Determinants

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In the chapter on matrices, we studied about the basics and algebra of matrices. Here we continue on that path and learn about the **determinant** of a matrix. Determinants have many applications, like finding the inverse of a square matrix, determining consistency or inconsistency of a system of linear equations and finding solutions of linear equations using the matrix inverse.

1 Determinant of a Matrix

> **Definition : To every square matrix A of order n, we can associate a number called determinant of the square matrix A. It is denoted by $|A|$.**

This may be thought of as a function which associates each square matrix with a unique number (real or complex). So $f : M \to K$ where M is the set of square matrices, K is the set of numbers and f is the function defined by $f(A) = k$. Where $A \in M$ and $k \in K$ and thus $f(A) = k = |A|$, the determinant of A.
If we have a square matrix :

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then, determinant } |A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Note : Only square matrices have determinants. We will see now how exactly the determinant for a square matrix is calculated.

1.1 Determinant for Matrix of order One

Let $A = [a]$ be a matrix of order one, then determinant of A is defined to be equal to a. In other words, $|A| = a$.

1.2 Determinant for Matrix of order Two

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, then the determinant of A is defined as :

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

For example, Let us evaluate $A = \begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$

$$\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = 2(2) - 4(-1) = 4 + 4 = 8$$

1.3 Determinant for Matrix of order Three