A Report On

Implied Volatility Estimation and Analysis for NIFTY 50 Options

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Abstract

The **Black Scholes formula** is one of the many fundamental methods used for the determination of option prices in the market.

In the **Black-Scholes** options pricing formulas one parameter that cannot be directly observed is the volatility of the stock price. If data of the actual market price of options are known, then the volatility can be viewed as unknown and can be calculated via the implicit inverse of the Black Scholes equation.

The volatility **'σ'** plays the role of the unknown parameter. The volatility determined in by using this process is called **implied volatility** and is the root of the inverse of the Black Scholes equation. Iterative methods such as Newton's method can then be used to find the root of such an equation.

This paper aims at implementing Newton's method to calculate the implied volatility for **NIFTY-50** options for a period of one year i.e 1st April 18 - 31st March 19. Further, the achieved statistics are combined and the relationship between implied volatility and underlying price is observed along with other results.

Literature Review

Various Volatilities and estimates :

- **India VIX :**

  India VIX is India's first volatility Index which is a key measure of market expectations of near-term volatility.India VIX is computed by NSE based on the order book of NIFTY Options. The best bid-ask quotes of near and next-month NIFTY options contracts which are traded on the F&O segment of NSE are used for computation of India VIX. It measures the degree of volatility or fluctuation that active traders expect in the Nifty 50 over the next 30 days. It was the Chicago Board Options Exchange which originally came up with the term VIX in 1993 and the NSE, with the CBOE's permission, kicked off the India VIX a few years ago.

  Need :

  Volatility is defined as the variation in price of any financial instrument. Thus, when the markets are highly volatile, they tend to move steeply up or down and during this time volatility index increases. Volatility indices are also referred to as the Fear Gauge because when the volatility index rises, one should become careful as the markets can fluctuate into any direction. Investors use volatility indices to capture the market volatility and make their future investment decisions.

- **IMPLIED VOLATILITY**

  Implied volatility represents the current market price for **volatility**, or the fair value of **volatility** based on the market's expectation for movement over a defined period of time. **Realized volatility**, on the other hand, is the actual movement that occurs in a given underlying over a defined past period.

- **Greek IV Index Construction and Properties**:

  George Skiadopoulos (2007) [9] in this research paper constructs a model to estimate the implied volatility for synthetic at the money (ATM) options in the greek market. Greek Volatility Index (GVIX) represents the implied volatility for ATM options with

22 trading days (30 calendar days) left. This gives us insights about the aggregate role of implied volatilities in a growing economy, which are important for portfolio management.

- **Realized Volatility and VIX :**

Ritesh Kumar and Ashok Banerjee (2012) [6] in their paper compared the performance of conditional volatility model (GARCH) and VIX in predicting underlying volatility of NIFTY 50 index.The main finding of the study is that the forecast error is minimum in case of VIX. This indicates that a model-free estimator of volatility captures underlying volatility better than an econometric model of volatility (GARCH).

Abhijeet Chandra , M. Thenmozhi (2015) [5] in their paper used VIX to derive relationships between the underlying and since VIX has been proven to be one of the best estimators for volatility the inferences made through this paper are of utmost importance as they relate to our study and broadly gives us various insights about relationships of volatility with other variables.

Estimation

In financial markets, volatility in returns are estimated using several approaches, including model-based estimation techniques such as conditional volatility models of ARCH/ GARCH family, and model-free measures of implied volatility such as CBOE VIX or India VIX.

- **Closed Form approximations of Implied Volatility:**

- **Brenner and Subrahmanyam:**

Focusing for simplicity on European options, Brenner & Subrahmanyam (1988)[1] provide a formula that approximates fairly well the volatility when the stock price is very close to the strike price, but it very quickly loses accuracy as it departs from it.

The Brenner & Subrahmanyam approximation in his first formulation is:

$$\sigma_{BS} = \sqrt{\frac{2\pi}{T}}\,\frac{C}{S}$$

When $S \neq X$ the formula is modified to the form as given below :

$$\sigma_{BS} = \sqrt{\frac{2\pi}{T}} \frac{C - \delta}{S}$$

where $\delta = (S - X)/2$.

- **Bharadia, Christofides and Salkin formula :**

  Bharadia, Christofides, and Salkin (1995) [2] suggest an improvement given by the following simplified formula:

$$\sigma_{BCS} = \sqrt{\frac{2\pi}{T}} \frac{C - \delta}{S - \delta}$$

  Also in this case, as well as for the Brenner and Subrahmanyam formula(2.4), the accuracy of the approximation worsens as soon as the option departs from the at-the-money position.

- **Chance model :**

  Chance (1996)[3] starts with the implied volatility $\sigma * = \sigma BS$ of an at-the-money-call C option obtained with the Brenner–Subrahmanyam approximation formula. He then applied a second taylor series to the expansion of Implied volatility and finds out that the formula has quadratic form and derived a closed form of his own using it.

- **Numerical Methods**

  Most of these methods revolve around mechanisms to invert one of the available theoretical option pricing models for instance black scholes model to find an approximate for the implied volatility.

  More commonly known as root-finding methods.

1. **Newton Raphson** :

The Newton-Raphson method is a powerful technique for solving equations numerically.The Newton Method, properly used, usually homes in on a root with devastating efficiency. The methodology section details on the algorithm.

- **The Secant Method**

  The Secant Method is the most popular of the many variants of the Newton Method. We start with two estimates of the root, x0 and x1. The iterative formula, for n ≥ 1 is :

$$x_{n+1} = x_n - \frac{f(x_n)}{Q(x_{n-1}, x_n)}, \quad \text{where} \quad Q(x_{n-1}, x_n) = \frac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n}.$$

2. **Genetic Algorithms** (Future Possibilities) :

Among other root finding/minimizing techniques genetic algorithms are another way to theoretically minimize the black scholes formula to derive an estimate of implied volatility.

