

Avalanches, Brains and Stocks: Simulating Self-Organised Criticality

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Abstract

Experiments with a granular bead pile have shown that the pile can model critical systems such as avalanches. In the experiments, a bead is dropped on the apex of the pile of beads. Eventually, one such bead causes several beads of the conical bead pile to avalanche. The experiments have studied the distribution of avalanches and how the distribution is affected by altering the bead type, bead cohesion, and bead drop height. In this study we present a computational simulation of the experiment. The simulation models each bead as independent particles with their own positions and velocities. The internal and external forces on each particle are accumulated to provide the new particle positions using Newton's second law of motion. The independence of each particle allows a natural way to express parallelism which we utilise by threading various processes of the particles to the graphical processing unit. This allows the simulation run in real-time while still having 60k particles on the pile.

With this simulation we can learn new information from the system that may have been challenging to study with the actual experiment. For example, we can vary the shapes and numbers of the beads. With the simulation it is also possible to record the velocity of each particle on the surface and even inside the pile.

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Chapter 1

Introduction

Chapter 2

Background

2.1 Per Bak and Sand in Brains

In 1999 a Danish Scientist by the name Per Bak explained the processes of a brain with a term he created - “self-organised criticality”. He suggested that the brain’s neural activity worked in a similar process as a sand pile which incurs avalanches of varying sizes to maintain stability. As more sand is added to the pile, more avalanches occur along the surface of the conical shape. Even with these unpredictable surface avalanches, the conical shape of the pile is maintained. Per Bak pointed out that although the individual avalanches were unpredictable in timing and size, the distribution of the timings and sizes of several avalanches demonstrate a regularity.

2.2 Self Organised Criticality

talk about connection between SOC, earthquakes, stockmarkets

2.3 The Experiment

2.4 Previous Simulation

2.5 Unified Particle Physics and Position Based Dynamics

2.6 Math & Citations

Examples of inline math are $\alpha = \sqrt{\gamma^2 + \Gamma^2}$ and $\vec{v} = 7\hat{x} - 5\hat{y}$ and $\vec{u} \times \vec{v}$ and $c = (2.99 \pm 0.01) \times 10^8$ m/s. One example of block (display) math is

$$\int_0^1 x^2 dx = \frac{1}{3}, \quad (2.1)$$

and a second example is

$$\xi = \alpha \left(\frac{1}{\omega_0^2 + \omega^2} \right). \quad (2.2)$$

Note how block math is punctuated like words in a sentence! The block math equations are automatically numbered. We can reference Eq. 2.1 or Eq. 2.2 by inserting labels in the block, but then we must compile \LaTeX twice.

We can readily cite articles [1] and books [2] and URLs [3] in our bibliography, but now we must compile \LaTeX , BibTeX , \LaTeX , \LaTeX .

2.7 Figures & Tables

We can also include figures, but first we need to use package “graphicx” under document class. We can reference Fig. 2.1 like equations. All figures should have captions.

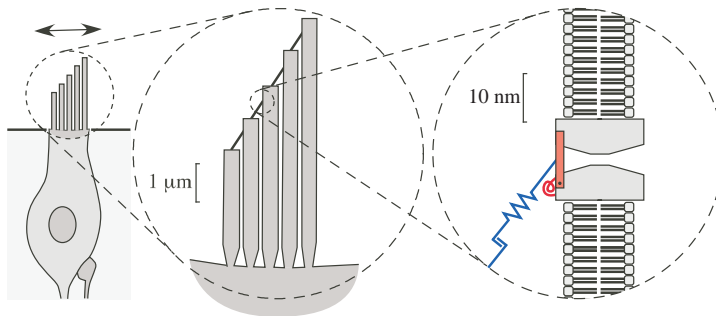


Figure 2.1: Figure captions go on bottom.

In the document class line, we can easily convert from “preprint” one-column, double-spacing for rough drafts to “twocolumn” single spacing for final drafts!

Table 2.1: Table captions go on top.

abscissa	ordinate
1.0 s	5.6 m
2.0 s	6.7 m
3.0 s	9.9 m

Chapter 3

Conclusions

3.1 Stuff

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Appendix A

Extra Stuff

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Appendix B

Extra Stuffing

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