Basic Stochastic Control Question

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1 Are J and $\tilde{J}$ the same thing?

Suppose we have a contolled SDE

$$dX = U(x, \alpha(x,t);)\,\mathrm{d}t + \sqrt{2D}dW \tag{1}$$

where $\alpha(x,t)$ is the feedback control, and are trying to optimize some criterion:

$$J(x_0, t_0; \alpha) = \mathbb{E}_X \left[\int_0^T r(X_t)\,\mathrm{d}t \right] \tag{2}$$

Is eq. (2) the same as: as $\tilde{J}$ is that the same as optimizing

$$\tilde{J}(x_0, t_0; \alpha) = \int_0^T \int_{\Omega_X} r(x) \cdot f(x,t|x_0,t_0)\,\mathrm{d}x\,\mathrm{d}t \tag{3}$$

where $f(x,t|x_0,t_0)$ is the transition density associated with the SDE, eq. (1)?

I.e. are J and $\tilde{J}$ the same thing?

Recall that f satisfies a Fokker-Planck PDE with delta initial conditions.

$$\begin{aligned} &\partial_t f(x,t) = -\partial_x[U(x;\alpha)\cdot f(x,t)] + D\partial_x^2 f(x,t) \\ &\left\{ \; f(x,t_0) \quad = \quad \delta(x - x_0) \quad \text{delta function at } x_0 \right. \end{aligned} \tag{4}$$