Relations & Functions - 2

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As seen in part 1, the concept of relation in mathematics has been drawn from the meaning of relation in English language, according to which two objects are related if there is a recognizable link between them. However, abstracting from this, we mathematically define a relation R from set A to B as an arbitrary subset of $A \times B$. If a is related to b under the relation R, we denote it as $(a, b) \in R$ or alternatively as $a\ R\ b$.

1 Types of Relations

We know that a relation in a set A is any subset of $A \times A$. It could be $\emptyset$ or $A \times A$ or some subset in between. This leads us to the following definitions :

1.1 Empty Relation

Definition : A relation R in a set A is called an empty relation, if no element of A is related to any element of A. That is $R = \emptyset$.

1.2 Universal Relation

Definition : A relation R in a set A is called universal relation, if each element of A is related to every element of A. That is $R = A \times A$.

Both the empty relation and the universal relation are sometimes called trivial relations.

1.3 Reflexive Relation

Definition : A relation R in a set A is called reflexive if $(a, a) \in R$ for every $a \in A$.

1.4 Symmetric Relation

Definition : A relation R in a set A is called symmetric if $(a, b) \in R$ implies that $(b, a) \in R$ for all $a, b \in A$.

1.5 Transitive Relation

Definition : A relation R in a set A is called transitive if $(a, b) \in R$ and $(b, c) \in R$ implies that $(a, c) \in R$ for all $a, b, c \in A$.

1.6 Equivalence Relation

Definition : A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

1.7 Example

Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by $R = \{(a, b)$: both a and b are either odd or even$\}$. Show that R is an equivalence relation.
Solution :
$\Rightarrow$ Given any element a in A, it should be either odd or even. So $(a, a) \in R$ for all $a \in A$. Thus R is reflexive.
$\Rightarrow$ Further, $(a, b) \in R$ implies both a and b must be either odd or even, thus $(b, a) \in R$ as well for any $a, b \in A$. Thus R is symmetric.
$\Rightarrow$ Finally, $(a, b) \in R$ and $(b, c) \in R$ implies all elements a, b, c must be either even or odd simultaneously, further implying $(a, c) \in R$ for any $a, b, c \in A$. Thus R is transitive.
Hence we have shown that R is an equivalence relation.

2 Types of Functions