Energy price jumps, fat tails and climate policy

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Abstract: Many authors who have analyzed key energy prices, such as crude oil and natural gas, have found that these prices exhibit "fat tails" – the feature that large percentage changes occur far more often that would be predicted by a conventional model. These fat tails can arise either because of time-varying volatility or because of rapid, unexpected changes — also known as jumps. Addressing global climate change is likely to require broad-based deployment of new infrastructure. This new infrastructure is likely to be both costly to build and difficult to reverse – suggesting the deployment of new infrastructure is an example of "investment under uncertainty" (Dixit and Pindyck, 1993). In this context, a key concept is the "option value of waiting," *i.e.*, the potential gain in value that arises from waiting to learn more about the evolution of some key underlying stochastic ingredient, such as a commodity price or the cost of a carbon permit. We argue that this option value of waiting is likely to be increased by the presence of jumps. Assuming there is some urgency in undertaking these investments, the increase in option value of waiting is worrisome, and motivates the deployment of a policy intervention that reduces this option value.

Keywords: Energy prices, jumps, fat tails, investment under uncertainty, climate policy

1 Introduction

Global climate change has become the central environmental concern of our – and perhaps any – time (IPCC 2014). To address this concern is likely to require broad-based deployment of new infrastructure, for example refineries capable of producing low-carbon transport fuels, replacing power plants that utilize dirty fuels (such as coal) with less dirty fuels (such as natural gas) or by broadly deploying renewable energy sources at a local level (*e.g.*, rooftop solar). Undertaking these changes will also require an expansion of delivery infrastructure, such as pipelines or export / import facilities capable of converting gases into liquids. This new infrastructure is likely to be quite costly, and the associated investments very difficult to reverse. To the extent the underlying financial variables, such as commodity prices, exhibit important uncertainties, investment in these new facilities becomes a classic problem of "investment under uncertainty" (Dixit and Pindyck, 1993).

The complication of this investment problem is likely to be exacerbated if the underlying commodity prices exhibit "fat tails" (Weitzman, 2009). There is evidence that key commodity prices, such as many energy prices, do exhibit fat tails; these fat tails could arise because of complex characterizations of the variance of price returns, as with the "Generalized Autoregressive Conditional Heteroscedasticity" (GARCH), or because commodity prices are subject to rapid and unexpected changes, known as "jumps" (Chevallier and Ielpo, 2014; Chevallier and Sévi, 2014; Gronwald, 2012; Mason and Wilmot, 2014).

A number of authors have analyzed key energy prices such as crude oil, natural gas and electricity. Table 1 provides a summary of a representative sample of papers in the literature that have analyzed energy commodity markets. A key observation that one may draw from the table is that many authors find that either jumps or volatility effects are important characteristics of these prices; both contribute to fat tails. But the impact of fat tails upon the sort of investment problem we outlined above has not received much attention in the literature. We address this lacuna in the current paper.

A central theme in our analysis is that the presence of jumps or fat tails increases the premium associated with delaying investment, often referred to as the "option value of waiting." To the extent that society sees urgency in undertaking these investments, identifying a policy intervention that would serve to offset this enhanced option value would then appear to be socially attractive.

We start by summarizing the evidence supporting the presence of fat tails for a variety of key energy-related prices in Section 2. In section 3 we consider a standard investment under uncertainty problem, where such investment will be difficult, if not impossible, to reverse. Here we first work through the problem when the uncertainty is characterized by geometric Brownian motion, and then when the uncertainty is subject to the possibility of abrupt changes – *i.e.*, jumps. The nature of the problem when jumps are possible is sufficiently complex that closed form solutions cannot be obtained, and so we undertake numerical simulations to flesh out the results; results from these simulations are discussed in Section 4. We discuss the broader implications of our analysis in section 5, and conclude in section 6.

2 Literature review: price jumps and volatility

There is a long literature that addresses movements in key energy prices, which commodities are relevant to climate policy. Predicting such prices has value for both private and public decisions, by informing investment decisions or providing important background information for policy choices. Initially, much attention was paid to relatively simple models, for example by assuming that changes, or percentage changes, in prices were normally distributed; these assumptions give rise to Brownian motion or geometric Brownian motion (GBM) models. Subsequent work often focused on volatility concerns. For example, Pindyck (2004) specifically studies volatility in natural gas and crude oil price returns; using daily data, he models these price returns with a "Generalized Autoregressive Conditional Heteroscedasticity" (GARCH) process. More recently, many analysts have allowed for jumps. Shafiee and Topal (2010) reviews price modeling techniques that include both GBM and mean reversion, while utilizing a binary method of introducing discontinuities (jump and dips) into the process; they conclude that empirical models should include a diffusion component, mean reversion and a jump diffusion component.[1]

The empirical importance of jumps has been documented for a variety of energy prices. Examples include oil (Askari and Krichene, 2008; Gronwald, 2012; Postali and Picchetti, 2006; Wilmot and Mason, 2013), natural gas (Benth et al., 2008; Mason and Wilmot, 2014), carbon permits in the European Union trading market (Alberola et al., 2008; Chevallier and Sévi, 2014; Daskalakis et al., 2009), and coal (Wilmot, 2016; Xiaoming et al., 2012). Indeed, Chevallier and Ielpo (2014) argue that many energy prices are subject to these effects.[2]

