- Propositional logic : $\neg \quad \wedge \quad \vee \quad \Rightarrow \quad \Leftrightarrow$
- Predicate logic : $\forall \quad \exists$

<u>Implication</u> : $p \Rightarrow q$ "p implies q"

"if p, then q"

p	q	$p \Rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

$\boxed{\text{"vacuous truth"}}$

if $\underbrace{\text{it rains}}_{p}$, then $\underbrace{\text{I bring my umbrella}}_{q}$

$$\underset{\text{hypothesis}}{p} \Rightarrow \underset{\text{conclusion}}{q}$$

- $q \Rightarrow p$ is the <u>converse</u> of $p \Rightarrow q$ — <u>not</u> equiv.
- $\neg q \Rightarrow \neg p$ is the <u>contrapositive</u> of $p \Rightarrow q$ — equivalent

- Given predicate $P: D \to \{T, F\}$

$\forall x \in D, P(x)$ — "for all x in D, $P(x)$ $\underbrace{\text{holds}}_{\text{is true}}$"

universal quantifier

$\forall y \in D, P(y)$

$\underbrace{\exists}_{\text{quant.}} \underbrace{x}_{\text{variable name}} \underbrace{\in D}_{\text{domain for } x} , P(x)$ — "for some (at least one) $x \in D$, $P(x)$ holds"

existential quantifier

intuitively $D = \{x_0, x_1, x_2, x_3, \ldots\}$

$\forall x \in D, P(x)$ means $P(x_0) \wedge P(x_1) \wedge P(x_2) \wedge \ldots$

$\exists x \in D, P(x)$ means $P(x_0) \vee P(x_1) \vee P(x_2) \vee \ldots$

EXAMPLE:

$D = \{$ all $\underline{\text{strings}}$ over alphabet $\underbrace{\{a,b,c\}}\}$

finite sequence of characters

$= \{\varepsilon, a, b, c, aa, ab, ac, ba, bb, bc, ca, cb, cc, aaa, \ldots\}$

~~()~~ ~~""~~

epsilon represents empty string

sequence (b, b) "bb"

$P: D \times D \to \{T, F\}$

$\underbrace{P(x,y)}_{\text{name \& arguments}}$: $\underbrace{\text{“}x \text{ and } y \text{ have the same first character”}}_{\text{body}}$, $\underbrace{\text{for } x, y \in D}_{\text{domain for arguments}}$

$\exists x \in D, \exists y \in D, P(x,y)$

$\underbrace{\exists x, y \in D}_{\text{shorthand}}, P(x,y)$

"there are two strings that have the same first character"

True: could pick $x = abc$ $y = abc$

$x = ab$ $y = acabaaa$

...

$\forall x \in D, \forall y \in D, P(x,y)$? $\forall y \in D, \forall x \in D, P(x,y)$

$$\left.\begin{array}{l}\forall x \in D, \exists y \in D, P(x,y) \\ \exists y \in D, \forall x \in D, P(x,y)\end{array}\right]$$ same or not?

More on Thursday...