Effect of cohesion on avalanches in granular piles

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Experiments and modeling show that cohesion significantly changes the avalanching behavior of a granular bead pile. In particular, cohesion can lead to catastrophically large avalanches that recur almost periodically. The effects of cohesion on the statistics and the time series properties of the experimental avalanches are matched by the effect of weakening on the slip avalanches in a simple model used for earthquakes and the plastic deformation of brittle solids. The results reflect a large universality class for avalanche statistics that depends neither on the material specifics nor on the system's geometry.

Introduction:

When a granular system responds to a slowly increasing driving force, it can switch from a stable structure to avalanching behavior, with a wide distribution of avalanche sizes, often extending to catastrophically large events. This is true both in natural settings, with hazards such as landslides and snow avalanches, and industrial situations involving the transport of granular materials—such as sand, agricultural grains, building materials—that require the grains to flow freely. High humidity or electrostatic forces can create cohesion between the grains, causing the grains to first interlock and then suddenly give way in catastrophically large avalanches. The results can be costly and even devastating. Understanding the effects of cohesion can provide key insights for industrial applications and the prevention of natural hazards.

The ultimate goal of this experimental and theoretical study is to understand and predict the effects of cohesion in order to determine whether a process needs to be modified or whether the effects of cohesion will be negligibly small or can even be suppressed. Cohesive forces between sand grains are known to drastically increase the stability of sandpiles, so that wet sand has a much larger maximum angle of stability or "critical angle" than dry [1]. While several studies have investigated the static properties of wet and dry sand piles, little is known about the effect of cohesive forces on the dynamics, such as slip avalanches and their statistical distributions. Our investigation of effects due to cohesion identifies the key parameters of granular dynamics with cohesion in the pile geometry. The results relate avalanche statistics in very different geometries, and they match the avalanche dynamics of granular piles to those of slowly sheared cohesive materials, solids, and jammed grains [2].

In particular we predict and measure the effects of controlled cohesion on the statistics of slip avalanches on bead piles - one of the simplest examples of the flow of granular materials. Past work [3-7] has mostly focused on the dynamics of non-cohesive granular materials; our results show that cohesion can lead to catastrophic effects that cannot be treated as a small perturbation to the non-cohesive studies. In this work we find excellent qualitative agreement between experimental results for a bead pile with cohesion and the predictions of an analytic model and related simulations for the effect of cohesion on slip avalanches in a sheared solid. The similar behavior observed in these two distinctly different geometries indicates that the universality class for avalanche statistics depends less on the system's geometry than expected.

Experiments:

We drop one bead at a time onto a conical bead pile, shown schematically in Fig. 1. The beads are soft steel spheres, 3 mm in diameter. The pile is constructed on a circular base 17.8 cm in diameter (equivalent to about 60 bead diameters). Depending on the cohesion of the system, the pile contains 15,000 to 20,000 beads. The pile rests on a balance with a precision of 0.01 g, which is sufficient to resolve the addition or loss of a single bead. After each bead drop, the control computer waits for a stable mass and records it, before dropping another bead. The avalanches in the system are characterized by measuring the mass that leaves the pile. In an individual data run, we drop approximately 60,000 beads onto the pile over the course of 165 hours. For all runs presented here, beads were dropped onto the pile from a height of 2 cm. This low drop height was chosen since our previous work [4] shows a higher drop height tunes the pile away from criticality and power law behavior.

