SOC End-Report

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1 Double Auctions

Double auctions are a type of market mechanism commonly used in economics and finance to facilitate the exchange of goods or financial assets between buyers and sellers. In a double auction, both buyers and sellers can submit bids and offers, respectively, indicating the quantity they are willing to buy or sell and the price they are willing to pay or accept.

Double auctions can be found in various contexts, including stock exchanges, commodity markets, and online platforms for buying and selling goods and services. They contrast with single auctions, where either the buyers or sellers are the only ones allowed to place bids or offers, limiting the market dynamics and potentially leading to less efficient outcomes.

In options trading, double auctions refer to the process of matching buy and sell orders for options contracts in a competitive marketplace. Options are financial derivatives that give the holder the right, but not the obligation, to buy or sell an underlying asset (such as stocks, commodities, or currencies) at a predetermined price (the strike price) within a specific period (until the expiration date). Options trading can occur through various exchanges or trading platforms, and double auctions are a prevalent mechanism used to facilitate these transactions.

2 Modelling The Price of an Option

2.1 Black Scholes Formula

The Black-Scholes formula, also known as the Black-Scholes-Merton model, is a mathematical formula used to calculate the theoretical price of European-style options. It was developed by economists Fischer Black and Myron Scholes in 1973, with contributions from Robert Merton. The formula revolutionized options pricing and significantly contributed to the development of modern financial derivatives markets.

The Black-Scholes formula for the theoretical price of a European-style call option is given by:

$\mathrm{C} = \mathrm{S}_0 N(d_1) - X e^{-rT} N(d_2)$

where: C is the option's theoretical price,
$S_0 is the current price of the underlying asset,$
$X is the option's strike price,$
$r is the risk-free interest rate,$
$T is the time to expiration of the option (in years),$
$N(\cdot) is the standard normal cumulative distribution function,$
$d_1 is calculated as: d_1 = \frac{\ln\left(\frac{S_0}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}},$
$d_2 is calculated as: d_2 = d_1 - \sigma\sqrt{T},$

and σ represents the volatility of the underlying asset's returns.

The Black-Scholes formula is used to determine the fair market value of a call or put option. However, it is essential to remember that the model has some limitations, including the assumptions of constant volatility and the efficient market hypothesis, which may not hold in real-world situations. As a result, traders and investors often use modified versions of the model or other option pricing models to reflect market realities more accurately.

2.2 Option Greeks

Option Greeks are a set of measures used in options trading and risk management to understand how an option's price may change in response to different factors. These factors include changes in the underlying asset's price, time to expiration, implied volatility, and changes in interest rates. The Greeks are essential tools for traders and investors to assess and manage the risk associated with options positions. There are five primary option Greeks:

- Delta (δ): Delta measures the sensitivity of an option's price to changes in the price of the underlying asset. It represents the percentage change in the option price for a $1 change in the underlying asset's price. For call options, delta ranges from 0 to 1, where a delta of 0.50 means the option's price moves approximately 50 cents for every $1 change in the underlying asset's price. For put options, delta ranges from 0 to -1, where a delta of -0.40 indicates that the option's price decreases by approximately 40 cents for every $1 increase in the underlying asset's price.

- Gamma (γ): Gamma measures the rate of change of the option's delta concerning changes in the underlying asset's price. It shows how quickly the delta of an option is changing. Higher gamma values mean the delta is more sensitive to changes in the underlying asset's price. Gamma is at its highest for at-the-money options and decreases as options move further in or out of the money.

- Theta (θ): Theta measures the rate of decline in an option's value with the passage of time, also known as time decay. It indicates how much an option's value is expected to decrease each day due to the diminishing time to expiration. Theta is usually expressed as a negative value, indicating that options lose value over time, all else being equal.

- Vega (V): Vega measures the sensitivity of an option's price to changes in implied volatility, which represents the market's expectation of future price fluctuations of the underlying asset. Vega indicates how much an option's price is expected to change for a 1% change in implied volatility. Higher Vega values mean the option's price is more sensitive to changes in volatility.

- Rho (ρ): Rho measures the sensitivity of an option's price to changes in the risk-free interest rate. It indicates how much an option's price is expected to change for a 1% change in the interest rate. Rho is more relevant for options that have a longer time to expiration since interest rate changes can have a more significant impact on long-term options.

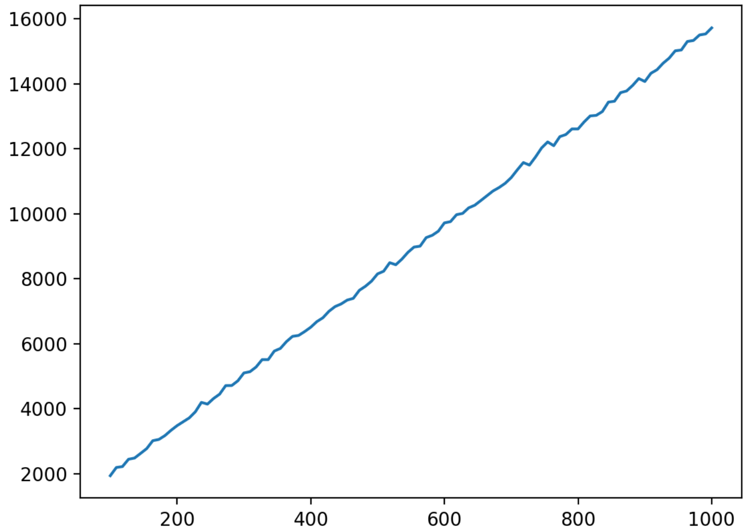
2.3 My Model

I have modeled the price at which the sellers trade a particular stock by using both black-scholes formula and option greeks. Each investor value these greeks differently , so I have randomised the percentage in which these greeks play a role in determining the price.

3 What am I doing here?

It is a basic theorem of auctions derived through game-theory that each bidder in a double auction gets the highest reward when he bids his true valuation. But while entering the double auction a bidder doesn't know his true valuation. So we have used Reinforced-Learning Technique applied using bandit systems through UCB-LCB algorithms.
There are three entities participating in this auction the bidder(N in number) , the seller (M in number) and auctioneer. Each buyer and seller bids and only receives his reward from the auction. Based on this reward the buyer follows UCB bounds to determine his next bid and the seller uses his LCB bound to determine his next selling price .

The auctioneer returns the rewards to each buyer and seller (with some normal random variable) and sets the market price for each auction. The methodology for setting the market price in double-auctions have been extensively researched and there are several methods to do so. But we are following a new mechanism which is similar to Mc-Affee method in which the market price is equal to the average price of the last two participating buyer and seller.
The role of auctioneer is also to determine the number of participating buyer and sellers which it does by plotting the demand-supply curve and eliminating all the buyers and sellers in that round which lie to the right of the intersection.
There are three regrets that we get in this simulation the regret on buyer side, regret on seller side ad the social regret. I am hereby attaching the social regret that we are getting because it encompasses both regret from seller's and buyer's



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4 Bibliography

For mathematical expressions used please follow the following research paper

Basu, Soumya, and Abishek Sankararaman. ”Double Auctions with Two-sided Bandit Feedback.” arXiv preprint arXiv:2208.06536 (2022).