Thermal Analysis of Satellite

Avish Wagde

June 2020

Contents

1 Abstract

The process of thermal control for a spacecraft involves managing the energy entering and leaving the spacecraft to ensure that the components of the spacecraft remain within an acceptable temperature range. Spacecraft perform optimally and have longer working lives when the temperature of their components remains within these boundaries, often near the temperature at which they were fabricated. Thus, the first step in the thermal control process is to establish the temperature specifications in which the spacecraft will be operated during its lifetime. The thermal design ensures that these specified values are not exceeded, particularly those in orbit. This is substantiated during the development process by analysis, similarity studies, and testing. The aim of this document is to learn various aspects that affect the thermals of a low orbit satellite. Starting with the basics of heat transfer by conduction and radiation this document document will discuss various factors contributing for the thermals of the satellite and how to analyse them. Later on a particular problem statement will be discussed as a example.

2 Introduction

The process of thermal control for a spacecraft involves managing the energy entering and leaving the spacecraft to ensure that the components of the spacecraft remain within an acceptable temperature range. Spacecraft perform optimally and have longer working lives when the temperature of their components remains within these boundaries, often near the temperature at which they were fabricated. The thermal design ensures that these specified values are not exceeded, particularly those in orbit. This is substantiated during the development process by analysis, similarity studies, and testing.

The aim of this document is to learn various aspects that affect the thermals of a low orbit satellite. The sections in order go through the basic concepts of heat transfer by conduction and radiation that were studied, where more focus is given to radiation part. Consecutively factors affecting thermals of a LEO satellite like solar radiation, Albedo, Earth IR and internal heat generation are explained after which the process of thermal analysis is explained right from basics of energy balance to Numerical approximation methods for solving thermal model of the satellite. In the next section we considered a cube satellite in a low earth orbit and analysis for its various special configurations is discussed.

3 Heat Transfer Concepts

For developing the understanding of heat transfer by conduction and radiation the online lecture series on NPTEL by Prof. S.P Sukhatme on Heat and Mass transfer is followed. The content of the lectures are which are briefly mentioned ahead.

- Conduction (in solids) Section discusses the following topics in brief :
  - One-D steady state situations
    - Infinite slab
    - Infinite long hollow cylinder
  - Concept of thermal resistance
    - Infinite composite slab
    - Infinite long hollow composite cylinder
  - Critical radius of insulation
  - Differential Equation of heat conduction
    - Cartesian coordinates
    - Cylindrical coordinates
    - Spherical coordinates
  - Heat generation 1D problems
  - Unsteady state heat conduction
    - With negligible internal temperature gradients $T = f(t)$
    - Internal gradients not negligible

        · Infinite Slab
        · Infinite Solid cylinder
        · Solid Sphere
        · 2D and 3D rectangular bar/block

- Thermal Radiation discusses the following :
  - Basic concepts
  - Emission characteristics of surfaces
  - Laws of Black body radiation
  - Radiation incident on a surface
  - Directional nature of Radiation
  - Heat exchange by radiation between black surfaces
  - Shape Factor
  - Radiant heat exchange in an enclosure black surfaces
  - Radiant heat exchange in the annular space between two tubes, two spheres, etc. And also case where $[A1/A2 \to 0]$

3.1 Conduction 1

.

- For a closed system the Work $W = 0$ and the first law of thermodynamics states that

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{dE}{dt}$$

  hence, $\frac{dQ}{dt} = \frac{dE}{dt}$ for a closed system.

- Conduction is governed by the Fourier's Law which in 1D states that

$$\frac{q}{A} = -K\frac{dT}{dx}$$

- Some general results :-
  - For a infinite plate of width b, one end at T_1 and other at T_2 with $T_1 > T_2$,the temperature a distace x, x being measured from surface at $T1$ is given by

$$\frac{x}{b} = -\frac{(T - T_1)}{(T_1 - T_2)}$$

  - For a infinite hollow cylinder with r_i, r_o and T_i, T_o ($T_i > T_o$) being the inner and outer radius and temperature respectively, the temperature T at any radius r is given by,

$$\frac{\ln\left(\frac{r}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)} = \frac{T - T_i}{T_o - T_i}$$

  - For a infinite hollow sphere with r_i, r_o and T_i, T_o ($T_i > T_o$) being the inner and outer radius and temperature respectively, the temperature T at any radius r is given by,

$$\frac{\left(\frac{1}{r} - \frac{1}{r_i}\right)}{\left(\frac{1}{r_o} - \frac{1}{r_i}\right)} = \frac{T - T_i}{T_o - T_i}$$

- **Thermal resistance** : it goes as an equivalent to electrical resistance in thermals and is the ratio of temperature difference and heat flux.

$$R_{ab} = \frac{T_1 - T_2}{q}$$

- Some simple results:-
  - For a infinite slab of width b and cross section area A and thermal conductivity K
$$R = \frac{b}{KA}$$
  - For heat flow from a surface to fluid R is given by Newton's law of cooling
$$R = \frac{1}{hA}$$
  - For infinite hollow cylinder with outer and inner radius r_i and r_o respectively, thermal conductivity K and long length L
$$R = \frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi KL}$$
- When insulation is added on top of a hollow metallic cylinder it is seen that at first the heat flow increases and then decreases. This happens because when we add insulation the surface area increases thus convection resistance decreases and the conduction resistance increases , overall at first the decrease in convection resistance is greater than the increase in conduction resistance hence the phenomenon. But this ours only when (K is thermal conductivity of insulation)
$$r_o < \frac{K}{h}$$

3.2 Conduction 2

- The general differential equation of Conduction for a Isotropic solid is
$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial T}{\partial z}\right) + \bar{q} = \rho c_p \frac{\partial T}{\partial t}$$
where $\bar{q}$ is the heat generation term. Now if thermal conductivity is constant with x, y, z coordinates then
$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \bar{q} = \rho c_p \frac{\partial T}{\partial t}$$
which can be rewritten in terms of Thermal diffusivity ($\alpha = \frac{K}{\rho c_p}$) as
$$\nabla^2 T + \frac{\bar{q}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$
And if there is no heat generation in steady state the equation takes the form of Laplace's equation,
$$\nabla^2 T = 0$$
- In cylindrical coordinates, for constant K the equation takes the form
$$K\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + \bar{q} = \rho c_p \frac{\partial T}{\partial t}$$
And in spherical coordinates
$$K\left(\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}\right) + \bar{q} = \rho c_p \frac{\partial T}{\partial t}$$
- There are two types of conditions to completely solve these partial differential equations
  - Initial Condition : generally corresponds to model at time $t = 0$.
  - Boundary conditions :
    * Prescribed Surface condition. For eg. $T = T_0$ at $x = L$.
    * Prescribed heat flux incident on the surface. For eg.
$$-K\left(\frac{\partial T}{\partial x}\right)_{x=L} = -\left(\frac{q}{A}\right)_0$$
    * Prescribed heat transfer coefficient at the surface. For eg.
$$-K\left(\frac{\partial T}{\partial x}\right)_{x=L} = h(T_{x=L} - T_f)$$

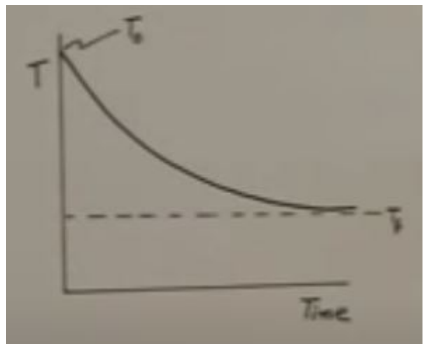
3.3 Conduction 3

- For bodies with negligible internal temperature gradient , the variation within the body are negligible, so for a body with heat transfer coefficient h, external fluid temperature T_f and initial temperature T_0, by first law we have

$$\frac{dE}{dt} = \rho c_p V \frac{dT}{dt} = hA(T_f - T)$$

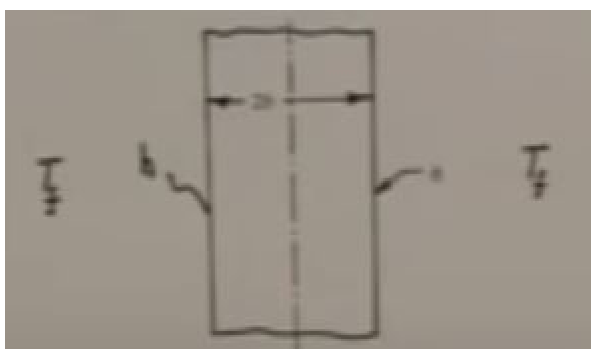
let $\theta = T - T_f$

$$\frac{\theta}{\theta_0} = e^{-\frac{hAt}{\rho c_p V}}$$



- In case where internal temperature gradient can't be neglected we have to go and solve the general partial differential equation

For eg. consider a infinite slab of width $2b$ initial temperature T_0, outside fluid temperature T_f, heat transfer coefficient h, Thermal conductivity K and thermal diffusivity as α.