A natural consequence of the potential presence if jumps in various energy commodity prices is that electricity spot prices can also exhibit jumps (Benth et al., 2008; Huisman and Mahieu, 2003). The jumps in these energy prices can be large: Huisman and Mahieu (2003) argue that daily jumps in electricity prices on the order of 30% are not uncommon. Similarly, Thompson et al. (2009, p. 227) observe "high [natural gas] price spikes far outside normal seasonal equilibrium levels;" likewise, Chen and Forsyth (2010, p. 359) argue it "is not uncommon to see spot gas price jumps ... as

[1] An alternative approach to allowing for large discrete changes in prices is to use a regime switching model. Papers that have adopted such a framework typically use a Markov switching model, wherein there are a small finite number of states (often 2), and where the probability of moving from one state to another becomes a focus of the investigation. Chen and Forsyth (2010) use a regime switching model in their investigation of optimal storage, and Chen and Forsyth (2007) employ such an approach to analyze Henry Hub NYMEX futures prices between February 2003 and July 2007. Their results indicate 22 switches during this period, and exhibit long periods of non-switching. While these authors provide a thorough analysis of the switching model, they also note that "[a]n obvious improvement would be to include price jumps within each regime" (p. 174).

[2] These energy markets can be correlated: Asche et al. (2006) and Panagiotidis and Rutledge (2007) demonstrate a link between UK natural gas prices and European crude prices. On the other hand, Brown and Yücel (2008, p. 48) argue that there has been a fundamental change in the relationship between oil and gas prices after 2000. And while the data from 2000 - 2006 is reflective of emerging technological trends, it predates the major burst in gas production that followed the widespread application of fracking.

much as 20% in a single day." Moreover, with prices subject to jumps, it is not unexpected that related derivatives would also exhibit jumps – for example, the convenience yield associated with holding crude oil stockpiles (Mason and Wilmot, 2020).

Understanding the nature of these price movements has important implications for large-scale investment decisions, particularly when these decisions entail up-front costs that are costly (or impossible) to reverse. Here too there is a long tradition in the literature of analyzing investment decisions when key ingredients, such as underlying prices, are stochastic. Much of this literature follows the seminal work of Dixit and Pindyck (1993). A key insight here is that there is a value to waiting – reflecting the fact that the gain from investing may increase over time; this value is referred to as the "option value" of delaying investment. The larger is the option value from delaying, the later will the investment be undertaken.

These results have clear implications for a broad range of decisions in energy markets, from smaller decisions (such as choosing when to shut down an oil refinery for maintenance, thereby temporarily foregoing a stream of earnings) to larger decisions (such as when to drill or complete an oil or gas well). The implications are particularly significant for very large decisions, such as the decision to build a new pipeline or deploy an LNG export facility, decisions whose cost can run into hundreds of millions, or perhaps billions, of dollars.

By extension, energy price jumps have an important potential to influence climate policy by virtue of the likely effect that jumps will have on the option value of waiting to invest. That carbon prices are inter-related to key energy prices is natural, since the value of a carbon permit would be partly determined by the flow of emissions (Hammoudeh et al., 2014). In turn, emissions are likely to depend on the rate of usage for the key fossil fuels, which one expects would be influenced by prices of those inputs.

3 Investment under uncertainty when jumps can occur

In light of the literature on energy prices summarized in Table 1 and in section 2, a natural question to ask is: what is the impact of including jumps in the stochastic specification of the price for a key commodity? In this section, we provide such an analysis. Our discussion here is mainly descriptive; we relegate the more technical details to the Appendix.

To illustrate the basic ideas, we start with a conventional investment under uncertainty problem, under which the key underlying stochastic process is geometric Brownian motion (Dixit and Pindyck, 1993). This underlying variable could be the price of some carbon-based fuel, such as coal or oil, or it could be the price associated with some policy-related element like a tradable carbon permit. The investment problem involves a one-time sunk expenditure K; making this expenditure allows the decision-maker to obtain a new payoff flow. The investment could reflect development of climate-friendly capacity, such as a renewable energy plant or a carbon-capture facility, which aim is the removal of CO_2 from emissions associated with a fossil-fuel based power plant. Alternatively, it could reflect development of some infrastructure that will facilitate reduced reliance on particularly dirty fuels; an example here would be the deployment of a liquefied natural gas (LNG) facility that will enhance the ability to ship natural gas to an erstwhile heavy user of coal (*e.g.*, a country like India). A key question here regards timing: when should the investment be taken? Answering this question requires a determination of the value associated with forestalling the investment, together with a determination of the value of investment.

We assume that there is a stochastically evolving component, X_t, that captures the benefits associated with investing at time t. While one could describe the evolution of X_t in terms of a relevant resource price, the interpretation of the model is more direct if one focuses on the benefits of the investment. Letting K denote the one-time investing cost, the net benefits of acting (investing) at t are equal to[3]

$$X - K.$$

These net benefits are compared against the value associated with waiting, and then building at the optimal time in the future; we refer to this as the "optimal value." The optimal value is functionally related to the stochastically evolving component through the optimal value function $F(X)$. We start by working through the problem when X follows a geometric Brownian motion (GBM) process. Later, we discuss the determination of $F(X)$ when X is also subject to the potential for jumps.

