Last time...

Prove $\forall d \in \mathbb{N}$, $\underline{\text{Atomic}(d)} \Rightarrow \underline{\text{Prime}(d)} \vee d \leq 1$

where

Atomic(d): $\forall a, b \in \mathbb{Z}$, $d \nmid a \wedge d \nmid b \Rightarrow d \nmid ab$

Prime(d): $d > 1 \wedge \forall k \in \mathbb{Z}^+$, $k \mid d \Rightarrow k = 1 \vee k = d$

"does not divide"

Rework the statement:

"is logically equivalent to"

$$\text{Atomic}(d) \Rightarrow \text{Prime}(d) \vee d \leq 1$$

$$\equiv \neg \text{Atomic}(d) \vee \text{Prime}(d) \vee \underline{d \leq 1}$$

$$\equiv \neg \text{Atomic}(d) \vee \underline{\neg(d > 1)} \vee \text{Prime}(d)$$

$$\equiv \neg(d > 1 \wedge \text{Atomic}(d)) \vee \text{Prime}(d)$$

$$\equiv d > 1 \wedge \text{Atomic}(d) \Rightarrow \text{Prime}(d)$$

Indirect proof (by contrapositive)

$$\underbrace{\forall d \in \mathbb{N}}, \quad \underbrace{\neg \mathrm{Prime}(d)} \Rightarrow \underbrace{d \leq 1 \vee \neg \mathrm{Atomic}(d)}$$

Proof header

Let $d \in \mathbb{N}$. Assume $\neg \mathrm{Prime}(d)$.

WTS: $d \leq 1 \vee \neg \mathrm{Atomic}(d)$

ROUGH WORK

KNOW	WANT
$d \in \mathbb{N}$	$d \leq 1 \vee \neg \mathrm{Atomic}(d)$
$\neg \mathrm{Prime}(d)$	?

expand definitions

$d \leq 1 \vee$
$\exists k \in \mathbb{Z}^+, k \mid d \wedge k \neq 1 \wedge k \neq d$

KNOW

$d \leq 1 \vee \exists a, b \in \mathbb{Z}, d \nmid a \wedge d \nmid b \wedge d \mid ab$

WANT

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* pause from rough work (and proof)

Proof by cases:

- When you know $A \vee B$
  (assumption, previous deduction, definition, fact)
- Break up proof into cases

— Case 1: Assume A.

IMPORTANT: no info about B

... prove conclusion ...

- Case 2: Assume B.

... prove conclusion ...

same conclusion!

Then, conclusion holds.

- -

back to rough work — also in final proof

- Case 1: Assume $d \leq 1$.

  Then, $d \leq 1 \vee \exists a, b \in \mathbb{Z}, d \nmid a \wedge d \nmid b \wedge d \mid ab$

- Case 2: Assume $\exists k \in \mathbb{Z}^+, k \mid d \wedge k \neq 1 \wedge k \neq d$

WTS: $d \le 1 \lor \exists a,b \in \mathbb{Z},\ d \nmid a \land d \nmid b \land d \mid ab$

WANT	KNOW
Let $a =$ ____	$k \in \mathbb{Z}^+$
Let $b =$ ____	$k \neq 1$
wts $d \nmid a$	$k \neq d$
$d \nmid b$	$k \mid d \Rightarrow \exists m \in \mathbb{Z},\ d = km$
$d \mid ab$	

DETAILS: Worksheet 7

Consider the converse of the original statement

$\forall d \in \mathbb{N},\ \text{Prime}(d) \Rightarrow d > 1 \land \text{Atomic}(d)$

Proof header

Let $\boxed{d \in \mathbb{N}.}$

Assume Prime(d) : $\boxed{d > 1} \land \boxed{\forall k \in \mathbb{Z}^+, k \mid d \Rightarrow k = 1 \lor k = d}$

KNOWN

WTS: $\underline{d > 1} \land$

$\forall a, b \in \mathbb{Z},\ d \nmid a \land d \nmid b \Rightarrow d \nmid ab$

Let $\boxed{a, b \in \mathbb{Z}.}$

Assume $\boxed{d \nmid a}$ and $\boxed{d \nmid b}$

WTS: $\boxed{d \nmid ab}$ WANTED

Need additional facts

Key idea

d is prime $\wedge$ $d \nmid a$

$\Rightarrow \exists r, s \in \mathbb{Z},\ rd + sa = 1$