CS220: Applied Discrete Mathematics

Spring 2025 Assignment 1 Due: Friday ,

Feb. 14 2025 on Gradescope

Topics: logic, predicates, statements, sets.

Questions

1. **Identifying Propositions:** Determine whether each of the following sentences is a proposition or not (mark X near yes or no). If the sentence is a proposition, then write its negation. Otherwise leave blank.

   (a) Have a nice day. Yes ______ No __X__ Negation ______

   (b) The soup is cold. Yes __X__ No ______ Negation The soup is not cold.

   (c) Do you like my new shoes? Yes ______ No __X__ Negation ______

   (d) It's a beautiful day. Yes __X__ No ______ Negation It's not a beautiful day.

   (e) The light is on. Yes __X__ No ______ Negation The light is not on.

2. **Logical Notation:** Express each English statement using logical operations $\wedge$, $\vee$, $\neg$ and the propositional variables t, n, and m defined below. The use of the word "or" means inclusive or. Write your answer on the line.

   t: The patient took the medication.
   n: The patient had nausea.
   m: The patient had migraines.

   (a) The patient had nausea or migraines. n V m

   (b) The patient had nausea and migraines. n /\ m

   (c) There is no way that the patient took the medication. ~ t

   (d) The patient did not have migraines. ~ m

3. **Applying Logical Operations:** Assume the propositions p, q, r, and s have the following truth values: p is false, q is true, r is false, s is true
   What is the truth value of the following compound propositions? (Mark X where applies):

   (a) $p \vee r$ True ______ False __X__

   (b) $p \wedge r$ True ______ False __X__

(c) $\neg q$ True ________ False __X____

(d) $q \oplus s$ True ________ False __X____

4. **Truth Values:** Indicate whether each statement is true or false, assuming that the "or" in the sentence means the inclusive or. Then indicate whether the statement is true or false if the "or" means the exclusive or. Mark T for True or F for false.

   (a) February has 31 days or the number 5 is an integer.

   Inclusive or __T____ Exclusive or __T____

   (b) The number π is an integer or the sun revolves around the earth.

   Inclusive or __F____ Exclusive or __F____

   (c) 20 nickels are worth one dollar or whales are mammals.

   Inclusive or __T____ Exclusive or __F____

   (d) There are eight days in a week or there are seven days in a week.

   Inclusive or __T____ Exclusive or __T____

5. **Compound Propositions:** The propositional variables, p, q, and s have the following truth assignments: p = T, q = T, s = F. Give the truth value for each proposition (mark X where applies):

   (a) $p \vee \neg s$ True __X____ False ________

   (b) $(p \wedge s) \vee q$ True __X____ False ________

   (c) $p \vee \neg(q \wedge s)$ True __X____ False ________

   (d) $(q \wedge \neg p \wedge s)$ True ________ False __X____

6. **Propositional Functions:**

   Let Takes(x, y) be the propositional function "x takes course y," Teaches(x, y) be the propositional function "x teaches course y," and Passes(x, y) be the propositional function "x passes course y". The universe of discourse is the set of all living people and all courses (i.e., you do not have to check this in your expressions). Write each of the following propositions symbolically in one expression:

   (a) Peter takes CS 220 and CS 410, but not CS 680.

   (b) Bob passes every course that he takes except CS 220.

   (c) Francesca passes every course that is taught by Prof. Einstein.

   (d) There is a course that both Aaliya and Peter took, but both of them failed it.

7. **Tautologies and Contradictions:** Find out for each of the following propositions whether it is a tautology, a contradiction, or neither (a contingency). Prove your answer.

   (a) $[(p \to q) \wedge (q \to p)] \to (p \leftrightarrow q)$

   (b) $(p \vee q \vee r) \to [(q \to r) \vee (p \to q)]$

8. **Rules of Inference:** Use the rules of inference to show that the arguments below are valid, i.e., that their conclusion follows from their hypotheses. First extract and name all relevant propositions, and then write down all hypotheses and the conclusion in propositional logic notation. Finally, apply the step-by-step method we used in class and list all those steps in your answer.

   (a) Hypotheses: If there is gas in the car, then I will go to the store. If I go to the store, then I will get a soda. There is gas in the car. Conclusion: I will get a soda.

(b) Hypotheses: When Prof. P. gets angry, he fails his entire class. When the entire class fails, the Chancellor gets complaints. When the Chancellor gets complaints, she will either fire Prof. P., cut his salary, or do both. Prof. P. got angry, and he was not fired. Conclusion: Prof. P.'s salary was cut.