Bruce K. Grace (2010) [7] used a genetic algorithm approach for the estimation purpose. Brief about genetic algorithm :

A typical genetic algorithm consists of three steps: (1) reproduction; (2) crossover; and (3) mutation. For a GA, reproduction corresponds to natural selection. Random selection is used in the search procedure, so no derivatives are needed. Each parameter set (population) consists of a group of, usually, binary strings. Each digit in the binary string is called a bit. Each string is decoded and given a fitness value based on the resulting performance of the function. The fitter strings are selected in proportion to their performance and, so, have a greater probability of siring o spring in the succeeding population. These selected strings are then paired with each other at random in the crossover procedure.

The paper demonstrated the usefulness of genetic algorithm over other methods and hence has bright future possibilities of replacing the trivial minimization techniques used by many traders.Researchers can use GAs alone or in combination with other, more traditional, methods. In other words, the GA could be used to find a starting value for calculus-type optimization methods.

3. Artificial Neural Networks

Tamal Datta Chaudhuri, Indranil Ghosh (2015) [8] used Artificial Neural Network models based on various back propagation algorithms to predict volatility in the Indian stock market through volatility of NIFTY returns and volatility of gold returns. The paper involves 3 experiments. All the three involve changing the training data and testing data to predict for different timelines. The purpose of this paper was to examine the efficacy of the ANN framework in predicting volatility in the Indian stock market.

DATA

- **Source :**
  - The data was taken from the official website of National Stock Exchange Of India (NSE).
  - The data was taken for NIFTY-50 OPTIONS from 1st April 18 to 31st March 19 for both call and put options for the above period.
  - Similarly the underlying data was also taken from the official website for the same period.
  - We used the 1 year treasury bill yield to maturity as a proxy for the risk free interest rate.

- Data Cleansing and Validation
  - Since Black Scholes formula works best for the ATM options the data for each day was filtered so as to include only the ATM option for the given day. And the calculation was done for the nearest ATM option for the corresponding day i.e for each of 247 trading days in the given period implied volatility was estimated for the option which had the minimum difference between its strike price and the underlying spot price.
  - As a part of data validation :
    - Since the data for options and risk free rate were taken from two different (though both authentic) sources the **put call parity** was verified.

- The put call parity was satisfied with the procured data and only after that the data was used to continue the work further.

The following graphs show the Put Call Parity :

- The put call parity is stated as below :

