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ABSTRACTS

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Index of nonlocal elliptic operators and fixed points of group actions

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Many problems in mathematical physics and noncommutative geometry are reduced to operators in the form of the linear combination

$$D = \sum_{g\in G} D_g T_g : C^\infty(M) \longrightarrow C^\infty(M) \qquad (1)$$

of the shift operators $(T_g u)(x) = u(g^{-1}(x))$ associated with the action of a group G on a smooth manifold M by diffeomorphisms, where the coefficients D_g are (pseudo)differential operators on M (e.g., see works by Antonevich and Lebedev, Connes, Skubachevskii, Nazaikinskii and Sternin). The equation $Du(x) = f(x)$ links values of the unknown function u and its derivatives at the points of the orbit of the group action and describes nonlocal interactions. Under very general assumptions about the group and its action on the manifold, conditions for the ellipticity of operators (1) are obtained, ensuring their Fredholm solvability in Sobolev spaces, and the Fredholm index of the operator is defined.

The index problem for elliptic operators (1) has been studied by many authors. In the case of a finite group G, the index was calculated by Antonevich in terms of the Lefschetz numbers of an auxiliary matrix operator. An index formula for operators on a non-commutative torus (where the group $G = \mathbb{Z}$ acts on the line by shifts) was found by Connes in terms of classes in cyclic cohomology (these and related classes were later used in studies of the quantum Hall effect). In a more general setting, an index formula was obtained by Nazaikinskii, Savin and Sternin for arbitrary isometric actions of groups of polynomial growth. For general actions (i.e., not necessarily isometric), an index formula for the group $\mathbb{Z}$ was obtained in [1] in cyclic cohomology. Related index problems were studied by Perrot and Rodsphon and also by Gorokhovsky, de Kleijn, and Nest.

In the present talk, we explain an index formula for operators (1) for the case of the group $\mathbb{Z} \times F$, where F is an arbitrary finite group. To solve this index problem, we explicitly write the contribution to the index of the fixed points of finite-order elements of the group.

This work is based on joint article [2]. It was financially supported by the Russian Science Foundation, project 24-21-00336, `https://rscf.ru/en/project/24-21-00336/`.

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Topological index of nonlocal elliptic operators on two-dimensional manifolds with cylindrical ends

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Let M be a manifold with cylindrical ends, i.e. a non-compact Riemannian manifold that is isometrically isomorphic, outside a compact set, to a cylinder $\Omega \times \mathbb{R}_+$ with a smooth compact base Ω. Pseudodifferential operators with symbols stabilizing at infinity are considered on M. For such operators, the ellipticity condition consists in the requirement of invertibility of the interior symbol, which is a smooth function on the cosphere bundle S^*M, and the invertibility of the conormal symbol, which is a smooth family of operators on the base Ω of the cylinder with parameter $p \in \mathbb{R}$. If the ellipticity condition is satisfied, then the operator is Fredholm, and the problem of calculating its index in topological terms arises.

We apply methods of Connes cyclic cohomology theory [1] and show that the topological index in the two-dimensional case is simply the Chern–Connes pairing of a class in the K-theory of the algebra of symbols and a special class in the cyclic cohomology of the algebra of symbols that we present.

In addition, we construct a topological index in the cyclic cohomology for nonlocal operators on the cylinder $\mathbb{S}^1 \times \mathbb{R}$ with coordinates $x \in \mathbb{S}^1, t \in \mathbb{R}$, generated by the action of the group $\mathbb{Z} \oplus \mathbb{Z}_2$ using the following diffeomorphisms:

$$\gamma = (g,k)\colon M \longrightarrow M, \quad \gamma(x,t) = \big((-1)^g x, t + 2\pi k\big). \tag{1}$$

These nonlocal operators have the form

$$A = \sum_{\gamma\in\Gamma} A_\gamma T_\gamma \colon L^2(\mathbb{S}^1 \times \mathbb{R}) \longrightarrow L^2(\mathbb{S}^1 \times \mathbb{R}), \tag{2}$$

where $A_\gamma \in \Psi(M)$ are pseudodifferential operators on M,

$$T_\gamma u(x,t) = u\left((-1)^g x, t - 2\pi k\right)$$

is the unitary representation of $\mathbb{Z} \oplus \mathbb{Z}_2$ by shift operators induced by the action on M, and only finitely many terms in (2) are nonzero.

This work is based on joint article [2] with Zhukov K. N. and Savin A. Yu.

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On the convergence of approximate solutions to the tracking problem

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In [1], the tracking problem was considered in non-linear vector optimization for oscillatory processes described by integro-differential equations in partial derivatives with Fredholm integral operator, in which the scalar functions representing external and boundary effects depends nonlinearly on several controls. An algorithm for constructing a complete solution to this problem has been developed. It has been established that the problem possesses specific features; in particular, the components of the optimal distributed and boundary vector controls satisfy a system of equal relations and are determined as the solution to a system of two nonlinear integral equations. Sufficient conditions for the unique solvability have been established. It has been shown that the presence of the Fredholm integral operator in the integro-differential equation has a significant impact on the solvability of the nonlinear optimization problem — in particular, when constructing a generalized solution to the boundary value problem of the controlled process and when proving the existence and uniqueness of a solution to a system of nonlinear integral equations with respect to the optimal controls

This article continues the investigation of the complete solution to the tracking problem developed in [1], specifically addressing the convergence of its approximations. Particular attention is given to the influence of the Fredholm integral operator on the convergence of the approximated solutions to the exact solution, and sufficient conditions for this convergence are established. It is shown that the presence of the integral operator necessitates the construction of three types of approximations of the optimal process: approximation via the resolvent of the kernel of the integral operator, approximation by optimal controls, and finite-dimensional approximation. Accordingly, three types of approximations of the minimum value of the functional are also introduced. Sufficient conditions are provided for the convergence of approximations of the distributed and boundary vector optimal controls, the three types of optimal process approximations, and the corresponding approximations of the minimum value of the functional.

