

Algebra Base

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The branch of mathematics where we study about quantities and numbers is called **arithmetic**. The branch of mathematics where we study about shapes, figures & dimensions is called **geometry**. The third fundamental branch of mathematics is called **algebra**. The main feature of this branch is the use of symbols & letters to define rules and formulas in a general way. For example, the commutative rule $a + b = b + a$, where by using letters, we can talk about any number and not just a particular number.

The letters a and b here, are called **variables**. A variable is a placeholder for a number that is either unknown or not fixed. For example, the length of a square can have any value, it is not fixed, it is a variable. Whereas the number of angles in a triangle has a fixed value three. It is not a variable.

Since, variables stand for numbers, operations can be performed on them as on numbers. This leads to the study of algebraic expressions and helps us develop powerful tools for problem solving in daily life.

1 Algebraic Expressions

Algebraic expressions are formed of **variables** and **constants**. For example, $4x + 5$ is made of two terms $4x$ and 5 .

Terms themselves are formed as product of **factors**. For example, the term $4x$ is the product of factors 4 and x .

In the term $4x$, x is the variable whereas the constant value 4 , is called the numerical coefficient or simply **coefficient**.

1.1 Like and Unlike Terms

Like terms are the ones with the same variable part, meaning the variable itself and its power are identical in both terms. For example, $7x$ and $14x$ are like terms, also $5x^2$ and $-9x^2$.

Unlike terms on the other hand have differing variables. For example, $7x$ and $7y$, $7x$ and $7xy$, $7x$ and $7x^2$ are all unlike terms.

2 Monomials, Binomials and Polynomials

An expression that contains only one term is called a **monomial**. Expression that contains two terms is called a **binomial**. An expression containing three terms is a **trinomial** and so on.

In general, an expression containing one or more terms with non-zero coefficients is called a **polynomial**.

3 Equations

An equation is a condition on a variable. The condition is that an expression with a variable is equal to a fixed number or that two expressions should have equal value. For example, $x - 3 = 10$.

An equation has two sides **LHS** (Left Hand Side) and **RHS** (Right Hand Side) with an equal (=) sign in between. The LHS of an equation is equal to the RHS only for certain values of the variable in the equation. These values are called the solutions of that equation.

4 Standard Identities

An **identity** is an equality that is true for every value of the variable in it. For example, consider the product $(a + b)^2$ or $(a + b)(a + b)$:

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b) \\ &= a(a + b) + b(a + b) \\ &= a^2 + ab + ba + b^2 \\ &= a^2 + 2ab + b^2\end{aligned}$$

Thus, we obtain our first standard identity :

$$(a + b)^2 = a^2 + 2ab + b^2 \quad (1)$$

Since, the expression on the RHS is obtained from the LHS by multiplication, the equality holds for any value of a and b .

Similarly, we also have the following standard identities :

$$(a - b)^2 = a^2 - 2ab + b^2 \quad (2)$$

$$(a + b)(a - b) = a^2 - b^2 \quad (3)$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab \quad (4)$$

The above four identities are useful in carrying out squares and products of algebraic expressions. They also allow easy alternative methods to calculate products of numbers and so on.

5 The Origin Story

The origin and growth of mathematics is invariably tied to the origin and growth of the first major civilizations that sprang up along different river basins. Egypt, Mesopotamia & Bharat are the first of these known cradles of civilization.

Bharat, along the fertile plains of rivers Indus and Ganga was the most expansive of the three. During that ancient period, the use of letters to denote unknowns and forming expressions from them was quite common in India. Many great Indian mathematicians like Aryabhatta, Brahmagupta, Mahavira and Bhaskara contributed a lot to the study of mathematics.

The Indian name for algebra, **Beejaganita**, dates back to these ancient Indian mathematicians.

6 References

1. Class 6 - Chapter 11 : Algebra.
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2. Class 8 - Chapter 9 : Algebraic Expressions and Identities.
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