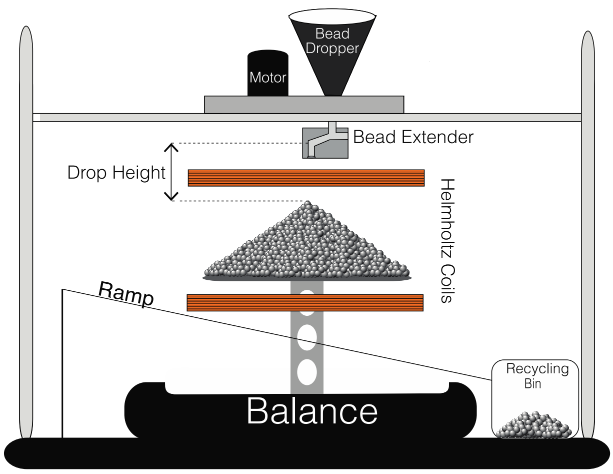

Fig. 1: Experimental bead pile apparatus. A single bead is dropped onto the pile; the balance measures the mass of the pile itself. Any beads that fall off the pile cause a decrease in mass; the beads fall onto a ramp and are returned to a bin for reuse. A pair of Helmholtz coils surrounds the pile in order to provide a magnetic field to cause cohesion between the soft steel beads. The beads do not retain their magnetization when no magnetic field is present.

We have previously studied the universal scaling behavior in avalanche distributions by changing the bead material and changing the height from which the beads are dropped [3,4]. Now we are adding cohesion to the system by inducing magnetization in soft steel beads through the application of an external magnetic field. The cohesive, dipole-dipole force between two beads is proportional to the product of the two dipoles, each of which is proportional to the applied magnetic field and thus proportional to the precisely controlled current applied to the Helmholtz coils. The cohesive force is thus proportional to the square of the applied current, and so we use the current squared as a measure of the cohesion in the system.

In our theoretical approach we use the power of universality to make predictions for a much larger class of systems, *i.e.* we focus especially on universal properties of the statistics. These are quantities such as scaling exponents and scaling functions that are independent of the microscopic details of the system or the precise nature of the cohesion. The theory of critical phenomena and the renormalization group have been used to show that these universal quantities are the same for a large class of systems which share the same fundamental properties, such as symmetries, dimensions, interaction ranges, etc. [8]. This means that the results of these studies are expected to apply not just to our piles of steel beads, but also to cohesion in sandpiles, powders, soils, and other avalanching systems with different geometries and different physics [8]. Similar far-reaching universality has been observed in liquid-gas, liquid-liquid, uniaxial ferromagnetic and polymer-solvent systems near their critical point where the precise nature of the interaction is irrelevant [9]. Universality has also been pointed out previously for avalanche studies in the absence of cohesion (see for example ref. [10]).

Here we apply methods from the theory of critical phenomena to the non-equilibrium critical scaling behavior observed in the avalanche statistics of our system. The results of this study then provide predictions for the universal effects of cohesion on a large (universality) class of avalanching systems with and without cohesion.

Model:

Many models have been proposed to describe avalanches in dry sand-piles in the absence of cohesion [11-18]. The models have been studied via simulations and analytical mean field theory. Here we go beyond the traditional non-cohesive case and develop an analytic approach that provides intuition and identifies the relevant control parameters that are then used to guide both experiments and simulations.

We base the approach on a simple analytic model for the statistics of slip-avalanches in slowly sheared granular materials [19]. The main idea is that weak spots in the material tend to slip by a random amount when the local stress exceeds some local failure threshold $\tau_{s,r}$. When they slip the released stress is redistributed to the other parts of the system and can trigger other sites to also slip through elastic interactions, resulting in slip avalanches. The size S of each avalanche is defined to be the integral of the slip distance over the failure area [19]. The model agrees with experiments on cohesionless (sheared) granular materials [20-22] and cohesionless bead piles [3,4]. We extended the model to include the effects of cohesion (between the grains) on the avalanche statistics [2]. Cohesion is modeled as a fast-healing, threshold weakening ε with that occurs upon slip, when a cohesive link breaks during a slip-avalanche. Once a location has slipped, this weakening reduces the slip threshold for that location by an amount proportional to ε. At the end of each avalanche the cohesion reheals the slip thresholds back to their static strengths. The mathematical description of the model is briefly reviewed in the supplementary material and in [2,19,23].