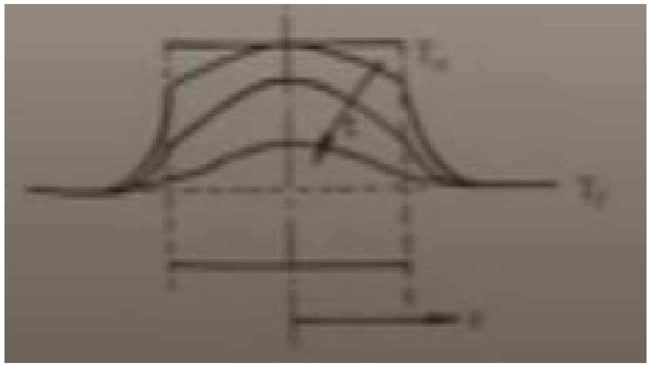
The model reduces to 1D and we solve it by using separation of variable method and get the following convergent series as solution with $\theta = T - T_f$

$$\frac{\theta}{\theta_0} = 2\sum_{n=1}^{\infty} \frac{\sin(\lambda_n b)}{\lambda_n b + \sin(\lambda_n b)\cos(\lambda_n b)} e^{-\lambda_n^2 \alpha t} \cos \lambda_n x$$

where λ_i are the eigenvalues and roots of the equation

$$\cot \lambda b = \frac{\lambda K}{h}$$

the result has the following looking solution,



3.4 Conduction 4

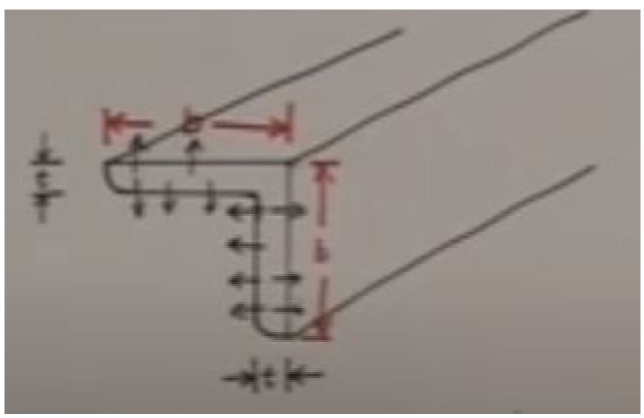
- When we make the solution discussed above dimensionless. We get a function which depends on $\lambda_n b$, $\frac{\alpha t}{b^2}$, $\frac{x}{b}$. And as we know $\lambda_n b$ only depends on $\frac{hb}{K}$

$$\frac{\theta}{\theta_0} = f\big(\lambda_n b, \frac{\alpha t}{b^2}, \frac{x}{b}\big) = f\big(\frac{hb}{K}, \frac{\alpha t}{b^2}, \frac{x}{b}\big)$$

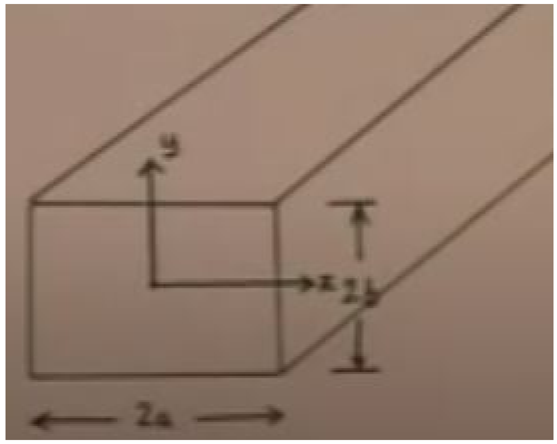
  Here $\frac{hb}{K}$ is the **Biot Number**(B) which is the ratio of thermal resistance of solid to thermal resistance of surface. And $\frac{\alpha t}{b^2}$ is the dimensionless measure of time called the **Fourier's Number**

- As Biot number approaches 0, the Internal Temperature Gradient are negligible. So we can say that if $B < 0.1$ the internal temperature gradient become negligible.

  For other Surfaces $B = \frac{hL}{K}$ where L is the characteristic dimension for solid body (the extent of distance to which heat flows). So in the below case $L = \frac{t}{2}$.



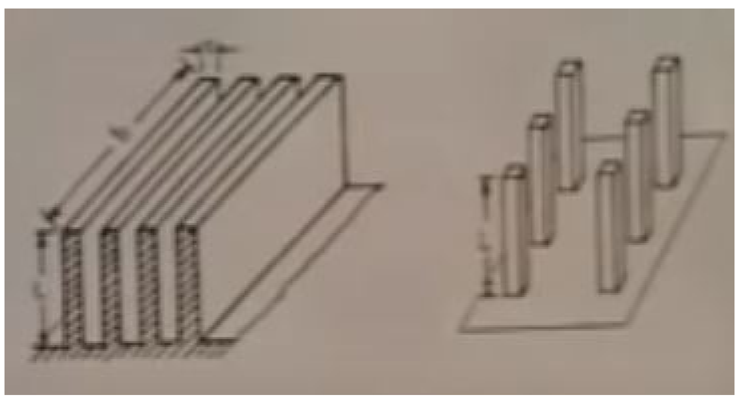
- Solution for 2D and 3D cases may also be given in similar terms. For eg. instead of just taking a slab of width $2b$, lets take a long rod of rectangular cross section with dimensions $2a$x$2b$, solution for such a case can be given as



$$\frac{\theta}{\theta_0} = (\frac{\theta}{\theta_0})_x(\frac{\theta}{\theta_0})_y$$

where $(\frac{\theta}{\theta_0})_x$ and $(\frac{\theta}{\theta_0})_y$ are solution for dimensionless temperature in infinite slab of width $2a$ and $2b$.

- **Fins :** they are used to reduce the thermal resistance at a surface and thereby increase the heat transfer rate from the surface to the adjacent fluid.

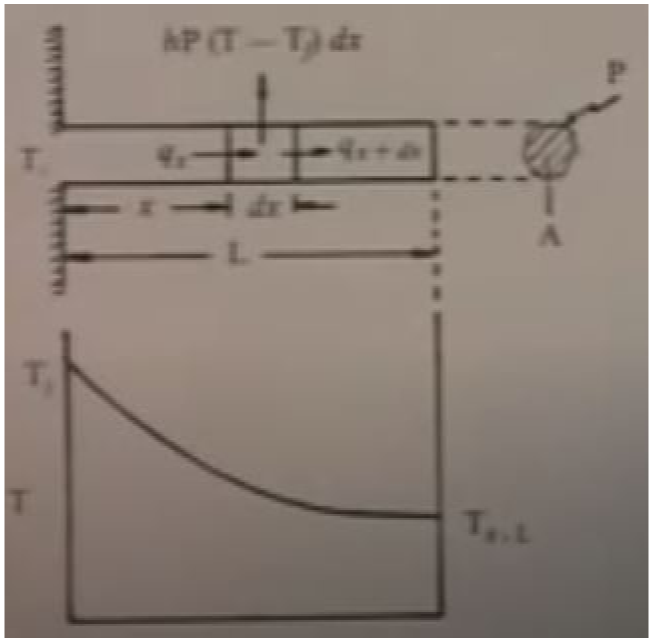


- Pin Fins are slender rod sticking out of a plane surface, as shown in second diagram above. Applying the first law and all the boundary conditions the temperature distribution along the length of pin fin is

$$\frac{\theta}{\theta_1} = \frac{\cosh m(L-x)}{\cosh mL}$$

where $m = \sqrt{\frac{hP}{KA}}$, $\theta = T - T_f$ and $\theta_1 = T_1 - T_f$ is the temperature difference with respect to T_f at the onset of the pin fin. The rate of heat flow from fin surface is given by

$$q = -KA(\frac{d\theta}{dx})_{x=0} = \sqrt{hPKA}\theta_1 \tanh mL$$



- **Fin effectiveness (ϕ)** : it is the ratio of rate of heat flow from fin surface to the rate of heat flow from fin surface if thermal conductivity is considered infinite, in which case the temperature everywhere in the fin will be T_1.

$$\phi = \frac{q}{q_{K=\infty}} = \frac{\tanh mL}{mL}$$

3.5 Radiation 1

- *Basic concepts:-*
  - Solid and liquids at all temp. Emit thermal radiation, given by Stefan-Boltzmann Law-

    $$(\frac{q}{A}) \propto T^4$$

    i.e. the rate of emission per unit area is proportional to fourth power of temperature. Thermal radiations are EMW, and are continuous spectrum of all wavelengths, i.e. all wavelengths exist but the major area under the radiations vs wavelength curve will lie in between the 0.1 - 10 μm range of wavelength.
  - Surfaces of all bodies have a capacity to absorb all or part of the radiation emitted by surrounding surfaces falling on it.
  - A surface emits radiation in all directions encompassed by a hemisphere (directional nature).
  - In all situations it is assumed that space between the surfaces is a vacuum or filled with gas that does not participate in the radiative exchange in any ways.
- *Black surface:-* ideal surface , absorbing all radiation falling on it regardless of its wavelength or direction. Emits maximum energy for a given temp and wavelength.
- *Terms:-*

  Total - over all wavelengths , hemispherical - over all directions

  - *Total hemispherical emissive power (e) :-* Radiant flux emitted from the surface of a body (W/m^2). For a black body it is denoted by e_b.
  - *Total hemispherical emissivity (ϵ) :-*it is the ratio of e and e_b at a particular temperature.
  - *Monochromatic or spectral hemispherical emissive power (e_λ):-*radiant flux emitted per unit wavelength.

    $$e_\lambda = \frac{de}{d\lambda} \text{ or } e = \int_0^\infty e_\lambda d\lambda$$

    i.e e_λ is the quantity which when integrated over all wavelengths yields e.
  - *Monochromatic or spectral hemispherical emissivity* $(\epsilon_\lambda) : -ratio of e_\lambda$ *and* $e_{b\lambda}$ *at the same temperature and wavelength.*
- *Gray surface :-* surface having the same value of ϵ_λ at all wavelengths. (idealisation)
- *Planck's law :-*

  $$e_{b\lambda} = \frac{2\pi C_1}{\lambda^5(\exp^{\frac{C_2}{\lambda T}} - 1)}$$

  $C_1 = 0.596x10^{-6}Wm^2$

  $C_2 = 0.014387\ mK$

  for non-black surface, $e_\lambda = \epsilon_\lambda e_{b\lambda}$
- *For graph of $e_{b\lambda}$ vs λ, :-*
  - It increases with wavelength , goes through a maximum and then asymptotically goes to zero.
  - For a particular wavelength it increases with temperature.
  - As temperature increases the value of wavelength at which maximum occurs decreases.

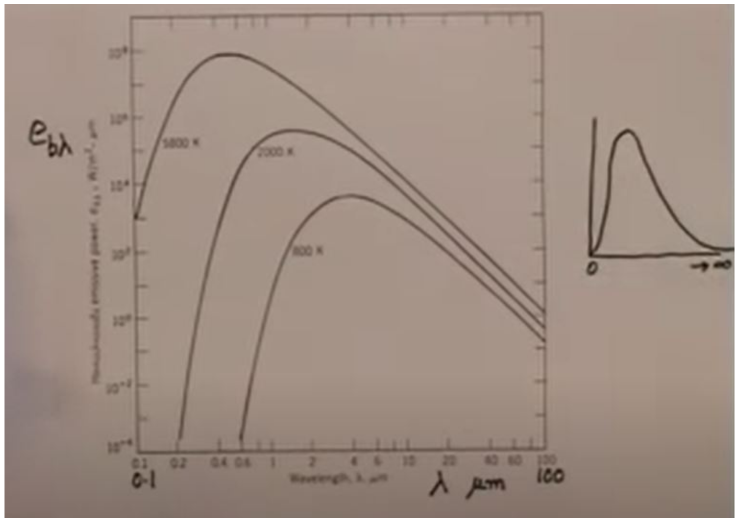


Figure 1: $e_{b\lambda}$ vs λ

- *Wien's law :-*

$$\lambda_m T = 0.00290mK$$

  λ_m is the wavelength at which maximum occurs.

