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Improved quantum ant colony algorithm for solving TSP problem

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***Abstract*—For the low efficiency and poor performance of the ant colony algorithm in solving TSP problems, a new quantum ant colony algorithm proposed. The models of probability selection and pheromone are redefined, integrated with the quantum information intensity factor; The factor updated by quantum rotating gate according to iteration process; Some important parameters are self-adapted controlled at the same time; And 3-opt is used to further local optimization. Stimulation shows the performance is greatly improved.**

Keywords-quantum ant colony algorithm; TSP; quantum computing; quantum evolution

I. INTRODUCETION

Because basic ant colony algorithm appears more obvious advantages on being integrated easily and strong robustness contrasted by other bionic algorithms, it has been widely applied in the fields of solving TSP problems and job scheduling problems. But the shortcoming of basic ant colony algorithm is also very obvious becauseof its own structural features. On the one hand, the mathematical model of algorithm decides that parameters will affect the final results greatly, and it's difficult to give universal values range through mathematical derivation. As a result, the algorithm parameters setting is based on the user experience in more cases. So, it is more easy to cause too long searching time and convergence stagnation phenomenon. On the other hand, When the solving problem scale enlarges, the performance of the algorithm reduces very sharply, and it is difficult to get ideal optimization results.

In order to make up those shortcomings, scholars from various countries made many improvements to the algorithm, some representative are: B.Bullnheimer et al. proposed Rank-based Ant System (RAS) [1], RAS sort the route after every iteration, chooses *m* ants, which are some shortest route, to update pheromone, so as to speed up the convergence, but it also easy to fall into local optimum; O.Cordon et al. proposed Best-Worst Ant System (BWAS)[2], the algorithm only increase pheromone on the optimal path, meanwhile minimizing pheromone on the worst's, when the algorithm achieves convergence stagnation, reset all pheromone, the performance of this algorithm is improved compared with RAS; Dorigo later proposed Ant Colony System (ACS) [3], the pheromone update strategy has been adjusted, divided into local update and global update, new strategy makes better balance between the new developed path and the explored path. T.Stutzle et al. proposed Max-Min Ant System(MMAS) [4], pheromone is restricted between $(\tau_{\max}, \tau_{\min})$, when algorithm falls into stagnation, reset all pheromone, so effectively restrain premature convergence.

Using quantum mechanism combined with ant colony algorithm to solve large-scale TSP problem has not been extensively studied yet, relevant literature[5-7] presented TSP solutions based on quantum ant colony, and they all get good optimization results.

II. THE PRINCIPLE OF IMPROVE QUANTUM ANT COLONY ALGORITHM

A. The encoding scheme

Different from function optimization, the qubit of ants no longer stands for the coordinate position information in space, but the intensity of node information. Using quantum probability amplitude coding, the qubits information can be expressed as:

$$p_i = \begin{pmatrix} \alpha_{i1} & \alpha_{i2} & \cdots & \alpha_{in} \\ \beta_{i1} & \beta_{i2} & \cdots & \beta_{i2} \end{pmatrix} \qquad (i = 1 \sim m)$$

Where, *m* is the number of ants, *n* is the number of nodes in the TSP problem. α_{ij} 、 β_{ij} are initialized to $\pm 1/\sqrt{2}$.

B. translocation strategy

Ants realize translocation in accordance with the following rules:

$$x_i = \begin{cases} \arg\max\limits_{j \in Allowedk} \{\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(\mathrm{t}) \cdot \left| 1/\alpha_{ij}(t) \right|^2\} & ,q \le q_0 \\ x_i & ,q > q_0 \end{cases} \tag{1}$$

Where, *q* is a random number within the range [0,1], q_0 is a variable within the range [0,1]. When $q > q_0$, the choice of x_i is determined by the transition probability formula:

$$P_{ij}^{k}(t)=\begin{cases}\dfrac{\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)\cdot\left|1\big/\alpha_{ij}(t)\right|^{2}}{\sum\limits_{j\in Allowedk}\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)\cdot\left|1\big/\alpha_{ij}(t)\right|^{2}} & j\in Allowedk\\ 0 & otherwise\end{cases} \tag{2}$$