9. **Set operations:** Let us take a look at the sets $A = \{x, y, z\}$, $B = \{1, 2\}$, $C = \{x, z\}$. List the elements of the following sets D, E, F, G, H, and I:

   (a) $D = (A \times B) - (B \times C)$

   (b) $E = 2^C - 2^A$

   (c) $F = 2^{(2^B)}$

   (d) $G = (A \times B \times C) \cap (C \times B \times A)$

   (e) $H = \{(a, b, c) | a, b, c \in B \wedge a \neq b \wedge a \neq c\}$

   (f) $I = \{(a, b, c) | a \in B \wedge b \in A \wedge c \in B \wedge a \neq c\}$

10. **Cardinality:** Are the following statements true for all sets A, B and C? Prove your answers.

   (a) $|A \cup B \cup C| = |A - B - C| + |B - A - C| + |C - A - B|$

   (b) $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

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Q6. Propositional Functions:

(a) $Takes(Peter, CS220) \wedge Takes(Peter, CS410) \wedge \sim Takes(Peter, CS680)$

(b) $\forall x\, [\cancel{Takes}\ Passes(Bob, x) \longrightarrow Takes(Bob, x) \wedge \sim Passes(Bob, CS220)]$

~~(b) ∀x[~~

(b) $\exists x\, [Takes(Bob, x) \longrightarrow \sim Passes(Bob, CS220)]$

(b) $\forall x\, [Takes(Bob, x) \wedge x \neq CS220 \longrightarrow Passes(Bob, x)] \wedge \sim Passes(Bob, CS220)]$

(c) $\forall x\, [Teaches(Prof.\ Einstein, x) \longrightarrow Passes(Francesca, x)]$

(d) $\exists x\, [Takes\ \cancel{Bob}\ (Aaliya, x) \wedge Takes(Peter, x) \longrightarrow \sim Passes(Peter, x) \wedge \sim Passes(Aaliya, x)]$

Q7. Tautologies and Contradictions

(a) $[(p \to q) \wedge (q \to p)] \to (p \leftrightarrow q)$

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$	$(p \leftrightarrow q)$
1	0	0	1	0	0
1	1	1	1	1	1
0	0	1	1	1	1
0	1	1	0	0	0

(Ans) $[(p \to q) \wedge (q \to p)] \to (p \leftrightarrow q)$

$[(p \to q) \wedge (q \to p)] \to (p \leftrightarrow q)$
1
1
1
1

Here, '0' means False and '1' means True.
Using Truth Table with all the possible outcomes of 'p' and 'q' we know that all the outcomes of $[(p \to q) \wedge (q \to p)] \to (p \leftrightarrow q)$ is True.
And Tautology is a statement that is always True.
Hence, this statement is always True (Tautology).

(b) $(p \vee q \vee r) \to [(q \to r) \vee (p \to q)]$
we can rewrite this as:
$((p \vee q) \vee r) \to [(q \to r) \vee (p \to q)]$ — (i)

p	q	r	$(p \vee q) \vee r$	$q \to r$	$p \to q$	$(q \to r) \vee (p \to q)$	(i)
1	1	1	1	1	1	1	1
1	1	0	1	0	1	1	1
1	0	1	1	1	0	1	1
1	0	0	1	1	0	1	1
0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1
0	0	1	1	1	1	1	1
0	0	0	0	1	1	1	1

Here, '0' means False and '1' means True.
Using Truth Table with all the possible outcomes of p, q and r we got all the outcomes of statement (i) : True.
Hence, this statement is Tautology.

Q8. Rules of Inference:

(a) Hypotheses: If there is a gas in the car, then I will go to the store. If I go to the store then I will get a soda. There is gas in the car. Conclusion: I will get a soda.

Propositions:

A = There is a gas in a car.

B = I will go to the store.

C = I will get soda.

Hypothesis:

(1) If there is a gas in the car, then I will go to the store.

$A \longrightarrow B$

(2) If I go to the store, then I will get a soda.

$B \longrightarrow C$

(3) There is a gas in the car.

A

Conclusion:

I will get a soda

Step-By-Step: $[(A \rightarrow B) \wedge (B \rightarrow C)] \rightarrow (A \rightarrow C)$.

(b) Hypothesis: When Prof. P. gets angry, he fails his entire class. When the entire class fails; the chancellor gets complaints. When the chancellor gets complaints, she will either fire Prof. P., cut his salary, or do both. Prof. P. got angry, and he was not fired. Conclusion: Prof. P.'s salary ~~g~~ was cut.

Propositions:
P = Prof. P gets angry
F = He fails the class
C = chancellor gets complaints
S = cut the salary
N = Prof. P. ~~g~~ was not fired.

~~Conclusion~~: Hypothesis:

① Prof. P gets angry, he fails his entire class.
$P \rightarrow F$

② when the entire class fails, the chancellor gets complaints.
$F \rightarrow C$

③ when the chancellor gets complaint, she will either fire Prof. P, cut his salary or do both.
$[C \longrightarrow (\sim N \vee S) \vee (\sim N \wedge S)]$

④ Prof. P. got angry and was not fired.
~~P∧N~~ $(P \wedge N)$

Conclusion:
Prof. P's salary was cut.