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Krasovskii's problem on calming a multidimensional non-stationary control system with delays of degenerate type

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In [1], N. N. Krasovskii considered damping problem for control system with aftereffect described by differential-difference equations of delay type. He has brought this problem to the boundary value problem for systems of differential-difference equations with deviating argument in lower order terms. In [2], the Krasovskii problem was generalized to the case when the equation describing the control system has neutral type. In [3], a model with constant matrix coefficients and several delays was considered, in [4–6] — with variable matrix coefficients and several delays.

We consider the problem of bringing linear non-stationary control system with delay described by the system of differential-difference equations of degenerate type with variable matrix coefficients and several delays to an equilibrium state.

The relationship between the variational problem for a nonlocal functional describing the multidimensional control system with delays and the corresponding boundary value problem for the system of differential-difference equations is established. A priori estimates of solutions are obtained. The existence and uniqueness a generalized solution to this boundary value problem are proved. For the operator describing a degenerate type, Friedrichs extension was constructed.

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On limit connection between two problems of complex heat exchange

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The problems of complex heat exchange, where the radiative heat transfer and the conductive heat transfer should be taken into account simultaneously, appears in various fields of science and engineering, especially at high temperatures. Fundamentally, thermal radiation is a volumetric phenomenon that is most accurately described by the radiation transfer equation, which takes into account the emission, absorption, and scattering of radiation. However, in practice, a body with high absorption capacity is often understood as an opaque (absolutely black or gray) body in which the emission and absorption of thermal energy occurs only on the surface.

Papers [1, 2] were devoted to the mathematical justification of the fact that when the absorption coefficient tends to infinity, the body semitransparent to radiation turns to opaque and absolutely black body. Note that in those papers the influence of radiation scattering inside the body was either not taken into account at all [2], or the scattering coefficient was considered infinitely small compared to the absorption coefficient [1].

The aim of this work is a rigorous mathematical proof of the fact that a body semitransparent to radiation, with simultaneous proportional tendencies of absorption and scattering coefficients to infinity, turns into an opague gray body with an emittance $0 < \varepsilon_* < 1$.

We consider the problem of complex heat transfer in a convex body $G \subset \mathbb{R}^3$ filled with a heat-conducting material that is semitransparent to radiation. The unknown functions $u_\varepsilon(x)$ and $I_\varepsilon(\omega, x)$ are defined on the sets G and $D = \Omega \times G$, where $\Omega = \{\omega \in \mathbb{R}^3 \mid |\omega| = 1\}$, and have the physical meaning of the absolute temperature at the point $x \in G$ and the intensity of radiation at the point $x \in G$, propagating in the direction $\omega \in \Omega$. These functions satisfy the following system of equations:

$$-\operatorname{div}(\lambda(x, u_\varepsilon)\nabla u_\varepsilon) + 4\varkappa_\varepsilon h(u_\varepsilon) = \varkappa_\varepsilon \int_\Omega I_\varepsilon \, d\omega + f, \quad x \in G, \tag{1}$$

$$\omega \cdot \nabla I_\varepsilon + (\varkappa_\varepsilon + s_\varepsilon) I = \frac{s_\varepsilon}{4\pi} \int_\Omega I_\varepsilon \, d\omega + \frac{\varkappa_\varepsilon}{\pi} h(u_\varepsilon), \quad (\omega, x) \in D. \tag{2}$$

Here $0 < \lambda(x, u)$ is the thermal conductivity coefficient, $0 < \varkappa_\varepsilon = \dfrac{1 - \varpi}{\varepsilon}$ and $0 \leqslant s_\varepsilon = \dfrac{\varpi}{\varepsilon}$ are the absorption and scattering coefficients, ϖ is the scattering albedo. The function $h(u) = \sigma_0 |u|^3 u$ at $u > 0$ corresponds to the intensity of hemispherical radiation according to the Stefan–Boltzmann law, where $0 < \sigma_0$ is the Stefan–Boltzmann constant.

If the body G is in a vacuum, then there is no convective heat flux at the boundary:

$$\lambda(x, u_\varepsilon) \frac{\partial u_\varepsilon}{\partial n} = 0, \quad x \in \partial G. \tag{3}$$

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We define the sets $\Gamma^- = \{(\omega, x) \in \Omega \times \partial G \mid \omega \cdot n(x) < 0\}$, $\Gamma^+ = \{(\omega, x) \in \Omega \times \partial G \mid \omega \cdot n(x) > 0\}$ and supplement equation (2) with the boundary condition describing internal diffuse reflection of radiation at the boundary ∂G:

$$I_\varepsilon|_{\Gamma^-} = \frac{\theta}{\pi} \int\limits_{\omega' \cdot n(x)} I_\varepsilon|_{\Gamma^+}(\omega', x)\, \omega' \cdot n(x)\, d\omega' + (1-\theta) g_*, \quad (\omega, x) \in \Gamma^-. \tag{4}$$

Here $0 \leqslant \theta < 1$ is the internal reflection coefficient.