From formula (1)-(2)we can see, The movement of ants also have some relationship with q_0. We expect that, in early stages of algorithm execution, ant colony can spread to explore the optimal solution in each node, with the optimization process carried out, effective pheromone continue to be accumulated, ants increasingly concentrated near the optimal route, and only in the vicinity of the optimal route to explore. So, in the early stage of algorithm, q_0 should be taken relatively small to ensure traversal for ant colony in each node; By contrast, in the middle and late stage, q_0 should be taken slightly larger to concentrated near the optimal route to explore. So, q_0 can be modified as:

$$q_0=q_1+(\mathrm{q}_2-\mathrm{q}_1)\cdot\frac{n-1}{n_{\max}} \qquad q_0\in[q_1,q_2] \tag{3}$$

In formula (3),$n_{\max}$ stands for maximum iterations in algorithm, n stands for current number of iterations. q_1、q_2 is the initial value and maximum value of q_0.

Similarly, for information inspiration factor α,We also hope that weaken the positive feedback effect in early stage of algorithm by change the value of α, and ensure the ergodic of algorithm; As algorithm carries out, increasing positive feedback gradually, and accelerating convergence. So, we can get the above purpose by making the value α larger gradually, as follows:

$$\alpha=\begin{cases}k\cdot\alpha_0 & \alpha<\alpha_{\max}\\ \alpha_{\max} & otherwise\end{cases} \tag{4}$$

In formula(4), α_0、α_{max} is initial and maximum values respectively set for inspired factor,k is the increment coefficient.

C. Pheromone update strategy

The pheromone update is divided into individual pheromone update and the global pheromone update. Every ant update individual pheromone firstly at the end of a cruise, according to the following formula:

$$\tau(t+1)=(1-\rho_1)\tau(t)+\rho_1\Delta\tau_{ij} \tag{5}$$

$$\Delta\tau_{ij}=\sum_{k=1}^{m}\Delta\tau_{ij}^{k}\ ,\Delta\tau_{ij}^{k}=Q\left|1\big/\alpha_{ij}(t)\right|^{2} \tag{6}$$

In formula (5) and (6), n is the total number of nodes, L_k is the total length of the route of k_{th} ant taken in this cycle. Only the ant whose route is the shortest can leave behind pheromone on the paths it just passes through, the global update rule is as follows:

$$\tau(t+1)=\begin{cases}(1-\rho_2)\tau(t)+\rho_2\Delta\tau_{ij} & (i,j)\in \text{best route}\\ (1-\rho_2)\tau(t) & \text{otherwise}\end{cases} \tag{7}$$

$$\Delta\tau_{ij}=\frac{\left|1\big/\alpha_{ij}(t)\right|^{2}}{L_{gbest}} \tag{8}$$

In formula(7-8), ρ_1、ρ_2 are pheromone evaporation coefficient, L_{gbest} is the current shortest path length. The simulation shows that if ρ enlarge or reduce according to iteration process, it will get unstable performance to the final optimization results for different TSP problems. So, we make ρfixed value while iteration.

In order to prevent searching stagnation phenomenon caused by gap of pheromone increased sharply between best and worst route, referenced from the principal of MMAS, we restrict pheromone of each side in the range $[\tau_{\min},\tau_{\max}]$. When $\tau_{ij}<\tau_{\min}$, $\tau_{ij}=\tau_{\min}$; And when $\tau_{ij}>\tau_{\max}$, $\tau_{ij}=\tau_{\max}$.

D. The intensity of quantum information update strategy

Algorithm uses quantum rotation gate to update intensity of quantum information, changing the probability of qubit, which stands for the intensity of quantum information of nodes, by quantum rotation gate operator, promoting quantum individual evolute to the direction of the optimal solution.

E. Local optimization strategy

Related literatures[7-8] about TSP algorithm show that simply improve the characteristics of inspiration of ant colony algorithm, will not improve the searching efficiency for TSP problem significantly. Therefore, combining local optimization strategy become an effective means to improve efficiency of ant colony algorithm. So, in the process of iteration, local optimization strategy can provide high-quality solutions and strengthen positive feedback effect for ant colony algorithm, which will significantly improve the quality of the solutions.

The method of side optimization is a simple and effective means widely be used in many local optimization strategy for solving TSP. This method can greatly improve the searching success rate, reduce algorithm trapped in local optimal and appear stagnation phenomenon; But at the same time, it will greatly increase the computational overhead. In this paper,the trilateral method (3 - opt) is adopted as the local optimization method. In order to improve the time efficiency of algorithm and reduce the computational complexity, we make the following appointment: After a traversal for ant colony, sorting all the optimization results of ants, then selecting only a certain number of optimal solutions, rather than the global optimal solution or the all solutions, to 3-opt optimization, so as to make balance with computational efficiency and population diversities.