Under GBM, one can express the stochastic evolution of X as

$$dX/X = \alpha dt + \sigma dz, \tag{1}$$

where dz is an increment of a Wiener process. This form implies that X is lognormally distributed.

At any moment where the decision to undertake the investment has yet to be made there are two possible decisions: either wait or build now. The decision to wait earns a flow payoff of zero (since nothing has been done), while the value associated with waiting, $F(X)$, is retained. The decision to build yields the immediate payoff $X - K$, as we noted above. If delaying is optimal, the fundamental equation of optimality requires that the optimal value function satisfy (Dixit and Pindyck, 1993):

$$\rho F(X) = \frac{1}{\mathrm{dt}} E\left[\mathrm{d}(F)\right], \tag{2}$$

where ρ is the decision maker's discount rate and the expression on the right-hand side is the so-called Itô operator. The left-hand side of eq. (2) measures the capitalized value from the investment (or, equivalently, the interest earned on the net returns), while the

[3] In the pursuant discussion, we will often suppress the time subscript so as to reduce notational clutter.

right-hand side measures the anticipated capital gains. We show in the Appendix that the solution to this equation takes the form

$$F(X) = aX^{\beta}.$$

The value function $F(X)$ can be interpreted as the value of an option to invest in the future (Dixit and Pindyck, 1993). Accordingly, it is optimal to invest when this value equals the net benefit from acting now; this implies a cutoff value X^* for the underlying stochastic ingredient, which is implicitly defined by

$$F(X^*) = X^* - K. \tag{3}$$

We show in the Appendix that the cutoff value is

$$X^* = \frac{\beta K}{\beta - 1}. \tag{4}$$

Now suppose the value X evolves according to the mixed jump-diffusion process. Here, we assume changes in X are composed of two types of changes: 'typical' fluctuations, represented through the GBM process, and 'abnormal' fluctuations, due to the arrival of new information or some unusual event. We model the arrival of these abnormal fluctuations as following a Poisson process.[4] Letting n_t denote the number of such events that have occurred as of time t, the change in n_t during the interval $(t, t+\Delta t)$ is described by

$$dn_t = \begin{cases} 0, & \text{with probability } 1 - \lambda dt \\ 1, & \text{with probability } \quad \lambda dt, \end{cases} \tag{5}$$

where $\lambda > 0$ is a parameter measuring the arrival frequency

[4] Some authors model price jumps using a Benth et al. (2008) applies a model including Lévy process, an approach that requires an *ex ante* definition of a jump. For example, Benth et al. (2008) define a jump as an observation that falls outside of 2 standard deviations from the mean. Other authors assume jumps follow a Poisson process; one advantage of this approach is that there is no need to arbitrarily define a jump *ex ante*.

We denote the size of a jump at time t, should one occur, is J_t. We regard the jump size as a random variable; for convenience, we assume J_t is independently and identically distributed as a lognormal random variable – *i.e.*, that ln(J) is Normally distributed with mean jump size θ and variance δ^2. If an abnormal fluctuation occurs, X jumps from X_t to $\exp(J_t)X_t$ (Askari and Krichene, 2008). The resultant stochastic process for the random variable X may then be written as

$$dX/X = \alpha dt + \sigma dz + Jdn. \tag{6}$$

Because the term capturing jumps will add something to the expected value of X, the drift term in the expressions for the evolution of X becomes (Dixit and Pindyck, 1993)

$$\frac{1}{\mathrm{dt}} E\left[\mathrm{d}(X)\right] = \alpha + \lambda\theta.$$

In this setting, the optimal value function is determined by the interaction between jump size, Y, and continuation value, V. The equation governing the optimal value function becomes

$$\frac{1}{2}\sigma^2 X^2 F''(X) + \tilde{\alpha} X F'(X) + \lambda \int_0^\infty F(XJ)G(J)dJ = 0, \tag{7}$$

where $G(J)$ is the probability density function governing jump size.

As before, the solution is governed by the value-matching condition ((3)), as well as a smooth-pasting condition. Unlike the GBM variant, however, this problem cannot be solved analytically. Accordingly, we employ numerical simulations in the pursuant discussion.

4 Simulation Results

To facilitate numerical simulations, we must first specify various parameters: the discount rate ρ, the mean α, and the standard deviation σ of the GBM formulation; we set these equal to $0.02, 0.04$ and .2, respectively. We specify the jump intensity associated with the Poisson process as $\lambda = 0.01$. The distribution governing Y, the magnitude of a jump (should it occur), is assumed to be lognormal – *i.e.*, $\ln(Y)$ is Normally distributed – with mean $\theta = 0$ and standard deviation $\delta = 1$.

