CS 220

* Assignment - 2

1. Set Questions:

(a) The set $S = \{(a,b), (c,d)\} - \{(a,b), (d,c)\}$ is empty.

Sol^n False, because $S = \{(c,d)\}$ as (c,d) and (d,c) are differents, they cannot be subtracted from S. (ordered pair)

(b) The power set of $S = \{a, b, c, d, e\}$ contains 25 elements.

Sol^n False, the power set of S contains 2^n elements. where n is the cardinality of S.
$\therefore$ power set of S contains: $2^5 = 32$ elements

(c) The cartesian product of $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$ has 12 elements.

Sol^n True, $|A| = 3$, $|B| = 4$, $|A \times B| = 3 \times 4 = \boxed{12}$.

2. Functions:

(a) False, The function is one to one but not onto as the range is not same as target (Q). So Function is Only Injective.

$f(n_1) = f(n_2)$ $\qquad \therefore f(n) = \frac{1}{n}$

$\frac{1}{n_1} = \frac{1}{n_2}$

$n_2 = n_1$

$\therefore f^n$ is injective

Here $n \in N^+$, Range does not have $\frac{2}{3}, \frac{5}{3}$, etc

$\therefore f^n$ is not surjective

classmate

Date
Page

(b) $S = \{(3,4), (2,5), (6,0), (9,1), (x,y)\}$

Sol^n True, For every ordered pair the first element is different and it maps with other elements. So Assuming that all the elements of set $A = \{3, 2, 6, 9, x\}$ are maped with set B of any number of elements. This represents that set $S: A \to B$.

(c) False, As f^n is not injective we can say that f^n is not bijective.

Let $z_1 = 0$, $z_2 = \pi$

For f^n to be injective: $z_1 = z_2$

$\therefore f(z_1) = f(z_2)$

$0 \times \sin(0) = \pi \times \sin(\pi)$

$0 = \pi \times 0$

$0 = 0$

As, the LHS = RHS but $z_1 \neq z_2$, f^n is not injective and not bijective.

3. Boolean Functions:

$x = 0, y = 0, z = 0 : G(x, y, z) = 1$

$x = 0, y = 1, z = 1 : G(x, y, z) = 1$

$x = 1, y = 0, z = 1 : G(x, y, z) = 1$

$\therefore G(x, y, z) = \bar{x} \cdot \bar{y} \cdot \bar{z} + \bar{x} \cdot y \cdot z + x \cdot \bar{y} \cdot z$

The function $G(x, y, z)$ is sum of all the minterms with outcome of '1'.

4. Relations

(a) The empty relation $R = \{\}$ defined on the natural numbers.

Sol^n Not Reflexive, As there is no set in relation R such that xRx.

Relation R is symmetric as there is no aRb and bRa.

Relation R is antisymmetric as there is no aRb, bRc, etc

Relation R is transitive as there is no aRb, bRc ~~such that~~ and aRc.

(b) The complete relation $R = N \times N$ defined on the natural numbers.

Sol^n Relation R is reflexive as xRx is in set R where $x \in N$.

Relation R is symmetric as xRy and yRx are in set R where $x, y \in N$.

Relation R is not anti-symmetric as $xRy \in R$ but $yRx \in R$ too. Ex: 2R3 and 3R2 $\in R$

Relation R is transitive as xRy, yRz and xRz belongs to set R.

(c) The relation R on the positive integers where aRb means $a|b$

Sol^n Relation R is reflexive as the number divides itself.

Relation R is not symmetric as the number divides other number but not the other way around.

Ex: $6|12 \in R$ but $12|6 \notin R$.

Relation R is anti-symmetric as $a|b \in R$ but $b|a \notin R$

Relation R is ~~not~~ transitive as $a|b \in R$, $b|c \in R$ then a definitely divides c, $a|c \in R$.

(d) The Relation R on $\{w, x, y, z\}$ where
$R = \{(w,w), (w,x), (x,w), (x,x), (x,z), (y,y), (z,y), (z,z)\}$.

Solⁿ Relation R is reflexive as R has (w,w), (x,x), (y,y), (z,z).

Relation R is not symmetric as $(x,z) \in R$ but $(z,x) \notin R$.

Relation R is not anti-symmetric as (x,w), $(w,x) \in R$.

Relation R is not transitive as (x,z), $(z,y) \in R$ but $(x,y) \notin R$.

