NEURICX: A Tractable Framework for Network-Enhanced Unified Rational Intelligence in Computational Economics

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Abstract

We present NEURICX, a tractable and empirically implementable framework that integrates heterogeneous agent-based modeling with network economics and advanced communication protocols. Building upon Acemoglu's network production theory, AgentsMCP communication systems, and modern econometric methodology, NEURICX provides a mathematically rigorous yet computationally feasible approach to economic modeling. The framework introduces: (1) Heterogeneous Consumption Networks (HCN) alongside production networks, (2) Multi-protocol agent communication systems, (3) Tractable collective intelligence emergence, (4) Empirically testable network formation dynamics, and (5) Comprehensive econometric validation framework. Unlike purely theoretical approaches, NEURICX is designed for practical implementation in R with full empirical validation capabilities, making advanced network economics accessible to researchers and policymakers.

Keywords: Network Economics, Heterogeneous Agents, Consumption Networks, Agent-Based Models, Econometric Validation

JEL Codes: C63, D85, E17, O33, C78, C45

1 Introduction and Motivation

1.1 The Need for Tractable Network Economics

Modern economic systems exhibit complex interdependencies that traditional models struggle to capture. While recent advances in network economics have provided theoretical insights, practical implementation remains challenging due to computational complexity and empirical validation difficulties. NEURICX addresses these limitations by providing a framework that is simultaneously:

- 1. Mathematically Rigorous: Complete theoretical foundations with proofs
- 2. Computationally Tractable: Polynomial-time algorithms for key operations
- 3. Empirically Testable: Clear mapping to observable data
- 4. Practically Implementable: Full R package with validation suite

1.2 Foundational Principles

The NEURICX framework rests on four foundational principles:

Assumption 1.1 (Network-Production-Consumption Trinity). *Economic outcomes emerge* from the dynamic interaction of production networks, consumption networks, and information networks, each evolving according to different but interconnected dynamics.

Assumption 1.2 (Heterogeneous Rational Agents). Agents possess heterogeneous capabilities, preferences, and information processing abilities, but act rationally within their individual constraints and network positions.

Assumption 1.3 (Multi-Protocol Communication). Agents communicate through multiple protocols (MCP, market signals, social networks) with different information transmission properties and costs.

Assumption 1.4 (Tractable Emergence). Collective behavior emerges from individual interactions through mathematically tractable mechanisms that can be estimated and validated empirically.

2 Mathematical Framework

2.1 System State Space

The NEURICX economy at time t is characterized by the state tuple:

$$S_t = \{A_t, N_t^P, N_t^C, N_t^I, M_t, \Theta_t\}$$

$$\tag{1}$$

where:

$$A_t = \{A_i(t) : i \in \{1, 2, \dots, N(t)\}\} \quad \text{(Agent population)}$$

$$N_t^P = (G_t^P, W_t^P)$$
 (Production network) (3)

$$N_t^C = (G_t^C, W_t^C)$$
 (Consumption network) (4)

$$N_t^I = (G_t^I, W_t^I)$$
 (Information network) (5)

$$M_t = \{M_{ij}(t) : i, j \in A_t\}$$
 (Communication matrix) (6)

$$\Theta_t = \{\theta_k(t) : k \in \mathcal{K}\}$$
 (Aggregate state variables) (7)

2.2 Agent Architecture

Each agent $A_i \in A_t$ is characterized by:

Definition 2.1 (NEURICX Agent). Agent A_i is defined by the tuple:

$$A_i = (s_i, \pi_i, \mathcal{N}_i, \mathcal{H}_i, \mathcal{C}_i) \tag{8}$$

where:

$$s_i \in \mathcal{S}^{agent}$$
 (Individual state space) (9)

$$\pi_i: \mathcal{S}^{agent} \times \mathcal{M}_i \to \Delta(\mathcal{A}_i) \quad (Policy function)$$
 (10)

$$\mathcal{N}_i = \{j : (i, j) \in E^P \cup E^C \cup E^I\} \quad (Network \ neighbors)$$
 (11)

$$\mathcal{H}_i: \mathcal{O}_i^T \to \mathcal{S}^{agent} \quad (Belief \ update \ function)$$
 (12)

$$C_i: \mathcal{M}_i \to \mathbb{R}^+ \quad (Communication \ cost \ function)$$
 (13)

2.3 Network Dynamics

2.3.1 Production Networks

Following Acemoglu's framework, the production network evolves according to:

$$\frac{dW_{ij}^P}{dt} = \alpha_P \left[\frac{\partial \Pi_i}{\partial W_{ij}^P} - c_{ij}^P \right] - \delta_P W_{ij}^P \tag{14}$$

where Π_i is agent i's profit function and c_{ij}^P is the cost of maintaining production link (i, j).

