Assignment 1

Exact and Parameterized Algorithms, 2021

Every question carries 10 marks

1. G is a cluster graph if every component of G is a clique. For a given graph G, the Cluster Editing problem is to decide whether at most k edges can be added or deleted to G to make G a cluster graph. Design a polynomial kernel for the Cluster Editing problem parameterised by k.
   Hint : Show that a Cluster Graph cannot contain a path of length 3 as an induced subgraph.

2. Complete the definition and analysis of the following branching algorithm for the Feedback Vertex Set problem parameterised by the solution size.

   —Preprocessing Steps–
   Show that there exists a cycle of length $O(\log n)$. (*Hint : Bound the min. degree and consider the BFS tree.*)
   ——-branching step—-
   Show that the running time is FPT.

3. Consider the Set Cover problem.

   Given a universe U, a family $\mathcal{F}$ of subsets of U, $(|U| = u, |\mathcal{F}| = f)$, does there exist a subset $\mathcal{F}'$ of $\mathcal{F}$ such that $|\mathcal{F}'| = k$ and every element in the universe is contained in at least one set in $\mathcal{F}'$?

   Give a branching algorithm for Set Cover parameterised by $u + f$.

   (*Hint : Design reduction rules such that every element in the universe is contained in at least two sets in $\mathcal{F}$. Branch on the largest set and argue that there is a drop of $(1, 4)$ in the measure.*)

4. Given a set of points $R \cup B$ in the plane where R is a set of "red" points and B is a set of "blue' points, find a set of lines such that

- every line is either horizontal or vertical
- every blue point is contained in at least one of the lines
- number of red points contained in the union of lines is at most k

(Note that a point (x_p, y_p) is contained in the horizontal line $y = y_p$ and the vertical line $x = x_p$).
Give a kernelization algorithm for this problem parameterised by k. i.e, bound the size of B and R by a (polynomial) function of k.