must **export** or **compile** documents to PDF, **convert** images into a supported format, and upload each file **individually**.

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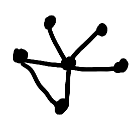
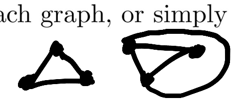
For all questions in this test, write your proofs *formally*, including a header and a proof body with justifications for each deduction. Remember that we are looking for evidence that you understand the conventions for writing correct proofs, so pay attention to the *structure* of your answers, in addition to their content!

**1. [8 marks] Short-Answer Questions**

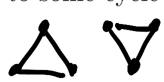
(a) **[2 marks]**
Give two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ such that $|V_1| = |V_2| = 6$ and $|E_1| = |E_2| = 7$ and $G_1$ is **not** connected and $G_2$ is **connected**. You may draw a picture of each graph, or simply list the elements in the sets $V_1, E_1$ and $V_2, E_2$ for each graph.

$G_1 =$ 

$G_2 :$ 

(b) **[2 marks]**
Prove or disprove the following statement: "There exists a non-empty graph $G = (V, E)$ such that every edge in $E$ belongs to some cycle in $G$, and $G$ contains at least two different cycles."

Pf:



$V = \{1,2,3,4,5,6\}$

$E = \{(1,2),(2,3),(3,1),(4,5),(5,6),(6,4)\}$

Let $G = (V,E)$ $\square$

(c) **[2 marks]**
Prove that $n + (1/n) \in \Theta(n)$.
In your answer, you **cannot** use facts from Theorems 5.1–5.9 in the Course Notes.

Pf: $\exists c_1, c_2, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow c_1 n \leq n + \frac{1}{n} \leq c_2 n$

$c_1 n - \frac{1}{n} \leq n$

Let $c_1 = 1$, $n_0 = 1$

$1 \leq n$

$n \leq n^2 + 1$

$n^2 \leq n^2 + 1$

$n^2 - 1 \leq n^2$

$n - \frac{1}{n} \leq n$

Let $c_2 = 3$

$1 \leq n$

$1 \leq n^2$

$\frac{1}{n} \leq n(3-1)$

$\frac{1}{n} \leq 3n - n$

$n + \frac{1}{n} \leq 3n$ $\square$

(d) **[2 marks]** Compute the value of each expression below. Write your answers in **decimal** notation and show your work.

i. $(12)_8 + (40)_{16}$ (1 0)
ii. $(401)_{10} + (1111)_2$

i) $(1 \cdot 8^1) + (2 \cdot 8^0) + (4 \cdot 16^1) = 74$

ii) $(4 \cdot 10^2) + (1 \cdot 10^0) + (1 \cdot 2^3) + (1 \cdot 2^2) + (1 \cdot 2^1) + (1 \cdot 2^0) = 416$