We prove that as $\varepsilon \to 0$ the first component u_ε of the solution to problem (1)–(4) tends to the solution to the problem

$$-\operatorname{div}(\lambda(x,u)\nabla u) = f, \quad x \in G, \tag{5}$$

$$\lambda(x,u)\frac{\partial u}{\partial n} + \varepsilon_* h(u) = \pi \varepsilon_* g_*, \quad x \in \partial G, \tag{6}$$

describing heat exchange in an opaque gray body G with a boundary ∂G having emissivity $0 < \varepsilon_* < 1$. Besides, $I_\varepsilon \to \frac{1}{\pi} h(u)$ in D, $I_\varepsilon|_{\Gamma^+} \to \widehat{\psi}_+ g_* + (1 - \widehat{\psi}_+)\frac{1}{\pi} h(u|_{\partial G})$ on Γ^+ and $I_\varepsilon|_{\Gamma^-} \to \psi_0 g_* + (1-\psi_0)\frac{1}{\pi} h(u|_{\partial G})$ on ∂G.

The "greyness" of the limiting opaque surface is a joint effect of surface reflectance and volumetric scattering properties. The emissivity ε_*, the value ψ_0, and the function $\widehat{\psi}_+$ are determined by the following formulas:

$$\varepsilon_* = (1-\theta)(1-\psi_*), \quad \psi_0 = \theta\psi_* + 1 - \theta, \quad \psi_* = 2\int_0^1 \psi(\mu, 0)\mu\, d\mu,$$

$$\widehat{\psi}_+(\omega, x) = \psi(\omega \cdot n(x), 0), \quad (\omega, x) \in \Gamma^+,$$

where $\psi(\mu, \tau)$ is the solution to the following spatially one-dimensional problem describing radiation transfer in the half-space with the condition of internal diffuse reflection of radiation:

$$-\mu \frac{d\psi(\mu,\tau)}{d\tau} + \psi(\mu,\tau) = \frac{\varpi}{2}\int_{-1}^1 \psi(\mu', \tau)\, d\mu', \quad \mu \in [-1, 0) \cup (0, 1], \quad \tau \in \overline{\mathbb{R}}^+,$$

$$\psi(\mu, 0) = 2\theta \int_0^1 \psi(\mu', 0)\mu'\, d\mu' + 1 - \theta, \quad \mu \in [-1, 0),$$

$$\psi(\mu, +\infty) = 0, \quad \mu \in (0, 1].$$

The result was published in [3] and was obtained as part of the implementation of the state assignments of the Ministry of Education and Science of Russia (project FSWF-2023-0012).

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Phase boundary properties in parabolic problems with hysteresis

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We study solutions of spatially one-dimensional parabolic equations with a discontinuous hysteresis operator described by a free interface boundary. The hysteresis operator is used in mathematical description of various chemical, physical, and biological processes such as thermoregulation, chemical reactors, ferromagnetism, self-organisation, etc.

It is established that for transversal initial data from the space $W_q^{2-2/q}$, $q > 3$, the problem is solvable in the space $W_q^{2,1}$, and the free (interface) boundaries are defined by monotone Hölder curves with the exponent $1/2$.

For the initial data from the space W_∞^2, it is proved that the interface boundaries satisfy the Lipschitz condition.

It is also shown that for non-transversal initial data, solutions with interface boundaries do not exist.

This talk is based on results [1–3] obtained jointly with N. N. Uraltseva and S. B. Tikhomirov.

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On a posteriori estimates for a thin obstacle problem

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Let Ω be a bounded connected domain in $\mathbb{R}^n$ with Lipschitz continuous boundary $\partial\Omega$. Let $\mathcal{M}$ be a smooth $(n-1)$-dimensional manifold in $\mathbb{R}^n$ dividing Ω into two Lipschitz subdomains Ω_+ and Ω_-.

Let $\tau\colon \partial\Omega \to \mathbb{R}$ and $\psi\colon \mathcal{M} \to \mathbb{R}$ be given functions such that $\tau \in H^{1/2}(\partial\Omega)$, ψ is

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smooth, and $\psi \leqslant \tau$ on $\mathcal{M} \cap \partial\Omega$. We consider the problem

$$\frac{1}{2}\int\limits_{\Omega} |\nabla v|^2 dx \to \min_{\mathbb{K}}. \tag{1}$$

Here

$$\mathbb{K} = \{v \in H^1(\Omega): \quad v = \tau \text{ on } \partial\Omega, \quad v \geqslant \psi \text{ on } \mathcal{M} \cap \Omega\} \tag{2}$$

is a closed convex set.

Problem (1), (2) is called the *thin obstacle problem* with the obstacle ψ. It has a unique smooth solution, see references in [1]. In the two-dimensional case this problem describes the equilibrium of an elastic membrane above a very thin object.

We call a function $v \in \mathbb{K}$ an approximate solution for problem (1), (2). We want to evaluate how close it is to the exact one. For that we will use functional a posteriori error estimates. They confine the distance between an approximation and the minimizer with some majorant $\mathfrak{M}$ depending on the approximate solution and on problem's data. Such estimates can be obtained via classical duality theory methods, see [1]. We can also get them using the so-called deviation identity method, see [2].

Differently obtained a posteriori estimates may or may not have the same form of the upper bounds $\mathfrak{M}$. In this talk, we will present two estimates and compare them with each other. We will make their validation on well-known examples and also will provide results of numerical simulation.

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Approximation of solutions of second-order differential equations containing cubic nonlinearity with the help of multidimensional Laplace transform

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We study the second-order differential equation containing the cubic nonlinearity

$$\frac{d^2u(t)}{dt^2} + \omega^2\left(\chi_{\{s>0\}}s(t)\right)u(t) + u^3(t) = \chi_{\{s>0\}}s(t), \qquad t > 0. \tag{1}$$

Here s and ω stand for an external impulse force and an oscillation frequency, respectively. While $\chi_{\{s>0\}}$ denotes the characteristic function of the set $\{t : s(t) > 0\}$, i.e.

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$\chi_{\{s>0\}} = 1$ if $s(t) > 0$ and $\chi_{\{s>0\}} = 0$ otherwise. In physics, equations of type (1) describe the processes underlie the echo phenomena.

