Formula Sheet—Calculus I

Derivatives of basic elementary functions:

$(a^x)' = a^x \ln a$ $\quad (e^x)' = e^x$ $\quad (\log_a x)' = \frac{1}{x \ln a}$ $\quad (\ln x)' = \frac{1}{x}$ $\quad (x^a)' = a x^{a-1}$

$(\cot x)' = -\frac{1}{\sin^2 x}$ $\quad (\tan x)' = \frac{1}{\cos^2 x}$ $\quad (\cos x)' = -\sin x$ $\quad (\sin x)' = \cos x$

$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$ $\quad (\arctan x)' = \frac{1}{1+x^2}$ $\quad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$ $\quad (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$(\cosh x)' = \sinh x$ $\quad (\sinh x)' = \cosh x$

Indefinite Integrals:

$$\int x^a dx = \frac{x^{a+1}}{a+1} + C \ , \quad (a \neq -1) \qquad \int \frac{dx}{x} = \ln|x| + C$$

$$\int \sin x dx = -\cos x + C \qquad \int \cos x dx = \sin x + C$$

$$\int \frac{dx}{\cos^2 x} = \tan x + C \qquad \int \frac{dx}{\sin^2 x} = -\cot x + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C \qquad \int e^x dx = e^x + C$$

$$\int \frac{dx}{1+x^2} = \arctan x + C \qquad \int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C \qquad \int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln | x + \sqrt{x^2+a^2} | + C \qquad \int \frac{dx}{\sqrt{x^2-a^2}} = \ln | x + \sqrt{x^2-a^2} | + C$$

$$\int \frac{dx}{1-x^2} = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + C \qquad \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$\int \sinh x dx = \cosh x + C \qquad \int \cosh x dx = \sinh x + C$$

(Trigonometric identities on next page)

<u>Trigonometric Identities</u>:

$$\sin(x+y)=\sin x\cos y+\sin y\cos x$$

$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y}$$

$$\sin 2x=2\sin x\cos x$$

$$\begin{aligned}\cos 2x&=\cos^2 x-\sin^2 x\\&=1-2\sin^2 x\\&=2\cos^2 x-1\end{aligned}$$

$$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$$

$$\sin x+\sin y=2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x-\sin y=2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$

$$\cos x+\cos y=2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\cos x-\cos y=-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$