3.6 Radiation 2

- Stefan-Boltzmann Law :- For emissive power of a black surface,

$$e_b = \sigma T^4$$

  $\sigma = 5.670x10^{-8} W/m^2K^4$ use $e_{b\lambda}$ from Plank's law and integrate it over all the λ to get e_b

- Radiation incident on a surface:-

  Radiation falling on a surface is absorbed,reflected and transmitted. The reflection is in all directions.

  - *Total hemispherical irradiation (H)* - Radiant flux incident on surface of body (W/m^2).
  - *Total hemispherical absorptivity (α)* - fraction of H absorbed at surface. For black surface $\alpha = 1$.

$$\alpha = \frac{H_\alpha}{H}$$

  - *Monochromatic hemispherical irradiation (H_λ)* - Radiant flux incident on the surface of a body per unit wavelength ($W/m^2 - \mu m$).

$$H_\lambda = \frac{dH}{d\lambda} \text{ or } H = \int_0^\infty H_\lambda d\lambda$$

  - *Monochromatic hemispherical absorptivity (α_λ)* - Fraction of monochromatic hemispherical irradiation absorbed at a surface.For black surface $\alpha_\lambda = 1$.

$$\alpha_\lambda = \frac{H_{\alpha\lambda}}{H_\lambda}$$

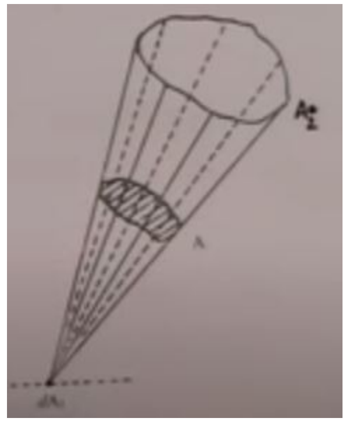
  - Similarly we can define terms like -
    - Total hemispherical reflectivity (ρ)
    - Monochromatic hemispherical reflectivity (ρ_λ)
    - Total hemispherical transmissivity (τ)
    - Monochromatic hemispherical transmissivity (τ_λ)

    Hence, $\alpha + \rho + \tau = 1$ and $\alpha_\lambda + \rho_\lambda + \tau_\lambda = 1$.

3.7 Radiation 3

- Solid Angle

  Construct a conical surface with vertex at dA_1 passing through the perimeter of A_2. The *Solid Angle* subtended by the area A_2 at the differential area dA_1 is numerically equal to the area A of the portion of the surface of a sphere of unit radius, center at dA_1, which is cut out by the conical surface. (unit: sr)



  Solid angle subtended by differential area dA-

$$d\omega = \frac{dA}{r^2} = \sin\psi d\psi d\phi$$

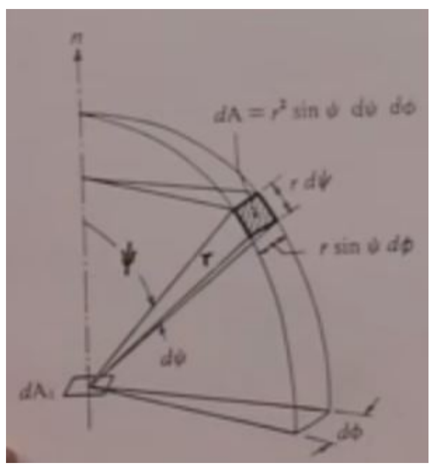


Figure 2: Spherical coordinate system with ψ as zenith angle and ϕ as azimuth angle.

- Radiation Intensity

  Total Radiation intensity in a given direction (i) - Radiant flux passing in the specified direction per unit solid angle

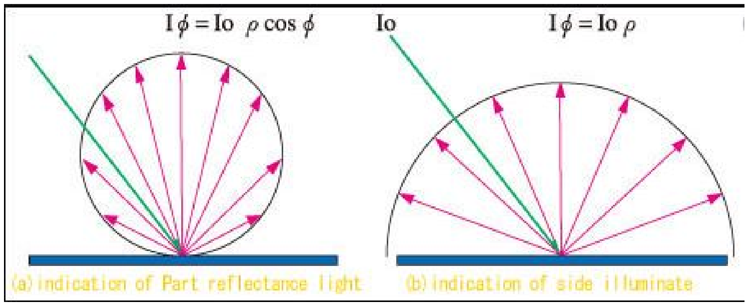
$$i = \frac{de}{d\omega} \text{ or } e = \int i d\omega$$

  integration is over all directions encompassed by a hemisphere.

  Thus, $e = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} i \sin\psi d\psi d\phi$

  Lambert's law - $i = i_n \cos\psi$,where i_n is the normal intensity, such a surface is called diffuse surface. So for a diffused surface we get

$$e = \pi i_n$$



- Kirchoff's Law

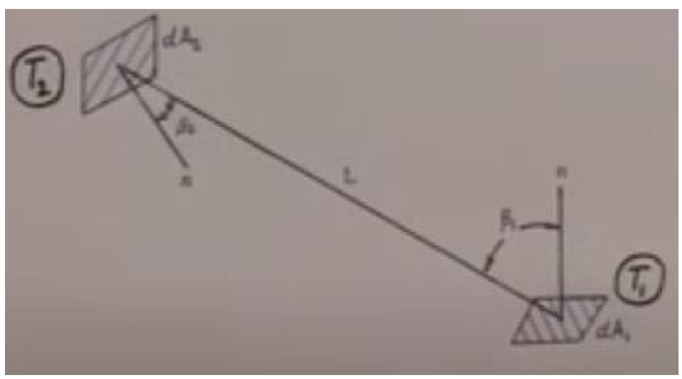
  If a surface emits in a diffuse manner, the monochromatic emissivity of the surface is equal to its monochromatic absorptivity.

  $$\epsilon_\lambda = \alpha_\lambda$$

  In addition if surface is Gray called 'diffuse-gray' surface we have,

  $$\epsilon = \alpha$$

- Heat Transfer by Radiation between two Arbitrarily placed black surface elements



Intensity of radiation i emitted by dA_1 in direction of dA_2 is

$$(\frac{\sigma T_1^4}{\pi})\cos\beta_1$$

Solid angle $(d\omega)$ subtended by dA_2 at dA_1 is

$$\frac{dA_2\cos\beta_2}{L^2}$$

Rate at which radiation emitted by dA_1 flows towards dA_2 is

$$\begin{aligned}
&= i.d\omega.dA_1 \\
&= (\frac{\sigma T_1^4}{\pi}\cos\beta_1)(\frac{dA_2\cos\beta_2}{L^2})dA_1 \\
&= (\frac{\cos\beta_1\cos\beta_2 dA_2}{\pi L^2})\sigma T_1^4 dA_1 \\
&= (dF_{1-2})(\sigma T_1^4 dA_1)
\end{aligned}$$

the term $dF_{1-2} = \dfrac{\cos\beta_1\cos\beta_2 dA_2}{\pi L^2}$ is called the **Shape factor** of dA_1 w.r.t dA_2.

Shape factor :- it's a number between 0 and 1 which stands for the fraction of radiant heat flow rate emitted from dA_1 that is intercepted by dA_2. It depends only on geometry and orientation of the two surfaces.

Similarly, rate at which radiation emiited by dA_2 flows towards dA_1 and is absorbed by it is

$$(dF_{2-1})\sigma T_2^4 dA_2$$

where $dF_{2-1} = \dfrac{\cos\beta_1 \cos\beta_2 dA_1}{\pi L^2}$

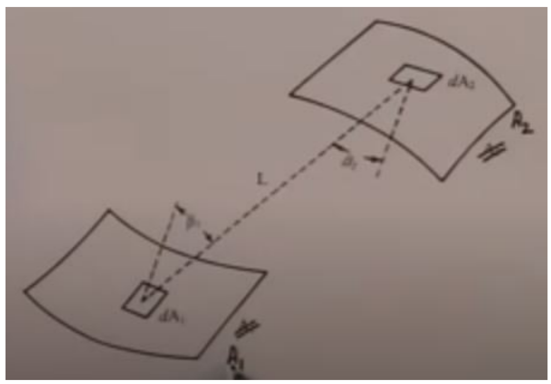
Thus, net radiative heat exchange rate (dq_{12}) between dA_1 and dA_2

$$\begin{aligned}
&= \frac{\sigma}{\pi}\frac{\cos\beta_1 \cos\beta_2 dA_1 dA_2}{L^2}(T_1^4 - T_2^4) \\
&= (dF_{1-2})\sigma(T_1^4 - T_2^4)dA_1 \\
&= (dF_{2-1})\sigma(T_1^4 - T_2^4)dA_2
\end{aligned}$$

Note that $(dF_{1-2})dA_1 = (dF_{2-1})dA_2$.