F. *Algorithm steps*

Step1 Initialization algorithm parameters, and put *m* ants scattered randomly on *n* city nodes;

Step2 Ants choose next city according to the translocation formula (1)-(3), then update tabu table;;

Step3 Update local pheromone according to formula (5)-(6)

Step4 Determine whether all the ants complete traversal, if true, turn to Step5, otherwise return to Step2 to continue to travel to the next city;

Step5 Calculate every ant's total path length and travel sequence;

Step6 Choosing some best route, then using 3-opt for local optimization

Step7 using quantum rotating gate to update quantum information intensity of each node;

Step8 According to formula (7) - (8),updating global pheromone, at the same time updating visibility matrix, inspiring factor and global pheromone volatilization coefficient;

Step9 Determine whether stagnation phenomenon happens for several iterations, if so, reset pheromone, otherwise turn to step10;

Step10 Determine whether reach convergence conditions or the maximum number of iterations, if so, turns to step11, otherwise clear tabu table and return to step1, until reach the convergence conditions or maximum number of iterations;

Step11Output the shortest route length and the tour sequence.

III. STIMULATION FOR ALGORITHM PERFORMANCE

Contrast our scheme(IQACA) with fast ant colony algorithm based on the public side optimization(FACA)[8] and standard ant colony algorithm(ACA), the simulation environment is matlab2012b。Set parameters for each algorithm: Ant colony size *m* is equal to the number of city nodes n, Q=100, β=2, $\tau_0 = Q \cdot ones(\mathrm{m},\mathrm{n})$; In IQACA, q_1=0.5,q_2=0.9, α_0=0.01, $\alpha_{\max}$=1,k=1.1, ρ_1=0.7, ρ_2=0.9, $\tau_{\min}$=10^{-3}, $\tau_{\max}$=Q, the step value of quantum rotating gate $\Delta\theta = 0.1\pi$, mutation probability $p_{\mathrm{m}} = 0.1m$, pheromone reset condition is the optimal solution don't update for 40 times, the maximum number of iterations $t_{\max}$=50; In FACA, $\rho_{\min}$=0.01, $\rho_{\max}$=0.9, decreasing factor λ'=0.99; In ACA, ρ=0.9; In FACA and ACA, q_0=0.5, α=1, $t_{\max}$=500. Other parameters are consistent with the literature in FACA and ACA. Choosing some typical TSP problem, every algorithm runs 10 times independently, the results of the optimal value, average optimal value and mean deviation are shown in table 1.

Tab 1 Typical TSP problem optimization results

Name	The known optimal value	IQACA			FACA			ACA		
		Optimal value	Average optimal value	Mean deviation	Optimal value	Average optimal value	Mean deviation	Optimal value	Average optimal value	Mean deviation
BAYG29	9074	9074	9077	3.3e-4	9216	9482	4.5e-2	9298	9514	4.8e-2
CTSP31	15381	15378	15403	1.4e-3	15780	16237	5.6e-2	15639	16436	6.9e-2
EIL51	426	428	436	2.3e-2	439	449	5.4e-2	456	480	0.13
BERLIN52	7544	7544	7544	0	7544	7782	3.2e-2	7549	7954	5.4e-2
ST70	675	677	687	1.8e-2	708	725	7.4e-2	706	731	8.3e-2
PR76	108159	108159	110096	1.8e-2	115095	117554	8.7e-2	117181	119891	0.11
KROE100	22068	22175	22668	2.7e-2	23111	23538	6.7e-2	24038	24616	0.12

Test results can be seen from table 1, From Table1 we can see, for the given parameters, our algorithm are better than other two algorithm in all indicators. For CTSP31, the result of our algorithm is even better than the known optimal value; For some TSP problems, although the optimal result of our algorithm isn't equal to announce results, it is very approximate; If we increase the number of iterations, solving accuracy of those TSP problems can be effectively improved.

IV. CONCLUSION

A new quantum ant colony algorithm is proposed. quantum information intensity factor is integrated into probability selection model and the pheromone updating the model of standard quantum ant colony algorithm; dynamic adjustment for heuristic information factors and compare constant of probability selection are realized; local optimization strategy is also given. Stimulation proves the algorithm has good performance.

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