At zero cohesion, modeled by zero weakening ($\varepsilon = 0$), our mean field model for sheared granular materials belongs to the same universality class as the branching model process suggested by Zapperi *et al.* [14,15] for cohesionless sandpiles. In other words, our mean field model for sheared granular materials at zero weakening and zero cohesion predicts the same critical exponents and scaling functions for the avalanche statistics as Zapperi *et al.'*s mean field model for avalanches in cohesionless sandpiles [18], and these models agree with experiments [3,4]. Here we focus on the effect of cohesion. We postulate that our model with a positive weakening parameter describes the behavior that would be expected in a sand pile at finite cohesion. Cohesion implies strong pinning before a bead is released from its hold of the neighboring beads. Once the bead is released, it is much easier to keep rolling, very much like the weakening effect introduced in our model for sheared granular materials with cohesion [19,23]. At the end of each avalanche the beads stick again to the pile and the bonds reheal to their original strength and the process repeats.

In the following we compare the predictions of this model to the experimental results. A system size of 10,000 cells was used in the simulations of the model presented here. In the simulations the driving force was slowly increased between the avalanches. Each avalanche amounted to a force drop in the simulation – very much like when a solid material is slowly sheared at a fixed strain rate (see supplementary material).

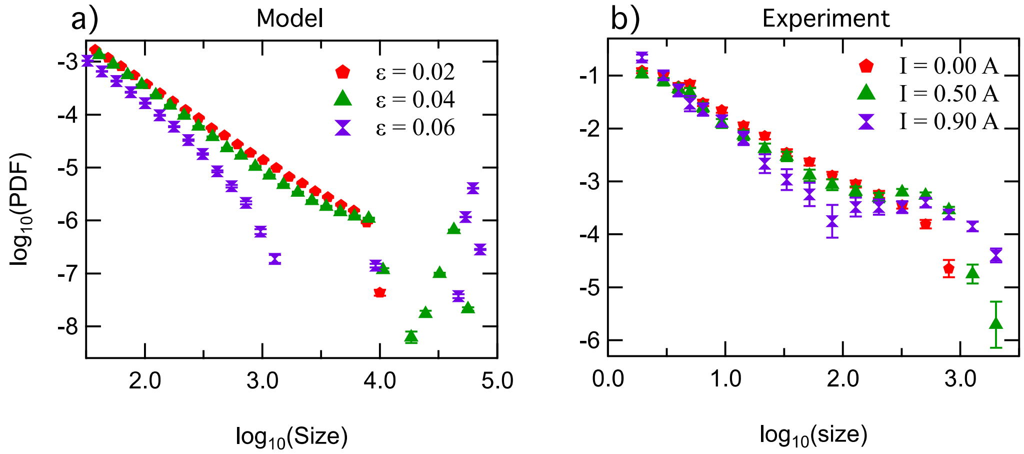

Fig. 2 Probability distribution functions for (a) the model and (b) experiment. In (a), cohesion is increased by increasing the weakening parameter ε, while in (b) the cohesion depends on the current I in Helmholtz coils surrounding the pile. In the $\varepsilon = 0.06$ simulation dataset, note that the PDF equals zero (and thus is not visible on the plot) for most sizes between 10^3 and 5×10^4. Each data set in (a) is the average of five simulation runs. In (b), the result for 0.00 A is a single run of ~60,000 bead drops; the data for 0.50 and 0.90 A show the average of two runs. In all cases, error bars are calculated assuming Poissonian statistics, using the square root of the number of avalanches in the bin as the uncertainty for the number of avalanches in the bin.

Results:

The change in avalanche dynamics due to the addition of cohesion is shown in the probability distribution functions (PDFs) plotted as a function of avalanche size in Fig 2. The PDF is calculated by binning the avalanches into logarithmically spaced bins. As fast-healing effective cohesion is added, the model predicts that the avalanche size distribution develops a bump at the largest sizes [2]. The bump implies that in addition to many small avalanches, there are almost periodically recurring very large avalanches, very similar to a system with stick-slip behavior. The bump moves to larger avalanche sizes S and grows in height for increasing cohesion ε, as seen in Fig. 2(a). This prediction agrees with our experimental data for avalanche size distributions at finite cohesion, shown in Fig. 2(b), where the experimental avalanche size s is defined as the number of beads leaving the pile.

