Project Plan

Avi Vajpeyi [1], Rory J. Smith [*2] and Jonah B. Kanner [†2]

[1]The College of Wooster, Wooster, OH 44691, USA
[2]LIGO Laboratory, California Institute of Technology, Pasadena, CA 91125, USA

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Abstract

On September 14th, 2015, the Advanced LIGO detected the first gravitational wave [2]. The detected wave had a very large Signal to Noise Ratio value, which made it stand out from the rest of the candidate events. This paper investigates an alternative detection statistic, involving the 'Bayes Factor.' This detection statistic might prove to be more robust than SNR, as it may be able better to discern between strains due to gravitational waves, and strains due to noise. This study of the new detection statistic is focused on binary black hole systems.

1 Introduction

The completion of the two Advanced Laser Interferometer Gravitational-Wave Observatories has led to the discovery of a gravitational-wave signal [2]. This paper deals with a new detection statistic, involving Bayesian statistics, to rank the different candidate events (the strains in the data sets that could potentially be due to gravitational waves). The candidate events we focus on are from binary black hole systems as they are believed to be fairly common in the Universe [1]. Before we study this new method, we will discuss how data is currently being recorded and analysed.

Each of the advanced LIGO observatories uses a modified Michaelson Interferometer that measures the difference in the length of the orthogonal arms of the observatory to detect the presence of a gravitational wave [2]. On passing, a gravitational wave induces a difference in the length of the arms which is measured as a phase shift in the circulating laser light..

*smith_r@ligo.caltech.edu
†jkanner@caltech.edu

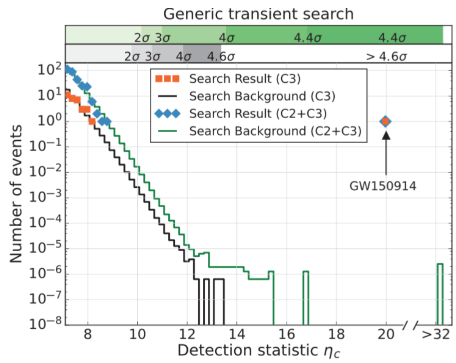
To determine if data recorded by LIGO stores gravitational wave information, the data is processed with two search techniques. One search looks for generic transient waveforms (unmodeled or unexpected waveforms) in data [3]. The second is a match filtered search that compares the data with templates of waveforms generated by general relativity [3].

Both the search processes are made challenging due to the background noise present in the data. This background noise can result from defects in a mirror, the uncertainty in the number of photons traveling in a the laser beam (shot noise), seismic activity, or even thermal noise generated by the Brownian motion of electrons inside circuits [4]. To separate strains caused by background noise from those caused by gravitational waves, the data from one LIGO observatory is compared with another LIGO observatory's data [2]. To compare the data sets, one of the them can be time-shifted so that it matches the data in the other detector over the light-travel time between the detectors. That is, one data set can be time-shifted so that both of the data sets lie along the same interval of time.

After making the necessary time shift, events present in one data set but not the other become apparent. This process cuts a majority of the strains that may have been present due to noise [2]. We attempt to discount any remaining noise-strains by implementing a detection statistic to rank the strains according to the likelihood that the strain resulted from a gravitational wave.

The detection statistic currently being used to rank strains according to the maximum likelihood of it being due to a gravitational wave is the signal-to-noise ratio (SNR) detection technique [3]. This method compares the power of the strain signal to the power of the remaining noise at a given point [4]. Although the power of the noise is difficult to calculate, we know that this ranking process can make some gravitational wave strains stand out, as seen from the Fig 1. However, we believe that this method lacks the sensitivity necessary to detect some gravitational waves that do not have SNR values as high as those of GW150914. Hence we would like to investigate an alternate detection statistic, specifically one that compares the relative probability that a data set contains a strain due to a gravitational wave to the probability that the data just contains noise. This ratio is known as the Bayes Factor.
Additionally, the templates that are used in the matched filtered searches account for only a small subset of the vast possibility of gravitational wave signals. This might result with some gravitational waves signals that do not have templates to be left undetected . In contrast, the LALInference program, which is what will be used to implement the new detection statistic, uses templates that can describe a far bigger set of signals. Hence, the Bayes factor might prove to be more sensitive than SNR, as we will have more information about the signal.