3.8 Radiation 4

- Heat exchange between two finite black surfaces



if A_1 and A_2 are at uniform temperature say T_1 and T_2 respectively, then

$$q_{1-2} = [\int_{A_1}\int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi L^2} dA_2 dA_1]\sigma(T_1^4 - T_2^4)$$

Defining the shape factor of surface 1 w.r.t 2

$$F_{1-2} = \frac{1}{A_1}[\int_{A_1}\int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi L^2} dA_2 dA_1]$$

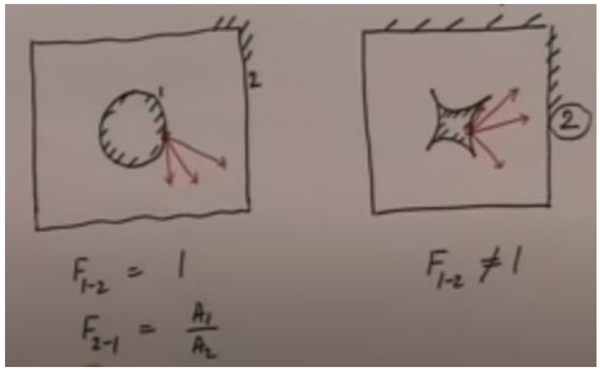
we have, $q_{12} = F_{1-2}\sigma(T_1^4 - T_2^4)$ also $F_{1-2}A_1 = F_{2-1}A_2$

If (A_1) is small compared to distance between them, then $dA_1 \to A_1$

$$\begin{aligned}
q_{12} &= [\int_{A_2} \frac{\cos\beta_1 \cos\beta_2}{\pi L^2} dA_2]\sigma A_1(T_1^4 - T_2^4) \\
&= F_{1-2}\sigma A_1(T_1^4 - T_2^4)
\end{aligned}$$

- Remarks on Shape factor

The shape factor (F_{1-2}) is unity when all radiation from 1 is intercepted by 2 i.e. 2 completely surrounds 1 and 1 is convex in shape, if its concave then some radiation from 1 will go back again to 1. So reciprocal relation is very useful when one of the shape factor is unity. A surface has a shape factor w.r.t itself if it is concave, so in second figure below $F_{1-1} \neq 0$



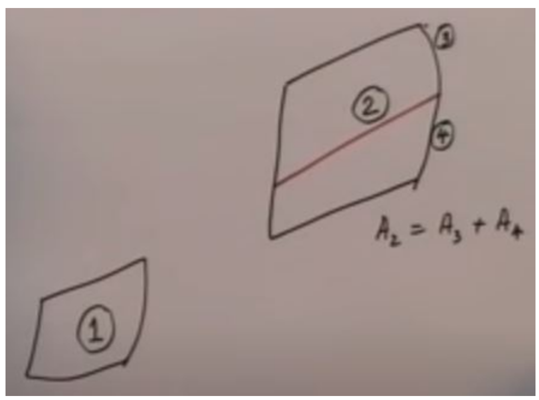
The value of Shape factor depends only on geometry and orientation, it has been calculated for a wide variety of situations in practice, and available to us in form of graphs, equations, tables etc.

If n surfaces make up an enclosure, then $\forall j \in (1, n-1)$

$$\sum_{i=1}^{n} F_{j-i} = 1$$

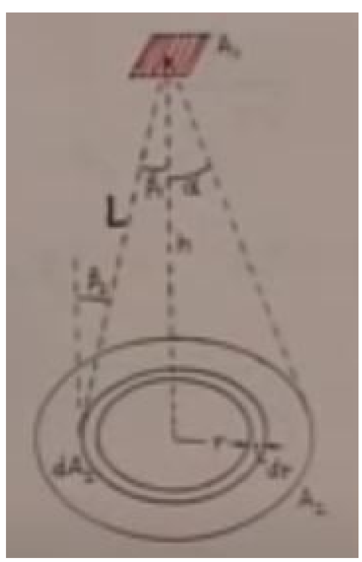
In the figure below we can say that by the defination of Shape factor,but the reverse is not true

$$F_{1-2} = F_{1-3} + F_{1-4} \text{ but } F_{2-1} \neq F_{3-1} + F_{4-1}$$

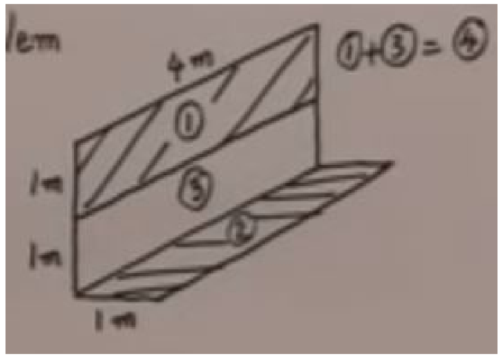


- Some Shape factors

  Between a small area $A-1$ and a parallel circular disk A_2, is calculated to be $F_{1-2} = \sin^2 \alpha$



For the figure below, we already know $F_{3-2}, F_{2-3}, F_{4-2}, F_{2-4}$

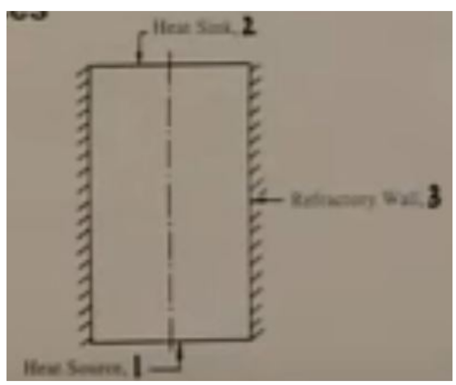


$$\begin{aligned} F_{1-2} &= \frac{A_2}{A_1} F_{2-1} \\ &= \frac{A_2}{A_1}(F_{2-4} - F_{2-3}) \\ &= 0.34 - 0.27 \\ &= 0.07 \end{aligned}$$

3.9 Radiation 5

- Radiant heat Exchange in an Enclosure Having Black surfaces

  Surfaces 1 and 2 are maintained at temperature T_1 and T_2 with q_1 and q_2 as the rates at which heat is supplied to them respectively. Surface 3 is insulated completely at temperature T_3.



The Solution for the following problem is found to be,

$$q_1 = -q_2 = (e_{b1} - e_{b2})[A_1 F_{1-2} + \frac{A_1 F_{1-3} F_{3-2}}{F_{3-2} + F_{3-1}}]$$

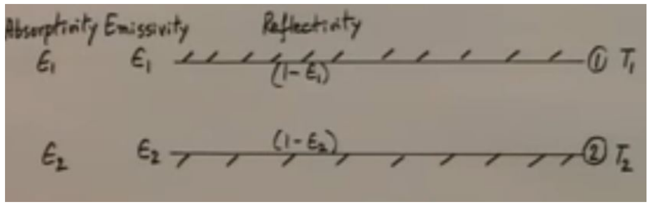
$$T_3 = [\frac{F_{3-2} T_2^4 + F_{3-1} T_1^4}{F_{3-2} + F_{3-1}}]^{\frac{1}{4}}$$

in addition if $F_{1-1} = F_{2-2} = 0$,

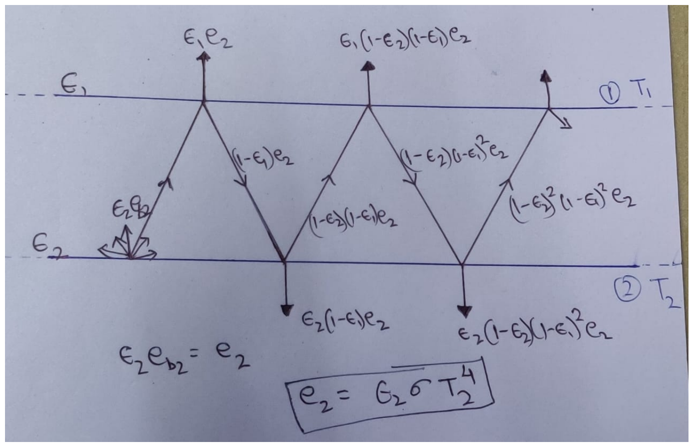
$$q_1 = -q_2 = (e_{b1} - e_{b2}) A_1 [\frac{A_2 - A_1 F_{1-2}^2}{A_1 + A_2 - 2A_1 F_{1-2}}]$$

$$T_3 = [\frac{(A_2 - A_1 F_{1-2}) T_2^4 + A_1 (1 - F_{1-2}) T_1^4}{A_1 + A_2 - 2A_1 F_{1-2}}]^{\frac{1}{4}}$$

- Heat Exchange by radiation between two infinite parallel Diffuse-Gray surfaces



Here both the surfaces are considered opaque. The radiation emitted by surface 2 reaches surface 1 and is partly absorbed and partly reflected, the reflected part again reaches 2 and is partly absorbed and partly reflected, and this goes on. The same is for emitted radiation from surface 1.



Hence, net exchange of radiant heat flux between 1 and 2 is

$$(\frac{q}{A})_1 = -(\frac{q}{A})_2 = \frac{\sigma(T_1^4 - T_2^4)}{(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1)}$$

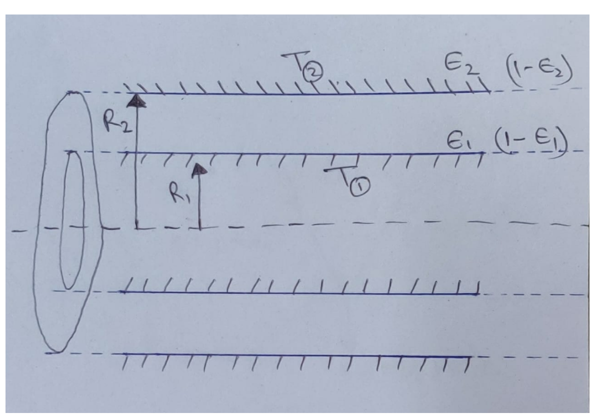
If n such similar plates are inserted between these two parallel plates such that emissivity for all the plates is same (ϵ) then,

$$(\frac{q}{A}) = \frac{1}{(n+1)} \frac{\sigma(T_1^4 - T_2^4)}{(\frac{2}{\epsilon} - 1)}$$

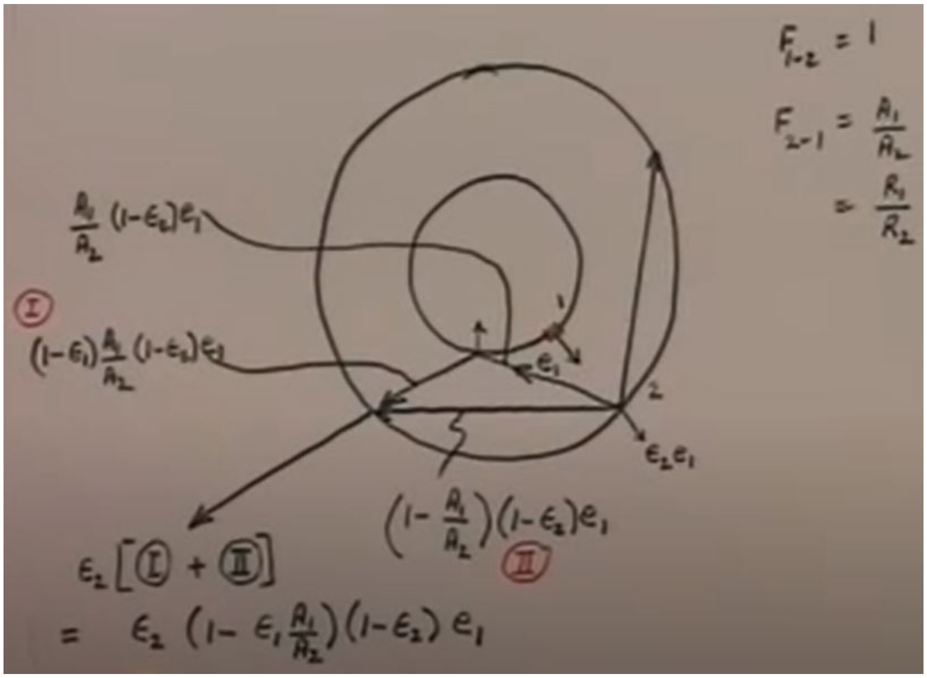
3.10 Radiation 6

- Heat exchange by radiation in the annular space between two infinitely long concentric cylinders

  Surfaces of both the cylinders 1 and 2 are maintained at T_1 and T_2, with emissivities ϵ_1 and ϵ_2 respectively. Surfaces are considered diffuse-gray.



