

Assignment 2

Problem 1

a) For boolean AND function the negative and the positive class are linearly separable.

b) AND Function is given by

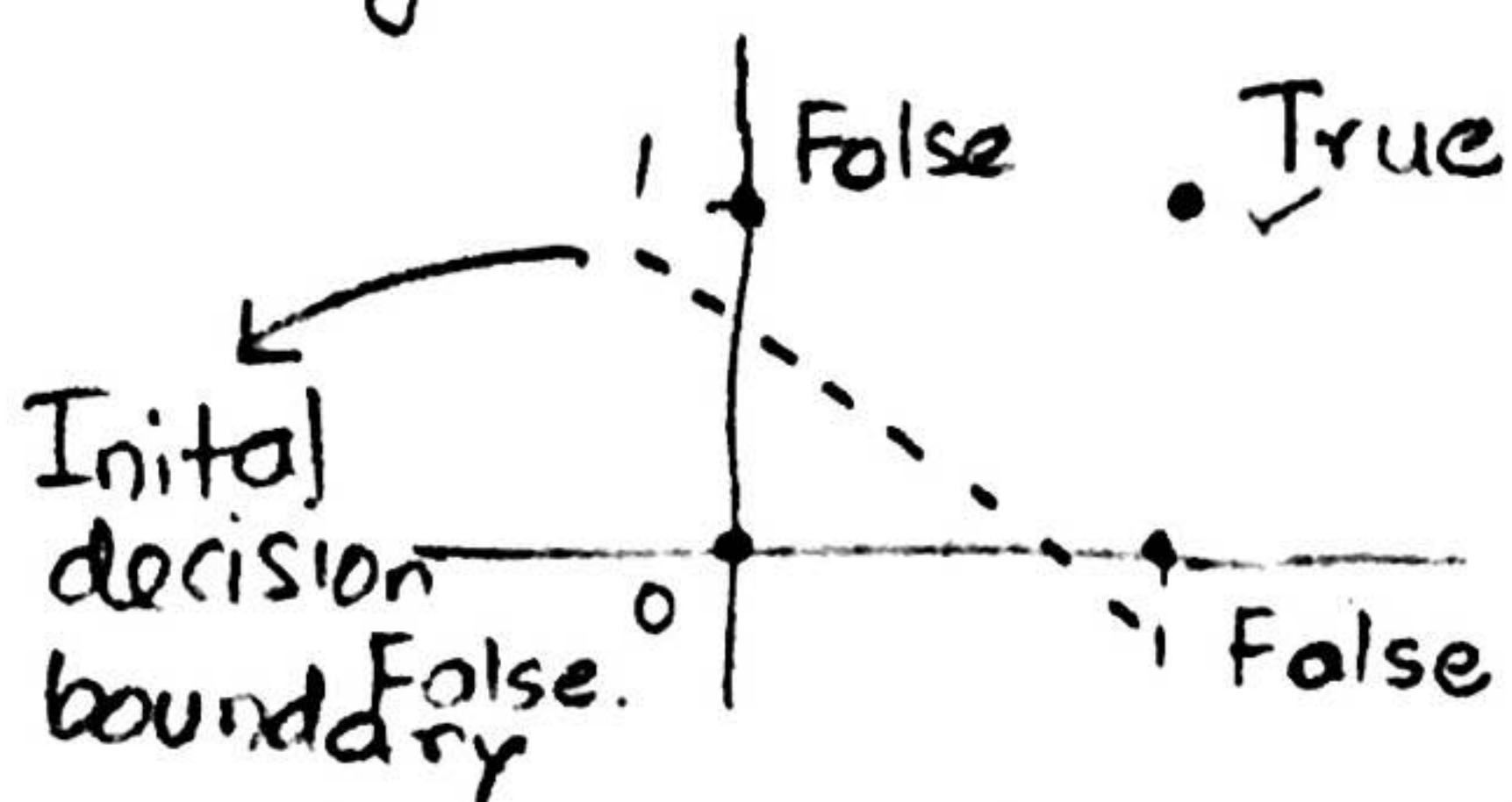
$$f(0,0) = \text{False}$$

$$f(0,1) = \text{False}$$

$$f(1,0) = \text{False}$$

$$f(1,1) = \text{True}$$

Plotting these AND Function



$$\text{let } w = \begin{bmatrix} 0 \\ 1 \\ -0.5 \end{bmatrix}$$

$$\text{learning rate} = 0.2$$

Equation of the initial decision boundary $\Rightarrow x_1 + x_2 - \frac{1}{2} = 0$.

No of misclassified points = 2

No of classified points = 2

If $w^T x > 0 \rightarrow \text{classify true}$.

$w^T x < 0 \rightarrow \text{classify False}$.