As the cohesion is increased by increasing the current I through each Helmholtz coil, the number of midsize avalanches decreases, while both the number and the size of the largest avalanches increases. At the highest level of cohesion, with 0.90 A in each coil, a pronounced hump has developed in the PDF, due to the decrease of mid-size avalanches and the enhanced probability of the largest avalanches. The model shows the same behavior – the largest avalanches increase in number with increasing weakening so that we observe an enhancement in the probability that these largest avalanches will occur. At the same time, we observe a decrease in probability for mid-size avalanches, those that involve about a factor of 10 fewer beads than the largest avalanches.

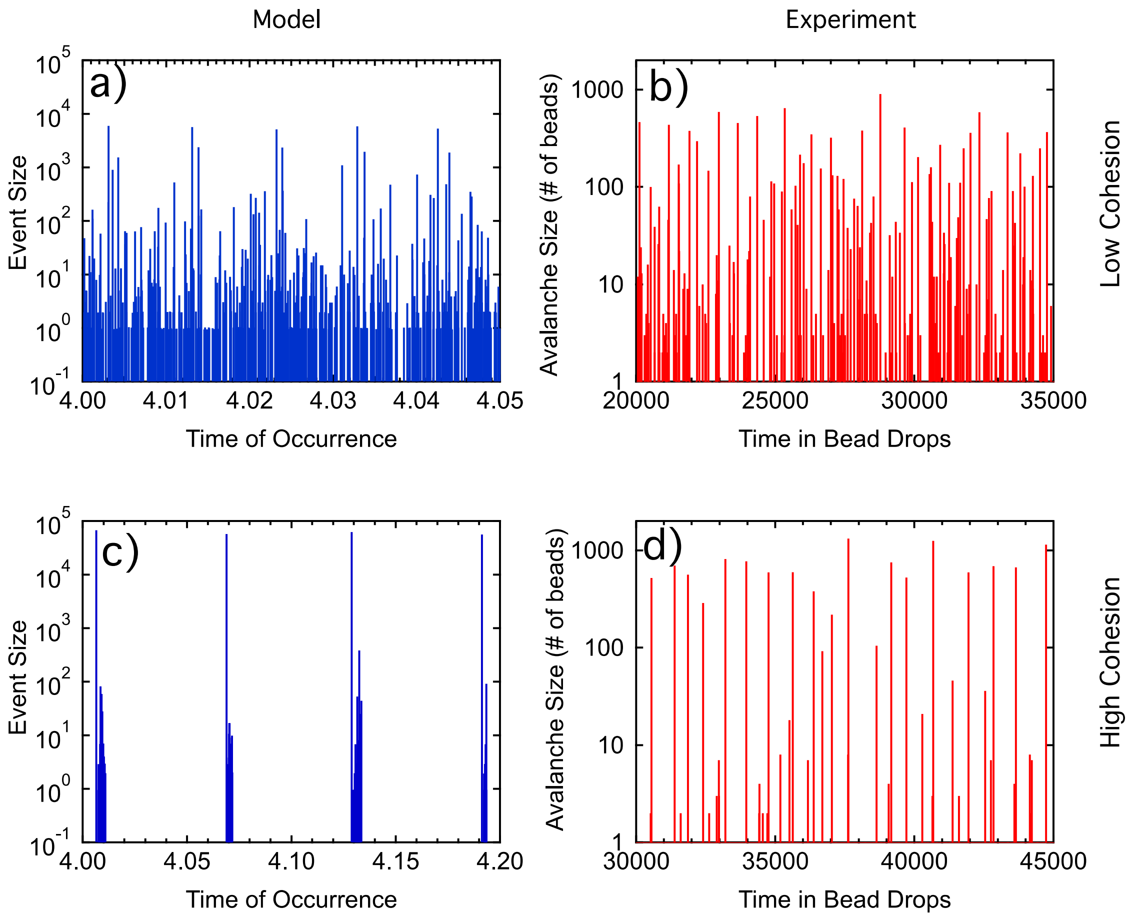

Fig. 3 Time series comparing the model (blue) and experiment (red) for low cohesion (top row) and high cohesion (bottom row). Note that the time scale for the higher cohesion simulation plot (c) has been extended to four times the range for the lower cohesion simulation plot (a), to better show the repeating cycle of large avalanches at high cohesion. Results shown are for weakening of (a) 0.02 and (c) 0.06, and experimentally for coil current of (b) 0.00 A and (d) 0.90 A.