$$c + Ke^{-rT} = p + S_0$$

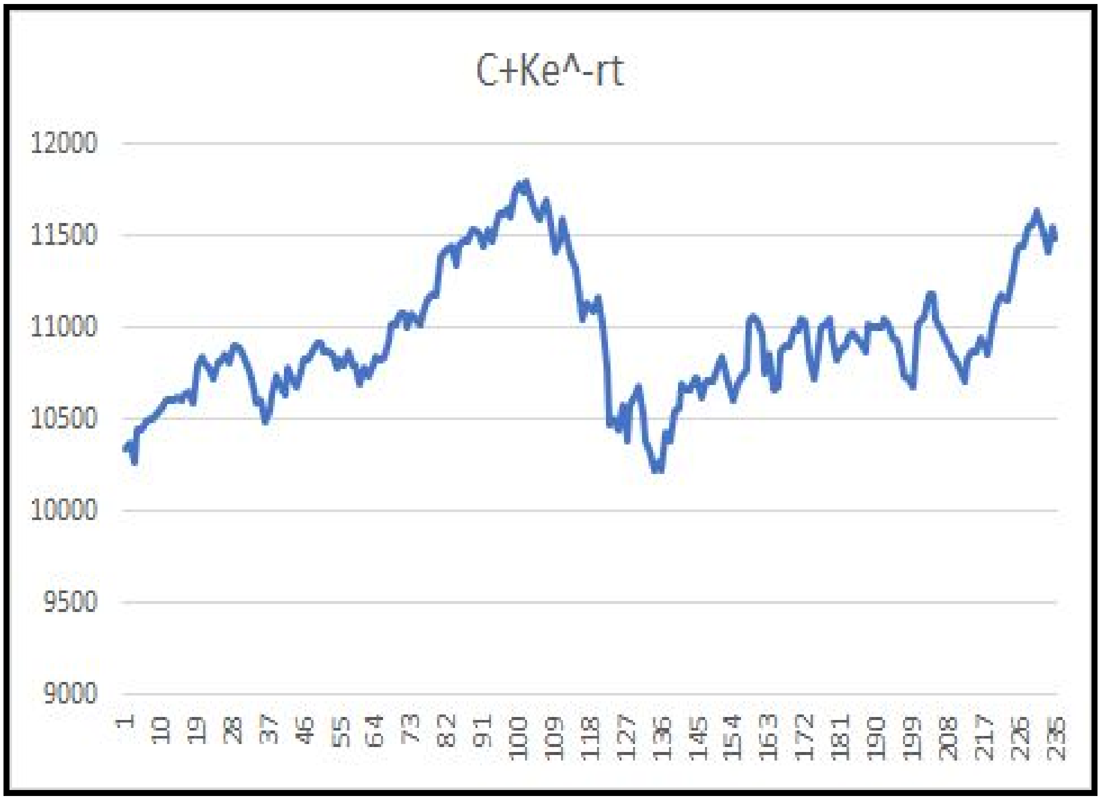

Figure 1

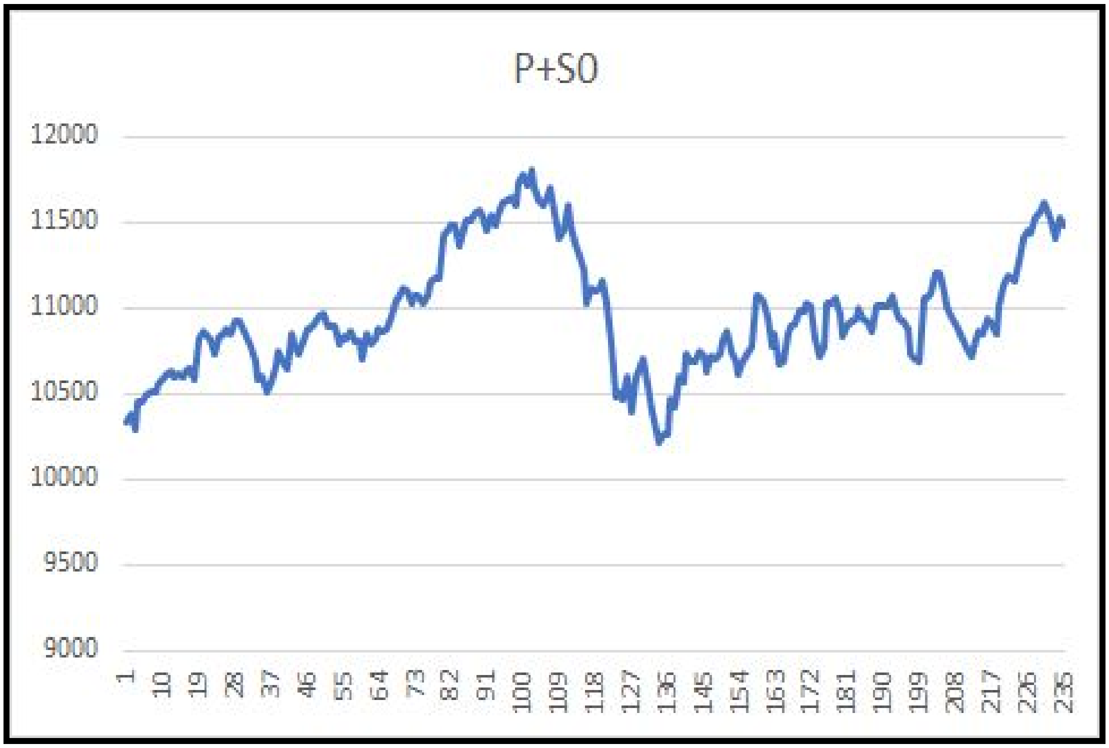

Figure 2

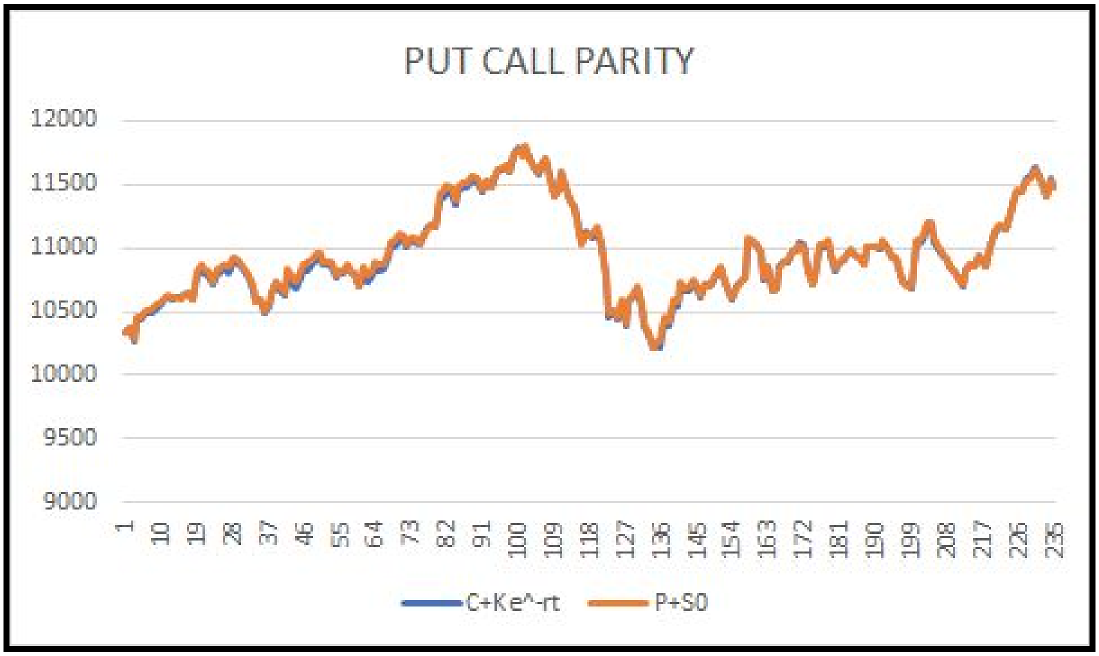

Figure 3

- **ATM Property:**

The following graph shows the relationship between underlying and strike price for the selected period:

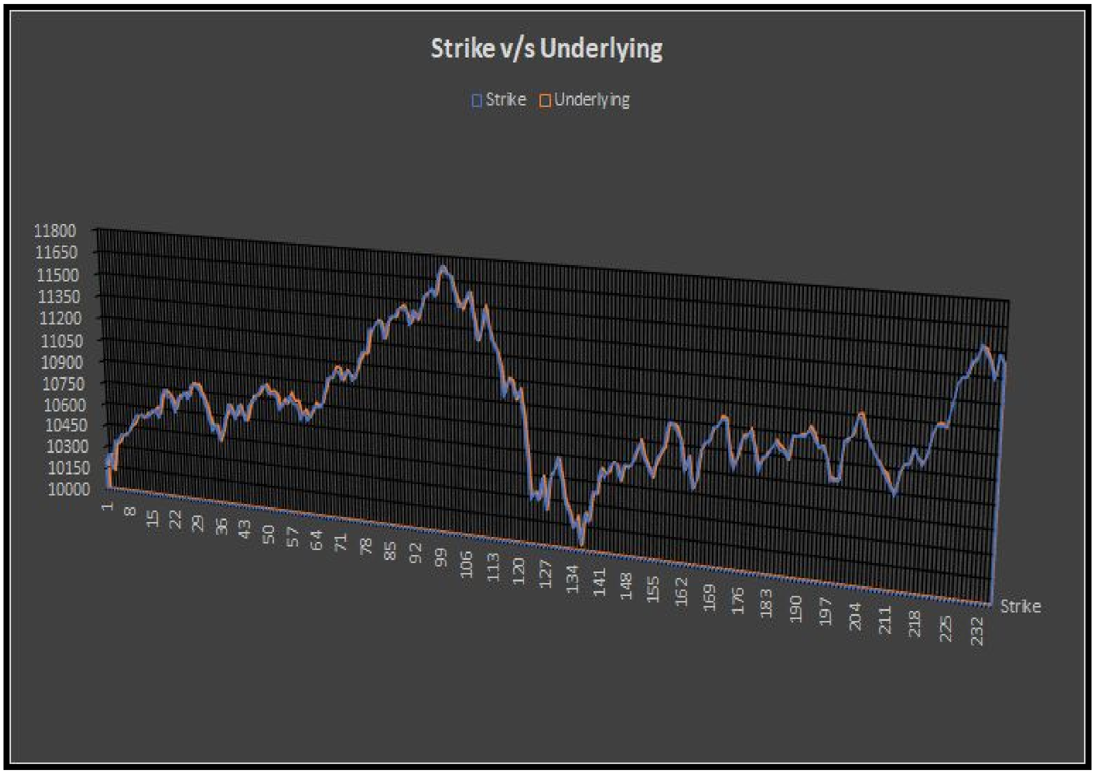

Figure 4

The above graph visualizes the ATM options that we have selected from the raw data for the 247 trading days in the given period.

Methodology

Newton Raphson Method

This method was created by Isaac Newton and Joseph Raphson. It is used in numerical analysis to find out root of real valued function. It was derived using Taylor series the details of which can be found in the following link : [Newton Raphson Method](). The important result of the method is the following :

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

After having an initial estimate of the parameter the above equation is recursively called to get an estimate for each iteration until we get a value for which f(x) is approximately equal to 0. The user can decide upto which decimal place the user needs the efficiency based on the error factor in the algorithm.

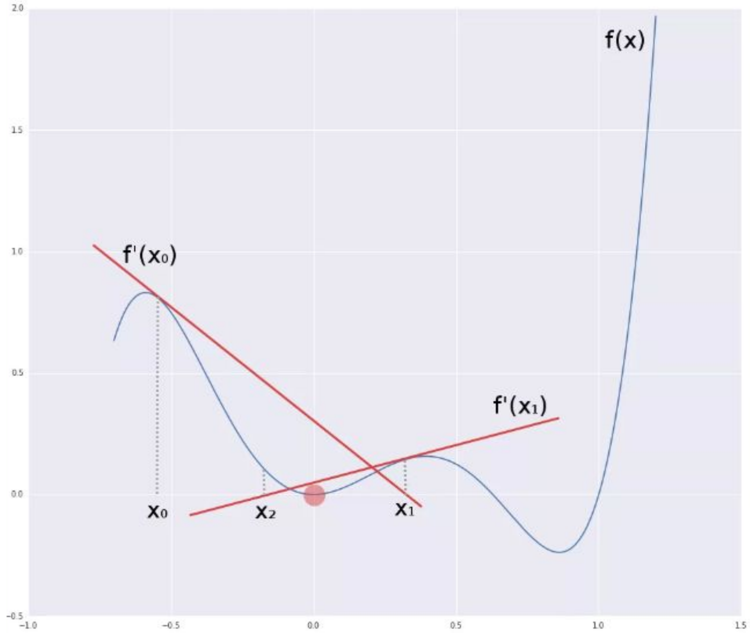

Figure 5: Newton Raphson Method

In case of option pricing the black scholes formula behaves as f(x) and implied volatility as x.

f'(x) on the other hand is the change in option price as implied volatility changes that is the vega of the given option.

The Black Scholes Formula

The Black & Scholes formula for pricing a European call is described by the following equation:

$$C = S\,N(d_1) - X\,N(d_2) \tag{2.1}$$

where

- C is the price of a call option;
- S is the value of the underlying;
- K is the strike price;
- $X = K\,e^{-rT}$ is the present value of the strike price;
- r is the interest rate;
- T is the time to maturity in terms of a year;
- $N(x)$ is the cumulative distribution function of the standard normal i.e.

$$N(x) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2}\,dt$$

- $d_1 := \frac{\log(S/K)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T}$ is the first parameter of probability i.e. "factor by which the present value of contingent receipt of the stock, contingent on exercise, exceeds the current value of the stock" [17];
- $d_2 := \frac{\log(S/K)}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T}$ is the second parameter of probability which represents the risk-adjusted probability of exercise;
- σ is the volatility.

Figure 6 : Black Scholes Formula

Dividend has been ignored in the formula.

The Greek 'Vega'

Vega is the first derivative of σ volatility and thus is an integral piece in the formulation of implied volatility. What follows is a quick derivation of Vega.

As Vega is the first derivative of volatility, its partial derivative takes the form ∂C/∂σ. Therefore, we take the partial derivative of the Black-Scholes formula with respect to σ.

As indicated by Shuaiqiang Liu 1, Cornelis W. Oosterlee , Sander M. Bohte (2019) [9] , However, the

Formula

$$\nu = S\phi(d1)\sqrt{t}$$

$$where: \phi(d1) = \frac{e^{-\frac{d1^2}{2}}}{\sqrt{2\pi}};$$

$$d1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

Legend

K	Option strike price
N	Standard normal cumulative distribution function
r	Risk free interest rate
σ	Volatility of the underlying
S	Price of the underlying
t	Time to option's expiry

Newton-Raphson method may fail to converge, either when the Vega is extremely **small** or when the **convergence stalls**. The near-flat function shapes appear in certain σ-regions, especially when the option is either deep in-the-money (ITM) or deep out-the-money (OTM). The reason for this can be attributed as follows :When the stock is far away from the strike, then the extrinsic value is low, and an increase in volatility would not affect the payoffs by much. Hence the extrinsic value will not increase significantly, so the vega is low. Alternatively, when the stock is near the strike, an increase in volatility has a direct effect on the payoffs. Hence the extrinsic value will increase significantly, so the vega is higher. This can be seen from the below figure.