Modifying approach from [1] and using the multidimensional Laplace transform in the similar manner as in [2], we justify that the inverse Laplace transform of the spectrum

$$U(p) := \sum_{n=0}^{N} a_n(p,\omega) S^n(p) \tag{2}$$

can be considered as an approximate solution. In representation (2), U and S are the spectra of u and s, respectively; while a_n are factors selected in a special way and determining the level of the corresponding components.

The numerical implementation of the proposed approximate solution is in a good agreement with a number of experimentally observed phenomena, both under the influence of sequences consisting of simple and complex perturbations, which determine the time dependence of the coefficients included in nonlinear equation (1).

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Hypergeometric systems arising in the representation theory

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At the end of the 20th century I.M. Gelfand and coauthors discovered a remarkable system of PDEs, now called the Gelfand-Kapranov-Zelevinsky system (GKZ for short). This system is satisfied by A-hypergeometric functions forming a class of multivariate hypregeometric functions, which is big enough (and includes in some sence almost all certain examples of multivariate hypregeometric functions) and also possesses numerous "good" properites analogous to properties of Gauss's hyperegeometric function.

The GKZ system is a system of PDEs for a function $F(z_1, ..., z_N)$, $z_i \in \mathbb{C}$, defined by a lattice $B \subset \mathbb{Z}^N$ and a vector $\gamma \in \mathbb{C}^N$. The GKZ system consists of equations of two types:

1. For all $a = (a_1, ..., a_N) \perp B$ one has

$$a_1 z_1 \frac{\partial F}{\partial z_1} + ... + a_N z_N \frac{\partial F}{\partial z_N} =< \gamma, a > F$$

with the standard $\mathbb{C}$-bilinear scalar product.

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2. For all $b \in B$ one writes $b = b^+ - b^-$, $b^\pm \in \mathbb{Z}^N_{\geqslant 0}$, and has the equation

$$\left(\left(\frac{\partial}{\partial z}\right)^{b^+} - \left(\frac{\partial}{\partial z}\right)^{b^-}\right) F = 0,$$

where the multi-index notation is used.

Gelfand, Kapranov, and Zelevinsky (see a review of their results in [1]) proved that this system has a finite-dimensional solution space and found some description for the dimension of this solution space for the singular locus of this system. Unfortunately, the last description is not explicit.

In [2], some certain examples of GKZ system were defined, coming from the problems of the representation theory of the Lie algebra $\mathfrak{gl}_n$. We will show in the talk that one can find the dimension of the solution space for these GKZ systems and describe the singular locus explicitly in combinatorial terms. Also, some facts about branching of solution in a neighbourhood of the singular locus will be presented.

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On weak solvability of operator Riccati integral equation in Banach space

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Let X be a Banach space with duality pairing $\langle f, x\rangle$ ($x \in X, f \in X^*$) and let $\mathcal{L}(X, X^*)$ be a space of bounded operators acting from X to its dual.

An operator function $\{U_{t,s}\}_{0\leqslant s\leqslant t\leqslant T}$ on a Banach space is called *forward (in time) evolution family* if $U_{t,t} = I$ and $U_{t,s} = U_{t,r}U_{r,s}$ for all $0 \leqslant s \leqslant r \leqslant t \leqslant T$. An operator function $\{V_{s,t}\}_{0\leqslant s\leqslant t\leqslant T}$ on a Banach space is called *backward (in time) evolution family* if $V_{t,t} = I$ and $V_{s,t} = V_{s,r}V_{r,t}$ for all $0 \leqslant s \leqslant r \leqslant t \leqslant T$. An evolution family is called *strongly continuous* if it is strongly continuous in t (for fixed s) and in s (for fixed t). Note that if $U_{t,s}$ is strongly continuous evolution family in space X then $V_{s,t} = U^*_{t,s}$ is a weakly-* continuous backward evolution family in X^*.

Operator $A_1 \in \mathcal{L}(X, X^*)$ is symmetric if $\langle A_1x, x\rangle \in \mathbb{R}$ for all $x \in X$. Symmetric A_1 is non-negative if $\langle A_1x, x\rangle \geqslant 0$ for all $x \in X$. Analogously operator $A_2 \in \mathcal{L}(X^*, X)$ is symmetric if $\langle y, A_2y\rangle \in \mathbb{R}$ for all $y \in X^*$. Symmetric A_2 is non-negative if $\langle y, A_2y\rangle \geqslant 0$ for all $y \in X^*$

By $C_{w*}(\mathcal{I}; \mathcal{L}(X, X^*))$ we denote a space of weakly-* continuous operator functions on a segment $\mathcal{I} = [0, T]$.

Theorem. *Let X be a Banach space and the following assumptions hold:*

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1. $\{U_{t,s}\}_{0\leqslant s\leqslant t\leqslant T}$ *is a strongly continuous and uniformly bounded forward evolution family in* $\mathcal{L}(X)$,

2. *operator functions* $C \in C_{w*}(\mathcal{I};\mathcal{L}(X,X^*))$ *is symmetric and non-negative,*

3. *operator functions* $B \in C_s(\mathcal{I};\mathcal{L}(X^*,X))$ *is symmetric and non-negative.*

Then for all symmetric non-negative $G \in \mathcal{L}(X,X^*)$ *the (backward) integral Riccati equation*

$$P(t) = U^*_{T,t} G U_{T,t} + \int_t^T U^*_{s,t}\{C(s) - P(s)B(s)P(s)\}U_{s,t} ds$$

has a unique symmetric non-negative solution $P \in C_{w*}(\mathcal{I};\mathcal{L}(X,X^*))$.

