CS220: Applied Discrete Mathematics

Spring 2025 Assignment 3

Topics: Computations, Recurrences, Induction

Questions

1. Given the following piece of code:

```
procedure mystery(int a1, int a2,..., int an)
{
   i = 1
   while (i < n and ai <= ai+1)
     i = i + 1
   if (i == n)
      print "Yes!"
   else print "No!"
}
```

   (a) What property of the input sequence a_n does this algorithm test?

   (b) What is the computational complexity of this algorithm, i.e., the number of comparisons being computed as a function of the input size n?

   (c) Provide a reasonable upper bound for the growth of the complexity function by using the big-O notation (no proof necessary).

2. Order of growth:

   (a) Order the following functions by growth rates:
   $N, \sqrt{N}, N^{1.5}, N^2, N\log N, N\log\log N, N\log^2 N, N\log(N^2), 2/N, 2^N, 2^{N/2}, 37, N^3, N^2\log N$.
   Indicate which functions grow at the same rate.

   (b) Rank the following three functions: $\log N$, $\log(N^2)$, $\log^2 N$. Explain.

3. Botanists at UMass Boston recently discovered a new local flower species that they named the Boston powerflower. It has beautiful, blue blossoms, and each plant lives for only one summer. During that time, each plant produces 11 seeds. Five of these seeds will turn into plants in the following year, and the remaining six seeds will turn into flowers the year after. As the name powerflower suggests, these seeds always turn into flower plants; there is no failure ever. During the year of this discovery (let us call it year one), the scientists found two powerflowers on the UMass campus, and in the following year, there were already five of them.

   (a) Devise a recurrence relation for the number of flower plants fn in year n, and specify the initial conditions.

   (b) Use the above recurrence relation to predict the number of plants on the UMass campus in years 3, 4, 5, and 6.

   (c) Find an explicit formula for computing the number of plants in any given year without requiring iteration, i.e., repeated application of a formula. You should (but do not have to) check the correctness of your formulas using some of the results you obtained in (b).

4. Induction:

   (a) Use mathematical induction to show that the following equation is true for all positive integers n: $1 + 3 + 5 + \cdots + (2n - 1) = n^2$

   (b) Show that every positive power of 6 ends in 6.

   (c) how that $2^{n+2} + 3^{2n+1}$ is divisible by 7 for all positive integers n (**Hint:** Define your inductive hypothesis expression as $7m$ for some m, and do a bit of algebraic manipulation).