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Dynamic Topology Optimization Based on Ant Colony Optimization

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***Abstract*—A modified ant colony optimization algorithm implementing a new definition of pheromone and a new cooperation mechanism between ants is presented in this paper. The study aims at improving the suitability and computational efficiency of the ant colony optimization algorithm in dynamic topology optimization problems. The natural frequencies of the structure must be maximized yet satisfying a constraint on the final volume. Optimization results obtained in three test cases indicate that modified ACO is more efficient and robust than ACO in solving dynamic topology optimization problems.**

Keywords-Ant colony optimization (ACO); Dynamic topology optimization; Natural frequency; Finite element method (FEM)

I. Introduction

The ant colony optimization (ACO) algorithm is a metaheuristic search method for global optimization. ACO was initially proposed by Dorigo [1] to find the optimal path in a graph. ACO mimics the behavior of ants seeking a path between their colony and a food source. This optimization technique was successfully applied to many engineering problems including structural optimization [2-4].

In this study, a modified ant colony optimization algorithm is developed in order to improve computational efficiency and suitability of ACO in topology optimization problems dealing with natural frequencies. A filtering scheme [5] is adopted to prevent the formation of checkerboard patterns in the optimization process. Optimal designs are compared with those obtained with soft-kill BESO in order to assess the applicability and the efficiency of the proposed ACO algorithm in dynamic problems.

II. Topology Optimization for Dynamic Problems

A. *Formulation of dynamic topology optimization problems*

Excessive vibration due to resonance occurs when the frequency of the dynamic excitation is close to one of the natural frequencies of the structure. Therefore, it is necessary to restrict the fundamental frequency or several of the lower frequencies of the structure to a prescribed range in order to avoid severe vibration.

In the finite element method, the dynamic behavior of a structure is represented by the following general eigenvalue problem [6];

$$\left([K]-\omega_i^2[M]\right)\{u_i\}=\{0\} \tag{1}$$

where, $[K]$ is the global stiffness matrix, $[M]$ is the global mass matrix, ω_i is the i-th natural frequency and $\{u_i\}$ is the eigenvector corresponding to ω_i. The natural frequency ω_i and the corresponding eigenvector $\{u_i\}$ are related by the Rayleigh coefficient:

$$\omega_i^2=\frac{\{u_i\}^T[K]\{u_i\}}{\{u_i\}^T[M]\{u_i\}} \tag{2}$$

Dynamic topology optimization problems where the objective is to maximize the i-th natural frequency of a continuum structures are considered in this research. For a solid-void design, the optimization problem can be stated as [7]:

$$\text{Maximize : } \omega_i$$

$$\text{Subject to : } V^* - \sum_{i=1}^{n} V_i x_i = 0 \tag{3}$$

$$x_i = x_{\min} \text{ or } 1$$

where, V_i is the volume of an individual element, and V^* is the prescribed volume. n is the total number of elements in the structure. The binary design variable x_i denotes the density of the i-th element; $x_{\min}$ is set equal to be a sufficiently small value to denote a void element.

B. *Material interpolation scheme*

To obtain the gradient information of the design variable, it is necessary to interpolate the material between $x_{\min}$ and 1. A popular material interpolation scheme is the so-called power-law penalization model (the SIMP model). For the solid-void design, the material density and Young's modulus are functions of the design variable x_i as [8]:

$$\rho(x_i) = x_i \rho^1 \tag{5}$$

$$E(x_i) = x_i^{\,p} E^1 \quad (0 < x_{\min} \le x_i \le 1) \tag{6}$$

where, ρ^1 and E^1 are the density and Young's modulus of solid material, respectively. p is the penalty factor, usually used as 3.

However, the SIMP model described above cannot be directly applied to dynamic topology optimization problems where the objective is to maximize target frequencies. This is because the very high ratio between penalization on mass and stiffness for small values of x_i may result in the presence of localized modes in the low-density [9].