Figure 1: Search results of the generic transit search in which the first gravitational wave signal, marked GW150914, was detected. The wave strain of GW150914 stands out as the strongest strain in the entire search. SNR was used in this search to calculate the background noise and the events. Figure taken from [2]



2 Objectives

To compute the Bayes factor between two hypotheses, we need to first define the models we are comparing. The models are descriptions of the data $d(t)$, which contains either only noise, or noise along with a gravitational wave signal, parameterized by a certain vector $\vec{\theta}$. The parameter vector $\vec{\theta}$ contains information on several quantities describing the binary black hole system such as the masses of the black holes, their spin vectors, and the distance of the black holes from Earth [5]. The models we will use can be written as:

- $\mathcal{H}_{null}$: the noise model, which corresponds to the hypothesis that $d(t)$ contains only noise, $\mathcal{H}_{null} : d(t) = n(t)$.
- $\mathcal{H}_{GW}$: the gravitational wave signal model, which corresponds to the hypothesis that the data contains noise, and a gravitational wave signal parameterised by $\vec{\theta}$. Hence the model is defined by $\mathcal{H}_{GW} : d(t) = h(\vec{\theta}, t) + n(t)$.

We can compare these two models by calculating the relative probabilities in the form of the posterior odds ratio $O_{GW,null}$ between the two of them [5],

$$\begin{aligned} O_{GW,null} &= \frac{P(d|\mathcal{H}_{GW})}{P(d|\mathcal{H}_{null})} \frac{P(\mathcal{H}_{GW})}{P(\mathcal{H}_{null})} \\ &= B_{GW,null} \frac{P(\mathcal{H}_{GW})}{P(\mathcal{H}_{null})}, \end{aligned} \tag{1}$$

where $B_{GW,null}$ is the ‘Bayes’ Factor,’ which is equal to [5]:

$$B_{GW,null} = \frac{P(d|\mathcal{H}_{GW})}{P(d|\mathcal{H}_{null})} \ . \tag{2}$$

It should be noted that the relative probability, Eq 1, and the Bayes factor, Eq 2, contain no reference to the signal parameters $\vec{\theta}$. Hence, the Bayes factor $B_{GW,null}$ can be calculated from our hypothesis, for any values for the parameters $\vec{\theta}$.

To be able to account for the set of parameters $\vec{\theta}$ for a gravitational wave, the likelihood of the model $\mathcal{H}_{GW}$ needs to be marginalised over all the parameters, weighted by their prior probability distribution, giving the marginal likelihood or evidence, given by $P(d|\mathcal{H}_{GW})$ [5].

To calculate $P(d|\mathcal{H}_{\mathcal{GW}})$, first we need to determine the *posterior probability density function* (PDF). For gravitational wave analysis, the PDF of the parameters $\vec{\theta}$ is [4]:

$$p(\vec{\theta}|d, \mathcal{H}_{GW}) = \frac{p(\vec{\theta}|\mathcal{H}_{GW}) \ p(d|\vec{\theta}, \mathcal{H}_{GW}}{p(d|\mathcal{H}_{GW})}, \tag{3}$$

or in other terms,

$$Posterior\ Probability\ Density\ Function = \frac{Prior \times Likelihood}{Evidence}.$$

In other words, in Eq 3, we have each model $\mathcal{H}_{GW}$, to have a vector of parameters $\vec{\theta}$, with which we can calculate a 'Prior' distribution of $P(\vec{\theta}|\mathcal{H}_{GW})$. This states what values the model $\mathcal{H}_{GW}$ might be expected to take from the data set d. We also have $p(d|\vec{\theta}, \mathcal{H})$, which is the 'likelihood' of the data, given that the model $\mathcal{H}_{GW}$ is true.