For a given parameterization, we solve for the critical value associated with investing; the interpretation is that when the expected value from investing meets or exceeds this critical value, the investment will be taken. This critical value will correspond to the sum of the investment cost itself and the option value of waiting. The difference between the critical value and the requisite up-front investment may then be interpreted as the option value of waiting. We also calculate the ratio of the critical value to up-front investment cost.

Our first set of comparisons investigates the role played by the jump intensity. Here, we vary λ between 0 and 0.2, by increments of 0.05; results from this set of simulations are summarized in Figure 1 and Figure 2. The first of these displays the option value associated with delaying investment, for various levels of up-front investment (*i.e.*, K) across the possible values of λ. The second figure displays the cutoff value associated with investment, as a fraction of the required up-front expense, again across various levels of K and possible values of λ. The first feature we observe is that the option value of delaying investment rises as the amount of money that must be invested increases. This is intuitive: because larger investments require risking more money, the decision-maker is more cautious about undertaking the investment. In these simulations, the tendency to delay investment tends to be more pronounced as the probability of a jump increases: while option value is largely insensitive to λ when the required investment is small, it does respond to increased jump intensity at larger investment levels.[5] Moreover, we note that the impact of increasing λ is most pronounced at small value of λ. In particular, the largest effect appears to occur when the probability of a jump occurring is increased from 0, *i.e.*, when the possibility of jumps is introduced. Indeed, this effect become ever-more important as the up-front cost is increased. Intuitively, allowing for jumps raises the option value because of the potential for a more dramatic future increase in the underlying price, which raises the expected future path of prices. This effect is more important the larger is the initial investment.

In the second set of simulations, we vary θ – the expected value of the (natural log of) jump size – allowing for values ranging from -0.2 to 0.2, by increments of 0.1. In this way we consider cases where abrupt movements in prices are negative on average as well as cases where jumps are positive on average. The results from this simulation are presented in Figure 3 and Figure 4; in both figures, we consider the same range of up-front investment as in the first set of simulations. As in those simulations, we note that option value of delaying the investment rises as the amount of money that must be invested increases; values as in the preceding simulations. While one might have expected option values to respond to the average value of a jump, our results indicate this does not emerge. Evidently, the average jump value exerts a less significant influence on the value of delaying investment than does the potential for a jump in the first instance.

The third set of simulations we consider varies δ, the standard deviation of the jump size; here we consider values ranging from 0.5 to 1.5, by increments of 0.25. Results from these simulations are presented in Figure 5 and Figure 6. Again, we consider the same range of up-front investment values as in the first set of simulations. Interestingly, while option value is generally unresponsive to changes in the underlying variance when that variance is less than one, that ceases to hold as the variance of the jump size increases above unity – and to a greater degree. Indeed, variations in the

[5] One should not make too much of the seeming equivalence of option values at the smallest level of K: The numerical grid we employ in the solution algorithm is not sufficiently granular to detect differences between option values at small levels of K.

potential size of the jump play an ever-larger role as the amount of money that must be invested increases. Again, this seems intuitive: when prices are subject to possible jumps with particularly large variation, the impact on the value of waiting increases to an ever-larger degree – generating an increasing motive. That is, greater variation in jump sizes make waiting more attractive, and hence raise the option value at the optimal investment time.

5 Discussion

As we noted above, there is a natural connection between greenhouse gas emissions and the prices of key fossil fuels. One point that seems likely to carry particular salience going forward is the role for natural gas. Over the past decade or so, particular with the onset of the natural gas boom in the United States (US) that followed the broad-based deployment of "fracking," the role of coal in generating electricity has diminished. Moreover, until inexpensive energy storage is widely available, there will be a need for a flexible fuel source that can easily be brought online, and just as easily removed from service. Natural gas would seem a logical choice here.

These point suggest a key future role for LNG. But the utilization of LNG requires very large up-front investments, both for de-gasification facilities at the point of export, and re-gasification facilities at the point of import. These facilities can be enormously expensive, perhaps running into the billions of US dollars. Justifying such a large investment requires an attractive stream of payoffs over time, and those payoffs will naturally vary as the price of the underlying commodity (here, natural gas) varies.

Similarly, the value of other climate-friendly investments, such as a large windfarm, a new nuclear plant or a carbon capture facility, will depend on the value of avoided carbon emissions; in turn, this value is reflected in the price of a carbon permit. As natural gas prices, or carbon permit prices, have been found to exhibit jumps or fat tails, our results suggest the value of waiting to invest in the project is likely to rise, inducing a delay in investment.

As a fallback policy, in the event that low carbon technologies are less broadly available to be deployment, society may need to address climate change by adaptation. Such adaptation could take the form of changing lifestyles, but it might also manifest as migration from relatively hotter to relatively cooler areas. Migration is a particular form of a costly investment – inasmuch as relocation imposes costs on those who migrate – and these costs are difficult to reverse. Our results suggest the option value associated with delaying "investment," hear interpreted as taking the decision to relocate, might be larger in the face of jumps in key attributes that influence the climate such as prices of fossil fuels. To the extent these sort of effects manifest, they could serve to mitigate a tendency towards migration; while this saves on the costs associated with relocation it also implies those who wait to move would be faced with larger damages from a hotter local climate.