Now here as 2 completely surrounds 1 and is convex $F_{1-2} = 1$ but $F_{2-1} \neq 1$, thus radiation emitted by 1 completely reaches 2 and it is partly reflected and partly absorbed. But now unlike the previous case of parallel plates as $F_{2-1} \neq 1$ part of reflected radiation reaches to itself and the left reaches 1, again it is partly absorbed and partly reflected. The reflected part again completely reaches 2 and is again partly absorbed and partly reflected, then again some part of the reflected radiation from 2 reaches itself and remaining to 1, and this phenomenon continues. The below figure describes the phenomenon pictorially.



$$q_1 = -q_2 = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}(\frac{1}{\epsilon_2} - 1)}$$

- While deriving the above equation we no-where used the fact that the geometry is cylindrical or that they are concentric, so it turns out that the above solution can be used at many places, doesn't matter what the geometry is or if it's concentric in nature. The only requirement for applying the above equation is that $F_{1-2} = 1$.

  Special case if $A_1 << A_2$, then we have $\frac{A_1}{A_2} \to 0$ thus,

$$q_1 = -q_2 = \sigma \epsilon_1 A_1 (T_1^4 - T_2^4)$$

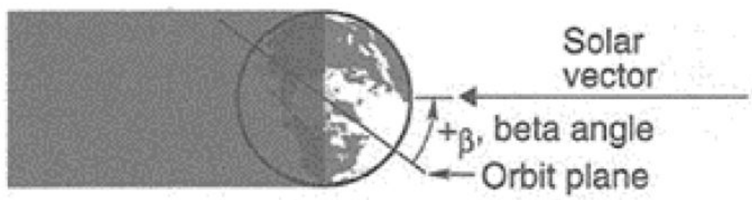
4 Factors Affecting Thermals of a LEO Satellite

[1]

Different heating fluxes are present during different stages of a satellite's life, including ground, ascent, and orbit. Heating in space is due mainly to satellite internal heat generation and radiation from the Sun and other celestial bodies. The time a satellite will spend in orbit is far longer than any other stage, so most of the focus of the thermal engineering effort is spent on ensuring the temperature stability of the satellite in space.

The altitude of a satellite in low Earth orbit (LEO) is between 150 km and 1000 km. This altitude lies above the outer limits of the atmosphere and below the Van Allen radiation belts. This means that the only external sources of heat that a satellite will be subjected to are direct solar radiation, Earth albedo radiation, and Earth infrared radiation. Effects from other celestial bodies, from elementary particle bombardment, and from space background are assumed to be negligible

Before analyzing environmental heating in LEO, it is important to define the orbit beta angle . The beta angle is the minimum angle between the orbit plane and the solar vector and can vary from -90 to +90 degrees, as shown in figure below. An orbit with a beta angle of zero will appear edgewise when viewed from the Sun. In an orbit like this a satellite will pass over Earth's subsolar point where the amount of radiation from the Sun reflected off the Earth will be highest, but this orbit also has the longest eclipse time. As the beta angle increases, the amount of radiation reflected off the Earth from the Sun decreases, but the eclipse time decreases as well. Eclipse time drops to zero for a circular orbit with a beta angle equal to 90 degrees, but there is also no radiation reflection off the Earth from the Sun in this type of orbit. Beta angles can be expressed as positive or negative. The beta angle is positive if the satellite is progressing in a counterclockwise direction around the orbit as seen from the Sun, and negative if clockwise.



4.1 Direct solar radiation

The radiant energy from the Sun is the most significant heat source in LEO. The radiation is nearly constant, and is equal along all directions. The lengthy distance of the Earth from the Sun allows for the assumption that the radiation propagates along parallel rays, leading to the term "solar vector," which is a vector with direction along the solar rays and a magnitude of the solar constant S.The Earth is farthest from the Sun during the northern hemisphere's summer, and the minimum value of the solar constant is about 1322 W/m^2. When the Earth is closest to the Sun, during the northern hemisphere's winter, the solar constant is about 1414 W/m^2.

The amount of radiation from the Sun impinging on a surface is characterized by the solar constant and the orientation of the object with respect to the Sun. The heat flux in W/m^2, as a result of direct solar radiation, absorbed by a surface depends upon the solar absorptivity α of the surface, the solar constant S, and the incident angle θ from the surface normal to the solar vector. Solar infrared radiation has much shorter wavelengths than those emitted by a body at room temperature, which is the typical satellite temperature. This allows for selection of surface finishes which have a low absorptivity in the short-wavelength part of the infrared spectrum, and a high emissivity in the long-wavelength part of the spectrum.

$$e_s = \alpha S A \cos\theta$$

4.2 Albedo radiation

The solar energy reflected off the Earth is referred to as "albedo radiation." The amount of albedo radiation incident upon a spacecraft is a function of spacecraft orientation and orbit and can be a significant source of radiation when the spacecraft is near the Earth. Albedo is considered to be in the same spectrum as solar radiation, and is calculated as a fraction of the solar constant. The heat flux due to albedo that is incident on a surface depends on the surface absorptivity α, the solar constant S, the albedo factor A_f, and the view factor F_{12} between the surface and the Earth.

$$e_{albedo} = \alpha S A_f F_{12}$$

The albedo factor is dependent upon the surface properties of the Earth. The amount of incident solar radiation reflected by the Earth varies between 25 % and 55%, depending on those surface properties.

The mean reflectivity calculated from the Earth Probe data is 30.4%. The results from the albedo model show that the received irradiance by a satellite in vicinity of Earth is dependent on altitude, solar angle, and position over Earth, including both latitude and longitude. The albedo at the 23 deg N latitude is calculated at 90 deg West and Greenwich Meridian. The results show that the albedo at 90 deg West (26.1%) is twice the amount of albedo at 0 deg (13.7%). The albedo at a position over the sunlit boundary is 6.0% at 500km altitude and 5.4% at 800km altitude. Due to the high solar angle to the polar regions, the albedo directly over the poles is between 33.8% and 36.7% even though the reflectivity is in excess of 90%. The maximum albedo is found over Greenland, where the irradiance is 49%. Finally, applying albedo compensation in attitude estimation, improves the maximum error from 9.9 deg to 1.9 deg. The standard deviation is reduced more than a factor three.[2]

4.3 Earth IR

The IR energy emitted by the Earth can vary with season, latitude, the local temperature of the Earth's surface, and the amount of cloud cover. These localized variations can be significant, but they are far less severe than variations in albedo. The Earth IR intensities of interest for satellites are the long-term averages, so the variations are not of great concern.

The Earth IR energy absorbed by a surface is a function of the Earth's temperature and the orientation of the surface with respect to Earth. The heat flux, in W/m^2, absorbed by the surface of a satellite depends on the Stefan-Boltzmann constant σ, the surface emissivity ϵ, the view factor between the Earth and the surface F_{12}, and the effective ideal radiator, or black body, temperature of the Earth T_E. The effective black body temperature of the Earth is on average 255 K.

Earth emitted radiation is long-wave infrared radiation, which is the same band of radiation normally emitted by satellites. This explains why the fraction of Earth IR radiation absorbed by a satellite is determined by its emissivity ϵ. This also means that a surface finish chosen in order to reflect Earth IR radiation will also reduce the surface's radiation emission ability. The average value of the Earth emitted infrared radiation is 230 W/m^2, but on a short time scale, this can vary between 150 and 350 W/m^2.[2]

4.4 Internal Heat Generation

The internal heat generation results from energy dissipation by components that are necessary for the satellite functions. Component heat dissipation may vary around the orbit and at different times during the mission due to the requirements of different mission phases. For satellites without moving parts, the electrical-power draw for components will be converted entirely to heat. The total amount of heat produced by a satellite will then depend only on the power consumption of the components.

5 Thermal Analysis

The purpose of satellite thermal analysis is to predict the temperatures of the satellite under known or assumed environmental conditions. The analysis starts with the identification of component temperature limits and heat dissipation. Thermal boundary conditions for each mission phase must also be identified, including spacecraft altitude and orientation relative to the Sun and Earth.

5.1 Thermal Energy Balance

Thermal control of a satellite on orbit is typically accomplished by balancing the energy absorbed from the environment and generated internally to the energy stored and emitted by the satellite as IR radiation, which follows the law of conservation of energy. This thermal energy balance is shown by first equation below. The heat entering the satellite comes from the external heating fluxes experienced in LEO. The heat flux in W/m^2 entering the satellite is a combination of all the external heat fluxes incident on the satellite in the LEO environment, as shown in the second equation below. The total heat flux incident on each of the satellite's external surfaces should be considered when using this approach to determine the total absorbed heat flux.

The heat generated is the total heat dissipated by the satellite components. Stored heat is a function of satellite mass and thermo-physical properties including density and specific heat. The energy stored is also called the heat capacity, thermal mass, or thermal capacitance. The heat storage capability of a material is represented by both the heat capacity and the specific heat c_p, where the heat capacity expresses it "per unit volume" and the specific heat expresses it "per unit mass." The heat stored in a volumetric element of a material in watts, which depends on the material density ρ and specific heat, is given by in third equation below. Heat rejected from the satellite occurs through radiation from the satellite external surfaces. It should be noted that this energy balance does not take into account effects due to conduction.

$$\dot{Q}_{in} + \dot{Q}_{generated} = \dot{Q}_{stored} + \dot{Q}_{out}$$

$$q_{in} = q_{solar} + q_{albedo} + q_{IR}$$

$$\dot{Q}_{stored} = \left(\rho c_p \frac{\delta T}{\delta t}\right) \Delta x \Delta y \Delta z$$

5.2 Thermal Extrema Analysis Cases

order to define upper and lower bounds on temperature predictions, and to account for errors and uncertainties, thermal engineers use hot and cold cases in their analyses. This analysis approach of designing to meet specified temperatures even under accumulated biases builds confidence in the model. The parameters used for these cases are chosen such that the resulting thermal loads are as extreme as the satellite will realistically experience during its lifetime. The temperatures reached during normal on-orbit operating conditions will lie between the temperatures reached during the hot and cold case operating conditions. Hot and cold case input parameters can include the solar vector, albedo factors, component dissipation, beta angle, and altitude.