This is one of the reasons for using ATM options in the analysis.

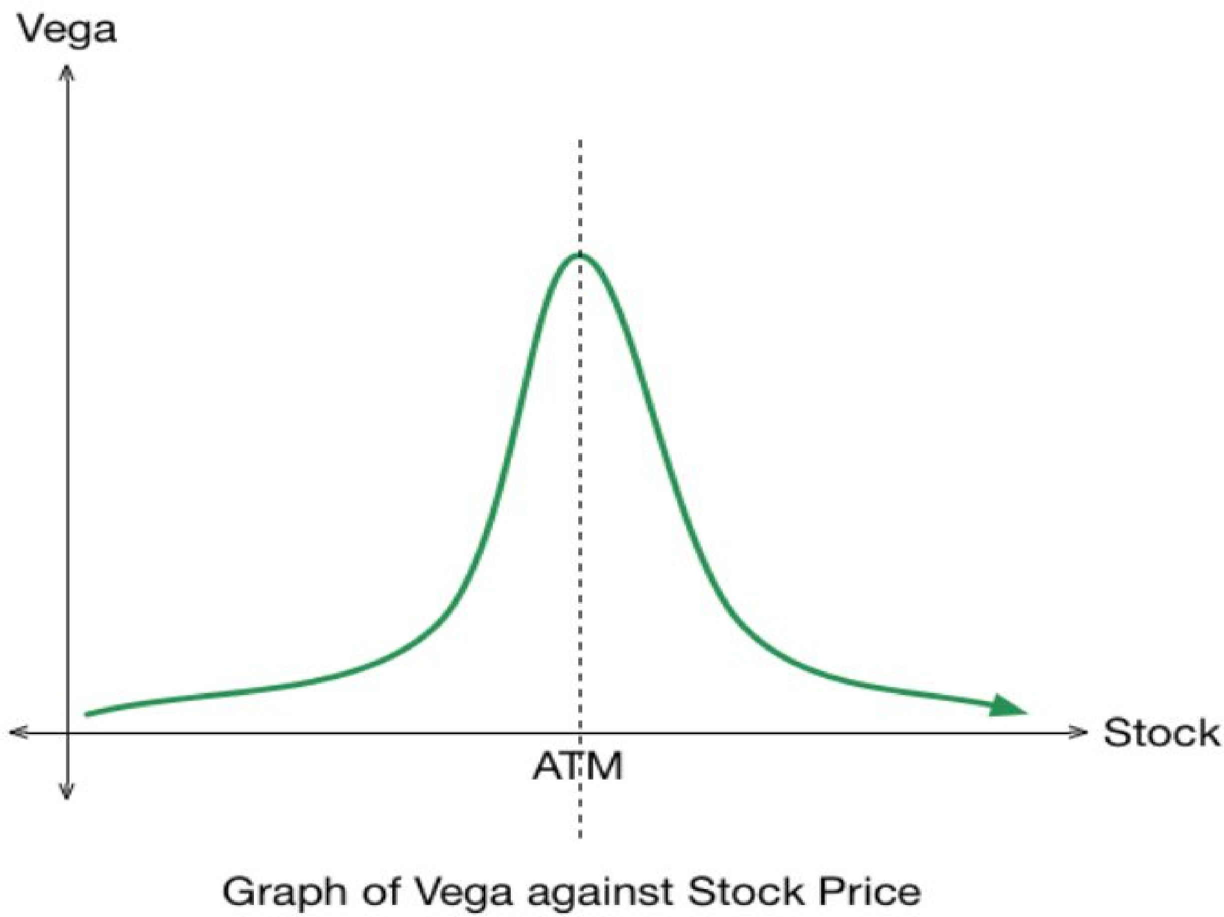

Figure 7

IMPLEMENTATION

As discussed above, Newton raphson's method was implemented in python for the data as described in the data section above.

Working of the algorithm :

Error was taken as : 10^(-6).

Initial Estimate : Brenner and Subrahmanyam (1988) provided a closed form estimate of IV, which was used as an initial estimate for the newton's method.

a. After having an initial estimate of the implied volatility the option price was calculated based on it and the error i.e. (Actual Market Price and Estimated Price) was checked against the pre defined error of 10^(-6).

If this difference was less than this error the current estimate of the implied volatility was returned by the program

Else

The next estimate was calculated using the newton's method and the process as described in **a.** Section above was repeated until the condition of error was met.

This procedure was done for all the 247 days as included in our datasets. The inference based on the calculated implied volatility can be found in the results section below.

The details of the python functions can be found in the Appendix section at the end of the report.

Results

This section shows the various graphs and relationships that were derived based on the calculated results.

1. **Underlying v/s Implied Volatility (CALL & Put)**

   The following graphs depict the relationships between the underlying and implied volatilities of call and put options.