To the extent that impending climate change is viewed as an urgent concern, this is a worrisome result. A natural policy response would be to take steps that raise the potential value of the investment, so as to mitigate the effect of jumps. One example could be to subsidize the investment; by lowering the up-front cost – say by offering some form of tax credit – the value of delaying investment is reduced. Another approach might be to add a fixed (*i.e.*, deterministic) bonus payment for actions that lower net carbon emissions. Such a policy would also lower the value of waiting, both by increasing the reward from the investment, and by lowering the relative importance of jumps.

6 Conclusion

A lengthy literature argues for the inclusion of "jumps" in key, climate-relevant, energy prices; these energy prices have clear relevance to the pattern of global use of fossil fuels. A natural question to ask is then: what implications do energy price jumps have for climate policy? Addressing carbon emissions will most likely require potentially irreversible investments in large infrastructure. In such a setting, there is often a value associated with delaying investment, as there is an "option value" with waiting (Dixit and Pindyck, 1993). A central theme in our analysis is that the presence of jumps or fat tails increases the premium associated with delaying investment, so as to capture this option value.

The model we used to sketch out the implications of jumps upon investment decisions assumed an exogenously fixed distribution governing a relevant price. As such, we abstracted from problems in which there is some important unknown factor that could change the distribution. For example, the price path of crude oil would likely be impacted in an important and permanent manner if a prohibition of some source of production – say fracking or oil sands – were executed on a broad basis; in such an event, the value of a large investment such as a major pipeline would be significantly and adversely impacted. One imagines that the potential for such an event would delay investment, much as the potential for jumps lowers the incentive to invest, and hence delays investment. In this way, our results are comparable to the precautionary principle that induces agents to "wait and see" in an environment where important learning may take place (Gollier and Treich, 2003).

This incentive to delay is all the larger if the decision-maker operates in multiple markets, where the prices in the two markets are likely to be influenced in a similar fashion by the potential event (Lange and Moslener, 2004). In the pipeline example we mentioned above, for example, the firm in question might operate in both oil and gas markets; in the event that a global tax were imposed it would impact both lines of business, which might conceivable alter the preferred sequence of investments – so that the firm opts out of building a pipeline, or redirects its investment dollars into some other option. A potential example is the decision to diversify into renewables; a handful of large oil and gas companies, such as BP, have recently announced their intention to undertake such investments in the near future.

7 Appendix

In this Appendix, we provide the mathematical details underlying our numerical simulations. We start by fleshing out the problem under geometric Brownian motion (GBM) process. As noted in eq. (2), the optimal value function satisfies

$$\rho F(X) = \frac{1}{\mathrm{dt}} E\left[\mathrm{d}(F)\right]. \tag{8}$$

We proceed by expanding the expression on the right-hand side, which gives rise to[6]

$$\frac{1}{\mathrm{dt}} E\left[\mathrm{d}(F)\right] = \alpha P F'(X) + \frac{1}{2}\sigma^2 X^2 F''(X).$$

As a result, the fundamental equation of optimality reduces to a log-linear second-order differential equation, the generic solution of which is the power function

$$F(X) = aX^{\beta}.$$

Inserting this form into the fundamental equation of optimality, taking derivatives as appropriate, and combining terms leads to the characteristic equation

$$\frac{\sigma^2}{2}\beta(\beta-1) + \alpha\beta\rho = 0, \tag{9}$$

which the parameter β. must satisfy. It is easy to see that there are two roots, one of each sign. We refer to the negative root as β_1 and the positive root as β_2. The complete

[6] One way to think about this step is that a Taylor's series expansion has been executed, with higher order terms dropped; see Dixit and Pindyck (1993) for details.

solution to the fundamental equation of optimality is then

$$F(X) = a_1 X(\beta_1) + a_2 X(\beta_2).$$

The solution must also satisfy a boundary condition, namely that the optimal value becomes negligible as the spot price becomes very large (since it will almost surely not be optimal to wait under those conditions, the option to wait is essentially worthless). Since $\beta_2 > 0$, the second term describing F would be unbounded if slope parameter a_2 were non-zero. Hence, the requirement that F tends to zero as X tend to zero implies $a_2 = 0$. Accordingly, the value function can be simplified to

$$F(X) = aX(\beta_1), \tag{10}$$

where we have substituted a for a_1 and β for β_1 to reduce notational clutter.

In addition, the solution must satisfy two other conditions, the value-matching condition and the smooth-pasting condition (Dixit and Pindyck, 1993). At X^*, the point at which it is optimal to build, the value-matching condition requires that the value of building equals the value of waiting:

$$F(X^*) = X^* - K. \tag{11}$$

Because the value function $F(X)$ can be interpreted as the value of an option to invest in the future (Dixit and Pindyck, 1993), this value must equal the difference between the value upon investing when the value of investing immediately, X^*, and the sunk cost of investing, K. The smooth-pasting condition requires that the slope of the optimal value function equals 1, the slope of the function reflecting the value of waiting (Dixit and

Pindyck, 1993). Accordingly,

$$a(X^*)^{\beta} = X^* - K; \tag{12}$$

$$a\beta(X^*)^{(\beta-1)} = 1. \tag{13}$$

Multiplying both sides of eq. (13) by X^*/β, substituting into eq. (12) and rearranging then yields eq. (4) in the text.