Since point (1,0) is misclassified

$$x = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

Taking the dot product $w^T x$
Since we want to classify it as False.

$$(-0.2)(1) + (0.8)x_1 + (-0.5)x_2 < 0$$

New equation of the line

$$0.8x_1 - 0.5x_2 - 0.2 = 0$$

$$-0.5x_2 = 0.2 - 0.8x_1$$

$$x_2 = \frac{-0.2}{0.5} + \frac{0.8}{0.5}x_1$$

$$x_2 = -0.4 + 1.6x_1$$

update step

$$w' = w + \eta dx_i$$

where η = learning rate

$$d = \begin{cases} 1 & \rightarrow \text{upper} \\ -1 & \rightarrow \text{lower} \end{cases}$$

$$w_0^T = w_0 + \eta dx_0$$

$$0 + (0.2)(-1)(1)$$

$$w_0^T = -0.2$$

$$w_1^T = w_1 + \eta dx_1$$

$$= 1 + (0.2)(-1)(1)$$

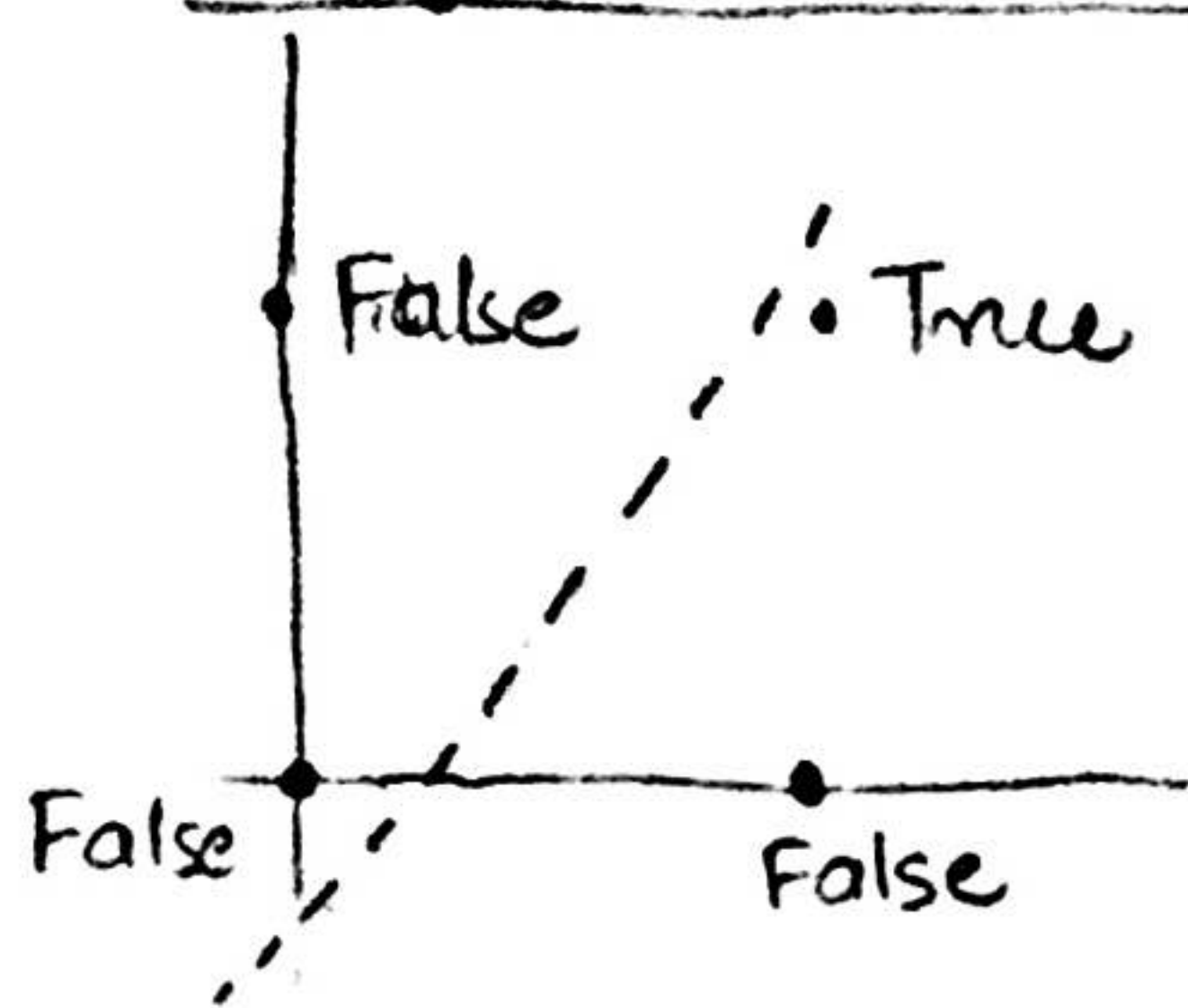
$$w_1^T = 0.8$$

$$w_2^T = w_2 + \eta dx_2$$

$$= -0.5 + (0.2)(-1)(0)$$

$$= -0.5$$

Plotting the new line



Considering (1,1) point

Since (1,1) is classified correctly
There will be no change in $w^T x$

Considering point (0,1)

Since (0,1) is misclassified.