The change in avalanche dynamics can also be observed using a time series of the avalanches in the system, as plotted in Fig. 3. This plot shows the size of the avalanche on a logarithmic scale versus the time at which the avalanche occurs. For the model, the time is set by a very slow driving rate (which is the rate at which the driving force is increased between the avalanches). This rate of increase is chosen to be sufficiently slow so that each avalanche is completed before the next one is triggered; the time values reported are proportional to the total amount of external force thus far added to the system. Experimentally, each bead drop is one unit of time, and again the time between the bead drops is chosen to be large enough to allow each avalanche to finish before a new bead is dropped on the pile.

In the system without cohesion (Fig. 3(b)), there are avalanches of all sizes, with a maximum avalanche size of about 900 beads. When the maximum amount of cohesion is added to the system (Fig. 3(d)), overall the number of avalanches decreases by just over a factor of 5 (comparing to Fig. 3(b)), the number of mid-range avalanches decreases especially, and the largest avalanches are larger in size than at low cohesion, up to about 1300 beads in size.

The same effect is seen in the simulation results, also shown in Fig. 3. Compared to the results at a lower weakening parameter, corresponding to lower cohesion (Fig. 3(a)), the size of the largest avalanches gets larger and more consistent in size with increased weakening (Fig. 3(c)). In the simulation, we also observe that the time between the largest avalanches becomes quite consistent so that these largest avalanches are almost periodic. This effect is also seen in the experimental results (though not quite as pronounced, due to the relatively low cohesion values accessible in the experiments) – the time between the largest avalanches becomes more regular with the addition of cohesion. We expect the effect to be even more pronounced for stronger cohesion in the experiments.

In both the simulation and the experiment, we also may calculate the time after an event of a given size until the next event of any size. These waiting times are shown as a function of the initial event size in Fig. 4. In the experimental measurements, there is a wide distribution of times after any particular avalanche, so the points plotted are the average waiting time for all avalanches within a size bin, with error bars representing the standard deviation of the mean for the distribution.

As can be seen in Fig. 4, for no cohesion, the average waiting time after an avalanche in the experimental system is relatively flat for a wide range of initial avalanche sizes, but the waiting time is longer for the largest avalanches. As the cohesion increases, the waiting time continues to be relatively flat for avalanches below about 50 beads in size, but then increases with avalanche size and with the amount of cohesion. The waiting time after the largest (>1000 beads) avalanches is, on average, nearly double when the Helmholtz coil current is 0.90 A (high cohesion) as when the current is 0.30 A (low cohesion). Since there is a greater number of these very large avalanches at high cohesion than at low cohesion, this difference in waiting time is due to a decrease in the number of small and mid-size avalanches.

In addition, an overall decrease in the total number of experimental avalanches as the cohesion increases can be seen qualitatively in Fig. 3 by comparing the number of events in Fig. 3(b) and Fig. 3(d), which both show a time period of 15,000 bead drops. Note that the effect has been divided out in Fig. 2 which shows merely the normalized distribution of avalanche sizes. The rate of avalanche occurrence (defined as the total number N of avalanches of any size occurring during a run, divided by the total elapsed time for the data run) is shown in Fig. 5. The rate decreases sharply as the cohesion in the system is increased, and the rate continues to decrease to a minimum at the highest levels of cohesion tested.

