PSO or TTO troubles? Please be patient — we are working through all the email...

Last time...

"Every natural number n greater than 20 satisfies $1.5n - 4 \geq 3$"

$\forall n \in \mathbb{N}, \; n > 20 \Rightarrow 1.5n - 4 \geq 3$

Proof: Let $n \in \mathbb{N}$, and assume $n > 20$

Then, $n \geq 5$

So $1.5n \geq 7.5$

So $\boxed{1.5n - 4 \geq 3.5 \geq 3}$ $\square$

Note:

S_1: $\forall n \in \mathbb{N},\ n > 20 \Rightarrow 1.5n - 4 \geq 3$ — CSC165 standard

S_2: $\forall n \in \mathbb{N},\ n > 20 \wedge 1.5n - 4 \geq 3$ — too "strong"

~~S_3~~: $\forall n \in \mathbb{N},\ n > 20,\ 1.5n - 4 \geq 3$ — syntax error

S_4: $\forall n,\ n \in \mathbb{N} \wedge n > 20 \Rightarrow 1.5n - 4 \geq 3$

↳ different style, NOT FOR CSC165!!
equivalent to S_1

$\forall$ (var. name) $\in$ (domain)

↑

set
- given in question
- standard number set ($\mathbb{N}, \mathbb{Z}, \mathbb{Z}^-, \mathbb{Z}^+, \mathbb{R}, \dots$)

"Some natural number n less than 20 satisfies $1.5n-4 \geq 3$."

S_5: $\exists n \in \mathbb{N},\ \underline{n<20} \wedge \underline{1.5n-4 \geq 3}$ ✓

S_6: $\exists n \in \mathbb{N},\ \underline{n<20} \not\Rightarrow 1.5n-4 \geq 3$ ✗

too "weak"

$\exists n \in \mathbb{N},\ n<2 \Rightarrow 1.5n-4 \geq 3$

$n=10$ $\quad 10<2$ is false

so $10<2 \Rightarrow 1.5\cdot 10-4 \geq 3$ is

vacuously true

Ex 2: Prove that for all integers x, if $x \mid x+5$, then $x \mid 5$.

$\cancel{\underbrace{(x \mid x)}_{T/F} + 5\ ?}$

$x \mid (x+5)$ ✓

1. Predicate notation / symbolic notation

$$\forall x \in \mathbb{Z},\ x \mid x+5 \Rightarrow x \mid 5$$

2. Proof header:

Let $x \in \mathbb{Z}$, and assume $\underbrace{x \mid x+5}$.

(WTP: $x \mid 5$)

... now what?

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go back and expand definition of "$\mid$"

1. $\forall x \in \mathbb{Z}, \left(\exists k_1 \in \mathbb{Z},\ x+5 = k_1 \cdot x\right) \Rightarrow \left(\exists k_2 \in \mathbb{Z},\ 5 = k_2 \cdot x\right)$

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back to proof:

2. Let $x \in \mathbb{Z}$, and assume $\exists k_1 \in \mathbb{Z},\ x+5 = k_1 \cdot x$.

ROUGH WORK:

KNOW

$x \in \mathbb{Z}$

$\exists k_1 \in \mathbb{Z},\ x+5 = k_1 \cdot x$

how to <u>use</u> this?

WANT

$\exists k_2 \in \mathbb{Z},\ 5 = k_2 \cdot x$

let $k_2 =$ ____

<u>Convention</u>: assumption of the form

$\exists$ (variable) $\in$ (domain) (property)

in a proof, we can use (variable) as the name of a value that satisfies (property)

Starting over... COMPLETE PROOF

Let $x \in \mathbb{Z}$, and assume $\exists k_1 \in \mathbb{Z}$, $x+5 = k_1 \cdot x$

Let $\boxed{k_2 = k_1 - 1.}$ $k_2 \in \mathbb{Z}$

Then, $x+5 = k_1 \cdot x$ ← start from KNOWN

$\Rightarrow \quad 5 = k_1 \cdot x - x$

$\Rightarrow \quad 5 = k_2 \cdot x$. $\square$ ← end with WANTED

Consider... In this last proof, did we rely on anything special about the constant 5?