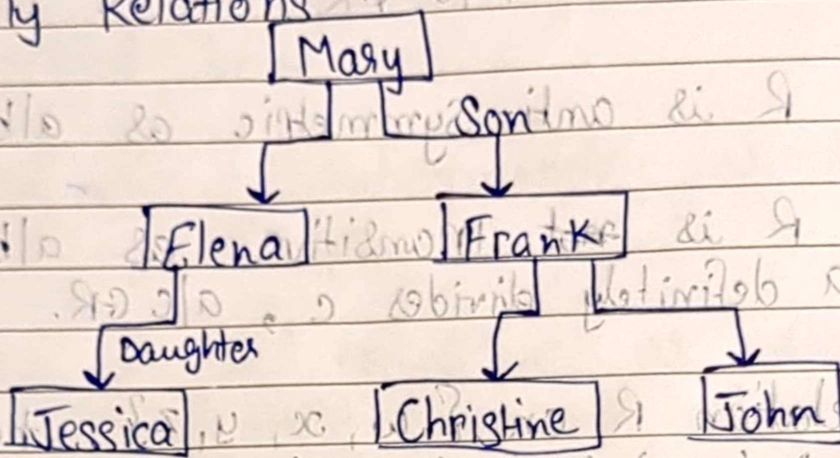
(e) The Relation R on the integers where aRb means $a^2 = b^2$.

Solⁿ R is reflexive as if xRx, $x^2 = x^2$

R is symmetric as if xRy : $x^2 = y^2$ then $y^2 = x^2$ so, yRx.

R is not anti-symmetric as xRy then yRx.
R is transitive as if $a^2 = b^2$, $b^2 = c^2$ then $a^2 = c^2$.

5. Family Relations



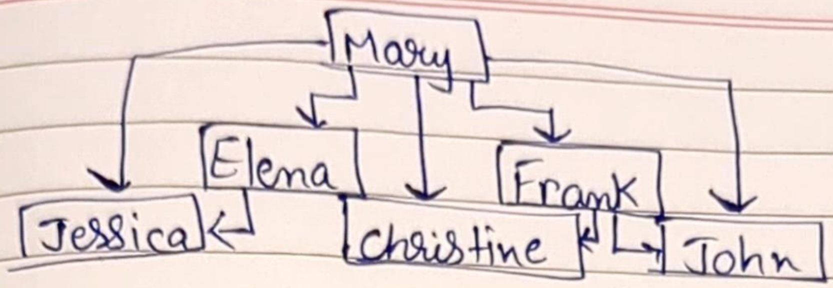
(a) $R = \{(a, b) \mid a \text{ is a parent of } b\}$

R = { (Elena, Jessica), (Frank, christine), (Frank, John), (Mary, Elena), (Mary, Frank)}

	Mary	Elena	Frank	Jessica	Christine	John
Mary	0	1	1	0	0	0
Elena	0	0	0	1	0	0
Frank	0	0	0	0	1	1
Jessica	0	0	0	0	0	0
Christine	0	0	0	0	0	0
~~Fr~~John	0	0	0	0	0	0

(b) The Transitive closure of R using boolean Powers method ~~using~~ including all direct and indirect relationships.

	Mary	Elena	Frank	Jessica	christine	John
Mary	0	1	1	1	1	1
Elena	0	0	0	1	0	0
Frank	0	0	0	0	1	1
Jessica	0	0	0	0	0	0
Christine	0	0	0	0	0	0
John	0	0	0	0	0	0



(c) The transitive closure of R specifies all possible parent relationships in family including grandparents, great-grandparents, etc. In short including Ancestors relation.

6. Count the Relations:

(a) Equivalence Relations on $A = \{c, d, e\}$.

~~set $A_1 = \{c\}$~~

~~$A_2 = \{d\}$~~

~~$A_3 = \{e\}$~~

~~$A_4 = \{(c,d), (e)\}$~~

~~$A_5 = \{(c), (d,e)\}$~~

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There are 5 Equivalence Relations: $\{\{c,d,e\}\}$, $\{\{c,d\},\{e\}\}$, $\{\{c,e\},\{d\}\}$, $\{\{d,e\},\{c\}\}$ and $\{\{c\},\{d\},\{e\}\}$.

(b) Partial Orderings on set $A = \{x, y\}$

There are 4 Partial ordering:

$\{(x,x), (y,y)\}$,

$\{(x,x), (x,y), (y,y)\}$

$\{(x,x), (y,x), (y,y)\}$

$\{(x,x), (x,y), (y,x), (y,y)\}$

(d) Total orderings on set $A = \{p, q\}$.

There are 2 Total orderings: $\{(p,p), (p,q), (q,q)\}$ and $\{(p,p), (q,p), (q,q)\}$.