Prep 6 Quiz Results for Frederick Meneses

Score for this attempt: **10** out of 10
Submitted Feb 22 at 7:48pm
This attempt took 2 minutes.

Question 1

1 / 1 pts

Consider this "proof" of the statement in Example 3.11 (p. 75) of the Course Notes: "the sum of the first n odd numbers is a perfect square."

Proof. We first define the following predicate:

$$P(n): \ \exists x \in \mathbb{N}, \ \sum_{i=0}^{n-1}(2i+1) = x^2, \ \text{where } n \in \mathbb{N}$$

We'll prove by induction that P(n) holds for all natural numbers n.

Base Case: Let $n = 0$. We'll prove that P(0) holds.

Let $x = 0$. We want to prove that $\sum_{i=0}^{n-1}(2i+1) = x^2$, which we can do by calculating both sides.

For the left-hand side, we substitute $n = 0$ to obtain $\sum_{i=0}^{-1}(2i+1) = 0$ (since this is an empty sum). For the right-hand side, we substitute $x = 0$ to obtain $0^2 = 0$.

Induction Step: Let $k \in \mathbb{N}$, and assume that P(k) holds. We'll prove that P(k+1) also holds.

By the induction hypothesis, we know that $\exists x_1 \in \mathbb{N}, \ \sum_{i=0}^{k-1}(2i+1) = x_1^2$. Let $x_1 = k$. We want to prove that $\exists x_2 \in \mathbb{N}, \ \sum_{i=0}^{k}(2i+1) = x_2^2$. Let $x_2 = k+1$. We will prove that $\sum_{i=0}^{k}(2i+1) = x_2^2$.

Then we can calculate starting with the left-hand side of the target equation:

$\sum_{i=0}^{k}(2i+1) = \sum_{i=0}^{k-1}(2i+1) + (2k+1)$ (pulling out the last term in the sum)

$\sum_{i=0}^{k}(2i+1) = k^2 + (2k+1)$ (by the I.H.)

$\sum_{i=0}^{k}(2i+1) = (k+1)^2$

$\sum_{i=0}^{k}(2i+1) = x_2^2$ (by our definition of x_2)

This proof has an error in it. What is the problem?

- One of the calculation steps is invalid.
- The value we choose for x_2 is not allowed to depend on the value of x_1.

Correct!

- (selected) Because x_1 is introduced by the assumption of an existential, we cannot choose a value for it in the proof.
- It defines the wrong predicate for using induction.
- The inductive proof structure is not correct.

Question 2

4 / 4 pts

We saw in lecture the basic inductive proof structure for proving a statement of the form $\forall n \in \mathbb{N},\ P(n)$:

- Prove P(0) and for all natural numbers k, P(k) implies P(k+1).

We also saw one variation of this structure, for proving a statement of the form $\forall n \in \mathbb{N},\ n \geq M \Rightarrow P(n)$:

- Prove P(M) and for all natural numbers k that are greater than or equal to M, P(k) implies P(k+1).

There are many more variations of the inductive proof structure that can be used to prove predicates for different subsets of not just the natural numbers, but the integers as well!

Your task is to match each inductive proof structure below to the statement it proves. (Assume that P is a predicate defined for all integers.) The same answer may be used more than once.

Correct!	**P(0) and for all natural numbers k, P(k) implies P(k+2)**	P(n) is true for all even naturi
Correct!	**P(0) and P(1) and for all natural numbers k, P(k) implies P(k+2)**	P(n) is true for all natural nur
Correct!	**P(0) and for all integers k, P(k) implies P(k-1)**	P(n) is true for all integers n <
Correct!	**P(0) and for all positive integers k, P(k-1) implies P(k)**	P(n) is true for all natural nur
Correct!	**P(0) and for all natural numbers k, P(k) implies P(k+5)**	P(n) is true for all natural nur
Correct!	**P(0) and for all integers k, P(k) implies both P(k-1) and P(k+1)**	P(n) is true for all integers n
Correct!	**P(0) and for all natural numbers k, (P(k+1) is False) implies (P(k) is False)**	P(n) is true for all natural nur

Other Incorrect Match Options:

- P(n) is true for NO integer n
- P(n) is true for all odd integers n

Question 3

1 / 1 pts

What is the decimal representation of the binary number 101101?

(Just write the number, with no spaces or other characters.)

Correct!

45

Correct Answers

45 (with margin: 0)

Question 4

1 / 1 pts

What is the binary representation of the decimal number 26?

(Just write the number, with no spaces or other characters.)

Correct!

11,010

Correct Answers 11,010 (with margin: 0)

Question 5

2 / 2 pts

If $b_{k-1}b_{k-2}\cdots b_1b_0$ is a binary representation of $n \in \mathbb{Z}^+$, then which of the following statements are true?

Select all that apply.

- [ ] $b_{k-1} = 1$
- [x] $n = \sum_{i=0}^{k-1} b_i 2^i$ — Correct!
- [ ] $b_{k-1}b_{k-2}\cdots b_1b_0 1$ is a binary representation for $n+1$
- [x] At least one of $b_{k-1}, b_{k-2}, \ldots, b_1, b_0$ is equal to 1 — Correct!

Question 6

1 / 1 pts

Suppose we have a binary representation $b_{k-1}b_{k-2}\ldots b_1b_0$ of some positive integer n. Which of the following is a correct binary representation of the number $4n+1$?

- (•) $b_{k-1}b_{k-2}\ldots b_1b_0 01$ — Correct!

- $b_{k-1}b_{k-2}\dots b_1b_0 00$
- $4b_{k-1}b_{k-2}\dots b_1b_0 1$
- $b_{k-1}b_{k-2}\dots b_1b_0 1$

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