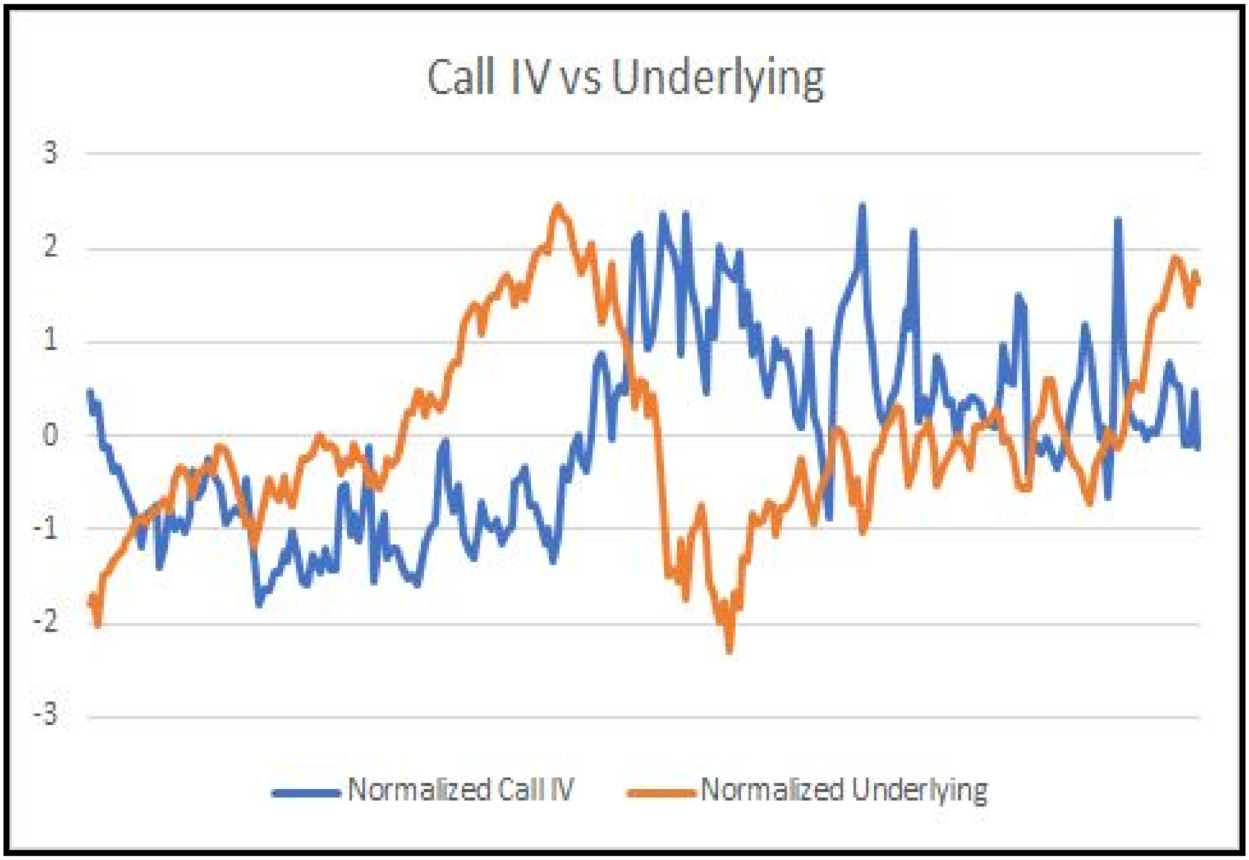

Figure 8

The above graph has been made with normalized values of underlying and call IV so as to fit them on a similar scale . This clearly shows the inverse relationship between the call IV and the underlying. An increasing underlying value leads to a decreasing implied volatility for call options.

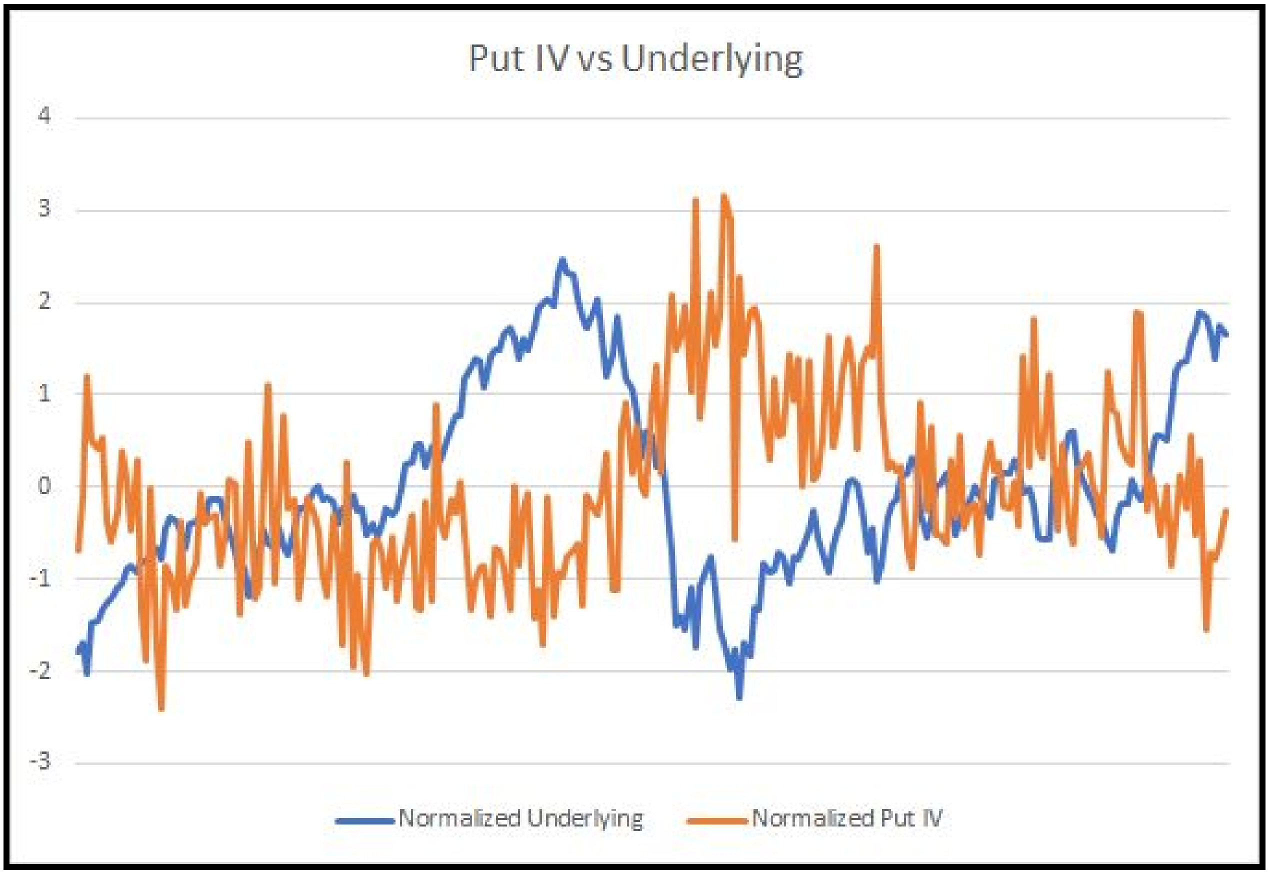

Figure 9:

The above graph has been made with normalized values of underlying and put IV so as to fit them on a similar scale . This clearly shows the asymmetric relationship between the put IV and the underlying. An increasing underlying value leads to a lower call implied volatility.

