Relations & Functions - 2

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1 Types of Relations

We know that a relation in a set A is a subset of A $\times$ A. It could be $\emptyset$ or A $\times$ A or some subset in between. This leads us to the following definitions :

1.1 Empty Relation

A relation R in a set A is called an empty relation, if no element of A is related to any element of A. That is R = $\emptyset$.

1.2 Universal Relation

A relation R in a set A is called universal relation, if each element of A is related to every element of A. That is R = A $\times$ A.

> Both the empty relation and the universal relation are sometimes called **trivial relations**.

1.3 Reflexive Relation

A relation R in a set A is called reflexive if $(a, a) \in$ R for every $a \in$ A.

1.4 Symmetric Relation

A relation R in a set A is called symmetric if $(a, b) \in$ R implies that $(b, a) \in$ R for all $a, b \in$ A.

1.5 Transitive Relation

A relation R in a set A is called transitive if $(a, b) \in$ R and $(b, c) \in$ R implies that $(a, c) \in$ R for all $a, b, c \in$ A.

1.6 Equivalence Relation

A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

1.7 Example

Let R be the relation defined in the set A = {1,2,3,4,5,6,7} by R = $\{(a, b)$: both a and b are either odd or even$\}$. Show that R is an equivalence relation.
Solution :
$\Rightarrow$ Given any element a in A, it will either be odd or even. So $(a, a) \in$ R for all $a \in$ A. Thus R is reflexive.
$\Rightarrow$ Further, $(a, b) \in$ R implies both a and b must be either odd or even, thus $(b, a) \in$ R as well for any $a, b \in$ A. Thus R is symmetric.
$\Rightarrow$ Finally, $(a, b) \in$ R and $(b, c) \in$ R implies all elements a, b, c must be either even or odd simultaneously, hence implying that $(a, c) \in$ R for any $a, b, c \in$ A. Thus R is transitive.
Hence, we have shown that R is an equivalence relation.

2 Types of Functions