Update steps

$$w_i' = w_i + \eta d x_i$$

$$x = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$w = \begin{bmatrix} 0 \\ 1 \\ -0.5 \end{bmatrix}$$

$$w_0' = 0 + (0.2)(1)(1)$$

$$= 0.2$$

$$w_1' = 1 + (0.2)(1)(0)$$

$$= 1$$

$$w_2' = 0.5 + (0.2)(1)(1)$$

$$= 0.7$$

Dot product

$$w_0' x_0 + w_1' x_1 + w_2' x_2$$

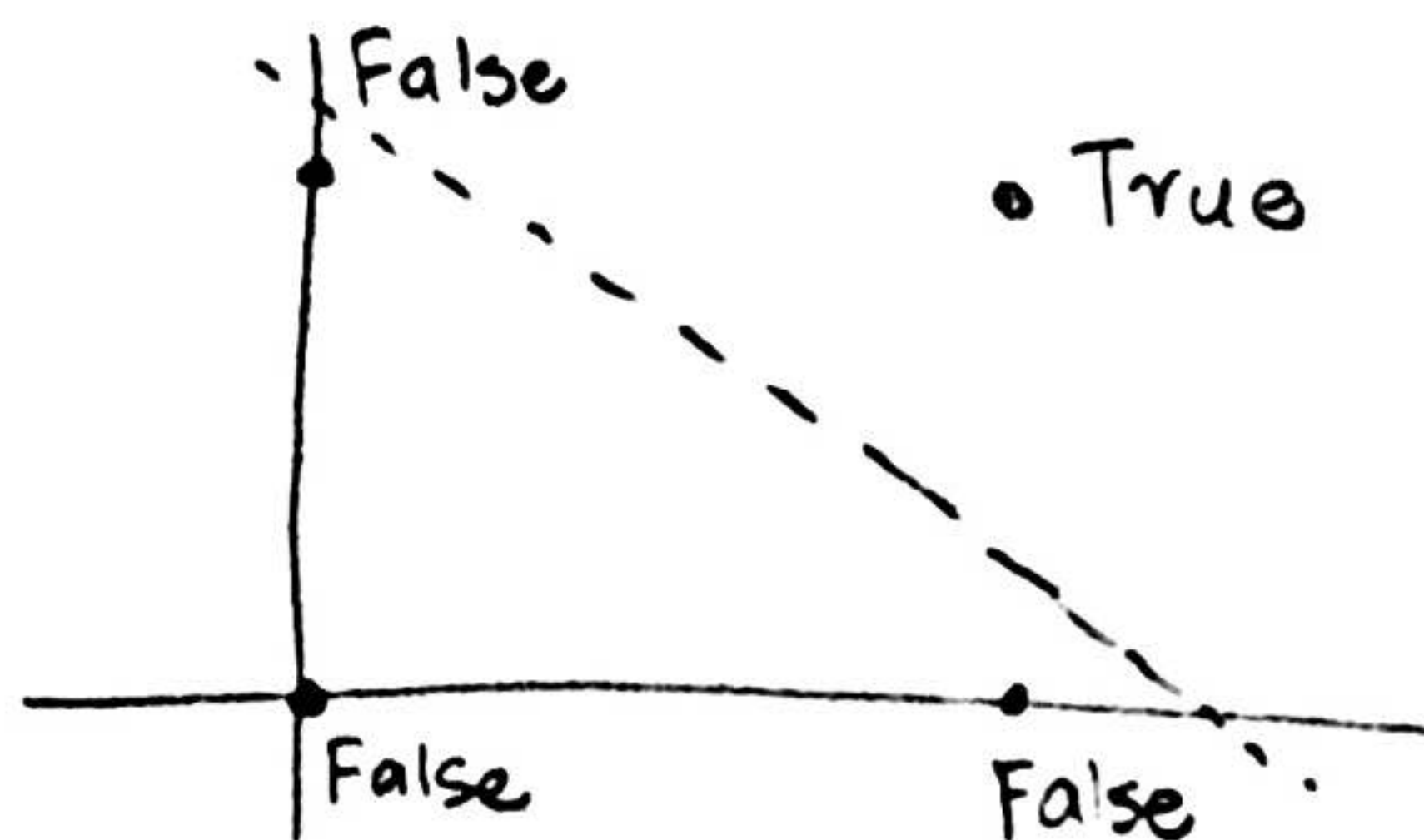
$$0.2 \times 1 + 1 x_1 + 0.7 x_2 = 0$$

$$0.2 + x_1 + 0.7 x_2 = 0$$

$$x_1 + 0.7 x_2 = -0.2$$

$$x_2 = \frac{-0.2}{0.7} - \frac{1}{0.7} x_1$$

Plotting the line



For point (0,0)

No updation since the point is already classified.