Multi-Place Predicate Logic Symbolization

Part II – Operations and Equality

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Subjects

1. Individual Constants or Names
   Lower case letters a-h

2. Variables
   Lower case letters i-z

3. Operation Letters
   Lower case letters a-h
   May have number places as superscript

Complex Particular Terms

‘Joe’s brother’s wife’

‘Sarah’s aunt’

‘David’s cousin’s dog’s vet’

Operations

F^1: *a* is Canadian. a^1: the cousin of *a*. e^0: Tom.

Tom is Canadian.

Fe

Tom's cousin.

$a(e)$

Tom's cousin is Canadian.

$Fa(e)$

The father of Jodie's dog walker's cousin is Canadian.

F^1: *a* is Canadian. a^1: the cousin of *a*.

b^1: the father of *a*. c^1: the dog walker of *a*.

f^0: Jodie.

Jodie's dog walker: c(f)
Cousin: a(c(f))
Father: b(a(c(f)))
... is Canadian: Fb(a(c(f)))

1. Use brackets with operations
    - Names have no brackets

2. Operations count as a single term each

3. Learn to read places/slots

How many places is each predicate and operation?

G(ab(cd))

G^2, a^0, b^2, c^0, d^0

Fa(b(c))

F^1, a^1, b^1, c^0

H(a(b)c(d)e(fg))

$H^3, a^1, b^0, c^1, d^0, e^2, f^0, g^0$

Predicates

1. Predicate Letters
   Upper case A-O
   May have number places as superscript

2. Identity Sign =

Identity Syntax

$\alpha = \beta$ where α, β are subjects

α does not equal β

$\sim \alpha = \beta$ or $\alpha \neq \beta$

Identity is a special 2-place predicate

Bad Identity Examples

$(c(a))=d$

$F(x=y)$

$x=G(ab)$

$a=\sim b$

$ab=y$

$Fa=b$

$(x=a)$

$\sim(x=y)$

Tom's cousin isn't Jodie's dog walker.

a^1: the cousin of *a*. c^1: the dog walker of *a*.

e^0: Tom. f^0: Jodie.

a(e) isn't c(f)

Tom's cousin isn't Jodie's dog walker.

a^1: the cousin of *a*. c^1: the dog walker of *a*.

e^0: Tom. f^0: Jodie.

$$a(e) \neq c(f)$$

or

$$\sim a(e) = c(f)$$

Frank and Carla cuddled different dogs.

D^1: *a* is a dog. C^2: *a* cuddled *b*. f^0: Frank. c^0: Carla.

$\exists x(Dx \wedge C(fx)) \wedge \exists y(Dy \wedge C(cy))$

Dogs could be same or different

$\exists x(Dx \wedge C(fx) \wedge C(cx))$

Dogs are the same

$\exists x(Dx \wedge C(fx) \wedge \exists y(Dy \wedge C(cy) \wedge x \neq y))$

Dogs are different

No singer except/besides Justin is awesome.

A^1: *a* is a awesome. B^1: *a* is a singer. a^1: Justin.

$\sim\exists x(Ax \wedge Bx \wedge x \neq a)$ $\forall x(Ax \wedge Bx \rightarrow x = a)$

Am I missing something?

Is Justin an awesome singer?

$\wedge(Aa \wedge Ba)$?

Implication

vs

Implicature

Only philosophers are happy.

A^1: *a* is a philosopher. H^1: *a* is happy.

$$\forall x(Hx \rightarrow Ax)$$

$$\forall x(\sim Ax \rightarrow \sim Hx)$$

Are there happy philosophers?

Rihanna is Drake's only friend.

F^2: *a* is the friend of *b*. a^0: Rihanna. d^0: Drake.

If anyone is Drake's friend, it's Rihanna.

$\forall x(F(xd) \rightarrow x=a)$ ✗

Is Rihanna Drake's friend?

Rihanna is Drake's only friend.

F^2: *a* is the friend of *b*. a^0: Rihanna. d^0: Drake.

