More Proofs

CSC165 Week 4 - Part 1

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Example: $\exists\ x \in \mathbb{Z},\ \forall\ y \in \mathbb{Z},\ x = y+1$

We want to show that $\forall\, x \in \mathbb{Z},\ \exists\, y \in \mathbb{Z},\ x \neq y+1$

Example: $\forall\ d \in \mathbb{Z}, \forall\ x \in \mathbb{Z},\ x \mid (x+d) \Rightarrow x \mid d$.

Rough Work:

Proof: We want to show that

$$\forall\, d \in \mathbb{Z},\ \forall\, x \in \mathbb{Z},\ x \mid (x+d) \Rightarrow x \mid d$$

Let $x \in \mathbb{Z}$. The value of $d \in \mathbb{Z}$ is fixed.
Assume $x \mid (x+d)$.

Since it has now been prove, we will let Fact 1 be:

$$\forall\, d \in \mathbb{Z},\ \forall\, x \in \mathbb{Z},\ \ x \mid (x+d) \Rightarrow x \mid d.$$

Example:

$\forall$ x, $\in \mathbb{Z}$, $\forall$ p $\in \mathbb{Z}$, Prime(p) $\wedge$ x | (x+p) $\Rightarrow$ x=1 $\vee$ x=p

Prime(p) = “p>1 $\wedge$ $\forall d \in \mathbb{N}$, d|p $\Longrightarrow$ d=1 $\vee$ d=p

Want to show:

$\forall$ x, $\in \mathbb{Z}$, $\forall$ p $\in \mathbb{Z}$, Prime(p) $\wedge$ x | (x+p) $\Rightarrow$ x=1 $\vee$ x=p

Assume, Prime(p) is true.
Also assume x | x+p.

Want to show:

$\forall\ x, \in \mathbb{Z}, \forall\ p \in \mathbb{Z},\ \ Prime(p) \wedge x \mid (x+p) \Rightarrow x=1 \vee x=p$

Assume, Prime(p) is true.
Also assume $x \mid x+p$.

$Prime(p) =$ "$p>1 \wedge \forall d \in \mathbb{N}, d|p \Longrightarrow d=1 \vee d=p$"

Want to show:

$\forall$ x, $\in \mathbb{Z}$, $\forall$ p $\in \mathbb{Z}$, Prime(p) $\wedge$ x | (x+p) $\Rightarrow$ x=1 $\vee$ x=p

Assume, Prime(p) is true.
Also assume x | x+p.

Prime(p) = “p>1 $\wedge$ $\forall d \in \mathbb{N}$, $d|p \implies d=1 \vee d=p$”

Fact 1 $\implies$ $x|p$

Want to show:

$\forall$ x, $\in \mathbb{Z}$, $\forall$ p $\in \mathbb{Z}$, Prime(p) $\wedge$ x | (x+p) $\Rightarrow$ x=1 $\vee$ x=p

Assume, Prime(p) is true.
Also assume x | x+p.

Prime(p) = "$p>1 \wedge \forall d \in \mathbb{N}, d|p \implies d=1 \vee d=p$"

Fact 1 $\implies$ x|p

$\exists\, k \in \mathbb{Z}$, $p = kx$

Want to show:

$\forall$ x, $\in \mathbb{Z}$, $\forall$ p $\in \mathbb{Z}$, Prime(p) $\wedge$ x | (x+p) $\Rightarrow$ x=1 $\vee$ x=p

Assume, Prime(p) is true.
Also assume x | x+p.

Prime(p) = "$p>1 \wedge \forall d \in \mathbb{N}, d|p \Longrightarrow d=1 \vee d=p$"

Fact 1 $\Longrightarrow$ x|p

$\exists$ k $\in \mathbb{Z}$, p = kx, but p is prime, if x|p then x = 1 or x = p

[redacted]

Example: $\forall$ a,b $\in \mathbb{Z}$, $2 \nmid a \wedge 2 \nmid b \implies 2 \nmid ab$

Generalization? $\forall\ d \in \mathbb{Z},\ \forall\ a,b \in \mathbb{Z},\ d \nmid a \wedge d \nmid b \implies d \nmid ab$

Example: $\forall\ d \in \mathbb{Z}\ (\forall\ a,b \in \mathbb{Z},\ d \nmid a \wedge d \nmid b \Longrightarrow d \nmid ab)$

$\Rightarrow \text{Prime}(d) \vee d \leq 1$