5.2.1 Hot Case

For a hot case analysis, input data will be chosen such that the resulting temperatures are as high as the satellite may experience during its mission. Such input data for the hot case will include the highest values for many parameters that are addressed here. The solar vector S may be chosen as the average value, but a more extreme and potential hot case for a satellite may occur when the Earth is closest to the Sun, with a solar constant of about 1419 W/m^2. Albedo factors may be chosen in a similar way, with those occurring during the warmest time of year chosen for this case, with the highest reasonable albedo factor measuring 0.55. Any other properties that are approximated in the analysis should also be chosen to result in higher temperatures. The power profile for components in a hot case analysis will correspond to the mission mode resulting in the greatest component heat dissipation.

5.2.2 Cold Case

Parameters chosen for a cold case should be selected to result in temperatures as low as the satellite may realistically experience during its mission. This will include the lowest solar vector S of about 1317 W/m^2 and the lowest albedo factors Af of 0.18. Any other approximations made in the analysis should also be chosen to result in low temperatures. The power profile for components in a cold case analysis should not be assumed zero, but should correspond to the mission mode resulting in the least amount of heat dissipation, likely a safety mode.

5.3 Numerical Approximation Methods

The more practical approach to solving the problem of predicting satellite temperatures is to use simplifications and numerical approximation methods applied to a thermal model of a satellite. The approximation methods of thermal modeling subdivide the satellite into nodes or elements that are connected by conduction and radiation. In order to create a thermal model, engineers must first configure nodes or elements to realistically represent the actual system. The total number of nodes or elements for a model will depend on the satellite size, complexity, and nodal resolution required. Engineers must also define heat flow paths between nodes or elements using conductors, and include heating or cooling rates at necessary locations in the model. Once the thermal model has been constructed, the numerical approximation methods are applied, usually using computer software, to calculate the temperatures.

5.3.1 Finite Difference Approximation

The finite difference approximation method (FDM) determines the solution to a finite difference model that approximates the actual satellite using nodes. Each node represents a concentration of parameters at a single point in the thermal system. The nodes are connected using conduction and/or radiation heat transfer principles. This method uses Taylor series approximation to construct a system of finite difference equations. The finite difference equations are then converted to a set of algebraic equations that can solved to find the temperatures using iterative techniques, matrix inversion schemes, or decomposition methods. This approach of using finite difference node meshes to make up the thermal model is also referred to as "lumped-parameter representation".

5.3.2 Finite Element Approximation

The finite element approximation method (FEM) utilizes elements to create the thermal model. Different element types that can be used have different shapes and a different number of nodes. Each element has nodes at its corners, where parameter values such as temperatures are calculated. The parameters can vary across the element, and can be found using interpolation functions. The ultimate purpose of this method is to create a set of algebraic equations for the temperatures of the elements and nodes, which can be done using the Galerkin method of weighted residuals. The accuracy of this method can be improved by using a mesh with more elements.

5.3.3 Steady State

In steady state analysis calculations, the heat flux entering the spacecraft and the heat flux leaving the spacecraft is constant. Steady state, by definition, means that the temperature and heat flux at a point in a body will not change with time. However, the temperature may vary from one point to another.

5.3.4 Transient

A realistic practical thermal design for a satellite is generally based on transient considerations. However, the starting point for a transient analysis calculation is the temperature distribution found from the steady state calculation. The transient calculation will result in the satellite temperature history at successive time intervals. Smaller time intervals will result in more accurate temperature calculations. A standard technique employed to verify a transient analysis in to repeat the calculations at smaller and smaller time steps and to observe the trend of convergence to an asymptote. For a transient model, the temperature will typically vary with time as well as location.

6 Cube Problem

6.1 No External heat load

Considering a cube, with $100W$ heat on each surface, the temperature of the cube is found to be $204K$ if the cube is considered a black body.

$$\sigma A T^4 = 100$$

here $\sigma = 5.670 * 10^{-8} W/m^2K^4$ and area is taken as $1m^2$

6.2 External Heat Load of Sun

Here sun is considered at special different orientations with respect to cube, such that the sun vector is incident normally on a face, incident on the edge and incident along the body diagonal. First let us consider that the sun vector makes angle ϕ with the normal at any surface, S is the solar constant and α is the absorptivity,

$$e_s = (\alpha S \cos\phi)A$$

And let the Total heat generation be J. (J = 100, here)

We solve the governing equations for temperature numerically using the python function **scipy.optimize.fsolve()** and get various desired results.

6.2.1 Normal Incidence on a face

Let the sun vector be incident on surface 1 (T_1) , intuitively all adjacent surfaces will have the same temperature T_2, and no incidence on surface 3 (T_3), intuitively we can say that $T_1 > T_2 > T_3$.

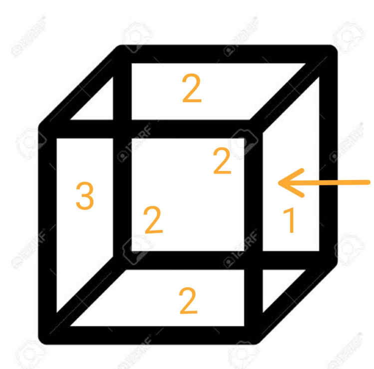


Figure 3: Normal incidence of Sun Vector

Noting that $\phi = 0$ here and in each conduction term we approximate

$$(\frac{\partial T}{\partial L})_{12} = \frac{T_1 - T_2}{L}$$

the governing equations for Surfaces 1,2 and 3 respectively will be as follows:

$$e_s + J = \sigma\epsilon A T_1^4 + 4F_{12}A\sigma\epsilon\alpha(T_1^4 - T_2^4) + F_{13}A\sigma\epsilon\alpha(T_1^4 - T_3^4) + 4KA'(\frac{\partial T}{\partial L})_{12}$$

$$F_{12}\sigma A\epsilon\alpha(T_1^4 - T_2^4) + KA'(\frac{\partial T}{\partial L})_{12} + J = \sigma\epsilon A T_2^4 + KA'(\frac{\partial T}{\partial L})_{23} + F_{23}\sigma A\epsilon\alpha(T_2^4 - T_3^4)$$

$$4F_{23}\sigma A\epsilon\alpha(T_2^4 - T_3^4) + F_{13}\sigma A\epsilon\alpha(T_1^4 - T_3^4) + 4KA'(\frac{\partial T}{\partial L})_{23} + J = \sigma A\epsilon T_3^4$$

Plugging in values for Aluminium and taking $\epsilon = 0.8$ and $\alpha = 0.15$, it yields

$$T_1 = 247.34K$$
$$T_2 = 231.01K$$
$$T_3 = 228.40K$$

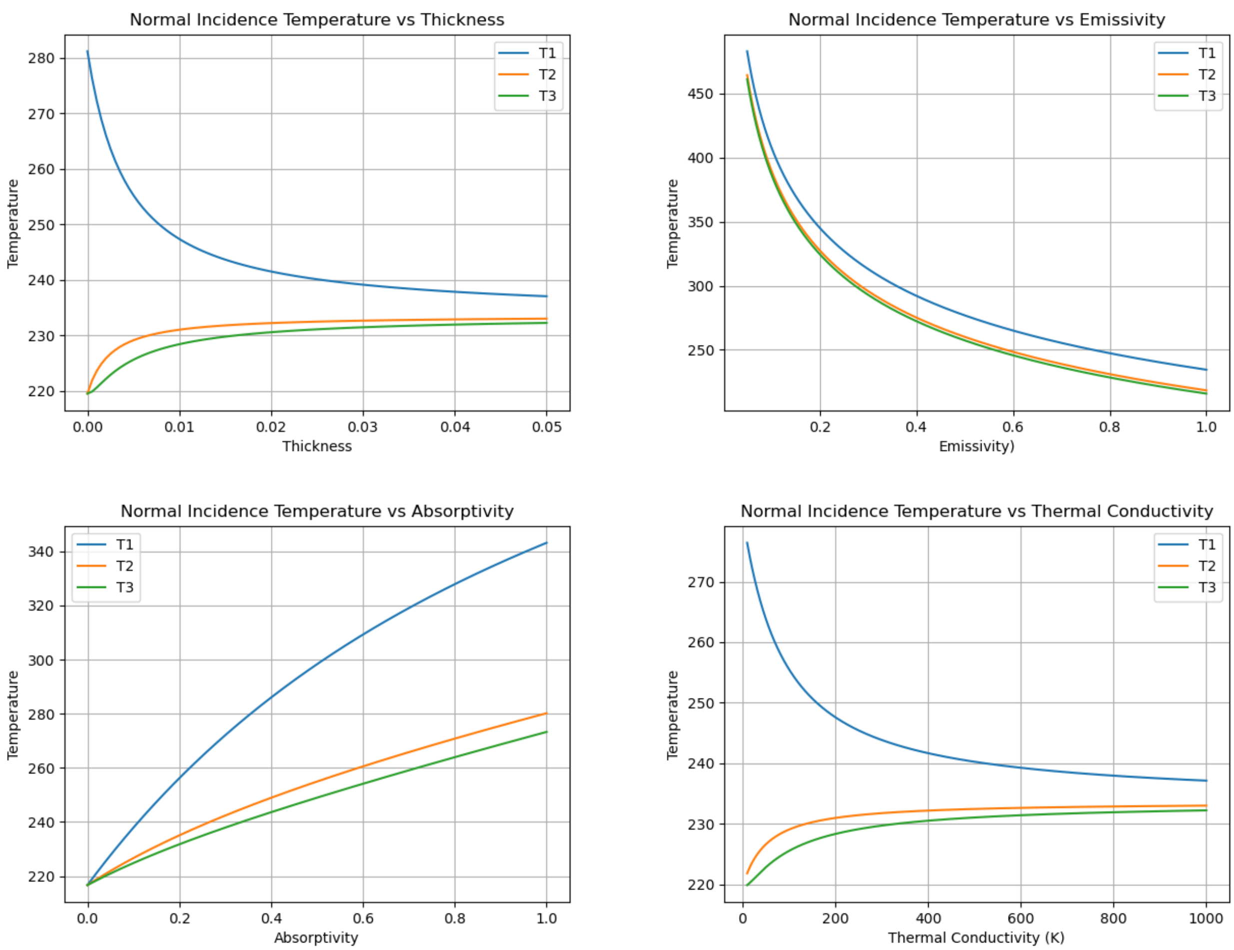
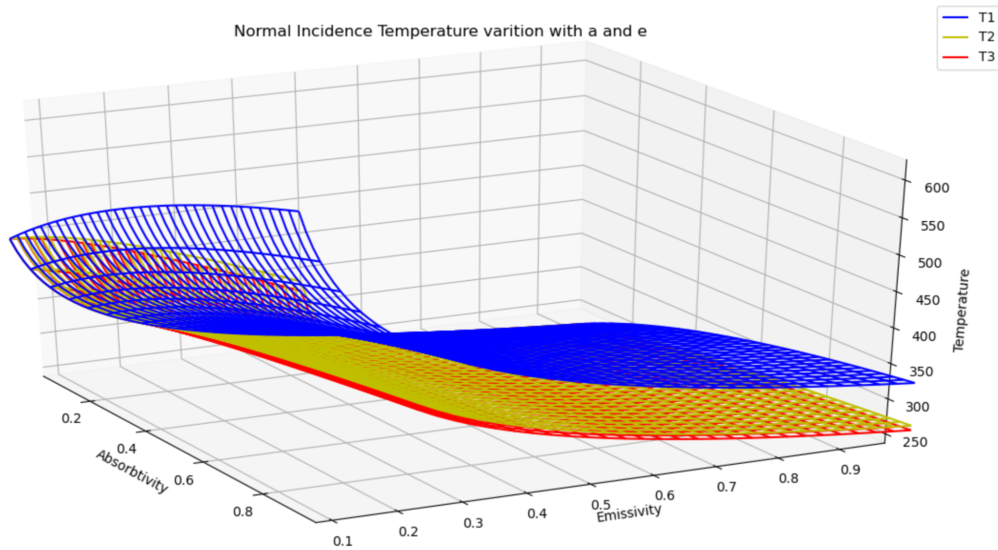


