

Set-Cover Formalization

Algebraic Analysis: L=15 vs L=17

PROBLEM: Minimum constraints to determine all 97 positions

SET-COVER INSTANCE:

- Universe $U = \{\text{unknown positions}\}$
- Constraint families $S = \{\text{single-position constraints}\}$
- Each constraint covers exactly 1 element

L=17 CASE:

- $|U| = 50$ (unknown positions)
- Each $s \in S$ covers 1 position
- Minimum cover = 50 constraints
- This is TIGHT (proven empirically)

L=15 CASE:

- $|U| = 21$ (unknown positions with anchors only)
- Slot reuse enables propagation
- Tail's 23 positions cover all 21 unknowns
- Achieves closure BUT wrong plaintext

Property	L=15	L=17
Slot mapping	Reuse (1.08/slot)	1-to-1 (unique)
Anchors only	76/97 derived	24/97 derived
Min constraints	21 additional	50 additional
With tail (23)	Complete (wrong PT)	Incomplete (50 remain)
Produces canonical	NO ✗	YES ✓ (where known)
Algebraic viability	YES	YES
Semantic correctness	NO	YES