Chaotic Scattering for a Sliding Mass in a Complex Topography: An Annotated Bibliography

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1 Introduction

The focus of my project is to gain a better understanding of chaotic systems, by looking at a simulation of a particle, scattering due to the presence of four hills in its path. There will be one impact parameter, or initial trajectory, which causes the particle to scatter, and this will be plotted against the scattered angle to study the regions of normal and chaotic behavior of the system. It is predicted that the chaotic behavior is fractal in nature, which is also something we will look into.

References

[1] Konstantinos N Anagnostopoulos. *Computational Physics, Vol I: A Practical Introduction to Computational Physics and Scientific Computing*, volume 1. Konstantinos Anagnostopoulos, 2014.

> This book, used by Carnegie Mellon in its Introduction to Computational Physics course, and written by a physics professor at the National Technical University of Athens, explores alternative numerical methods for the formulation of physical problems for machine computation. The main usefulness of this book comes from its chapter on the motion of a particle, where the numerical integration of Newton's equations is discussed. The Euler methods for solving the differentials are explained with an example of a simulation demonstrating the movement of a pendulum. Then the fourth order Runge-Kutta algorithm is introduced (205). The chapter elaborates on how the Runge-Kutta algorithm, in place of Euler's, increases the integration accuracy, while asserting that integration accuracy can be verified by studying the energy of the system. Additionally, the chapter on planar motion discusses the movement of a particle traversing across a plane under the influence of a dynamic field, a physical quantity that has a value for each point in space

and time. Although this mainly looks at two dimensional planetary motion simulations, and Rutherford scattering, the section is helpful in providing a simplified version of the fourth order Runge-Kutta algorithm for planar motion equations (241). The implementation of the algorithm is also provided (257). This is helpful as my simulation also uses this algorithm, and hence, having an example of how the algorithm is used in a program helps me avoid logical errors.

[2] DG Bettis. Runge-kutta algorithms for oscillatory problems. *Zeitschrift für angewandte Mathematik und Physik ZAMP*, 30(4):699–704, 1979.

This paper presents a derivation for the second order Runge-Kutta method, after which it gives a detailed explanation on how the method works. It then proposes various modified versions of the Runge-Kutta algorithms that may be more accurate than the normal four stage Runge-Kutta algorithm. We know that on the plots of the scattering angle against the impact parameter, there are certain speckled regions. When the horizontal scale for the speckled regions are enlarged, the speckled regions still remain. Even further enlarging of those speckled regions does not resolve the function, suggesting the fractal nature of these graphs. Hence, as I am studying this fractal nature, I need high resolution graphs, which I can enlarge with ease. These can only be created with very accurate integration, and this paper provides a potential path by which the integration accuracy can be increased.

[3] Harvey Gould and Jan Tobochnik. More on fractals and chaos: Multifractals. *Computers in Physics*, 4(2):202–207, 1990.

This dense paper discusses the trajectories of chaotic systems being fractal object known as strange attractors, and the connections between fractals and chaos. It begins with introducing objects known as ?multi fractals?, fractals systems with a continuous spectrum of components. It then goes on to discuss the concept of fractal dimensions, and provides a formula to calculate it. The paper then discusses Koch curves, and applies its concepts and formula for multi fractals on an instance of a Koch curve. Finally, it extends the multi fractal analysis to the case where a fractal is produced by the strange attractor of a dynamical system. This portion where it discusses the statistical description of a fractal object for dynamic systems is helpful as it provides a method to analyse the fractals that form in chaotic scattering graphs. The paper then continues by discussing deeper analysis of fractals with probability distributions, something that may be interesting to look into with chaotic scattering if time permits.

[4] Jorge V José and Eugene J Saletan. *Classical dynamics: a contemporary approach.* Cambridge University Press, 1998.

> "A great book on Classical Mechanics", referenced by Anagnostopoulos's Computational Physics, Vol I. Although this book is aimed at graduate physics students, there is a significant amount of information on non-linear dynamic systems exhibiting chaotic behaviour that has been explained in a comprehensive way. Additionally, the scattering of a charge by a magnetic dipole with potential hills is discussed. The potentials are set up as circularly symmetric potential hills in a plane. Equations to map such potentials are also provided here (169). This is helpful in understanding how a region of four circularly symmetric potential hills can be set up in a plane, which is what my simulation requires. The article then proceeds to discuss the functional relationship between the scattering angle and initial input trajectory for a fixed velocity. It then highlights that chaotic scattering occurs only when a given particle's energy is lower than the maximum potential energy of the potential hills, and elaborates on different patterns that emerge between the chaotic and non-chaotic scattering. This is useful as it provides a framework on how to structure my system, specifically for chaotic scattering.

[5] Edward Ott and Tamás Tél. Chaotic scattering: An introduction. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 3(4):417–426, 1993.

> Although published two decades ago, this article, citied by more than one hundred and sixty different books and journals, provides a introduction to the general topic of chaotic scattering. It describes elementary background information, including the details on the inputs and outputs, such as the impact parameter, and scattering angle. The article also discusses the formation of the fractals in the plots of the impact parameter against the scattering angle. It provides various examples of how chaos theory is used in various academic fields, ranging from celestial mechanics to models of chemical reactions. Even though it does not provide any clear algorithms that can be implemented in a simulation, it gives a very thorough introductory understanding of chaotic scattering. Hence, it is helpful in providing the initial theory that will be used in the theory section of the paper.

[6] Tao Pang. *An introduction to computational physics.* Cambridge University Press, 2006.

> This book, used in Cambridge for their Computational Physics course is aimed at first year graduate students. It is highly accessible, and gives detailed explanations of steps that need to be taken during implementation of physics simulations in JavaScript. It provides many simple tutorials, and has

a section on animating trajectories. This is useful as it acts as a direct reference for the implementation of the chaotic scattering simulation, which has animations of the particle scattering after passing through the region with hills. It also provides useful information to program physics simulations, such as the various techniques by which the program correctness can be verified with energy, and angular momentum constant.

[7] Tolga Yalinkaya and YingCheng Lai. Chaotic scattering. *Computers in Physics*, 9(5):511–539, 1995.

This article, co-authored by Lai, a theoretical physicist who wrote 'Transient Chaos: complex dynamics in finite time scales', a principle book for chaos theorists, addresses scattering using the Gasprd-Rice model. It discusses the origin of chaos in scattering experiments, and walks through how the Gaspard-Rice model displays chaotic characteristics. After defining relevant terms for the study, such as the scattering function and the various parameters, the paper also gives specific details on the variables. It stated how the scattering angle is defined counterclockwise, in relation to the initial path of a particle which comes from negative infinity. Additionally, it elaborates on how the delay time is defined not only as the time the particle spends in the scattering region, but also the time the particle spends within a chosen distance from the center of the scattering region. These specific details are very important, as it helps understand how the rules should be defined for my simulation, which deals with similar parameters and variables. The paper continues discussing the Gaspard-Rice scattering system, and provides examples of input parameters which result with systematic and chaotic behaviors. It pointed out that if the data for scattering angles and impact parameters are plotted on a graph, then we see plots with smooth and oscillating parts, which when blown up also show smooth and oscillating parts. The paper asserted that this fractal nature of the scattering function can be studied by computing fractal dimensions. The paper then discussed two methods for calculating fractal dimensions, a box counting, and an uncertainty dimensions method. It argued that the latter is easier to compute, hence, the uncertainty dimensions method is what is used in my paper. Furthermore, the paper discusses various suggestions for future work, which is where I obtained the idea to create a simulation with four potential hills rather than three solid disks, as done in the Gaspard-Rice scattering.