

CS220: Applied Discrete Mathematics

Spring 2025 Assignment 2

Due: **Friday, March. 7 2025** on Gradescope

Topics: Sets, Functions, Boolean Algebra, Relations,. Questions

1. **Set Questions:** Determine whether the following statements are true or false and explain briefly:

- (a) The set $S = \{(a, b), (c, d)\} - \{(a, b), (d, c)\}$ is empty.
- (b) The power set of $S = \{a, b, c, d, e\}$ contains 25 elements.
- (c) The Cartesian product of $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$ has 12 elements.

2. **Functions:** Determine whether the following statements are true or false and explain briefly:

- (a) The function $f(n) = 1/n$ with $f : \mathbb{N}^+ \rightarrow \mathbb{Q}$ (natural positives to rationals) is bijective.
- (b) The set of ordered pairs $S = \{(3, 4), (2, 5), (6, 0), (9, 1), (x, y)\}$ represents a function.
- (c) The function $f(z) = z * \sin z$ with $f : \mathbb{R} \rightarrow \mathbb{R}$ is bijective.

3. **Boolean Functions** Use the minterm method to find a Boolean expression that computes the following function $G(x, y, z)$:

X	Y	Z	$G(X, Y, Z)$
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

4. **Relations:** Determine whether the following relations are reflexive, symmetric, antisymmetric, and/or transitive:

- (a) The empty relation $R = \{\}$ defined on the natural numbers.
- (b) The complete relation $R = \mathbb{N} \times \mathbb{N}$ defined on the natural numbers.
- (c) The relation R on the positive integers where aRb means $a|b$ (a divides).
- (d) The relation R on $\{w, x, y, z\}$ where $R = \{(w, w), (w, x), (x, w), (x, x), (x, z), (y, y), (z, y), (z, z)\}$.
- (e) The relation R on the integers where aRb means $a^2 = b^2$.

5. **Family Relations:** Mary has two kids, Elena and Frank. Elena has a daughter named Jessica, and Frank is the father of Christine and John.

- (a) Write down the relation $R = \{(a, b) | a \text{ is a parent of } b\}$ defined on the set P of the six people, so that it reflects the family structure specified above. Use the set notation, the digraph notation, and the matrix notation.

- (b) Use the matrix notation as your starting point for computing the transitive closure of R . Apply the Boolean power method we discussed in class. Once you have derived the matrix representing the transitive closure of R , also translate it into set notation and digraph notation.
- (c) What does the transitive closure of R specify? What name could you give it, if you considered even larger family trees including many generations?

6. Count the Relations:

- (a) How many different equivalence relations can we define on the set $A = \{c, d, e\}$?
- (b) How many different partial orderings can we define on the set $A = \{x, y\}$?
- (c) How many different total orderings can we define on the set $A = \{p, q\}$?