One idea to avoid this problem is to keep the ratio between mass and stiffness constant when $x_i = x_{\min}$ by requiring that:

$$\rho(x_{\min}) = x_{\min} \rho^1 \tag{7}$$

$$E(x_{\min}) = x_{\min}^{\,p} E^1 \tag{8}$$

Therefore, an alternative material interpolation scheme can be expressed by [8]:

$$\rho(x_i) = x_i \rho^1 \tag{9}$$

$$E(x_i) = \left[\frac{x_{\min} - x_{\min}^p}{1 - x_{\min}^p}(1 - x_i^p) + x_i^p \right] E^1 \quad (0 < x_{\min} \le x_i \le 1) \tag{10}$$

The change in natural frequency $\Delta\omega_i$ caused by the removal of the generic i-th element from the structure indicates the extent to which the global structural response is sensitive to that element. Therefore, the change in natural frequency can be taken as the quantity of pheromone in the ant colony optimization process. The change in natural frequency $\Delta\omega_i$ can be defined as follows [10]:

$$\Delta\omega_i = \begin{cases} \dfrac{1}{2\omega_i} u_i^T \left(K_i - \dfrac{\omega_i^2}{p} M_i\right) u_i \ , & (x = 1) \\ -\dfrac{\omega_i}{2p} u_i^T M_i u_i \ , & (x = x_{\min}) \end{cases} \tag{11}$$

where, M_i is the mass matrix of each element, K_i is the stiffness matrix of each element, and u_i is the element eigenvector which are related to the removed each element.

III. Ant Colony Optimization Formulations for Dynamic Topology Problems

A. Ant colony optimization algorithm for dynamic topology optimization

As ants are randomly located in the elements of the structure during optimization process, their positions are expressed as "solid" (1) or "void" (0) elements in the design domain. The best topology found in each iteration includes only solid elements. This allows to obtain a stable topology if the target volume is large: in such a case, symmetry of stiffness matrix can be preserved as there are enough solid elements to represent the general topology of the structure. However, this is not true if the target volume is set to a small value. In such a case, computation efficiency decreases either in terms of stiffness matrix ill-conditioning (values of natural frequencies are not accurate) and high CPU time.

In order to solve the above mentioned problems, design variables must be defined in fashion of a continuous distribution of density [7,9]. For this reason, a ant colony optimization algorithm formulation was developed in this research. The main difference between ACO and modified ACO is the introduction of a continuous variable, called "fitness", in the finite element analysis. This new variable serves to replace the positions of ants in ACO. It serves to assess and monitor the importance of each element in the optimization process. If an element is always recognized as "solid" it contributes significantly to the structural response and concurs to driving the search process towards the optimum design. Hence, it will always be included in the population of ants used to update the distribution of material in the structure.

For each element, fitness variable is defined as the ratio between the total number of ants passing through each element and the inner loop number. That is:

$$fitness_i = \frac{\sum_{iter=1}^{N} (A_{iter})_i}{N}, \ (A = 1 \text{ or } 0) \tag{12}$$

Basically, the $(A_{iter})_i$ term indicates if an ant was passing (if the ant "exists", A=1; if the ant "does not exist", A=0) through the i-th element over the N optimization iterations completed up to this moment. Convergence behavior depends on the ant decision index as well as on the values assigned to the ACO internal parameters α, λ and γ. It has been suggested that convergence rate can also be accelerated by resizing pheromones newly added to the solution found by rank-based ant system or elite ants [11-13]. The change in natural frequency $\Delta\omega_i^{new}$ for the i-th element was defined as:

$$\Delta\omega_i^{new} = fitness_i \times \Delta\omega_i \tag{13}$$

IV. TEST PROBLEMS AND RESULTS

The ACO algorithm developed in this study was tested in three dynamic optimization problems of 2D structures under plane stress conditions. ACO was compared with soft-kill BESO. A filter scheme to prevent noise effects in optimized topology was utilized.

A. Test problem 1: short beam optimized for first or second natural frequency

The topology of the 5 *m* x 1 *m* x 0.01 *m* short beam clamped on both sides shown in Fig. 1 must be optimized in order to maximize the first or second natural frequency. The volume of the structure must be 90% of the original volume. Material properties are as follows: Young's modulus E=10 *MPa*, Poisson's ratio v=0.3 and mass density ρ =1 kg/m^3. The design domain was meshed in 8000 (i.e. 200 x 40) 4-nodes bilinear elements. The filter radius is r_{min}=3 while the penalty factor is p=3. Elimination ratio (ER) is the ratio of elements to be eliminated to total elements per each iteration in BESO, and used as 2% in this test problem. The ant colony optimization internal parameters are: λ=1, α=1, γ=1 and convergence error limit is 0.01%, respectively.