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On the stability of hyperbolic differential and difference equations with unbounded delay term

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The stability of problems for hyperbolic equations containing time delay terms with unbounded operator coefficients has not been sufficiently studied. Some researchers are interested in this kind of problems. Bounded solutions of nonlinear one-dimensional hyperbolic equations with bounded operators of time delay terms were investigated in earlier papers [1–4]. In papers [5, 6], the existence and uniqueness of bounded solutions to hyperbolic differential and difference equations with bounded operator coefficients of time delay terms were established. In general, solutions to hyperbolic differential equations with unbounded operators of time delay terms are not bounded. There are blow-up solutions to hyperbolic differential equations with unbounded operators of time delay terms [7]. In the present paper, we study the initial value problem

$$\begin{cases} \frac{d^2v(t)}{dt^2} + A^2v(t) = a\left(\frac{dv(t-\omega)}{dt} + Av(t-\omega)\right) + f(t), \quad t > 0, \\ v(t) = \varphi(t), \ -\omega \leqslant t \leqslant 0 \end{cases}$$

for a hyperbolic equation in a Hilbert space H with a positive definite unbounded operator A. The first and second order of accuracy difference schemes for the numerical solution of the differential problem are presented. The main theorems on stability estimates for the solutions of these problems are established. In practice, the stability

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estimates for solution of four problems for hyperbolic differential and difference equations with time delay are proved. Numerical results and explanatory illustrations are presented to show the validation of theoretical results.

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Asymptotic formulas and uniform difference schemes for solving hyperbolic perturbation problems with nonlocal conditions

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The abstract nonlocal boundary value problem

$$\begin{cases} \varepsilon^2 u''(t) + Au(t) = f(t), 0 < t < T, \\ u(0) = \alpha u(T) + \varphi,\ u'(0) = \beta u'(T) + \psi \end{cases}$$

for a hyperbolic equation in a Hilbert space H with a self adjoint positive definite operator A and with an arbitrary multiplier $\varepsilon \in (0, \infty)$ of the derivative term is considered. An asymptotic formula for the solution of this problem for small values of ε is established. Two-step uniform difference schemes of high order of accuracy are presented for solving this problem. The convergence estimates for the solution of these difference schemes are established.

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Method of maximal monotonic operators in the theory of nonlinear singular integral and integro-differential equations

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In this work, we consider singular integral and integro-differential operators in weighted real Lebesgue spaces, where the integral is understood in the sense of the Cauchy–Lebesgue principal value. In this case, unless otherwise specified, the weight function $\rho(x)$ is an arbitrary nonnegative measurable function on $[a,b]$, finite almost everywhere, nonzero almost everywhere. We denote by $L_p(\rho)$, $1<p<\infty$, the set of all measurable functions $u(x)$ on $[a,b]$ with the finite norm $\|u\| = \left(\int_a^b \rho(x)|u(x)|^p dx\right)^{1/p}$. It is known that $L_p(\rho)$ is a reflexive Banach space and $L_{p'}(\rho^{1-p'})$, $p'=p/(p-1)$, is the dual space. We say that a function $k(x,s)$ belongs to H_δ if it is defined on $[a,b]\times[a,b]$ and satisfies the *Hölder* condition:

$$\left|k(x_1,s_1)-k(x_2,s_2\right| \leqslant M\cdot\left(|x_1-x_2|^\delta+|s_1-s_2|^\delta\right),\ \textit{where}\ M>0,\ \delta\in(0,1].$$

We consider the singular integral operator with the Cauchy kernel

$$\left(\mathbb{K}u\right)(x)=\frac{1}{\pi}\int_a^b \frac{k(x,s)u(s)}{s-x}\,ds,\quad \textit{where}\ k(x,s)=k(s,x)\in H_\delta.$$

It is known that the operator $\mathbb{K}$ acts from $L_p(a,b)$ to $L_p(a,b)$ and is bounded for any $p>1$ and $\delta\in(0,1]$ (the detailed proof is given in [1]). In addition, it has the following important property: if $k(x,s)=k(s,x)\in H_\delta$, then one has $(\mathbb{K}u,u)=0$ for the inner product in $L_2(a,b)$.

Based on this property of the operator $\mathbb{K}$, we use the method of maximal monotone (in the Browder–Minty sense) operators to prove global theorems on the existence and uniqueness of a solution for various classes of nonlinear equations containing singular integral and integro-differential operators with kernels of the form (compare with [2])

$$\frac{b(x)w(s)-b(s)w(x)}{s-x}k(x,s),\ \ [b(x)w(s)-b(s)w(x)]k(x,s)\operatorname{ctg}\frac{s-x}{2},$$

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and others.

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Qualitative and asymptotic properties of singular solutions to nonlinear differential equations

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We consider the differential equation

$$y^{(n)} + \sum_{j=0}^{n-1} a_j(x) y^{(j)} = p(x)|y|^k \operatorname{sgn} y, \tag{1}$$

where $k > 1$, $n \geqslant 2$, and the functions $p, a_0, \dots, a_{n-1}$ are continuous on $[0; \infty)$, and some of its special cases, for example, the equation

$$y^{(n)} = p(x)|y|^k \operatorname{sgn} y. \tag{2}$$

Qualitative and asymptotic properties of solutions and some methods for their study are discussed (see, e.g., [1–5]). If we consider equation (1) as a perturbation of the corresponding linear equation ($p = 0$), then we can assert that under appropriate conditions on the coefficients of the equation, the solutions of equation (1) are in some sense close to the solutions of the linear equation. Namely, the following theorem on the asymptotic equivalence of solutions is valid.

