Propositions and Predicates

CSC165 Week 2 - Part 1

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Propositions are statements that have a true or false value.
The value of a proposition is fixed.

$f : A \longrightarrow \{T, F\}$

Predicates are functions that have codomain {True, False}
The value of a predicate depends on one or more input values.

$\overset{F}{(2 < 1)} \vee \overset{T}{(2 \in \mathbb{Z})}$ vs. $(x < 1) \vee (x \in \mathbb{Z})$

This is a proposition because we know that it is $\underline{\quad T \qquad\qquad}$

This is a predicate because its truth value depends on the value of x.

$(x < 1) \implies (x \geq 2)$ Proposition ?? Predicate ?

18 January 2021 is a Monday. Proposition ? Predicate ?

Today is Monday. Proposition ?? Predicate ?

x

5 is an even number Proposition ? Predicate ?

All dogs are animals. Proposition ? Predicate ??

$\forall$ If x is a dog, then x is an animal. Proposition ? Predicate ?

Quantifiers

A quantifier tells you the quantity of something.

$\forall$ is called the "universal quantifier".

It means "for all", "each", "every" or "all".

Example: $\forall\, x \in \mathbb{N},\, x \geq 0$

means "Every natural number x is greater than or equal to zero."

$\exists$ is called the "existential quantifier".

It means "there exists", "at least one", "for some element".

Example: $\exists\, x \in \mathbb{N},\, x \geq 5$

means "There exists a natural number x that is greater than or equal to five."

Reading quantifiers in precise and colloquial language

Statement 1: All dogs are animals.

Statement 2: If x is a dog, then x is an animal.

Statement 3: Let U = the set of living things on Earth.

$\forall$ x $\in$ U If x is a dog, then x is an animal.

$\forall$

For all x in U, $\boxed{D(x) \implies A(x)}$

where D(x) = "x is a dog" and A(x) = "x is an animal"

Which quantifiers are hidden in these sentences?

Example 1: All rational numbers are real numbers.

$\forall x \in \mathbb{Q},\ x \in \mathbb{R}$

Example 2: At least one prime number is even.

$\exists x \in \mathbb{N},\ \text{prime}(x) \wedge (\exists k \in \mathbb{Z},\ x = 2k)$

Example 3: Someone in this class is wearing a purple t-shirt.

$\exists x \in \{\text{people in this class}\},\ p(x)$ where p(x) = "x is wearing a purple t-shirt"

Example 4: It is always possible to add 1 to any integer to get another integer.

$\forall n \in \mathbb{Z},\ n+1 \in \mathbb{Z}$

$S(x,y)$

$\{x \in \mathbb{R} \mid \ldots\}$

$\vdots$

How to symbolize “There are infinitely many primes”

→ Let U = ℕ and P(x) = “x is prime”

There are prime numbers that get infinitely large.

$$\forall n \in \overset{U}{\mathbb{N}}, \exists p \in \overset{U}{\mathbb{N}}, \underbrace{(P(p) \wedge (n < p))}_{f(n,p)}$$

“we can find” = $\exists$

Double Quantifiers

Let U = {students in this course}, and $x \in U$, $y \in U$

Let $S(x,y)$ = "x studies with y"

$\forall x \in U, \forall y \in U, S(x,y) =$ Every student in the class studies with every student in the class.
Everyone studies with everyone.

$\forall x \in U, \exists y \in U, S(x,y) =$ For each student, they have someone they study with.
"Everyone has a study buddy"
"Everyone studies with someone"

$\exists x \in U, \forall y \in U, S(x,y) =$ Some student in the class studies with all students.
Someone studies with everyone.
(super studier)

$\exists x \in U, \exists y \in U, S(x,y) =$ A student studies with a student.
Someone studies with someone.