Now suppose the value X evolves according to the mixed jump-diffusion process

$$dX = \tilde{\alpha} X dt + \sigma X dz + (Y-1)dq,$$

where as above dz is an increment of a standard Weiner process; here, we interpret Y as the magnitude of a jump if it occurs, and dq as an increment of a Poisson process (which captures the probability that a jump occurs). Because the term capturing jumps will add something to the expected value, the mean term (α in the GBM representation) must be adjusted. Denoting the arrival rate under the Poisson process by λ, and the mean value of a jump (should one occur) by θ, the drift term in have (Dixit and Pindyck, 1993)

$$\frac{1}{\mathrm{dt}} E\left[\mathrm{d}(X)\right] = \tilde{\alpha} = \alpha + \lambda\theta.$$

In this setting, the optimal value function is determined by the interaction between jump size, Y, and continuation value, V. The equation governing the optimal value function becomes

$$\frac{1}{2}\sigma^2 X^2 F''(X) + \tilde{\alpha} X F'(X) + \lambda \int_0^{\infty} F(XY)G(Y)dY = 0. \tag{14}$$

As before, the solution is governed by the value-matching condition ((3)), as well as a smooth-pasting condition. Unlike the GBM variant, however, this problem cannot be solved analytically. Accordingly, we employ numerical simulations – as discussed in the main body of the paper above.

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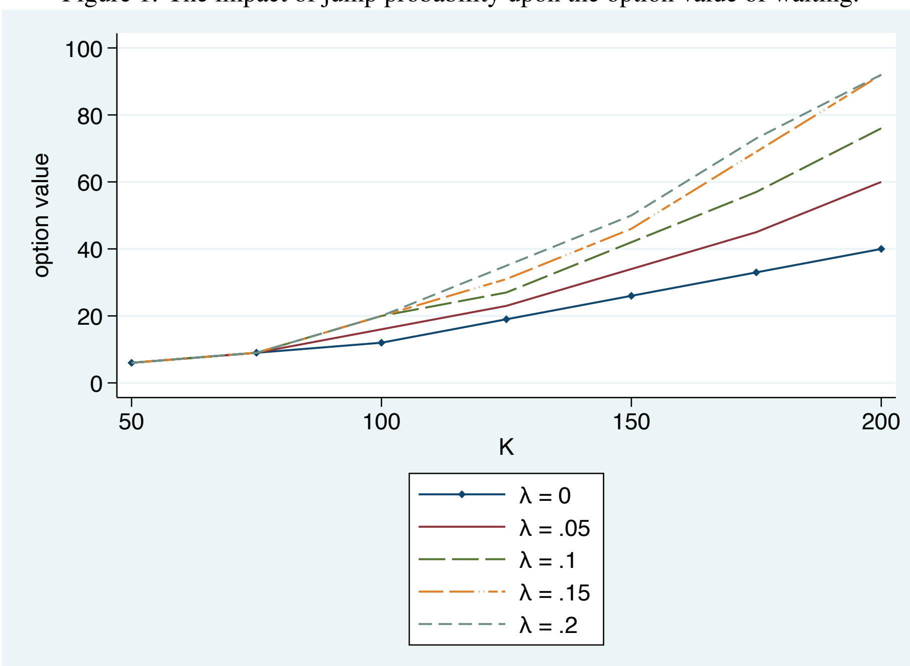
Table 1: Summary: results from literature on energy commodity prices