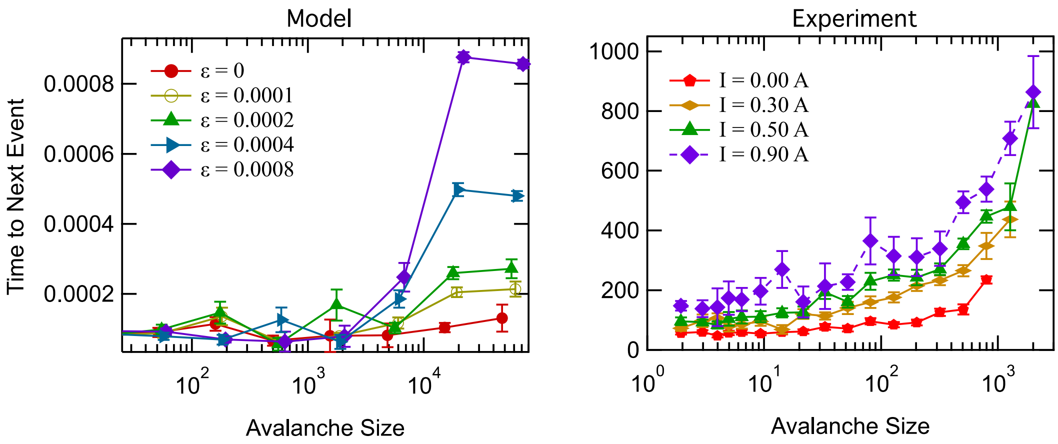

Fig. 4 The time from an avalanche of size *s,* until the next event of any size for both the model and the experiment. The results shown in (a) are each from a single simulation run. In (b), the result for 0.90 A shows the average of two data runs; the other three data runs shown are each single runs.

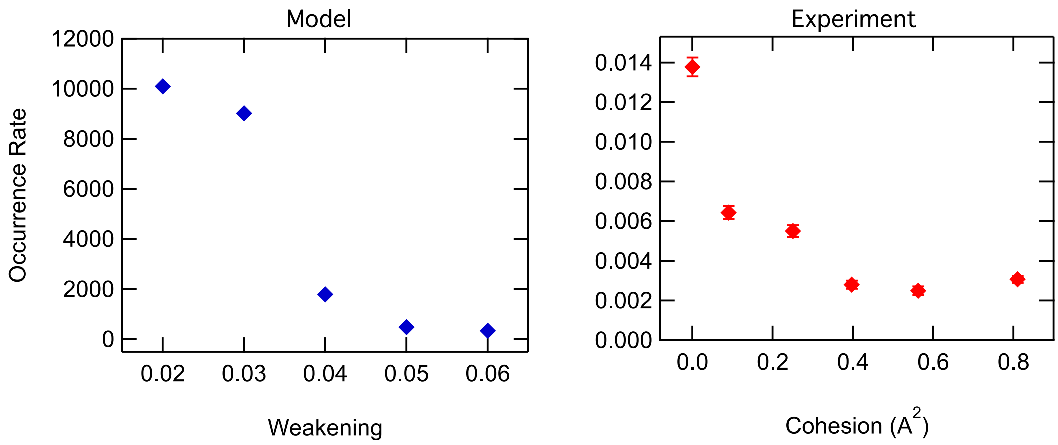

Fig. 5 Comparison of the occurrence rate of avalanches in the model (left) and the experimental system (right) as a function of cohesion. For the model, the rate is shown as a function of the weakening parameter; for the experiment, the rate is shown as a function of the square of the current. The uncertainty is calculated using as the uncertainty in *N*; for most points, the calculated uncertainty is smaller than the size of the plotted points.

Conclusion:

We have shown that both the avalanche statistics and the time series properties of avalanches on a bead pile with and without cohesion are well mimicked by a simple slip avalanche model with and without weakening. The model and the experiments both show that adding cohesion to the system leads to the formation of large, system spanning avalanches, that occur almost periodically, with smaller avalanches in between. This effect gives a distribution of avalanche sizes similar to what is observed for stick-slip behavior in other systems such as sheared granular materials and frictional systems. In other words, adding cohesion dramatically increases the probability of triggering catastrophically large avalanches. However, because these large avalanches recur almost periodically they also become more predictable than in the cohesionless case where large avalanches occur at random times.

For future work it will be interesting to compare these findings to the statistics of real landslides for soils with varying degrees of water content, and thus varying degrees of cohesion.

Acknowledgments:

We thank Dmitry Denisov and Peter Schall for helpful conversations. We thank the National Science Foundation for supporting this work through the collaborative NSF grants CBET 1336116 (SL) and 1336634 (KD).

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