Step by step: $[(P \rightarrow F) \wedge (F \rightarrow C)] \rightarrow (P \rightarrow C)$.
$C \longrightarrow (\sim N \vee S) \vee (\sim N \wedge S)$
$C \longrightarrow (\sim N \vee S)$

$\therefore (\sim N \vee S) \vee N = S \vee (\sim N \wedge S)$

For the statement to be true we need the statement 'S' to be true.

Q9. Set Operations:

$A = \{x, y, z\}$

$B = \{1, 2\}$

$C = \{x, z\}$

(a) $D = (A \times B) - (B \times C)$

$\therefore A \times B = \{(x,1), (x,2), (y,1), (y,2), (z,1), (z,2)\}$

$B \times C = \{(1,x), (2,x), (1,z), (2,z)\}$

$\therefore D = (A \times B) - (B \times C)$

$= \{(x,1), (x,2), (y,1), (y,2), (z,1), (z,2)\} - \{(1,x), (2,x), (1,z), (2,z)\}$

$D = \{(x,1), (x,2), (y,1), (y,2), (z,1), (z,2)\}$

(b) $E = 2^C - 2^A$

$\therefore 2^C = \{\phi, \{x\}, \{z\}, \{x, z\}\}$

$2^A = \{\phi, \{x\}, \{y\}, \{z\}, \{x,y\}, \{y,z\}, \{x,z\}, \{x,y,z\}\}$

$E = 2^C - 2^A$

$E = \phi$

(c) $F = 2^{(2^B)}$

$2^B = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

$2^{(2^B)} = \{\phi, \{\phi\}, \{\{1\}\}, \{\{2\}\}, \{\{1,2\}\}, \{\phi, \{1\}\}, \{\phi, \{2\}\}, \{\phi, \{1,2\}\}, \{\{1\}, \{2\}\}, \{\{1\}, \{1,2\}\}, \{\{2\}, \{1,2\}\},$

$\{\phi, \{1\}, \{2\}\}, \{\phi, \{1\}, \{1,2\}\}, \{\{1\}, \{2\}, \{1,2\}\},$
$\{\phi, \{2\}, \{1,2\}\}, \{\phi, \{1\}, \{2\}, \{1,2\}\}\}$

(d) $G = (A \times B \times C) \cap (C \times B \times A)$
when in doubt use parentheses:

$G = ((A \times B) \times C) \cap ((C \times B) \times A)$

$(A \times B) \times C = \{(x,1,x), (x,1,z), (x,2,x), (x,2,z),$
$(y,1,x), (y,1,z), (y,2,x), (y,2,z),$
$(z,1,x), (z,1,z), (z,2,x), (z,2,z)\}$

$(C \times B) \times A = \{(x,1,x), (x,2,x), (z,1,x), (z,2,x),$
$(x,1,y), (x,2,y), (z,1,y), (z,2,y),$
$(x,1,z), (x,2,z), (z,1,z), (z,2,z)\}$

$G = \{(x,1,x), (x,2,x), (z,1,x), (z,2,x),$
$(x,1,z), (x,2,z), (z,1,z), (z,2,z)\}$

(e) $H = \{(a,b,c) \mid \{a,b,c\} \in B \wedge a \neq b \wedge a \neq c\}$

$H = \{(1,2,2), (2,1,1)\}$

(f) $I = \{(a,b,c) \mid a \in B \wedge b \in A \wedge c \in B \wedge a \neq c\}$

$I = \{(1,x,2), (1,y,2), (1,z,2),$
$(2,x,1), (2,y,1), (2,z,1)\}$

Q10 Cardinality:
Let us assume : $A = \{1,2\}, B = \{1,3\}, C = \{4,3\}$

(a) $|A \cup B \cup C| = |A - B - C| + |B - A - C| + |C - A - B|$

$A \cup B \cup C = \{1, 2, 3, 4\}$
$A - B - C = \{2\}$
$B - A - C = \cancel{\{3\}}\ \phi$
$C - A - B = \{4\}$

Cardinality means the number of distinct elements in a set.

$\therefore |A \cup B \cup C| = 4, \quad |A - B - C| = 1, \quad |B - A - C| = 0, \quad |C - A - B| = 1$

$\therefore |A \cup B \cup C| = |A - B - C| + |B - A - C| + |C - A - B|$

$4 = 1 + 0 + 1$

$4 = 2$

$4 \neq 2$

Hence, the statement is false.

(b) Let us assume: $A = \{1, 2, 3\}$, $B = \{2, 4, 5\}$, $C = \{2, 5, 6\}$.

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

$\therefore A \cup B \cup C = \{1, 2, 3, 4, 5, 6\}$
$A \cap B = \{2\}$
$A \cap C = \{2\}$
$B \cap C = \{2, 5\}$
$A \cap B \cap C = \{2\}$

$\therefore 6 = 3 + 3 + 3 - 1 - 1 - 2 + 1$

$6 = 6$

Hence, the statement is true