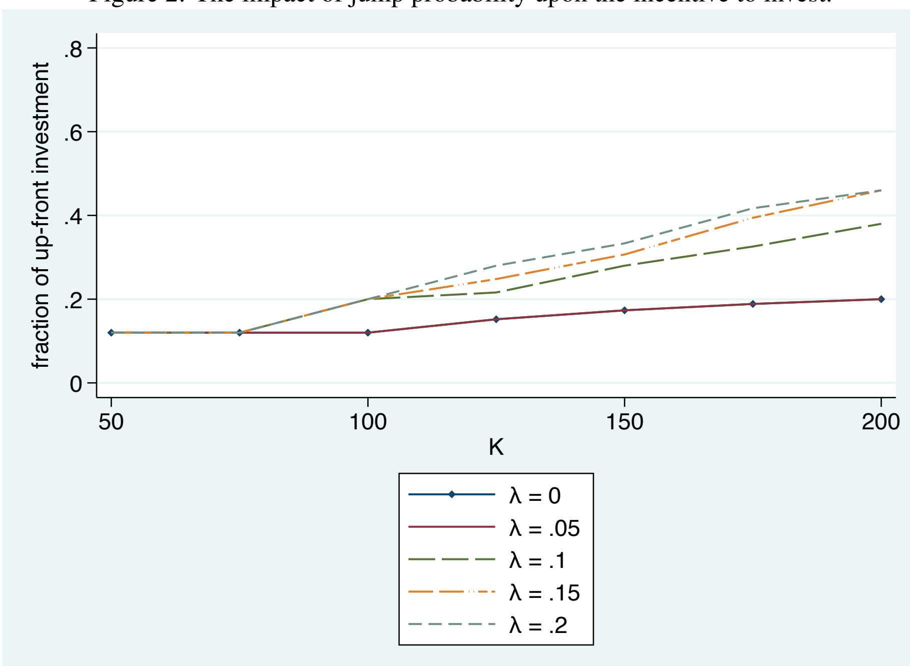
Author(s)	Prices Studied	Preferred Specification
Sadorsky (1999)	Real Oil Prices	GARCH
Xiaoming et al. (2012)	Australian BJ steam, Datong Coal	GARCH
Huisman and Mahieu (2003)	Electricity Prices	Jumps
Postali and Picchetti (2006)	(annual) International Oil Prices	GBM
Askari and Krichene (2008)	Brent Future Prices	Jumps, fat-tails
Benth et al. (2008)	Electricity, Natural Gas Prices	Jumps, fat-tails
Daskalakis et al. (2009)	EU emission allowance spot prices	Jumps
Gronwald (2012)	WTI Crude Oil Prices	Jumps, GARCH
Wilmot and Mason (2013)	Crude Oil Spot, Futures Prices	Jumps, GARCH
Chevallier and Ielpo (2014)	Various Commodities	Jumps
Chevallier and Sévi (2014)	CO_2 Futures Price	Jumps
Mason and Wilmot (2014)	US , UK Natural Gas Spot Prices	Jumps, GARCH
Wilmot (2016)	NYMEX Coal Futures Prices	Jumps, GARCH
Mason and Wilmot (2020)	Crude oil convenience yields	Jumps, GARCH

Figure 1: The impact of jump probability upon the option value of waiting.



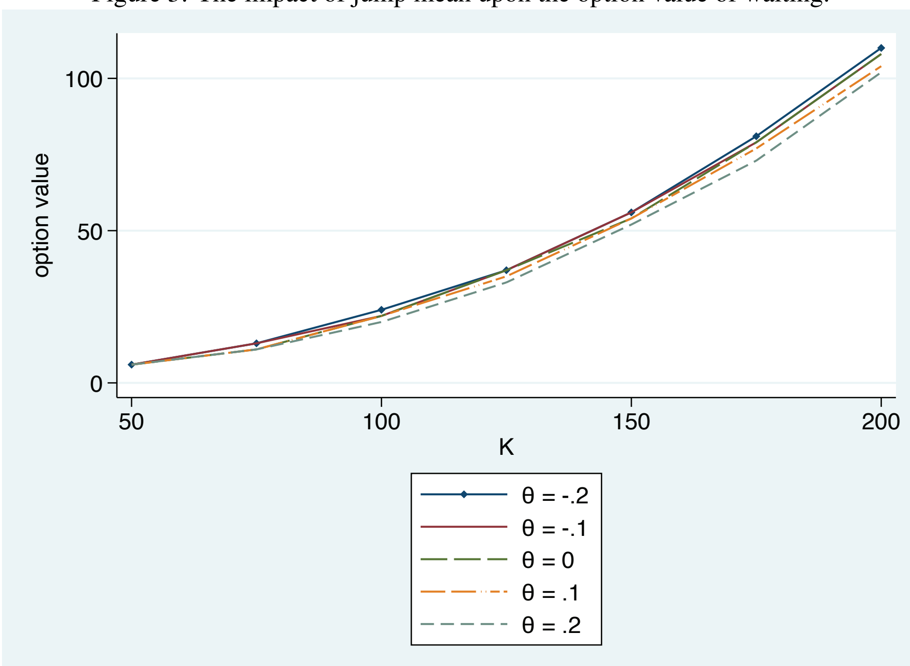
Source: Authors' calculations.

Figure 2: The impact of jump probability upon the incentive to invest.



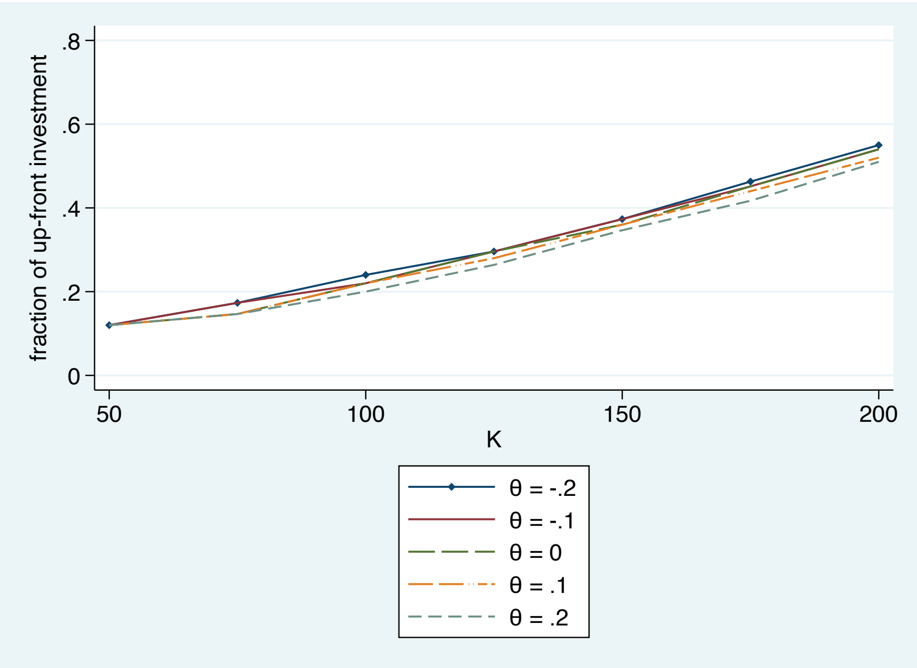
Source: Authors' calculations.

