TT2-Q2

Monday, March 1, 2021   7:13 PM

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Aids Allowed: Your *own notes* taken during lectures and office hours,
the lecture *slides and recordings* (for all sections), and the *Course Notes* (textbook).

Submission Instructions

- Submit your work directly on **MarkUs**—even if you are late!

- You may type your answers or hand-write them *legibly*, on paper or using a tablet and stylus.

- You may write your answers directly on the question paper, or on another piece of paper/document.

- You may submit your answers as a single file/document or as multiple files/documents. Each document may contain answers for only part of one question, an entire question, or multiple questions, but *please label each part of your answers* to make it clear what you are answering.

- You may name your file(s) any way you want (there is no "required file").

- You **must** submit your answers in PDF or as photos (JPEG/JPG/GIF/PNG/HEIC/HEIF). **Other formats** (e.g., Word documents, LaTeX source files, ZIP files) **are NOT accepted**—you must **export** or **compile** documents to PDF, **convert** images into a supported format, and upload each file **individually**.

For all questions in this test, "proof" means a *formal* proof that includes a header, and a proof body with justifications for each deduction. Each question can be answered correctly in less than one (1) page, and some questions have *very* short correct answers. You will NOT be penalized directly if you use more space for your answer, but longer answers increase the chance of errors... Remember that we are looking for evidence that you understand the conventions for writing correct proofs—NOT just how to answer these specific questions—so pay close attention to the *structure* of your answers!

2. [**5 marks**] This question tests you on "proof by induction". Even if there is a simple proof using another technique, *you will receive **at most** half the marks if you do NOT use a proof by induction*.

    In your answer, you may use the facts that $e = 2.71828\ldots$ and $1/e = 0.367879\ldots$ Also, if you want to look at the graph of any function in this question, you may use **https://www.desmos.com/calculator**—*but NO other online resource is allowed*.

    Use induction to prove the following statement. As part of your answer, make sure to provide an explicit definition for your predicate $P(n)$, and to state clearly what you are proving in each section of your proof.

$$\forall n \in \mathbb{N}, (n \geq 3) \Rightarrow (e^{-n+2} + 4 < n^2 + 1)$$

$P(n) : (n \geq 3) \Rightarrow (e^{-n+2} + 4 < n^2 + 1)$

We will show by induction that $\forall n \in \mathbb{N}, P(n)$.

Pf: <u>Base Case</u>

Let $n = 3$.

$e^{-3+2} + 4 < 3^2 + 1$

$\frac{1}{e} + 4 < 10$

$\frac{1}{e} + 4 \approx 4.37 < 10$ Base Case holds.

*Don't forget: this test contains **four** separate questions (plus the Academic Integrity statement)!*

<u>Induction Step!</u>

Let $n \in \mathbb{N}$. Assume $n \geq 3$ and $P(n)$. We will show $P(n+1)$

$\frac{1}{e}e^{-n+2} + 4 < \underbrace{e^{-n+2} + 4 < n^2 + 1}_{IH} < (n^2+1) + (2n+1)$ By induction hypothesis

$\frac{1}{e}e^{-n+2} + 4 < (n^2+1) + (2n+1)$

$\frac{1}{e}e^{-n+2} + 4 < (n+1)(n+1) + 1$

$e^{-(n+1)+2} + 4 < (n+1)^2 + 1$

□