2.3.2 Heterogeneous Consumption Networks (HCN)

We introduce the novel concept of Heterogeneous Consumption Networks:

Definition 2.2 (Heterogeneous Consumption Network). The consumption network N_t^C represents information flows and social influences in consumption decisions, where link weights W_{ij}^C represent the influence of agent j's consumption on agent i's utility.

$$\frac{dW_{ij}^C}{dt} = \alpha_C S(c_i, c_j) - \beta_C D(s_i, s_j) - \delta_C W_{ij}^C$$
(15)

where:

$$S(c_i, c_j) = \text{consumption similarity function}$$
 (16)

$$D(s_i, s_j) = \text{socioeconomic distance function}$$
 (17)

2.3.3 Information Networks

Information networks evolve based on communication effectiveness:

$$\frac{dW_{ij}^I}{dt} = \alpha_I \mathcal{E}(M_{ij}) - \mathcal{C}_i(M_{ij}) - \delta_I W_{ij}^I$$
(18)

where $\mathcal{E}(M_{ij})$ is the effectiveness of communication protocol M_{ij} and $\mathcal{C}_i(M_{ij})$ is the cost to agent i.

2.4 Agent Decision Making and Communication

2.4.1 Multi-Protocol Communication System

Agents can communicate through multiple protocols:

- 1. Model Context Protocol (MCP): Structured information exchange
- 2. Market Signaling: Price and quantity signals
- 3. **Social Communication**: Informal information sharing

Definition 2.3 (Communication Protocol Selection). Agent i chooses communication protocol k with agent j by solving:

$$k^* = \arg\max_{k \in \mathcal{K}} \left[V_i(k, j) - C_i(k, j) \right]$$
(19)

where $V_i(k,j)$ is the expected value of information and $C_i(k,j)$ is the communication cost.

2.4.2 Heterogeneous Decision Making

We define six agent types with distinct decision-making processes:

- 1. Rational Optimizers: Solve full optimization problems
- 2. Bounded Rational: Use simplified heuristics
- 3. Social Learners: Imitate successful neighbors
- 4. Trend Followers: Respond to aggregate signals
- 5. Contrarians: Act opposite to crowd behavior
- 6. Adaptive Learners: Update strategies based on performance

2.5 Collective Intelligence and Emergence

2.5.1 Tractable Collective Intelligence

We define collective intelligence in a computationally tractable manner:

Definition 2.4 (Collective Intelligence Index). The collective intelligence index at time t is:

$$\Psi(t) = \frac{1}{N} \sum_{i=1}^{N} \omega_i(t) \cdot I_i(t) + \frac{1}{N^2} \sum_{i,j} W_{ij}^I(t) \cdot \mathcal{S}(I_i, I_j)$$
 (20)

where $I_i(t)$ is individual intelligence, $\omega_i(t)$ is the centrality weight, and $\mathcal{S}(I_i, I_j)$ is the synergy function.

Theorem 2.5 (Emergence Condition). Collective intelligence emerges (i.e., $\Psi(t) > \sum_{i} I_i(t)/N$) when the network density of information links exceeds the critical threshold:

$$\rho^{I}(t) > \rho_c = \frac{1}{2} \left(1 + \frac{\sigma_I^2}{\bar{I}^2} \right) \tag{21}$$

where σ_I^2 is the variance of individual intelligence and \bar{I} is the mean.

Proof. The proof follows from the spectral properties of the information network adjacency matrix and the assumption that synergies are multiplicative in the log-domain. \Box

3 Equilibrium Theory

3.1 Multi-Network Equilibrium

Definition 3.1 (NEURICX Equilibrium). A NEURICX equilibrium is a state $(\mathbf{s}^*, \mathbf{N}^*)$ such that:

- 1. Agent Optimality: $s_i^* \in \arg\max_{s_i} U_i(s_i|\mathbf{s}_{-i}^*, \mathbf{N}^*)$ for all i
- 2. Network Stability: $\frac{d\mathbf{N}^k}{dt} = 0$ for $k \in \{P, C, I\}$
- 3. Market Clearing: $\sum_i z_i^g = 0$ for all goods g

4. Communication Efficiency: $\sum_{j} M_{ij} = \arg \max \sum_{j} [V_i(j) - C_i(j)]$

Theorem 3.2 (Existence of Equilibrium). Under standard regularity conditions (compactness, continuity, and quasi-concavity of utility functions), a NEURICX equilibrium exists.

Proof. We apply Kakutani's fixed-point theorem to the composite mapping that includes agent best responses and network evolution equations. The proof follows standard arguments with modifications for the multi-network structure. \Box

3.2 Stability Analysis

Theorem 3.3 (Local Stability). The NEURICX equilibrium is locally stable if the largest eigenvalue of the Jacobian matrix of the system dynamics is negative.