Rihanna is Drake's friend, and no one else is.

$F(ad) \wedge \sim \exists x(x \neq a \wedge F(xd))$

Rihanna is Drake's friend, and if anyone is Drake's friend they are actually just Rihanna.

$F(ad) \wedge \forall x(F(xd) \rightarrow x = a)$

Rihanna is Drake's only friend.

F^2: *a* is the friend of *b*. a^0: Rihanna. d^0: Drake.

✓ $F(ad) \wedge {\sim}\exists x(x \neq a \wedge F(xd))$

✓ $F(ad) \wedge \forall x(F(xd) \rightarrow x = a)$

✓ $\forall x(F(xd) \leftrightarrow x = a)$

The Prime Minister of Canada
has the best hair (of people).

a^0: Canada. b^1: The Prime Minister of {1}.
h^2: {1} is the hair of {2}. F^1: {1} is a person.
A^2: {1} is better than {2}.

A(Prime Minister of Canada's Hair ____)

A(h(b(a)) ____)

$\forall x(Fx \rightarrow A(h(b(a))h(x)))$ ✗

The Prime Minister of Canada
has the best hair (of people).

a^0: Canada. b^1: The Prime Minister of {1}.
h^2: {1} is the hair of {2}. F^1: {1} is a person.
A^2: {1} is better than {2}.

The Prime Minister of Canada has
better hair than any other person.

Superlatives

The Prime Minister of Canada has the best hair (of people).

a^0: Canada. b^1: The Prime Minister of {1}.
h^2: {1} is the hair of {2}. F^1: {1} is a person.
A^2: {1} is better than {2}.

If you're a person other than the Prime Minister of Canada, then the Prime Minister of Canada has better hair than you.

✓ $\forall x(Fx \wedge x \neq b(a) \rightarrow A(h(b(a))h(x)))$

The Prime Minister of Canada
has the best hair (of people).

a^0: Canada. b^1: The Prime Minister of {1}.
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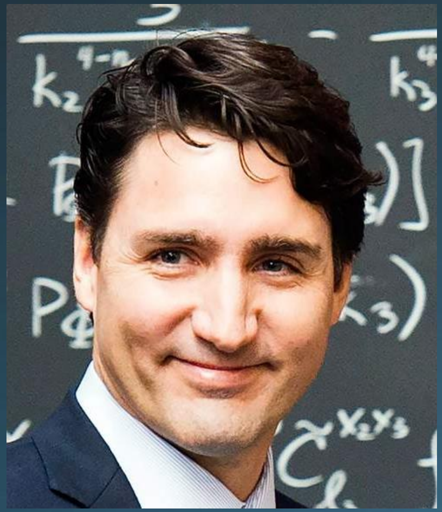


There does not exist a person who has better hair than the Prime Minister of Canada.

✓ $\sim\exists x(Fx \wedge A(h(x)h(b(a)))$

The Prime Minister of Canada has the best hair (of people).

a^0: Canada. b^1: The Prime Minister of {1}.
h^2: {1} is the hair of {2}. F^1: {1} is a person.
A^2: {1} is better than {2}.

If you're a person, then the Prime Minister of Canada has better hair than you as long as you're not the Prime Minister of Canada.

$\forall x(Fx \rightarrow A(h(b(a))h(x)) \wedge x \neq b(a))$

All sports are fun, except for Golf.

All sports, except for Golf, are fun.

A^1: *a* is a sport. F^1: *a* is fun. g^0: Golf.

✗ $\forall x(Ax \rightarrow Fx \land x \neq g)$

✓ $\forall x(Ax \land x \neq g \rightarrow Fx)$

C^1: *a* is a carrot. A^2: *a* ate *b*. a^0: Avery.

Avery ate at least one carrot.

$$\exists x(Cx \wedge A(ax))$$

Avery ate at least two carrots.