Theorem. *If continuous functions $a_0, \dots, a_{n-1}$ and p satisfy the conditions*

$$\int_{x_0}^{\infty} x^{n-j-1} |a_j(x)|\, dx < \infty \textit{ for all } j \in \{0, \dots, n-1\}$$

and, for some $m \in \{0, \dots, n-1\}$, the condition

$$\int_{x_0}^{\infty} x^{n-1+(k-1)m}\, |p(x)|\, dx < \infty,$$

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then for any constant $C \neq 0$ there exists a solution to equation (1) *with the following asymptotics at infinity:*

$$y^{(j)}(x) \sim \frac{C\, m!\, x^{m-j}}{(m-j)!} \quad \textit{for} \ \ 0 \leqslant j \leqslant m,$$

$$y^{(j)}(x) = o\left(x^{m-j}\right) \ \ \textit{and} \ \int^{\infty} s^{j-m-1} \left|y^{(j)}(s)\right| ds < \infty \ \textit{for} \ \ m < j < n.$$

Equation (1) can also be considered as a perturbation of nonlinear equation (2), so that knowledge of the asymptotic behavior of solutions of this equation gives us information about the asymptotic behavior of solutions to equation (1). In this connection, the asymptotic behavior of regular and singular solutions of equation (2) will be discussed, in particular, the typicality and atypicality of the power behavior of such solutions of equation (2) (see, e.g., [5]) and similar equations of a more general form with potential p, depending on x, y, y', $\cdot$, $y^{(n-1)}$.

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Construction and analysis of a mathematical model for a non-classical bending problem of an orthotropic plate with variable thickness

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This paper addresses the bending problem of an orthotropic plate with variable thickness subject to a uniformly distributed load on its upper surface.

Consider an orthotropic rectangular plate of variable thickness, with edges of lengths a and b, positioned within a Cartesian coordinate system $Oxyz$. Assume that

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the plate thickness changes linearly,

$$h = h_0 + h_1 x + h_2 y, \tag{1}$$

where h_0, h_1, and h_2 are constants.

The plate is elastically clamped at one end and is rigidly fixed at the other. The elastic clamped condition at $x = 0$ is given by

$$\frac{\partial w}{\partial x} - a_{55}\varphi_1 = -D \cdot M_x, \quad w = B \cdot N_x, \quad \frac{\partial w}{\partial y} - a_{44}\psi_1 = D_{xy} \cdot M_{xy}. \tag{2}$$

Here, w denotes the deflection, while M_x and M_{xy} represent the bending moment and the shear force, respectively. The quantities a_{55} and a_{44} are well known mechanical parameters of the plate material. The functions φ_1 and ψ_1 describe the variation of transverse shear stresses that characterize the properties of the elastically clamped support.

The resulting system written for $h_2 = 0$ takes the following form:

$$\begin{cases} 2h\dfrac{\partial \varphi}{\partial x} + 4\dfrac{\partial h}{\partial x}\varphi = -3q, \\ h^2 B_{11}\dfrac{\partial^3 w}{\partial x^3} + 2hB_{11}\dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial h}{\partial x} - h^2 a_{55} B_{11}\dfrac{\partial^2 \varphi}{\partial x^2} - 2ha_{55}B_{11}\dfrac{\partial h}{\partial x}\dfrac{\partial \varphi}{\partial x} + 8\varphi = 0. \end{cases} \tag{3}$$

The system was solved using the collocation method and conclusions were drawn on the basis of the numerical results obtained.

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About qualitative properties of the Vlasov–Poisson equations

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The Vlasov–Poisson system is a super classic well travelled subject. However, some progress is regularly achieved in particular on the qualitative behaviour of the solutions. Along this line starting from the classical Landau result, I will compare the short time quasilinear approximation and the long time one related to the Balescu–Lenhard and the Landau equations.

A priori estimate of solutions of an even-order ordinary differential equation with integral conditions

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We consider an ordinary differential equation of even order with a spectral parameter and "purely integral conditions," i.e. conditions containing only Lebesgue integrals of the unknown function and its derivatives with some weights. Such problems were first considered by A. Sommerfeld in 1908. The main difficulty in studying these problems lies in the fact that the domain of the corresponding differential operator is not dense in $L_2(0,1)$. Under certain conditions on the weight functions, an a priori estimate of solutions in Sobolev norms depending on the spectral parameter is obtained for sufficiently large values of the parameter. The discreteness and the sectorial structure of the spectrum of the corresponding operator are also proved.

This talk is based on the joint work with A. L. Skubachevskii [1].

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On the geometry of solvable Lie groups

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In this work, we consider the five-dimensional solvable Lie group, equipped with any left-invariant metric, either Riemannian or Lorentzian. The existence of non-trivial (i.e., not Einstein) Ricci solitons on both Riemannian and Lorentzian five-dimensional solvable Lie group is proved. Moreover, we show that there are no gradient Ricci solitons.

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Mathematical methods of managing a retail store as a dynamic system: the promotional pricing aspect

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The purpose of this study is to develop a framework that can address issues arising while managing a retail store as a complex system with stochastic elements. Specifically, we will focus on promotional pricing, which is a temporary reduction in price aimed at changing customer behavior. Promotional pricing is reflected in the category management paradigm, where each category is treated as a separate business entity. However, customers utility maximization strategy has a form [1]:

$$P(f(\cdot), A) = \arg\max_{y \in A} f(y),$$

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where A is a set of different categories.

