ON MARKUS

TT0: Term Test 0

Timed Assessment

You may start this timed assessment after: Monday, January 25, 2021, 09:00:00 AM

You must start this timed assessment before: Monday, January 25, 2021, 09:00:00 PM

This timed assessment is not available yet.

Duration: 0 hours, 15 minutes

Late submission policy

- A deduction of 0.0% will be applied every 0.25 hours up to 0.25 hours after the assignment is due.
- Thereafter, a deduction of 1.0% will be applied every 0.0166666666666667 hours up to a maximum of 0.0833333333333333 hours. — 1 minute — 5 minutes
- Thereafter, a deduction of 5.0% will be applied every 0.0166666666666667 hours up to a maximum of 0.0833333333333333 hours.

Working with definitions

Def: Let $d, n \in \mathbb{Z}$.
We say **d divides n** when $n = d \cdot k$ for some $k \in \mathbb{Z}$

(d is a divisor of n; n is a multiple of d)

Formally,

Divides(d, n): $\exists k \in \mathbb{Z},\ n = d \cdot k$

where $d, n \in \mathbb{Z}$ → NOT a quantifier ✓

WARNING!

Divides(d, n): ~~$\forall d, n \in \mathbb{Z},$~~ $\exists k \in \mathbb{Z},\ n = d \cdot k$

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def fun(x):
    x = ....
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def fun(x: int)
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Shorthand:

We write $d \mid n$ instead of Divides(d, n)

$d \mid n \neq d/n$

d/n: arithmetic expr. value is a number

$d \mid n$: predicate: value is T/F

EX: Write predicate formula that corresponds to the statement

"Every integer that divides 10 also divides 100"

$\forall z \in \mathbb{Z}$, ← ? → wait! I want $\forall z \in$ {integers that divide 10}

2 options:

① define a new set

$\mathbb{Z}_{10} = \{x \in \mathbb{Z} \mid x \mid 10\}$

$\forall z \in \mathbb{Z}_{10}$, $z \mid 100$

} okay but not the preferred method

* ② use implication to restrict domain

$\forall z \in \mathbb{Z}$, $z \mid 10 \Rightarrow z \mid 100$

"for all integers that divide 10, " ...

$\mathbb{Z}$ / $z|10 : z|10 \Rightarrow z|100$ True iff $z|100$ is True

$z \nmid 10 : \overline{z|10} \Rightarrow z|100$ vacuously true

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Back to predicate definition

$d|n : \cancel{\forall d, n \in \mathbb{Z},}\ \exists k \in \mathbb{Z},\ n = dk$

$3|10 : \forall 3, 10 \in \mathbb{Z}$

?

$\forall x \in D, P(x)$

$\forall y \in D, P(y)$

no variable! $\longrightarrow$ $\forall \_ \in D, P(\_)$

Expanding definitions

$$\forall z \in \mathbb{Z},\ z \mid 10 \Rightarrow z \mid 100$$

same as:

$$\forall z \in \mathbb{Z},\ (\exists k \in \mathbb{Z},\ 10 = zk) \Rightarrow (\exists k \in \mathbb{Z},\ 100 = zk)$$

↳ not the same k!

preferred:

$$\forall z \in \mathbb{Z},\ (\exists k_1 \in \mathbb{Z},\ 10 = zk_1) \Rightarrow (\exists k_2 \in \mathbb{Z},\ 100 = zk_2)$$

Define

Prime(x): $x > 1 \wedge$

where $x \in \mathbb{N}$

Reminder: x is prime iff $x > 1$ and the only positive divisors of x are 1 and x

$$\forall d \in \mathbb{Z}^+,\ d \mid x \Rightarrow d = 1 \vee d = x$$

$$\forall d \in \mathbb{Z}^+,\ d \neq 1 \wedge d \neq x \Rightarrow d \nmid x$$

$$\neg \exists d \in \mathbb{Z}^+,\ d \mid x \wedge d \neq 1 \wedge d \neq x$$