P(n): $\forall$ sets S, $|S| = n \Rightarrow$ S has exactly $\frac{n(n-1)(n-2)(n-3)(n-4)}{120}$ subsets of size 5.

subsets of size 4 → $\frac{n(n-1)(n-2)(n-3)}{24}$ "

$P(n) = \frac{n(n-1)(n-2)(n-3)(n-4)}{120}$ subset of size 5

Base Case: $|S| = 0 \rightarrow$ has 0.

Inductive Steps: I.H. $P(k) = \frac{k(k-1)(k-2)(k-3)(k-4)}{120}$ → Subset of size 5

WTP $P(k+1) = \frac{(k+1)(k)(k-1)(k-2)(k-3)}{120}$ → size 5.

$S' = (S_0, S_1 \ldots S_k)$ $S = S' \cup S_{k+1}$

P.1. Subset of 5 contains S_{k+1}

needs to have 4 elements from $S' \rightarrow$ Subset of $4 + S_{k+1}$

there are $\frac{k(k-1)(k-2)(k-3)}{24}$ subsets.

P.2. Subsets of 5 no S_{k+1}

$$\frac{k(k-1)(k-2)(k-3)(k-4)}{120}$$

then

$$\frac{k(k-1)(k-2)(k-3)(k-4)}{120} + \frac{k(k-1)(k-2)(k-3)}{24}$$

$$= \frac{k(k-1)(k-2)(k-3)(k-4) + 5k(k-1)(k-2)(k-3)}{120}$$

$$= \frac{k(k-1)(k-2)(k-3)((k-4)+5)}{120}$$

$$= \frac{k(k-1)(k-2)(k-3)(k+1)}{120}$$ Q