Symbol	Date	Expiry	Underlying	Strike	Close_Call	IV_CALL	Close_Put	IV_PUT
NIFTY	2018-04-02 00:00:00	2018-04-26 00:00:00	10211.8	10200	185.1	13.96500029	123.35	8.473639533
NIFTY	2018-04-03 00:00:00	2018-04-26 00:00:00	10245	10250	163.1	13.23671189	130.7	10.27485055
NIFTY	2018-04-04 00:00:00	2018-04-26 00:00:00	10128.4	10150	150.25	13.59059421	157.1	14.2566769
NIFTY	2018-04-05 00:00:00	2018-04-26 00:00:00	10325.15	10350	130.65	12.09905034	131.15	12.14830716
NIFTY	2018-04-06 00:00:00	2018-04-26 00:00:00	10331.6	10350	128.65	12.12259265	125.95	11.84744198
NIFTY	2018-04-09 00:00:00	2018-04-26 00:00:00	10379.35	10400	115.55	11.39234989	123.6	12.23975769
NIFTY	2018-04-10 00:00:00	2018-04-26 00:00:00	10402.25	10400	123	11.45660338	103.95	9.455297597
NIFTY	2018-04-11 00:00:00	2018-04-26 00:00:00	10417.15	10400	120.2	10.83786808	100.7	8.777617273
NIFTY	2018-04-12 00:00:00	2018-04-26 00:00:00	10458.65	10450	108.35	10.6372727	99.65	9.713850889
NIFTY	2018-04-13 00:00:00	2018-04-26 00:00:00	10480.6	10500	84.1	10.18622409	97	11.80251943
NIFTY	2018-04-16 00:00:00	2018-04-26 00:00:00	10528.35	10550	73.95	9.736174267	83.1	10.94664473
NIFTY	2018-04-17 00:00:00	2018-04-26 00:00:00	10548.7	10550	72.5	8.865395661	74.2	9.159167433
NIFTY	2018-04-18 00:00:00	2018-04-26 00:00:00	10526.2	10550	61.7	9.9553078	78	11.46612929
NIFTY	2018-04-19 00:00:00	2018-04-26 00:00:00	10565.3	10550	75.6	10.02587385	53.35	6.442605898
NIFTY	2018-04-20 00:00:00	2018-04-26 00:00:00	10564.05	10550	67.85	10.17082535	38.7	4.846731898
NIFTY	2018-04-23 00:00:00	2018-04-26 00:00:00	10584.7	10600	34.65	8.163093913	52.05	10.52798524
NIFTY	2018-04-24 00:00:00	2018-04-26 00:00:00	10614.35	10600	44.25	8.841573324	30	5.221056785
NIFTY	2018-04-25 00:00:00	2018-04-26 00:00:00	10570.55	10550	39.55	10.23918138	20.35	3.215793085

***Figure 10*: Implied volatilities for call and put options respectively, derived from our model (Excel Snapshot)**

MONTHLY STATISTICS

Month	Avg IV Call	Avg IV Put	Std Dev IV Call	Std Dev IV Put
April	10.86231628	9.489781854	1.64463504	2.93169189
May	9.645582114	8.90743232	1.36958666	1.92591756
June	9.077111956	8.371274092	1.20791402	2.34113635
July	9.281211914	8.814890653	1.40178987	1.68470068
August	9.723321263	7.941207651	0.88448344	1.45407694
September	14.11301767	10.25594813	2.39396585	2.10418947

October	**17.12008543**	**15.404536**	**1.72953075**	**3.01695017**
November	**14.80537502**	**13.73678707**	**2.14698407**	**2.00320547**
December	**15.97396404**	**12.54164091**	**2.00822448**	**2.73308338**
January	**13.87211354**	**10.97435712**	**1.27728223**	**1.98206143**
February	**13.24206242**	**11.72908077**	**2.04459572**	**2.19936318**
March 19	**13.34185316**	**9.566977418**	**0.99974085**	**1.53410854**

TABLE 1

OVERALL STATISTICS (ALL MONTHS INCLUDED)

Overall Average Call	Overall Average Put	Overall Std Dev Call	Overall Std Dev Put
12.5881679	**10.64449283**	**3.088908704**	**3.057618687**

TABLE 2

Relationship between underlying and closing price

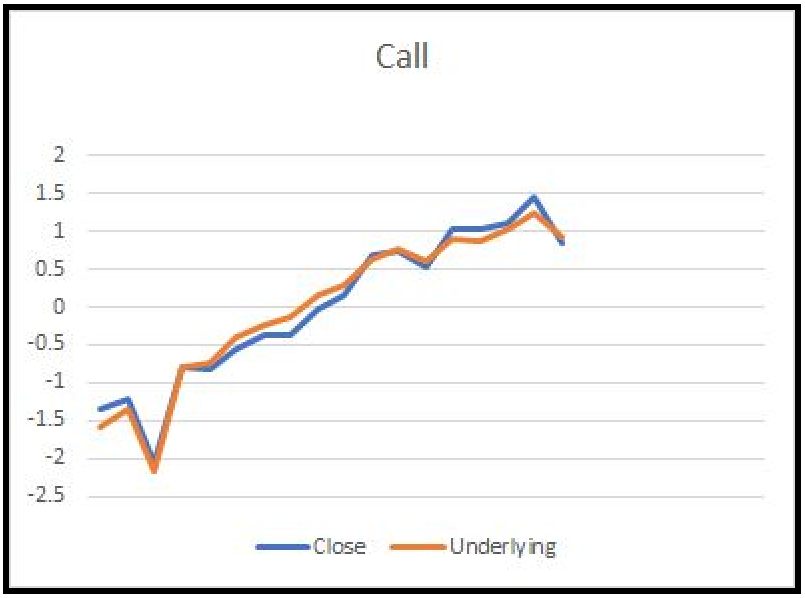

Figure 11: The above graph shows the change in price for a call option having the same strike price but different time to maturity and underlying price.