So managing category independently much likely does not satisfy to the optimal control [1]:

$$\Phi(u) \longrightarrow \max_{y \in A} f(y).$$

This problem of managing the disbalance between category-level and store-level objectives can be formally defined as a stochastic setting where the Center does not control agent actions directly, so the Center chooses control actions to respond to agents actions in multi-stage game in order to optimize its objectives:

$$\max_{u_t \in \mathcal{U}} \mathbb{E}_{a_t \sim \mathcal{P}_t} \left[R_{\text{global}}(a_t, u_{t-1}) + \gamma \cdot V_{t+1}(s_{t+1}) \right]$$

where:

$$\begin{aligned}
a_t &\text{ — agents' (categories) actions at time } t, \text{ drawn from } \mathcal{P}_t, \\
u_{t-1} &\text{ — Center's response chosen at previous step } t-1, \\
R_{\text{global}}(a_t, u_{t-1}) &\text{ — revenue at time } t, \text{ depending on } a_t \text{ and prior Center response}, \\
s_{t+1} &\text{ — system state after actions } (a_t, u_t), \\
V_{t+1}(s_{t+1}) &\text{ — expected future revenue starting from next state}, \\
\gamma &\text{ — discount factor (optional).}
\end{aligned}$$

The proposed solution combines game-theoretical formulations and optimization problems.

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On the equivariant mixed problem for the wave equation in the spherical cylinder

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We consider the wave equation

$$u_{tt} = \Delta u, \quad x \in B, \ t \in (0, T), \tag{1}$$

where $B = \{x \in \mathbb{R}^3 : |x| < 1\}$, with the initial conditions

$$u|_{t=0} = u_0(x), \quad u_t|_{t=0} = u_1(x) \tag{2}$$

and the boundary condition of the following form:

$$u|_{\partial B} * \alpha + u_\nu|_{\partial B} * \beta = 0. \tag{3}$$

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Here we assume that the functions $\alpha(x)$ and $\beta(x)$ are in $L_2(\partial B)$, and

$$\alpha = \sum_{l=0}^{\infty} \sum_{k=-l}^{l} \alpha_l^k Y_l^k, \quad \beta = \sum_{l=0}^{\infty} \sum_{k=-l}^{l} \beta_l^k Y_l^k$$

are their decomposition into Fourier series by the spherical functions Y_l^k.

Let $u_0 \in H^1(B)$ and $u_1 \in L_2(B)$ be given functions.

The symbol $*$ denotes the convolution on ∂B, which can be written as

$$\psi * \alpha = \sum_{l=0}^{\infty} \sum_{k=-l}^{l} \psi_l^k \alpha_l^k Y_l^k.$$

It is well known that (3) is invariant with respect to the group of rotations. It means that the relation $T_V(\alpha * u) = \alpha * T_V u$ holds for any $V \in SO(\mathbb{R}^n)$, where $T_V(f(x)) = f(Vx)$.

The assumption of α and β to be in $L_2(\partial B)$ is not a loss of generality. Furthermore, we can assume that these functions are smooth, as only the ratio of the Fourier coefficients of α and β is important.

The general theory of boundary value problems as well as boundary value problems with invariant boundary conditions are considered in monograph [1].

The talk is devoted to the solvability of problem (1)–(3). Under certain assumptions regarding the functions from the boundary condition, existence and uniqueness of the generalized solution of the problem are proved.

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Asymptotic behavior of eigenfunctions of elliptic operators in div-form

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Let $\Omega' \subset \mathbb{R}^d$, $d = 1, 2, \ldots$ be an open bounded smooth domain, and

$$\Omega = \Omega' \times (0, H) \subset \mathbb{R}^d \times \mathbb{R}_+.$$

The coordinates in Ω are designated as $x = (x', y) \in \Omega' \times (0, H)$.

The talk deals with the concentration (and non-concentration) properties (in sectors of Ω) of the eigenfunctions of the self-adjoint second-order elliptic operator

$$A = -\nabla \cdot \tilde{c}\,\nabla \text{ in } L^2(\Omega; dx) \text{ with domain } D(A) = \{v \in H_0^1(\Omega); \tilde{c}\,\nabla v \in H^1(\Omega)\}.$$

The coefficient $\tilde{c} > 0$ is assumed to be bounded, but no continuity assumption is imposed. It is analogous to the square of the speed of sound in the wave equation,

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and $\sqrt{\tilde{c}}$ is commonly known in the physical literature as the *celerity.* This study deals with *layered media*, namely, $\tilde{c}(x)$ depends only on the single spatial coordinate $y \in (0, H)$, so that $\tilde{c}(x) = \tilde{c}(x', y) = c(y)$.

The eigenvalues of A are partitioned (apart from a small residual set) into two disjoint infinite sets. The corresponding eigenfunctions are $\mathfrak{F}_G$ (guided) and $\mathfrak{F}_{NG}$ (non-guided). Their asymptotic properties are expressed by suitable estimates as the associated eigenvalues tend to infinity. The eigenfunctions in $\mathfrak{F}_G$ concentrate in "wells" of $c(y)$, subject to polynomial rate of decay away from the concentration sector. The non-concentrating eigenfunctions in $\mathfrak{F}_{NG}$ are oscillatory in every sector with non-decaying amplitudes. These results hold uniformly for families of celerities with a common bound on their total variation.

The paper leaves as an open problem the question of non-concentration in the case of a function $c(y)$ which is continuous but not of bounded variation.

Based on joint work with Yves Dermenjian.

On the solvability of a linear fourth-order boundary value problems with accretive operators

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The main purpose of this talk is to investigate the solutions of the following fourth-order abstract linear differential equation under various sets of assumptions:

$$u^{(4)}(x) - 2Bu''(x) - Cu(x) = f(x), \quad x \in (a, b), \tag{1}$$

where $u(x)$ is a vector-valued function taking values in a suitable Hilbert space $\mathcal{H}$ (finite or infinite dimensional), B and C are linear (bounded or unbounded) accretive operators on $\mathcal{H}$, and the function f belongs to $L^p(a, b; \mathcal{H})$ for some $1 < p < +\infty$, with $a < b$.

