TODAY!

- average-case analysis
- intro. to graphs

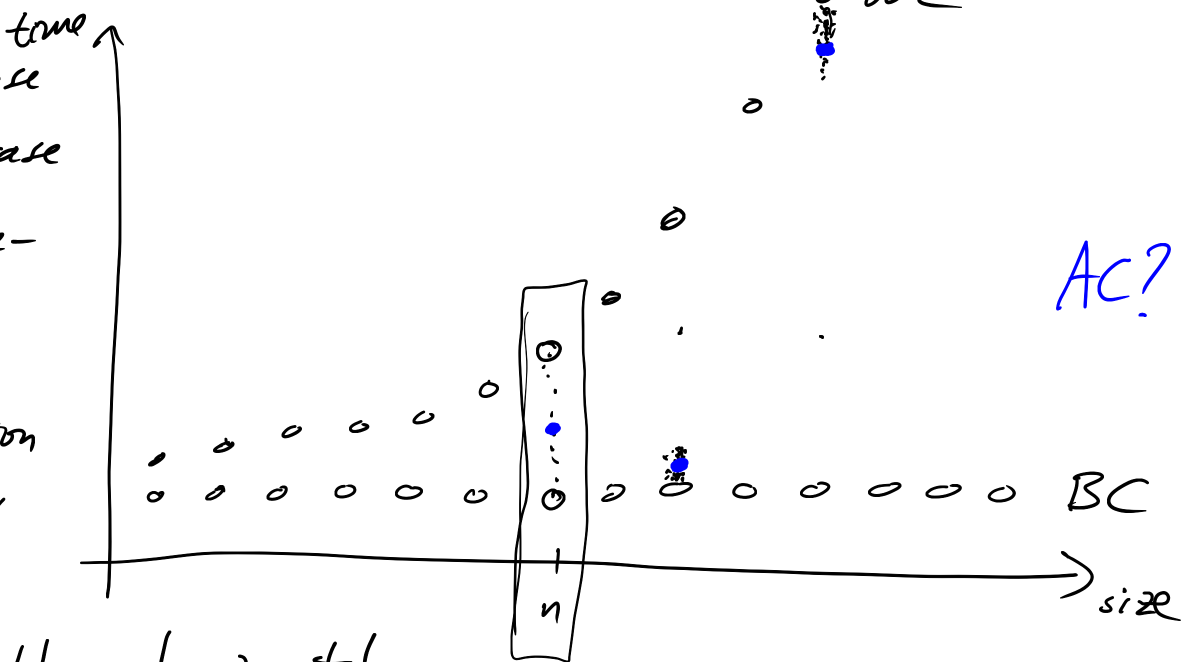
Average-case analysis

RT(x): running time on input x

WC(n): worst-case

BC(n): best-case

AC(n): average-case



NOTES:

- NO distinction between WC, BC, AC for algorithms where n is the only input!

- $AC(n) \neq \dfrac{BC(n) + WC(n)}{2}$

$AC(n)$ = average of $RT(x)$ over all inputs x of size n

Generally, start by defining

$I_n = \{\text{all inputs of size } n\}$

input of size n — another input of size n

imagine $I_n = \{x_0, x_1, x_2, \ldots, x_m\}$

$$\frac{RT(x_0) + RT(x_1) + RT(x_2) + \ldots + RT(x_m)}{m+1}$$

$$AC(n) = \frac{1}{|I_n|} \sum_{x \in I_n} RT(x)$$

runtime on input x

add up over all $x \in I_n$

size of I_n, i.e., how many inputs of size n we have

<u>Ex</u>: def search(L: list, x: int) → bool:

```
    for item in L:
        if item == x:
            return True
    return False
```

$WC(n) \in \Theta(n)$

$BC(n) \in \Theta(1)$

<u>One</u> input to algo consists of list L <u>and</u> int x

<u>(L, x)</u>.

$\boxed{\text{For WC lower bound, use input family } L = [1, 2, \ldots, n],\ x = 0.}$

Q: How to define $\mathcal{I}_n$?

What does "all possible inputs" really mean?

<u>Key insight</u>: need at least one input for each behaviour of the algorithm

For our example, this means inputs where x appears in L at each possible location.

- <u>Idea 1</u>:

$$\underset{\sim}{I}_n = \{ (L, x) \mid L \text{ is a permutation of } [1, 2, \dots, n] \text{ and } \underline{x = 1} \}$$

Permutation? e.g. $[1,2,3]$, $[1,3,2]$, $[2,1,3]$, $[2,3,1]$, $[3,1,2]$, $[3,2,1]$ for $n = 3$

Details → see notes...

- <u>Idea 2</u>:

$$\mathcal{I}_n' = \left\{ (L, x) \;\middle|\; L = [1, 2, \ldots, n],\ x \in \{\underline{0}, 1, 2, \ldots, n\} \right\}$$

Note: this includes one possibility $([1, 2, \ldots, n], 0)$ with x not in L.

$$AC(n) = \frac{1}{|\mathcal{I}_n'|} \sum_{(L,x) \in \mathcal{I}_n'} RT(L, x)$$

$$= \frac{1}{n+1} \left(RT([1, 2, \ldots, n], 0) + \sum_{x=1}^{n} RT([1, \ldots, n], x) \right)$$

$$= \frac{1}{n+1} \left((n+1) + \sum_{x=1}^{n} x \right)$$

$= \ldots$ arithmetic $\ldots$

• <u>Idea 3:</u> $\mathcal{I}''_n = \{(L,x) \mid L=[1,2,\dots,n],\ x \in \{1,2,\dots,n,\underline{\dots,n^2}\}\}$

<u>Graphs</u> — prove a few simple theorems about graphs

$\boxed{1}$ Prove $\forall \underline{G=(V,E)},\ |E| \leq \frac{|V|(|V|-1)}{2}$

<u>NOTES:</u>

"$\forall G=(V,E)$" introduces 3 related variables

- graph G
- vertex set (V) of G
- edge set (E) of G

$V = \{a,b,c,d\}$

$E = \{\{a,b\},\{a,c\},\{b,d\}\}$

<u>Proof:</u> every elem. in E is a subset of V

of size 2 (<u>by def.</u>)

- <u>From worksheet #10</u> — there are exactly $\frac{|V|(|V|-1)}{2}$ many such subsets possible. $\square$