Probability - 1

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1 Probability - An Experimental Approach

We get glimpses of uncertainty in our daily lives. Come to think of it, any game has an element of uncertainty in it. In Tennis for instance, where you cannot be sure of the other player's moves, or a team sport like Football which has got multiple variables.

Even in simpler games, there is always an element of uncertainty. For instance, when tossing a coin or throwing a dice. Here, we will utilize the coin-toss game as an experiment, and measure the chance of occurrence of various outcomes, numerically.

1.1 Coin Toss Experiment

In this **experiment**, we toss a coin and observe its outcome. Each toss of a coin is called a trial. A trial is simply an action which results in one or more outcomes.

Further, an **event** in an experiment is a collection of one or more outcomes. For instance, in the coin toss experiment, "getting a tail" is an event consisting of outcome "tail". And when throwing a dice, "getting an even number" is an event consisting of outcomes 2, 4 and 6.

1.2 Definition

Based on what we observe as the outcomes of our trials, we find the **experimental** or **empirical probability**. The experimental probability of an event E is defined as follows :

$$P(E) = \frac{\text{Number of trials in which E happened}}{\text{Total number of trials}}$$

So, the experimental probability is based on the total number of trials undertaken, and the number of times our required event comes up.

2 Examples

2.1

A coin is tossed 1000 times, and the following outcomes are observed :
Head : 455 times, Tail: 545 times.
Compute the probability for both events "getting a head" and "getting a tail".
Solution :

Let us call the event of getting a head as H and that of getting a tail as T. Then as per the definition of experimental probability :

$$P(H) = \frac{455}{1000} = 0.455$$

Similarly, the probability of getting a tail will be :

$$P(T) = \frac{545}{1000} = 0.545$$

2.2

Two coins were simultaneously tossed 500 times and the following events were recorded based on the outcomes observed :
Two Heads (Head-Head) = 105
One Head (Head-Tail or Tail-Head) = 275
No Head (Tail-Tail) = 120
Find the probability of occurrence of each of these events.
Solution :
Denoting the events of getting two heads, one head and no head as HH, H and T respectively, we can calculate the experimental probability as follows :

$$P(HH) = \frac{105}{500} = 0.21$$

$$P(H) = \frac{275}{500} = 0.55$$

$$P(T) = \frac{120}{500} = 0.24$$

Note : The probability of an event always lies between 0 and 1, that is :

$$0 \leq \mathbf{P(E)} \leq 1$$

This is natural, because in the formula for experimental probability, you can observe that the denominator is always going to be greater than or equal to the numerator.

3 References

1. Class 9 - Chapter 15 : Probability.
   NCERT Mathematics Textbook, Version 2020-21.
   As per Indian National Curriculum Framework 2005.