

Aids Allowed: Your *own notes* taken during lectures and office hours, the lecture *slides and recordings* (for all sections), and the *Course Notes* (textbook).

Submission Instructions

- Submit your work directly on [MarkUs](#)—even if you are late!
- You may type your answers or hand-write them *legibly*, on paper or using a tablet and stylus.
- You may write your answers directly on the question paper, or on another piece of paper/document.
- You may submit your answers as a single document or as multiple documents.
- You may name your file(s) any way you want (there is no “required file”).
- You must submit your answers in PDF or as photos (JPEG/JPG/GIF/PNG/HEIC/HEIF).
Other formats (e.g., Word documents, L^AT_EX source files) **are NOT accepted**—you must **export** or **compile** documents to PDF, and **convert** images into a supported format.

1. [8 marks] **Short answer questions.** No justification is required for any part of this question.

(a) [2 marks] Let $S = \{2, 3, 5, 7\}$. Find a set $S_1 \subseteq \mathbb{N}$ such that:

- $6 > |S_1| > |S|$, and
- $\forall x \in S, \exists y \in S_1, x \cdot |S_1| = y$

(b) [2 marks] Recall that $\mathcal{P}(S)$ is the set of all subsets of S . Express the following set **without** using \mathcal{P} :

$$\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$$

(c) [2 marks]

p	q	Student Number	Truth Value
False	False	6	True
False	True	7	False
True	False	8	True
True	True	9	False

Create a table like the one on the left, where you will write the last 4 digits of your student number, in the order they appear, from top to bottom in the column labelled “Student Number”. In the column labelled “Truth Value” write **True** if the digit from your student number is even, and write **False** if the digit from your student number is odd. For example, if your student number were 123456789, your truth table would look like the one on the left.

Then, write a propositional formula using only the symbols p , q , \wedge , \vee , and \neg (you may use each symbol any number of times) that is logically equivalent to your truth table (in other words, the truth table for your formula is the same as the truth table you generated from your student number).

(d) [2 marks] Suppose we want to **disprove** the following statement:

$$\forall n \in \mathbb{N}, \exists n_0 \in \mathbb{N}, n_0 > n \wedge \text{Prime}(n_0) \wedge \text{Prime}(n_0 + 2)$$

Write the complete *proof header* for a proof, introducing all variables and assumptions. You may write statements like “Let $d = \underline{\hspace{1cm}}$ ” without filling in the blank. The last statement of your proof header should be “We will prove that ...” where you clearly state what remains to be proved. **Careful: we are NOT asking you to write a proof!** Only the proof *headers*.

*Don't forget: there are **two more questions** in this test, in separate documents!*