Formula Sheet

Equations and Inequalities:

Binomial powers: $(a+b)^2 = a^2 + 2ab + b^2$, $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Difference of squares: $a^2 - b^2 = (a-b)(a+b)$

Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Difference of *n*-th powers: $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + \ldots + b^{n-1})$

Quadratic equation, completing the square: $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c = 0$

Roots of a quadratic equation: If $b^2 - 4ac \geq 0$ then $x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Factorization: If roots x_1 and x_2 exist then $ax^2 + bx + c = a(x - x_1)(x - x_2)$.

The graph of $y = ax^2 + bx + c$ is called a parabola.

The vertex of a parabola is at the point (x_0, y_0), where: $x_0 = -\frac{b}{2a}$, $y_0 = -\frac{b^2}{4a} + c$.

Powers and Logarithms:

$a^n \cdot a^m = a^{n+m}$, $a^{-n} = \frac{1}{a^n}$, $\frac{a^m}{a^n} = a^{m-n}$, $\sqrt[n]{a} = a^{\left(\frac{1}{n}\right)}$, $(a^n)^m = a^{n \cdot m}$, $(a \cdot b)^n = a^n \cdot b^n$, $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.

Definition of logarithm: If $a^x = b$ then $x = \log_a b$. a is the **base** of the logarithm.

Change of basis: $\log_a b = \frac{\log_c b}{\log_c a}$. Product Rule: $\log_a (b \cdot c) = \log_a b + \log_a c$.

Quotient Rule: $\log_a \left(\frac{b}{c}\right) = \log_a b - \log_a c$. Power Rule: $\log_a (b^n) = n \log_a b$

Trigonometry:

Basic definitions: $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$, $\cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{1}{\tan\alpha}$, $\sec\alpha = \frac{1}{\cos\alpha}$, $\csc\alpha = \frac{1}{\sin\alpha}$

Basic properties: $\sin(-\alpha) = -\sin\alpha$ $\cos(-\alpha) = \cos\alpha$ $\sin\left(\frac{\pi}{2} \pm \alpha\right) = \cos\alpha$

Periodicity: $\sin(\alpha + 2\pi) = \sin\alpha$ $\cos(\alpha + 2\pi) = \cos\alpha$ $\tan(\alpha + \pi) = \tan\alpha$

Pythagorean Theorem: $\sin^2\alpha + \cos^2\alpha = 1$ $1 + \tan^2\alpha = \frac{1}{\cos^2\alpha}$

Sum of angles:

$$\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \cos\alpha \cdot \sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \cdot \tan\beta}$$

Double Angles:

$\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha \qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha \qquad \tan(2\alpha) = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

$\cos 2\alpha = 2\cos^2\alpha - 1 \qquad \cos 2\alpha = 1 - 2\sin^2\alpha$

$\sin^2\alpha = \frac{1}{2}(1-\cos 2\alpha) \qquad \cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha) \qquad \tan\alpha = \dfrac{1-\cos 2\alpha}{\sin 2\alpha} = \dfrac{\sin 2\alpha}{1+\cos 2\alpha}$

Products of sines and cosines:

$\cos\alpha \cdot \sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$, $\sin\alpha \cdot \cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$

$\sin\alpha \cdot \sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$, $\cos\alpha \cdot \cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$

Sums and Differences of sines, cosines:

$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$, $\sin\alpha - \sin\beta = 2\sin\frac{\alpha-\beta}{2}\cos\frac{\alpha+\beta}{2}$

$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$, $\cos\alpha - \cos\beta = -2\sin\frac{\alpha-\beta}{2}\sin\frac{\alpha+\beta}{2}$

Solving Equations:

$\sin x = a$ (for $-1 \le a \le 1$) $\Rightarrow$ $x = \arcsin a + 2\pi \cdot n$ or $x = (\pi - \arcsin a) + 2\pi \cdot n$

$\cos x = a$ (for $-1 \le a \le 1$) $\Rightarrow$ $x = \arccos a + 2\pi \cdot n$ or $x = -\arccos a + 2\pi \cdot n$

$\tan x = a$ (for $-\infty < a < \infty$) $\Rightarrow$ $x = \arctan a + \pi \cdot n$

Triangles

The area of a triangle is $S = \frac{1}{2}h \cdot a$ (half height times base), or $S = \frac{1}{2}a \cdot b \cdot \sin\gamma$.

An inscribed angle in a circle is equal to half to central angle intercepting the same arc.

Law of Sines: $,\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma} = 2R$;

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$;

Analytic Geometry

Distance between points (x_1, y_1) and (x_2, y_2) : $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Equation of a line: $y = mx + n$ or $ax + by + c = 0$.

Equation of a line through the point (x_1, y_1) with slope m: $y - y_1 = m(x - x_1)$.

Slope of a line through the points (x_1, y_1) and (x_2, y_2) : $m = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)$.

Slope of a line perpendicular to a line of slope m (for $(m \neq 0)$) is: $\left(-\frac{1}{m}\right)$.

Equation of a circle with radius R and center (a, b): $(x-a)^2 + (y-b)^2 = R^2$.

Sequences

Arithmetic sequence: $a_n = a_1 + (n-1)d \quad (n = 1, 2, \ldots)$; also $a_{n+1} = a_n + d$.

Sum formula of an arithmetic sequence: $S_n = a_1 + a_2 + \ldots + a_n = (a_1 + a_n) \cdot \frac{n}{2}$

Geometric sequence: $a_n = a_1 \cdot q^{(n-1)} \quad (n = 1, 2, \ldots)$; also $a_{n+1} = a_n \cdot q$.

Sum formula of a geometric sequence: $S_n = a_1 + a_2 + \ldots + a_n = \frac{a_1 \cdot (1 - q^n)}{1 - q}$

Sum formula of an infinite geometric sequence: $S = \frac{a_1}{1-q}$, when $-1 < q < 1$.

Sets

Distributive laws: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Complement: $A^c = \Omega - A$. De Morgan's laws: $(A \cap B)^c = A^c \cup B^c$, $(A \cup B)^c = A^c \cap B^c$

Basic Functions and Graphs

First degree function: $y = f(x) = mx + n$. Graph is a straight line.

Second degree function: $y = f(x) = ax^2 + bx + c$. Graph is a parabola.

If $f(-x) = f(x)$ then f is called even. If $f(-x) = -f(x)$ then f is called odd.

Composition of functions: $(f \circ g)(x) = f(g(x))$.

Inverse function: $y = f^{-1}(x)$. Satisfies: $(f \circ f^{-1})(x) = f(f^{-1}(x)) = x$.

Binomial Theorem: $(a+b)^n = a^n + na^{n-1}b + \ldots + \binom{n}{k} a^{n-k}b^k + \ldots + nab^{n-1} + b^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}b^k$

(Term in position $k+1$ is $\binom{n}{k} a^{n-k}b^k$, where $\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$.)