- TT2? still not quite finished, updates on Piazza...
- TT3? Details on Quercus; office hours TBA
- PS4? Later today...

So far... examples of runtime analysis have been for algorithms with <u>one</u> input n.

Worst-case running time

Example:

```
def is_pal(s: str) -> bool:
    for i in range(len(s)):
        if s[i] != s[len(s)-1-i]:
            return False
    return True
```

Note: runtime depends not only on $n = len(s)$, but also on contents of s itself

Q: what is "the" runtime as a function of n?

A: no single answer...

Definitions & notation: For any algorithm A

- $RT_A(x)$: runtime (# steps) of A on input x

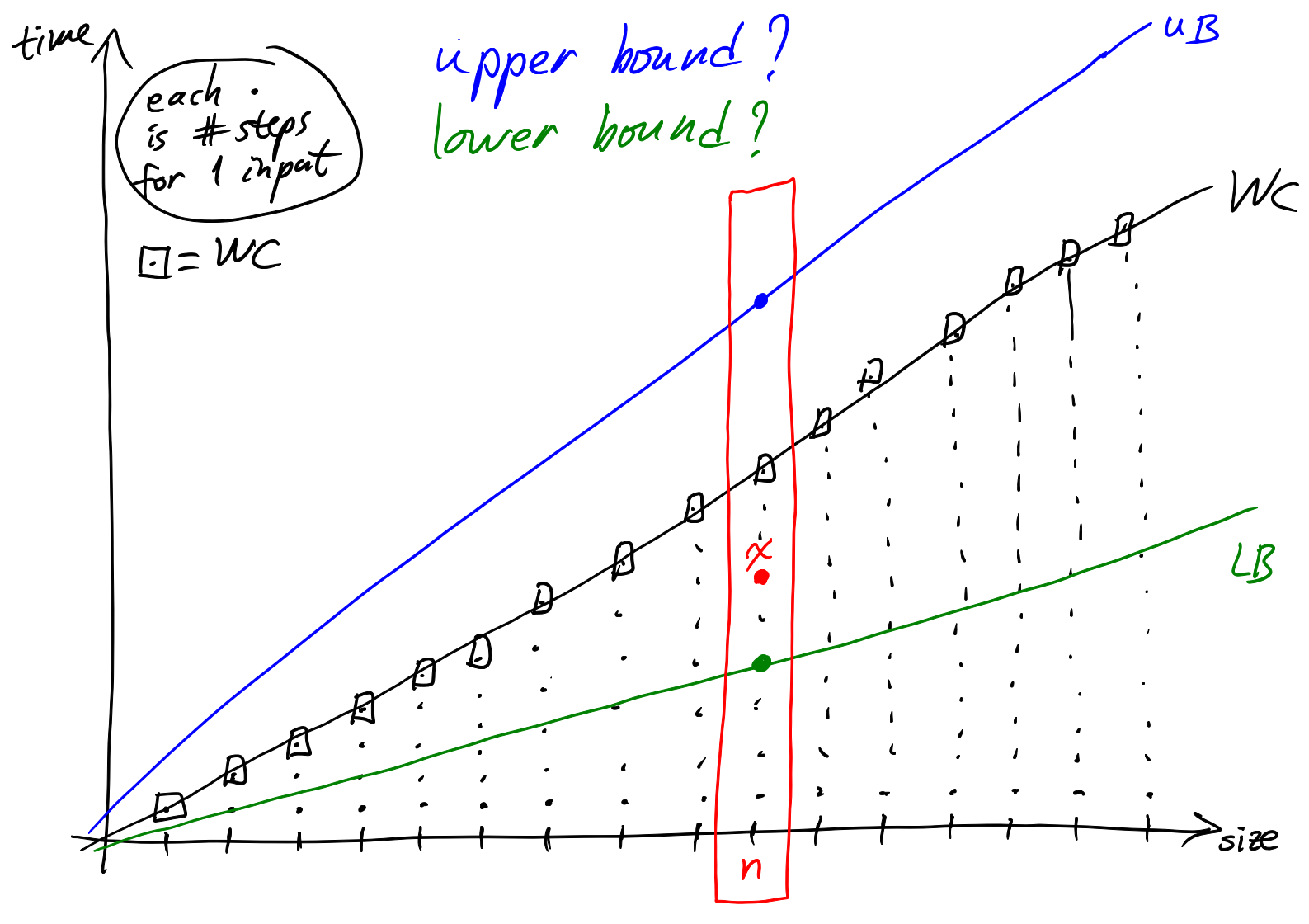
  we want function of n = size of x

- $\mathcal{I}_n = \{x \mid x \text{ is an input of size } n\}$

- worst-case runtime

$$WC_A(n) = \max\{RT_A(x) \mid x \in \mathcal{I}_n\}$$

intuitively: largest runtime of A for all inputs of size n


time
each · is # steps for 1 input
· = WC
upper bound ?
lower bound ?
UB
WC
LB
x
n
size

- g is an upper bound on WC ($WC(n) \in O(g)$):
  $\exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow \forall x \in \mathcal{I}_n, RT(x) \leq c \cdot g(n)$
- f is a lower bound on WC ($WC(n) \in \Omega(f)$):
  $\exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow \exists x \in \mathcal{I}_n, RT(x) \geq c \cdot f(n)$

WARNING: lower bound on worst-case does NOT mean "find an input that takes as little time as possible"

Back to is_pal example...

- Upper bound:
  WANT: - general argument that applies

to all inputs, for all sizes.

- okay to overestimate
- Let $n \in \mathbb{N}$, s be any string of length n
- loop performs at most n iterations
- each iteration takes time at most 1
- total is $\leq n \Rightarrow O(n)$

Lower bound

- Let $n \in \mathbb{N}$.
- Let $s =$ "$\underbrace{aa\ldots a}_{n \text{ times}}$" — not the only choice!

input family — concrete input for each n

- condition of if-statement is never true for input s — loop executes all n iterations
- runtime $\geq n \Rightarrow \underline{\Omega(n)}$

Conclusion: $WC_{is\_pal}(n) \in \Theta(n)$