Number Systems

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Arithmetic is the branch of mathematics that deals with the study of numbers and basic operations on them such as addition, subtraction, multiplication and division. As human civilization developed, they had to learn to handle large numbers. Naturally, there was a greater need for development of mathematics. Their path was not easy and they struggled all along the way. In fact, the development of mathematics can be understood as the story of development of the human civilization itself.

1 Natural Numbers

As we know, we use the numbers 1, 2, 3, 4... when we begin to count. They come naturally when we start counting. Hence, we call them natural numbers, denoted as **N**.

2 Whole Numbers

If we add zero to the collection of natural numbers, we get the collection of whole numbers. Thus, the numbers 0, 1, 2, 3... form the collection of whole numbers, denoted as **W**.

> **Note :** The common decimal numeral system from 0 to 9 in use worldwide today and the arithmetic around it, was developed in Bharat long ago. They are hence called the Hindu numerals.

3 Integers

Numbers less than zero are called negative numbers. They are denoted with a "$-$" sign such as -5. Some examples of their use can be in temperature measurement, water level measurement in a lake and so on.
If we put the collection of whole numbers and negative numbers together like this ...-3, -2, -1, 0, 1, 2, 3... we obtain the collection of integers, denoted as **Z**.

4 Rational Numbers

Rational comes from the word "ratio" and the collection of rational numbers is denoted by **Q**.
As per the definition of rational numbers, a number r is called a rational number,

if it can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
Notice that all the numbers up until now can be written in that form. For instance, -25 can be written as $\frac{-25}{1}$. Therefore rational numbers include the collection of natural numbers, whole numbers and integers.
Also, while representing any rational number, we choose p and q such that they are co-prime, that is, they have no common factors other than 1. So, among the infinitely many fractions equivalent to $\frac{1}{2}$ like $\frac{2}{4}$, $\frac{5}{10}$ and so on, we will choose $\frac{1}{2}$ to represent them all.

5 Irrational Numbers

A number s is called irrational, if it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Some examples are $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, π and so on.

> **Note :** Aryabhatta the great mathematician and astronomer of Bharat, was able to discover the correct value of π up to four decimal places. Also, in the Sulbasutra, a mathematical treatise of the Vedic period, you find an approximation of $\sqrt{2}$.

6 Real Numbers

The collection of all rational and irrational numbers together make up the collection of real numbers, denoted by **R**. Therefore, a real number is either rational or irrational.

> **Note :** An Interesting point of convergence between arithmetic and geometry is the fact that, there is a unique real number corresponding to every point on the number line. Also, corresponding to each real number, there is a unique point on the number line.

Now, looking at real numbers from the point of view of their decimal expansion, we find out that the decimal expansion of a **rational number** is either **terminating** or **non-terminating repeating**.

Whereas, the decimal expansion of an **irrational number** is **non-terminating non-repeating**. For example, a number like s = 0.101101110... is irrational.

6.1 Example

The following is an example of a rational number with **terminating** decimal expansion like $\frac{1}{2} = 0.5$ or $\frac{7}{8} = 0.875$.

6.2 Example

In the examples $\frac{10}{3} = 0.3333...$ and $\frac{1}{7} = 0.142857142857...$ we notice that the decimal expansion repeats after a certain stage. This is example of a rational number

with a **non-terminating repeating** decimal expansion.
The usual way of representing a repeating block of digits in a decimal expansion is with a bar above the repeating digits, as follows $\frac{10}{3} = 0.\overline{3}$ and $\frac{1}{7} = 0.\overline{142857}$.

6.3 Example

Recall $s = 0.101101110...$ from the previous sections. Notice that it is a **non-terminating non-repeating** decimal and therefore irrational. You can generate infinitely many irrationals similar to s. Other examples are :

$$\sqrt{2} = 1.4142135...$$
$$\pi = 3.1415926...$$

6.4 Example

Find an irrational number between $\frac{1}{7}$ and $\frac{2}{7}$.

Solution : We see that $\frac{1}{7} = 0.\overline{142857}$ and $\frac{2}{7} = 0.\overline{285714}$. To find an irrational number between them, we find a number which is non-terminating non-repeating. Of course you can find infinitely many numbers like that, for example $s = 150150015000...$

7 References

1. Class 9 - Chapter 1 : Number Systems.
   NCERT Mathematics Textbook, Version 2020-21.
   As per Indian National Curriculum Framework 2005.