Probability - 2

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1 Probability - A Theoretical Approach

In part 1, we looked into the experimental approach to probability, which is based on the results of actual experiments.
This experimental interpretation of probability can be applied, when it is possible to repeat an experiment a large number of times. The requirement of repeating an experiment has some limitations, as it may be very expensive or unfeasible in many situations. For example, launching a satellite in space and computing the probability of failure during launch.
In such experiments, the repetition can be avoided by calculating the theoretical probability instead. The theoretical probability of an event E is defined as :

$$\text{P(E)} = \frac{\text{Number of outcomes consistent with E}}{\text{Number of all possible outcomes of the experiment}}$$

It is important to **note** here that when dealing with theoretical probability, we do make certain assumptions and ignore certain possibilities.
For example, when tossing a coin, we assume that it is an unbiased fair coin that is not tampered with and is equally likely to land on head or tail. We also dismiss the negligible possibility of the coin somehow landing on its edge, which may happen, for example, if the coin falls in the sand.

2 Coin Toss Example

Let us understand the concept of theoretical probability through the coin toss example.
The possible outcomes of the coin toss experiment are two - Head and Tail. Now, let E be the event "getting a head". What would be $P(E)$?
The number of outcomes consistent with E is 1 (getting a head), and we already know that there are 2 possible outcomes (assuming a fair coin and ignoring the edge case). Therefore :

$$\text{P(E)} = \frac{\text{Number of outcomes consistent with E}}{\text{Number of all possible outcomes of the experiment}} = \frac{1}{2}$$

2.1 Elementary Events

An event consisting of only one outcome is called an elementary event. In the above example, E is an elementary event. Also, the **sum of probabilities of all elementary events is 1**.

This is natural because each elementary event would cover one possible outcome of the experiment. And when you sum them up, you get all possible outcomes of the experiment, which is same as the denominator in the formula for theoretical probability above. Hence proved.

3 Dice Throw Example

Suppose we throw a dice once. What is the probability of getting a number greater than 4 and what is the probability of getting a number less than or equal to 4 ?

Let E be the event, "getting a number greater than 4". The number of possible outcomes is six : 1, 2, 3, 4, 5, 6. And the number of outcomes consistent with E are 5 and 6. Therefore,

$$\text{P(E)} = \frac{2}{6} = \frac{1}{3}$$

Now, let F be the event "getting a number less than or equal to 4". Outcomes consistent with F are 1, 2, 3, 4. Therefore,

$$\text{P(E)} = \frac{4}{6} = \frac{2}{3}$$

3.1 Impossible Event

What is the probability of getting the number 8 in a throw of dice ?

We know that there are only six possible outcomes in a dice throw and no face of the dice is marked 8. Hence there are no outcomes that match with the above event. In other words, it is an impossible event with zero probability of occurring :

$$\text{P(getting 8)} = \frac{0}{6} = 0$$

3.2 Certain Event

What is the probability of getting a number less than 7 in a throw of dice ?

Since every face of a dice is marked with a number less than 7, it is certain that we will always get a number less than 7. So all possible outcomes are consistent with the event mentioned above. Therefore,

$$\text{P(getting a number less than 7)} = \frac{6}{6} = 1$$

Note : From the definition of theoretical probability above, we see that the numerator will always be less than or equal to the denominator. Therefore,

$$\mathbf{0 \leq P(E) \leq 1}$$

4 Complementary Events

Two Players Aviral and Omkar, play a tennis match. It is given that the probability of Aviral winning the match is 0.62. What is the probability of Omkar winning the match ?

Let A and O be the events that Aviral wins the match and Omkar wins the match respectively.
The probability of Aviral winning = $P(A)$ = 0.62 (given)
Naturally in a tennis match between Aviral and Omkar, only one can win. And Omkar winning is equivalent to Aviral not winning. Thus, A and O are called complementary events.
Therefore, the probability of Omkar winning = $P(O)$ = 1 - $P(A)$ = 0.38

5 Infinitely Many Outcomes

A missing helicopter is reported to have crashed somewhere in a rectangular region which is 9km long and 5km wide. This area also contains a lake that is 3km long and 3km wide. What is the probability that the helicopter crashed inside the lake ?

It is clear from the question that we cannot proceed the usual way here by counting the number of outcomes. As the Helicopter could have crashed anywhere in the rectangular region and the possible number of outcomes are infinitely many. As there are infinitely many points on a plane. In such cases, we find **probability as a ratio of favourable area to the total area**.
Area of the region where the helicopter can crash = 9×5 = 45 km^2
Area of the lake = 3×3 = 9 km^2. Hence,

$$P(\text{helicopter crashed in the lake}) = \frac{9}{45} = \frac{1}{5}$$

6 References

1. Class 10 - Chapter 15 : Probability.
   NCERT Mathematics Textbook, Version 2020-21.
   As per Indian National Curriculum Framework 2005.