Figure 3: The impact of jump mean upon the option value of waiting.



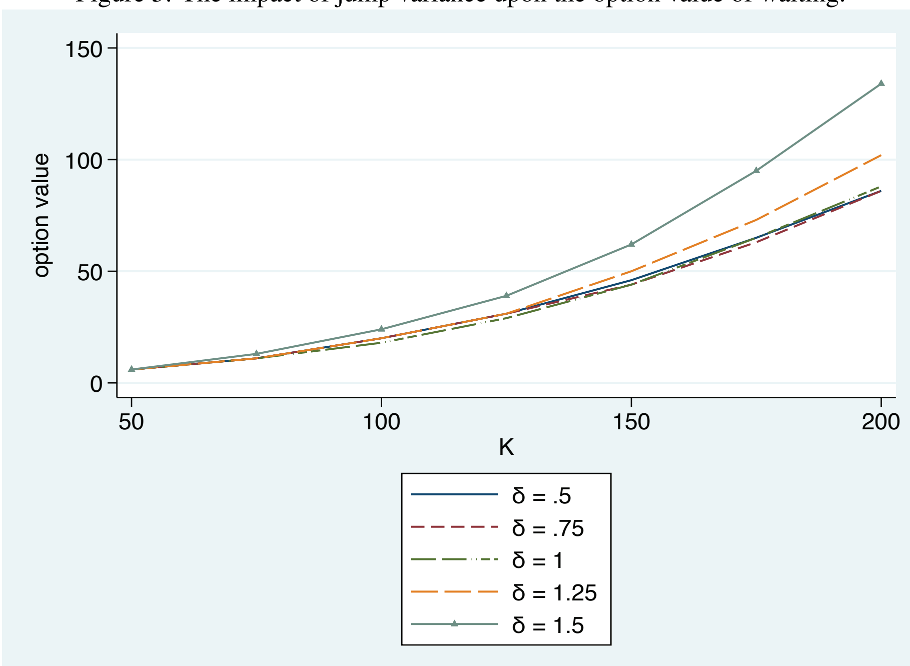
Source: Authors' calculations.

Figure 4: The impact of jump mean upon the incentive to invest.



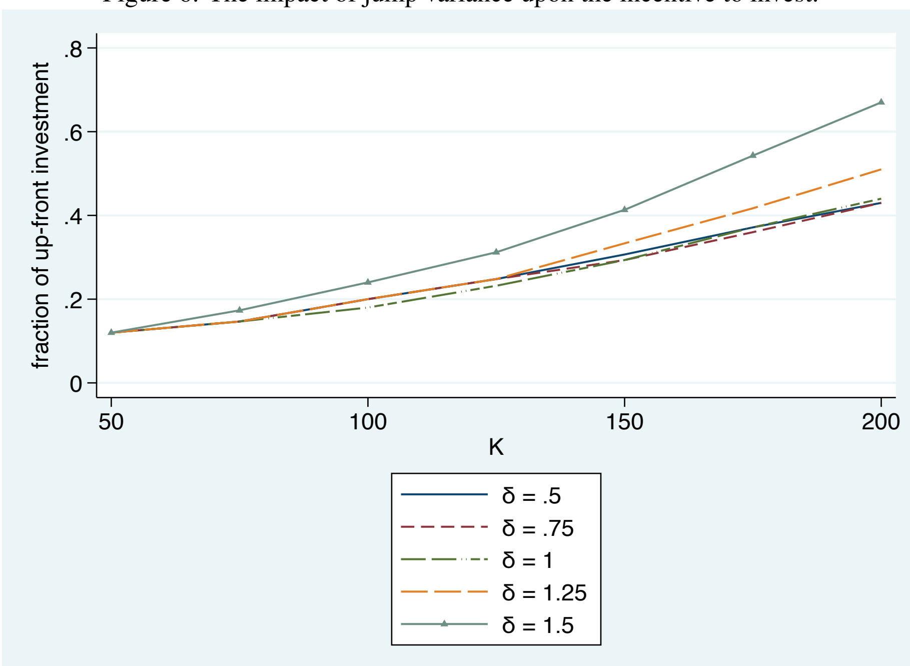
Source: Authors' calculations.

Figure 5: The impact of jump variance upon the option value of waiting.



Source: Authors' calculations.

Figure 6: The impact of jump variance upon the incentive to invest.



Source: Authors' calculations.