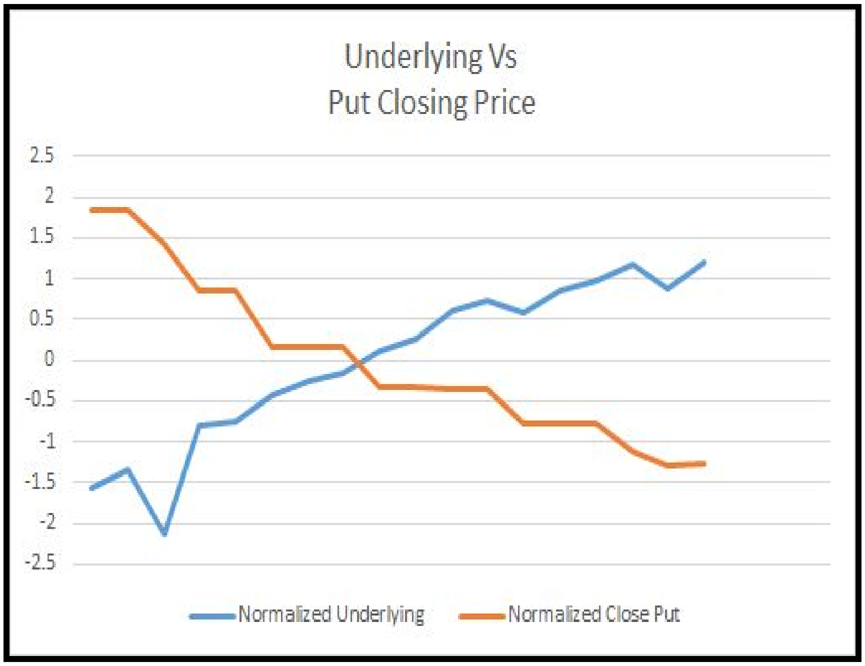

Figure 12: The above graph shows the change in price for a put option having the same strike price but different time to maturity and underlying price.

Conclusion

Let us now draw together the threads of our results. The model so implemented was successful in implementing the newton rampson's method to calculate the implied volatility of Nifty 50 options. The key findings are as follows.

The implied volatility of a option varies inversely with the underlying value, as observed from the *figures 8 and 9*. This relationship is observed because of the historical market evidence demonstrating that markets fall much more quickly than they rise. Due to this phenomenon the demand for put options either increase or its supply decreases leading to higher fluctuations in the put option price and similarly, for call options their demand decreases or supply increases due to this downward movement in the underlying price. These factors increase the implied volatilities for these options as proven by our results. These findings can be seen in *figures 11 and 12*.

To conclude , we can say that there are a number of methods available to estimate the implied volatility. Every method has its benefits and flaws and hence it should properly evaluated before applying a method, for instance the ATM constraint for Newton Raphson method as described above,.

Appendix

The following section gives a brief overview of the functions implemented in calculating the implied volatility of nifty index options.

1. Name : black_scholes

--- Arguments : Strike Price(K), Spot Price(S0), Time_to_Maturity(T), Risk_free_rate(r), Implied_Volatility(IV), Type_of_option(boolean).

--- Return Value : Price of option.

This function implements the black scholes formula and calculates the option price for the given variables.

2. Name : vega

--- Arguments : S0,K,T,r,IV

--- Return Value : Vega of the option.

This function returns the vega that is change in option price w.r.t implied volatility.

3. Name : volatility_estimate

--- Arguments : S0, K, T, r , Market_Price,Type_of_option(boolean)

--- Return Value : Implied Volatility of the option.

Calls newton_raphson procedure to calculate the implied volatility of the option.

4. Name : newton_raphson

--- Arguments : S0, K , T, r, Market_price, Last_Estimate_of_IV, flag

--- Return Type : Returns the implied volatility after each iteration of the method.

Implements Newton's Raphson method for implied volatility calculation.

References

[1] M. Brenner, M. Subrahmanyam, a simple formula to compute implied volatility, Financ. Anal. J. 44 (1998) 80–83.

[2] M. Bharadia, N. Christofides, G. Salkin, Computing the Black-Scholes implied volatility: generalization of a simple formula, Adv. Futures Options Res.8 (1995) 15–30.

[3] D.M. Chance, A generalized simple formula to compute the implied volatility, Financ. Rev. 31 (4) (1996) 859–867.

[4] Newton Rampson's Method (Link)

[5] Abhijeet Chandra ,M. Thenmozhi ,On asymmetric relationship of India volatility index (India VIX) with stock market return and risk management (2015).

[6] Ashok Banerjee , Ritesh Kumar ,Realized Volatility and India VIX (2012)

[7] BRUCE K. GRACE , Black±Scholes option pricing via genetic algorithms (2010)

[8] Tamal Datta Chaudhuri, Indranil Ghosh ,Forecasting Volatility in Indian Stock Market using Artificial Neural Network with Multiple Inputs and Outputs (2015)

[9] George Skiadopoulos, The Greek implied volatility index: construction and properties (2007)

[10] Option historical data (Nifty 50 options) (Link)

[11] Shuaiqiang Liu 1, Cornelis W. Oosterlee , Sander M. Bohte , Pricing Options and Computing Implied Volatilities using Neural Networks (2019)