Figure 4: Variation with different parameters for Normal Incidence



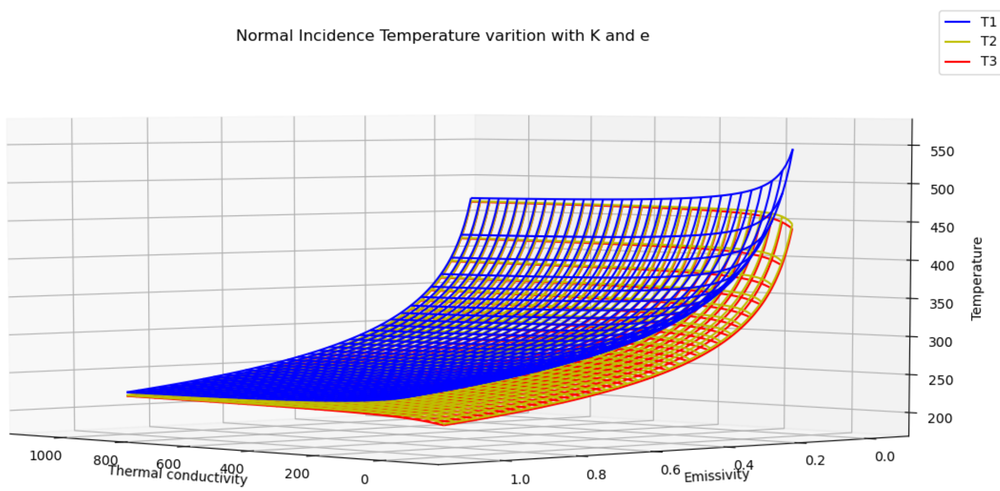


Figure 5: 3D plot

6.2.2 Incidence on a Edge

In this case $\phi = 45deg$ and also intuitively $T_1 > T_3 > T_2$.

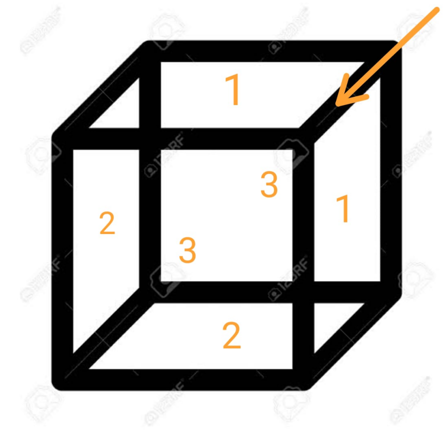


Figure 6: Sun Vector incident on edge

The governing equations for temperature of Surfaces 1,3 and 2 respectively will be:

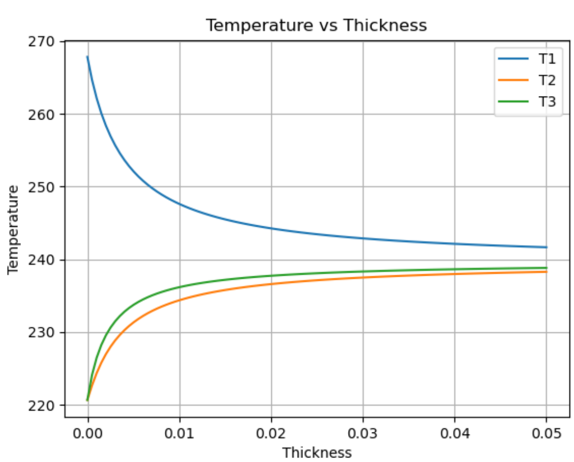
$$e_s + J = 2KA'(\frac{\partial T}{\partial L})_{13} + KA'(\frac{\partial T}{\partial L})_{12} + 2F_{13}\sigma A\epsilon\alpha(T_1^4 - T_3^4) + 2F_{12}\sigma A\epsilon\alpha(T_1^4 - T_2^4) + \sigma A\epsilon T_1^4$$

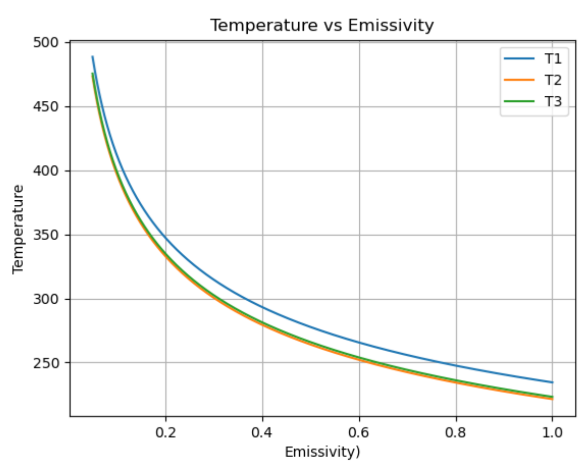
$$2KA'(\frac{\partial T}{\partial L})_{13} + 2F_{13}\sigma A\epsilon\alpha(T_1^4 - T_3^4) + J = \sigma A\epsilon T_3^4 + 2F_{32}\sigma A\epsilon\alpha(T_3^4 - T_2^4) + 2KA'(\frac{\partial T}{\partial L})_{32}$$

$$2F_{12}\sigma A\epsilon\alpha(T_1^4 - T_2^4) + 2F_{23}\sigma A\epsilon\alpha(T_3^4 - T_2^4) + KA'(\frac{\partial T}{\partial L})_{12} + 2KA'(\frac{\partial T}{\partial L})_{32} + J = \sigma A\epsilon T_2^4$$

Plugging in values for Aluminium and taking $\epsilon = 0.8$ and $\alpha = 0.15$, it yields

$$T_1 = 247.63K$$
$$T_2 = 234.36K$$
$$T_3 = 236.16K$$





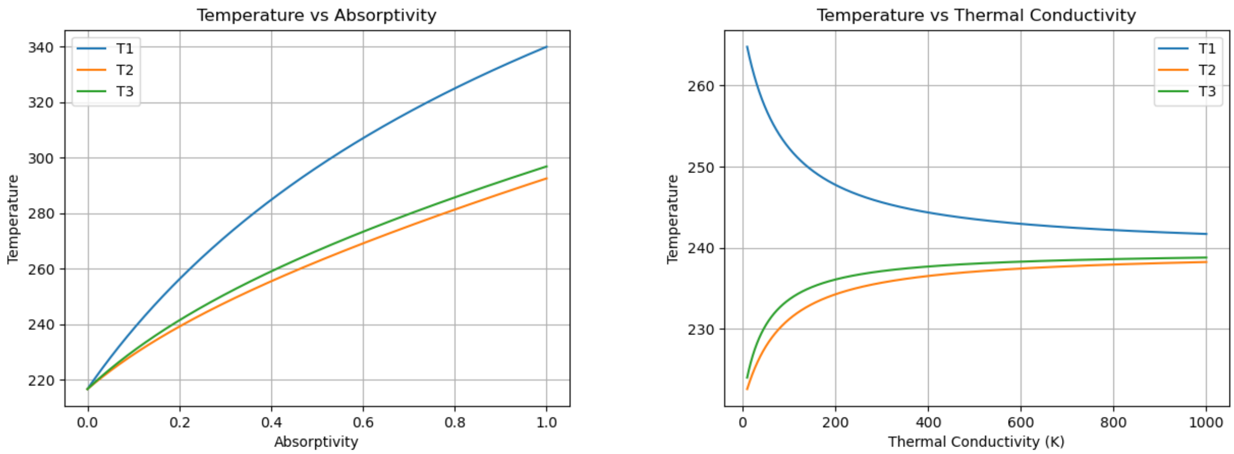


Figure 8: Variation with different parameters for incidence on edge

6.2.3 Incidence along Body Diagonal

Here $\phi = 54.74deg$ and also intuitively $T_1 > T_2$.

