

Correcting a proof in the InfoGAN paper*

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The InfoGAN paper¹ has the following lemma:

Lemma 5.1. *For random variables X, Y and function $f(x, y)$ under suitable regularity conditions: $\mathbb{E}_{x \sim X, y \sim Y|x}[f(x, y)] = \mathbb{E}_{x \sim X, y \sim Y|x, x' \sim X|y}[f(x', y)]$.*

The statement is correct, but the proof in the paper is confused – here’s a step where x mysteriously becomes x' :

$$\begin{aligned} & \int_x \int_y P(x, y) f(x, y) \int_{x'} P(x'|y) dx' dy dx \\ &= \int_x P(x) \int_y P(y|x) \int_{x'} P(x'|y) f(x', y) dx' dy dx \end{aligned}$$

After consulting with others, we weren’t able to fix that step of the proof. Instead, Nic Ford found this alternative proof. Hopefully this could help others reading the paper.

Proof.

$$\begin{aligned} \mathbb{E}_{x \sim X, y \sim Y|x}[f(x, y)] &= && \text{make expectations explicit...} \\ \mathbb{E}_{x \sim P(X)} [\mathbb{E}_{y \sim P(Y|X=x)}[f(x, y)]] &= && \text{by definition of } P(Y|X=x) \dots \\ \mathbb{E}_{x, y \sim P(X, Y)}[f(x, y)] &= && \text{by definition of } P(X|Y=y) \dots \dots \\ \mathbb{E}_{y \sim P(Y)} [\mathbb{E}_{x \sim P(X|Y=y)}[f(x, y)]] &= && \text{rename } x \text{ to } x' \dots \\ \mathbb{E}_{y \sim P(Y)} [\mathbb{E}_{x' \sim P(X|Y=y)}[f(x', y)]] &= && \text{by the law of total expectation...} \\ \mathbb{E}_{x \sim P(X)} [\mathbb{E}_{y \sim P(Y|X=x)} [\mathbb{E}_{x' \sim P(X|Y=y)}[f(x', y)]]] &= && \text{make expectations implicit...} \\ \mathbb{E}_{x \sim X, y \sim Y|x, x' \sim X|y}[f(x', y)] \end{aligned}$$

□

*Read and comment on the latest version of this note at <http://aoliver.org/correct-proof-of-infogan-lemma>

[†]<http://nicf.net>

¹<https://arxiv.org/pdf/1606.03657.pdf>