To finally solve for $p(d|\mathcal{H}_{GW})$, we rearrange Eq 3 and integrate over $\vec{\theta}$, to get

$$p(d|\mathcal{H}_{GW}) = \int_{\Theta} p(\vec{\theta}|\mathcal{H}_{GW}) \ p(d|\vec{\theta}, \mathcal{H}_{GW}) d\vec{\theta}, \tag{4}$$

since the integral of the PDF, by definition of a probability density, is

$$\int_{\Theta} p(\vec{\theta}|d, \mathcal{H}_{GW}) \ d\vec{\theta} = 1.$$

We can now use the value for $p(d|\mathcal{H}_{GW})$ from Eq 4, and substitute it into our equation for the Bayes Factor, Eq 2. Using this Bayes' Factor, may be able to better highlight the strains due to gravitational waves in our data.

We will demonstrate the use of the Bayes factor by generating a figure, like Fig 1. However, instead of using SNR to calculate the background noise and rank the cadidate events, the Bayes factor will be used.

3 Approach

The main objective of this project will be to asses the sensitivity of the Bayes' Factor to rank gravitational waves that might have been missed using SNR as the detection-statistic. This will be done with a procedure that we will write, with the help of the LALInterference program.

We will begin by establishing the search background of the noise, with the help of the Bayes factor. This search background is made after time-shifting the data, as discussed in Section1. We will then inject gravitational wave signal into data sets, and use the new detection statistic to study if it can detect the waves. If the Bayes Factor is unable to detect the gravity wave signal, we will study the thresholds at which the the gravity wave signals can finally be detected with the Bayes factor.

We will then try to study past data and see if the Bayes Factor detection statistic can help extract more gravitational waves signals from the noise in the various data sets. We will also like to study if it will be able to detect gravity waves from quieter events, that would appear on the background of SNR, as seen in Fig 1.

4 Project Schedule

WEEK 1: Restructure the problem statement, and understand the past programing done for Bayesian statistics. Develop an approach to write the program to create the detection statistic that will rank the events using the Bayes factor defined in Eq 2.
WEEK 2-3: Analyse noise and injected gravity wave signals to determine sensitivity of the Bayes Factor.
WEEK 4: Study the results in detail, and restructure the computer program. Test the thresholds and compare the Bayes Factor detection statistic to the SNR detection statistic.
WEEK 6-7: Understand the results and collect more data. Begin compiling figures and writing to make the work more presentable.
WEEK 8-9: Further documentation of the lab notebook, and program, for future researchers. Begin retaking data if required for the final paper.

References

[1] Jea Abadie, BP Abbott, Richard Abbott, Matthew Abernathy, Timothee Accadia, Fausto Acernese, Carl Adams, Rana Adhikari, Parameswaran Ajith, Bruce Allen, et al. Predictions for the rates of compact binary coales-

cences observable by ground-based gravitational-wave detectors. *Classical and Quantum Gravity*, 27(17):173001, 2010.

[2] BP Abbott, R Abbott, TD Abbott, MR Abernathy, F Acernese, K Ackley, C Adams, T Adams, P Addesso, RX Adhikari, et al. Observation of gravitational waves from a binary black hole merger. *Physical Review Letters*, 116(6), 2016.

[3] Jonah B Kanner, Tyson B Littenberg, Neil Cornish, Meg Millhouse, Enia Xhakaj, Francesco Salemi, Marco Drago, Gabriele Vedovato, and Sergey Klimenko. Leveraging waveform complexity for confident detection of gravitational waves. *Physical Review D*, 93(2):022002, 2016.

[4] Rory James Edwin Smith. *Gravitational-wave astronomy with coalescing compact binaries: detection and parameter estimation with advanced detectors*. PhD thesis, University of Birmingham, 2013.

[5] John Veitch and Alberto Vecchio. Bayesian coherent analysis of in-spiral gravitational wave signals with a detector network. *Physical Review D*, 81(6):062003, 2010.