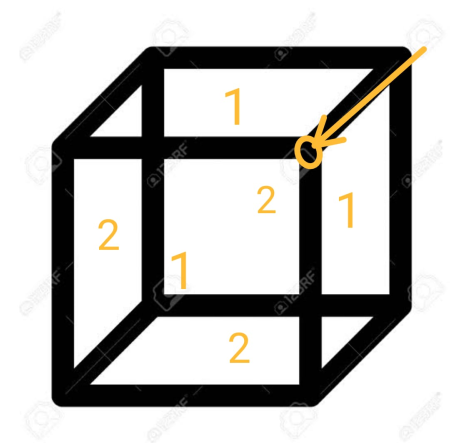


Figure 9: Sun Vector along body diagonal

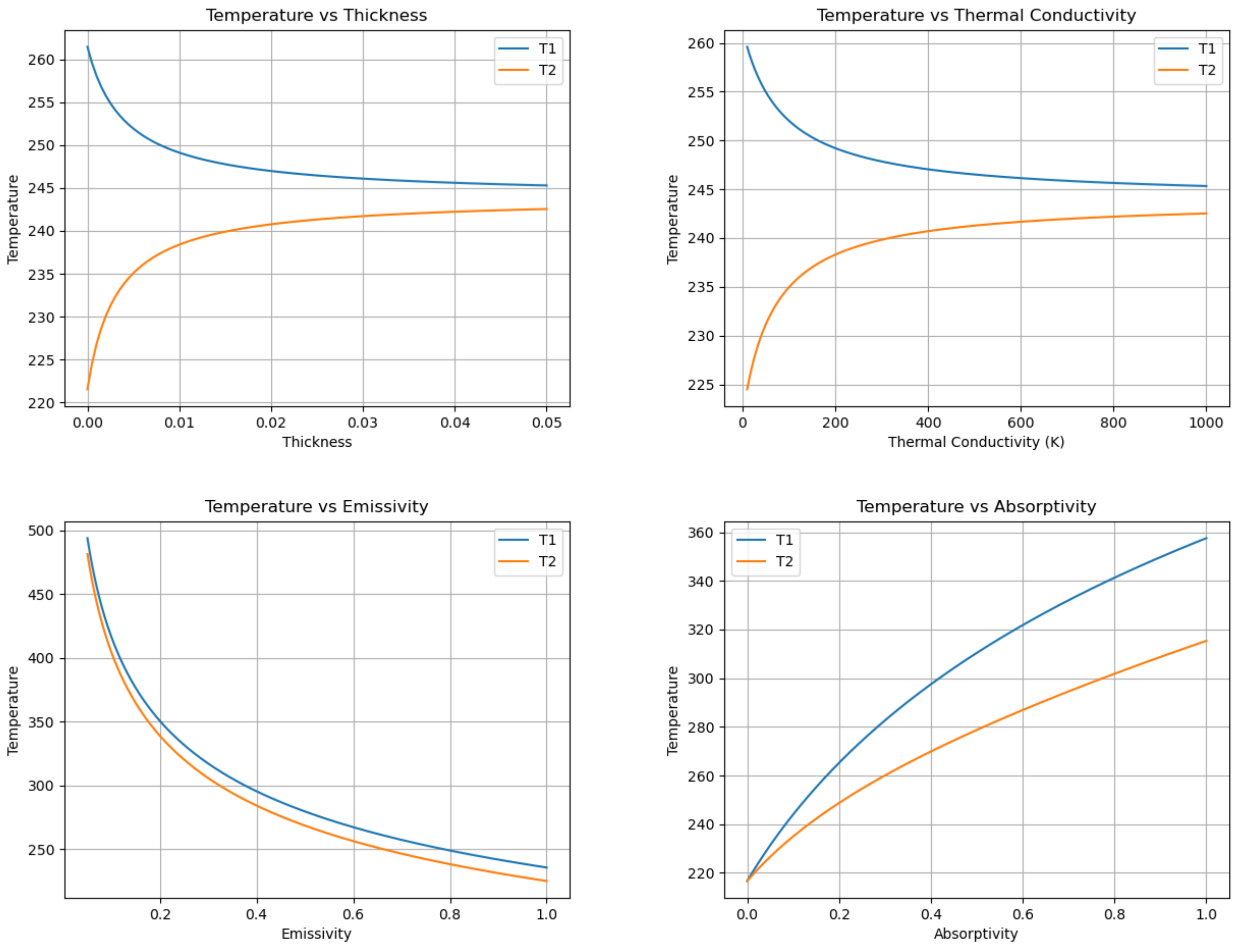
The governing equations for temperature of surfaces 1 and 2 respectively are:

$$e_s + J = 3F_{12}\sigma A\epsilon\alpha(T_1^4 - T_2^4) + \sigma A\epsilon T_1^4 + 2KA'(\frac{\partial T}{\partial L})_{12}$$

$$3F_{12}\sigma A\epsilon\alpha(T_1^4 - T_2^4) + 2KA'(\frac{\partial T}{\partial L})_{12} + J = \sigma A\epsilon T_2^4$$

Plugging in values for Aluminium and taking $\epsilon = 0.8$ and $\alpha = 0.15$, it yields

$$T_1 = 249.12K$$
$$T_2 = 238.39K$$



6.2.4 Results and Conclusion

After analysing all the graphs giving the variation of temperature with various parameters the following can be said:

- Variation with Thickness : It is intuitive that if we increase the thickness the conduction will increase, so we expect a decrease in larger temperature, and increase in lower temperatures of surfaces. The increase in T_2 in normal incidence can be said because of temperature difference between T_1 and T_2 which is far greater than difference between T_3 and T_2.

- It can be said that thickness doesn't affect T_2 and T_3 much as the increases in both of them is nearly 14 K, but there is nearly 40 K decrease in T_1, so increase in conduction helps largely in decreasing the temperature of surface exposed to sun vector.

- Variation of Temperature with Emissivity : As emissivity increases we have more heat energy going out of all surfaces so we expect a decrease in temperature of all the surfaces.

- The effect of change in emissivity seems similar for all the surfaces i.e. the curves have similar amount of decrease. The differences of temperature isn't changing much. For instance see at 0.4 (≈ 20) and 0.8 (≈ 23).

- Variation with Absorptivity : With increase in absorptivity every surface will absorb more amount heat energy, we expect all surface temperatures to increase.

- The surface exposed to solar flux seems to be affected greater by change in α than others which is evident by the great amount of increase in T_1 as compared to T_2 and T_3. For T_2 and T_3 the curves are nearly linear.

- Variation with Thermal Conductivity : Increase in K means more conduction, so the variation is pretty similar to variation with thickness case.

- Variation of Temperature with α and ϵ : We expect that for each value of α the 3D plot should have a contour line similar to the plot of Temperature vs Emissivity. And similarly for each value of ϵ a contour line similar to plot of Temperature vs absorptivity. So for every α the best solution should be when $\epsilon = 1$ and for every ϵ when $\alpha = 0$. So the best solution will be corresponding to α =0 and ϵ=1.
- Variation of Temperature with K and ϵ : For each value of K we expect the temperature to go down with increase in ϵ. Also for every value of K the contour line should be similar to plot of temperature vs emissivity and similarly for each ϵ the contour line should look similar to Temperature vs K.

6.3 Worst case Scenario

Considering everything from direct sun radiation, albedo, earth IR and internal heat generation the worst situation will be the one in more area of the cube is exposed to various radiations. So sun vector should be along the body diagonal ($\phi = 54.73\,\text{deg}$), and the same for albedo and Earth's IR. also the distance between earth and sun should be shortest so as to approach the Hottest case.

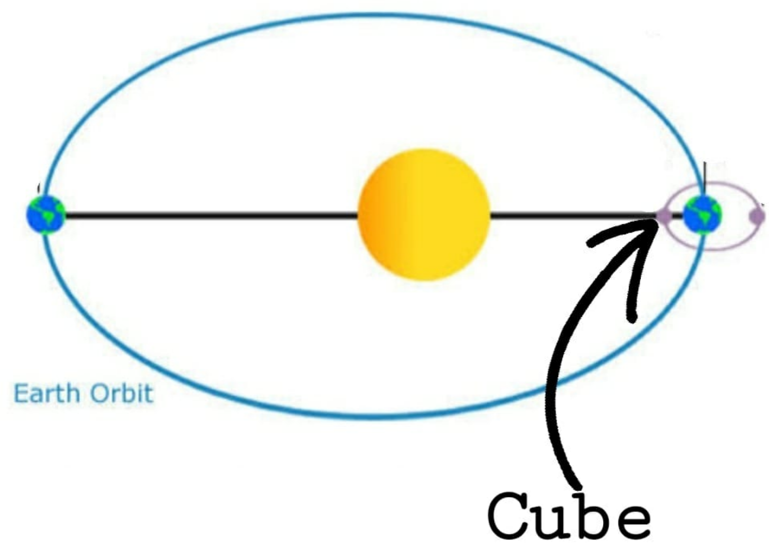


Figure 11: Position of cube for the worst case

References

[1] K. E. Boushon. (2018). Thermal analysis and control of small satellites in low earth orbit, [Online]. Available: `https://scholarsmine.mst.edu/cgi/viewcontent.cgi?article=8755&context=masters_theses`.

[2] D. Bhanderi and T. Bak. (2005). Modeling earth albedo for satellites in earth orbit, [Online]. Available: `https://www.researchgate.net/publication/268554976_Modeling_Earth_Albedo_for_Satellites_in_Earth_Orbit`.

A Appendix 1

Here are the general values of the constants that were used during analysis of the cube. The Values are in correspondence to Aluminium.

Sr. No.	Quantity	Symbol	Units (SI)	Value
1	Shape Factor	x	Unit less Quantity	0.2
2	Length of Edge	L	Meters (m)	1
3	Area of Face ($A = L^2$)	A	Meter sq. (m^2)	1
4	Stefan Boltzmann Constant	σ	$Jm^{-2}s^{-1}K^{-4}$	5.67x10^{-8}
5	Emissivity	ϵ	Unit less Quantity	0.8
6	Absorptivity	α	Unit less Quantity	0.15
7	Thermal Conductivity	K	$Wm^{-1}K^{-1}$	205
8	Thickness of Cube	w	Meters (m)	0.01
9	Area for Conduction ($A_0 = L * W$)	A_0	Meter sq. (m^2)	0.01
10	Solar Constant	S	Wm^{-2}	1400
11	Internal Heat Generation	J	W	100