Convergence curves obtained for BESO, ACO in the case of variant 1 where the objective is to maximize the first natural frequency and in the case of variant 2 where the objective is to maximize the second natural frequency, respectively. Optimal topologies obtained by the above mentioned three algorithms for variants 1 and 2 are presented in Figs. 2 and 3, respectively. The upper part of each sub-figure shows the topology found in the first optimization cycle while the bottom part of each sub-figure shows the optimized topology. The objective value is converged to 8.208 Hz.

B. Test problem 2: simply supported beam optimized for first natural frequency

The topology of the 8 *m* x 1 *m* x 0.01 *m* simply supported beam shown in Fig. 4 must be optimized in order to maximize the first natural frequency. The volume of the structure must be 50% of the original volume. Material properties are as follows: Young's modulus E=10 *MPa*, Poisson's ratio v=0.3 and mass density ρ = 1 kg/m^3. The design domain was meshed in 11200 (i.e. 280x40) 4-nodes bilinear elements. The filter radius is r_{min}=3 while the penalty factor is p=3. The internal parameters are: λ=2, α=1, γ=0.8 and convergence error limit is 0.01%, respectively.

In this test problem, ACO was not able to find an optimal topology. The reason is that ACO and hard-kill BESO use only solid or void elements. If intermediate topology is not consisted of a structure, the eigenvalue calculation cannot be carried out because of highly localized vibration mode. Therefore, ACO can be compared only with soft-kill BESO. Optimal topologies obtained by the above mentioned two algorithms are presented in Fig. 5. While the first intermediate designs are totally different (BESO did not change significantly topology with respect to the initial design), optimal topologies are much more similar.

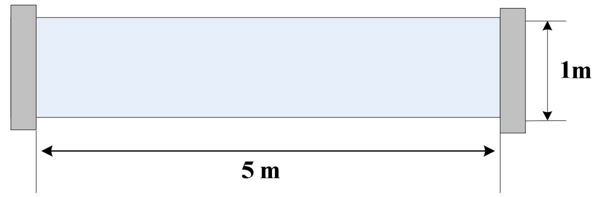

Figure 1. Schematic of the short beam under plane stress conditions in test problem 1

(a) BESO

(b) ACO

Figure 2. Comparison of topologies optimized by different algorithms for test problem 1 (first natural frequency)

(a) BESO

(b) ACO

Figure 3. Comparison of topologies optimized by different algorithms for test problem 1 (second natural frequency)

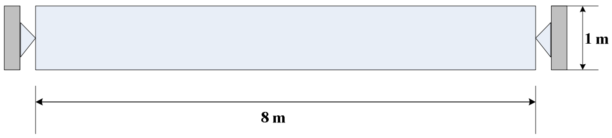

Figure 4. Schematic of the simply-supported beam in test problem 2

(a) BESO

(b) ACO

Figure 5. Comparison of topologies optimized by different algorithms for test problem 2 (first natural frequency)

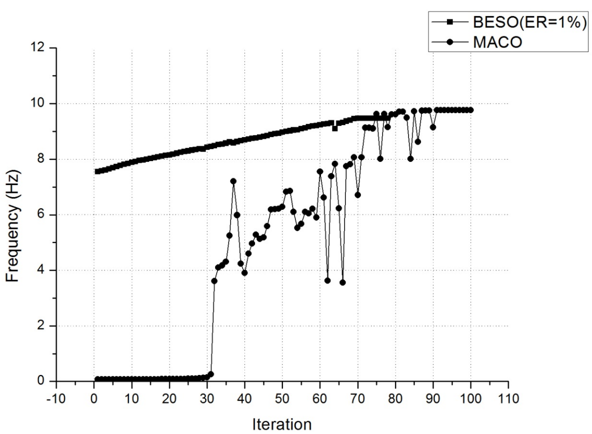

Figure 6. Convergence curves for test problem 2 (first natural frequency)

V. CONCLUSIONS

This paper presented a ant colony optimization algorithm for dynamic topology optimization of 2D structures. ACO implemented a novel ant colony optimization formulation where the concept of ant position is reformulated in terms of the sensitivity of structural response to the presence of each element.

From the comparison of the results obtained by ACO with those produced by BESO, the followings can be concluded:

(1) The ACO is a robust and stable algorithm in topology optimization for dynamic problems.

(2) It is verified that the modified ACO is more effective and suitable for dynamic topology optimization comparing with the former ACO.

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