# Relations & Functions - 2

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## 1 Types of Relations

We know that a relation in a set A is a subset of A $\times$ A. It could be $\emptyset$ or A $\times$ A or some subset in between. This leads us to the following definitions :

### 1.1 Empty Relation

A relation R in a set A is called an empty relation, if no element of A is related to any element of A. That is R = $\emptyset$.

### 1.2 Universal Relation

A relation R in a set A is called universal relation, if each element of A is related to every element of A. That is R = A $\times$ A.

> Both the empty relation and the universal relation are sometimes called **trivial relations**.

### 1.3 Reflexive Relation

A relation R in a set A is called reflexive if $(a, a) \in$ R for every $a \in$ A.

### 1.4 Symmetric Relation

A relation R in a set A is called symmetric if $(a, b) \in$ R implies that $(b, a) \in$ R for all $a, b \in$ A.

### 1.5 Transitive Relation

A relation R in a set A is called transitive if $(a, b) \in$ R and $(b, c) \in$ R implies that $(a, c) \in$ R for all $a, b, c \in$ A.

### 1.6 Equivalence Relation

A relation R in a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

### 1.7 Example

Let R be the relation defined in the set A = {1,2,3,4,5,6,7} by R = $\{(a, b)$ : both $a$ and $b$ are either odd or even$\}$. Show that R is an equivalence relation.
**Solution** :
⇒ Given any element $a$ in A, it will either be odd or even. So $(a, a) \in$ R for all $a \in$ A. Thus R is reflexive.
⇒ Further, $(a, b) \in$ R implies both $a$ and $b$ must be either odd or even, thus $(b, a) \in$ R as well for any $a, b \in$ A. Thus R is symmetric.
⇒ Finally, $(a, b) \in$ R and $(b, c) \in$ R implies all elements $a, b, c$ must be either even or odd simultaneously, hence implying that $(a, c) \in$ R for any $a, b, c \in$ A. Thus R is transitive.
Hence, we have shown that R is an equivalence relation.

## 2 Types of Functions

### 2.1 One-One

A function $f$ : X → Y is said to be one-one (or **injective**) if distinct elements of X under $f$ have distinct images in Y. That is, for every $x_1$, $x_2 \in$ X, $f(x_1) = f(x_2)$ implies $x_1 = x_2$.
Otherwise, if multiple elements in X have the same image in Y, then $f$ is called **many-one**.

### 2.2 Onto

A function $f$ : X → Y is said to be onto (or **surjective**) if every element of Y is the image of some element of X under $f$. That is, for every $y \in$ Y, there exists an element $x$ in X such that $f(x) = y$.
This implies that $f$ : X → Y is onto if and only if range of $f$ = Y = codomain.

### 2.3 One-One & Onto

A function $f$ : X → Y is said to be **bijective** if it is both one-one and onto.

### 2.4 Example 8 9 13 14