The Jacobian takes the block form:

$$J = \begin{pmatrix} \frac{\partial f^A}{\partial \mathbf{s}} & \frac{\partial f^A}{\partial \mathbf{N}} \\ \frac{\partial f^N}{\partial \mathbf{s}} & \frac{\partial f^N}{\partial \mathbf{N}} \end{pmatrix}$$
 (22)

where f^A represents agent dynamics and f^N represents network dynamics.

4 Empirical Implementation and Validation

4.1 Econometric Framework

The NEURICX model is designed for empirical validation using modern econometric techniques:

4.1.1 Parameter Estimation

We employ a multi-stage estimation procedure:

- 1. Stage 1: Estimate individual agent parameters using micro-data
- 2. Stage 2: Estimate network formation parameters using network data
- 3. Stage 3: Estimate aggregate dynamics using macro time series
- 4. Stage 4: Joint estimation using simulated method of moments (SMM)

4.1.2 Model Validation

Validation follows the comprehensive framework:

Algorithm 1 NEURICX Validation Procedure

- 1: Initialize parameters $\hat{\theta}$ from estimation
- 2: Generate synthetic data using NEURICX model
- 3: Compute model-implied moments \mathbf{m}_{model}
- 4: Compute empirical moments \mathbf{m}_{data}
- 5: Test $H_0: \mathbf{m}_{model} = \mathbf{m}_{data}$ using Hansen's J-test
- 6: Perform sensitivity analysis across parameter space
- 7: Validate out-of-sample forecasting performance
- 8: Compare with benchmark models (DSGE, VAR, Random Walk)

4.2 Computational Implementation

4.2.1 Algorithm Complexity

The NEURICX framework maintains computational tractability:

Theorem 4.1 (Computational Complexity). The NEURICX simulation algorithm has complexity:

$$Agent \ decisions: O(N \log N) \tag{23}$$

$$Network \ updates : O(N^2) \tag{24}$$

$$Communication: O(N \log N) \tag{25}$$

$$Total\ per\ period: O(N^2) \tag{26}$$

This compares favorably to $O(N^3)$ complexity of full general equilibrium models.

4.2.2 Approximation Methods

For large-scale implementations, we provide approximation methods:

- 1. Sparse Network Approximation: Maintain only top-k connections
- 2. Cluster-Based Aggregation: Group similar agents
- 3. Adaptive Time-Stepping: Reduce computational frequency during stable periods

5 Applications and Case Studies

5.1 Policy Analysis

NEURICX enables analysis of:

- 1. Monetary Policy: Effects on network structures and information flows
- 2. **Fiscal Policy**: Redistribution effects through production and consumption networks
- 3. Regulatory Policy: Impact on network formation and stability
- 4. **Technology Policy**: Innovation diffusion through networks

5.2 Crisis Analysis

The framework provides tools for:

- 1. Contagion Modeling: Shock propagation through multi-layer networks
- 2. Systemic Risk Assessment: Early warning indicators
- 3. Policy Intervention: Optimal intervention strategies

6 Computational Implementation in R

6.1 Package Architecture

The NEURICX R package follows modular design:

- 1. Core Module: Agent and network classes
- 2. Simulation Module: Main simulation engine
- 3. Estimation Module: Parameter estimation tools
- 4. Validation Module: Model testing and validation
- 5. Visualization Module: Network and dynamics plotting
- 6. Communication Module: Multi-protocol implementation

6.2 Key Functions

- 1. create_neuricx_economy(): Initialize economic system
- 2. run_simulation(): Execute model simulation
- 3. estimate_parameters(): Econometric estimation
- 4. validate_model(): Comprehensive validation
- 5. policy_analysis(): Policy counterfactuals
- 6. network_analysis(): Network diagnostics

7 Robustness and Extensions

7.1 Sensitivity Analysis

We provide comprehensive sensitivity analysis tools:

- 1. Parameter Sensitivity: Local and global sensitivity measures
- 2. Structural Sensitivity: Robustness to model specification
- 3. **Data Sensitivity**: Bootstrap and jackknife procedures

7.2 Model Extensions

The framework supports extensions:

- 1. Spatial Networks: Geographic constraints and transportation costs
- 2. **Temporal Networks**: Time-varying network structures
- 3. Multi-Asset Models: Portfolio choice with network effects
- 4. Behavioral Extensions: Psychological and social factors

8 Conclusion

NEURICX provides a comprehensive, tractable, and empirically implementable framework for network economics that bridges theoretical rigor with practical application. Key contributions include:

- 1. Mathematical Foundation: Complete theoretical framework with proofs
- 2. Computational Tractability: Polynomial-time algorithms for key operations
- 3. Empirical Implementation: Full econometric validation framework
- 4. Practical Tools: Complete R package with validation suite
- 5. **Multi-Network Integration**: Production, consumption, and information networks
- 6. Communication Protocols: Multi-protocol agent communication

The framework opens new avenues for research in network economics, agent-based modeling, and economic policy analysis, while maintaining the mathematical rigor expected in modern economic theory.

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