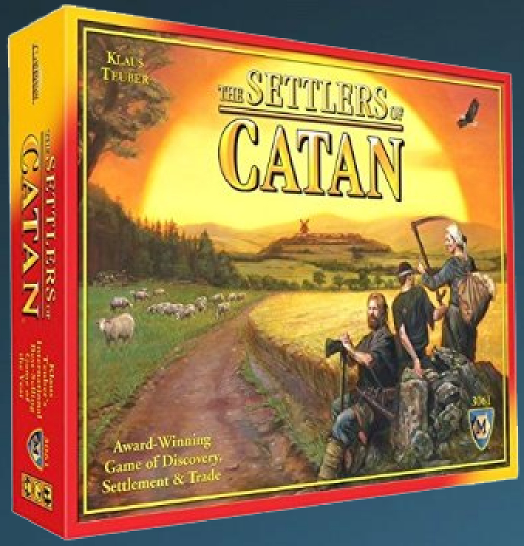
$$\exists x(Cx \wedge A(ax) \wedge \exists y(Cy \wedge x \neq y \wedge A(ay)))$$

Avery ate at least three carrots.

$$\exists x(Cx \wedge A(ax) \wedge \exists y(Cy \wedge x \neq y \wedge A(ay) \wedge \exists z(Cz \wedge z \neq x \wedge z \neq y \wedge A(az))))$$

Beth played at most one board game.

B^1: *a* is a board game. A^2: *a* plays *b*. b^0: Beth.



$$\forall x(Bx \wedge A(bx) \rightarrow \forall y(By \wedge A(by) \rightarrow x=y))$$

$$\forall x \forall y(Bx \wedge By \wedge A(bx) \wedge A(by) \rightarrow x=y)$$

$$\sim \exists x(Bx \wedge A(bx) \wedge \exists y(By \wedge A(by) \wedge x \neq y))$$

$$\sim \exists x \exists y(Bx \wedge By \wedge A(bx) \wedge A(by) \wedge x \neq y)$$

Beth played at most two board games.

B^1: *a* is a board game. A^2: *a* plays *b*. b^0: Beth.

$$\forall x(Bx \wedge A(bx) \to \forall y(By \wedge A(by) \to \forall z(Bz \wedge A(bz) \to x=y \vee x=z \vee y=z)))$$

$$\forall x \forall y \forall z(Bx \wedge By \wedge Bz \wedge A(bx) \wedge A(by) \wedge A(bz) \to x=y \vee x=z \vee y=z)$$

…

Sarah ate exactly one piece of fried chicken.

C^1: {1} is a piece of chicken. F^1: {1} is fried.

A^2: {1} ate {2}. a^0: Sarah.

Sarah ate a piece of fried chicken, and if she ate any other piece of fried chicken it was actually the same as the first.

Sarah ate a piece of fried chicken, and there does not exist a different piece of fried chicken that she also ate.

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$$\exists x(Cx \wedge Fx \wedge A(ax)$$

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Sarah ate a piece of fried chicken, and if she ate any other piece of fried chicken it was actually the same as the first.

$$\exists x(Cx \wedge Fx \wedge A(ax)) \wedge \forall y(Cy \wedge Fy \wedge A(ay) \rightarrow$$

Sarah ate exactly one piece of fried chicken.

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Sarah ate a piece of fried chicken, and if she ate any other piece of fried chicken it was actually the same as the first.

$$\exists x(Cx \wedge Fx \wedge A(ax) \wedge \forall y(Cy \wedge Fy \wedge A(ay) \rightarrow x=y))$$

$$\exists x \forall y(Cx \wedge Fx \wedge A(ax) \wedge \forall y(Cy \wedge Fy \wedge A(ay) \rightarrow x=y))$$

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Sarah ate a piece of fried chicken, and there does not exist a different piece of fried chicken that she also ate.

$$\exists x(Cx \wedge Fx \wedge A(ax) \wedge {\sim}\exists y(Cy \wedge Fy \wedge A(ay) \wedge x \neq y))$$