Homework 4

Part 2

Objective function is given by,

$$F(U,V) = \frac{1}{2}\sum_{(i,j)\in\Omega_1} (M_{ij} - U_i V_j^T)^2 + \frac{\lambda}{2}\left(\|V\|_F^2 + \|V\|_F^2\right)$$

Calculating $\frac{\partial F(U, V)}{\partial U}$

Since

$$\frac{\partial F(U,V)}{\partial U} = \left[\frac{\partial F(U_1 V_j}{\partial U_1} \ldots , \frac{\partial F(U_n V_j)}{\partial U_n}\right]$$

Consider the first term

$$\frac{\partial F(U_1, V_j)}{\partial U_1} = \left[\frac{1}{2}\sum_{(1,j)} (M_{1j} - U_1 V_j^T)^2\right]$$

All other terms will be zero.

$$\frac{\partial F(U_1, V_j)}{\partial U_1} = -\left(M_{1j} - U_1 V_j^T\right) V_j + \lambda U_1$$

$$= (1+\lambda) U_1 - M_{1j} \cdot V_j$$

$$\therefore \frac{\partial F(U_i, V_j)}{\partial U_i} = \left(U_i V_j^T - M_{ij}\right) \cdot V_j + \lambda U_i$$

To find $\frac{\partial F(U,V)}{\partial V}$

we know that,

$$\frac{\partial F(U,V_1)}{\partial V} = \begin{bmatrix} \frac{\partial F(U,V_1)}{\partial V_1} \\ \vdots \\ \frac{\partial F(U,V_r)}{\partial V_r} \end{bmatrix}$$

For $\frac{\partial F(U_i,V_1)}{\partial F(V_1)}$ $= -\left(M_{i1} - U_i V_1^T\right) U_i + \lambda V_1$

$$= (1+\lambda) V_1 - M_{i1}^T \cdot U_i$$

$$\frac{\partial F(U_i,V_j)}{\partial V_j} = \left(U_i V_j^T - M_{ij}\right) \cdot U_i + \lambda V_j$$