When this problem is supplemented with appropriate nonhomogeneous boundary conditions, we establish existence, uniqueness, and maximal regularity of the solution under necessary and sufficient conditions on the data. We also provide an explicit representation formula using tools such as analytic semigroups, sectorial operators with bounded imaginary powers, the theory of strongly continuous cosine operator functions, and the perturbation theory of m-accretive operators.

Using the splitting method introduced by Krein [5], the equation is transformed into a system of two coupled second-order equations. The first equation is elliptic, while the second one is hyperbolic. These equations are not directly amenable to standard methods. The main difficulty lies in analyzing these two types of problems simultaneously.

In our analysis, the coupled second-order hyperbolic and elliptic equations are solved in order to obtain an explicit solution to problem (1) under suitable boundary conditions. Our approach is primarily based on fractional powers of m-accretive operators, their perturbation theory, the techniques involving holomorphic semigroups generated by such operators, and the theory of strongly continuous cosine operator functions.

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Asymptotic analysis of a viscoelastic problem with long-term memory and the Tresca friction law

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This paper examines the asymptotic behavior of solutions of the three dimensional viscoelastic problem with long-term memory and the Tresca friction law in a thin domain Σ^ε. We study the asymptotic behavior of this problem when the thickness ε tends to zero and we prove a convergence theorem for the displacement and velocity in appropriate functional spaces. Besides, the limit problem with the limit of Tresca free boundary conditions and a specific Reynolds limit equation is obtained. To formulate the system of the linear viscosity with long-term memory, we begin with the following notations. Let Σ^ε be a bounded domain in $\mathbb{R}^3$, where $0 < \varepsilon < 1$ is assumed to be small. For the boundary of Σ^ε, one has $\partial\bar{\Sigma}^\varepsilon = \partial\bar{\Sigma}_1^\varepsilon \cup \partial\bar{\Sigma}_L^\varepsilon \cup \bar{\Lambda}$, where $\partial\bar{\Sigma}_1^\varepsilon$ is the upper surface of the equation $x_3 = \varepsilon q(x_1, x_2)$, $\partial\bar{\Sigma}_L^\varepsilon$ is the lateral boundary, Λ is a bounded domain in $\mathbb{R}^3$ of the equation $x_3 = 0$ which constitutes the bottom of the domain Σ^ε. Here $q\,(.)$ is a C^1-function defined on Λ such that

$$0 < \underline{q} = q_{min} \leqslant q\,(x_1, x_2) \leqslant q_{max} = \bar{q},\ \forall\,(x_1, x_2) \in \overline{\Lambda}.$$

For all $x = (x', x_3) \in \mathbb{R}^3$, where $x' = (x_1, x_2) \in \mathbb{R}^2$, the physical domain Σ^ε is given by

$$\Sigma^\varepsilon = \{(x', x_3) \in \mathbb{R}^3 : (x_1, x_2) \in \Lambda,\ 0 < x_3 < \varepsilon q(x')\}.$$

Let $u^\varepsilon(x, t)$ be the displacement at a point x at a moment $t \in [0, T]$. We denote by $r\,(.)$ the strain tensor given by

$$r_{ij}(u^\varepsilon) = \frac{1}{2}\left(\frac{\partial u_i^\varepsilon}{\partial x_j} + \frac{\partial u_j^\varepsilon}{\partial x_i}\right),\ 1 \leqslant i, j \leqslant 3.$$

The viscosity law with long-term memory is given by

$$\chi_{ij}^\varepsilon\,(u^\varepsilon) = 2\mu r_{ij}\,(u^\varepsilon) + 2\int_0^t g\,(t-s)\, r_{ij}\,(u^\varepsilon)\,(s)\,ds,\ 1 \leqslant i, j \leqslant 3,$$

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where μ is the Lamé coefficient and $g\,(.)$ is a relaxation function satisfying the following conditions:

1) $g:\mathbb{R}_+\to\mathbb{R}_-$ is a function of class C^2 such that

$$0<l\leqslant\frac{\mu}{2}+\int_0^t g\,(s)\,ds,\ \forall t\in[0,T]\,.$$

2) $g\,(.)$ is increasing, and there are positive constants G_1 and G_2 such that

$$0<g'\,(t)\leqslant G_1 \text{ and } |g''\,(t)|\leqslant G_2,\ \forall t\in[0,T]\,.$$

The viscosity equation with long-term memory is has the form

$$\frac{\partial^2 u^\varepsilon}{\partial t^2}-div\,(\chi^\varepsilon\,(u^\varepsilon))=p^\varepsilon\ in\ \Sigma^\varepsilon\times\,]0,T[\,, \tag{1}$$

where $p^\varepsilon=(p_1^\varepsilon,\,p_2^\varepsilon,\,p_3^\varepsilon)$ is the external force.

Let n be the unit outward normal vector to $\partial\Sigma^\varepsilon$. The normal and the tangential components of u^ε on the boundary are given by

$$u_n^\varepsilon=u^\varepsilon\cdot n,\quad u_\tau^\varepsilon=u^\varepsilon-u_n^\varepsilon n.$$

Similarly, we denote by χ_n^ε and χ_τ^ε the normal and the tangential components of χ^ε,

$$\chi_n^\varepsilon=(\chi^\varepsilon\cdot n)\cdot n,\quad \chi_\tau^\varepsilon=\chi^\varepsilon\cdot n-(\chi_n^\varepsilon)\,n.$$

The boundary conditions are as follows:

- The displacement u^ε is known on $\partial\Sigma_1^\varepsilon\times]0,T[$ and on $\partial\Sigma_L^\varepsilon\times]0,T[$ and satisfies the Dirichlet condition
$$u^\varepsilon=0. \tag{2}$$

- On $\Lambda\times]0,T[$, the velocity is supposed to be unknown and satisfies the condition
$$\frac{\partial u^\varepsilon}{\partial t}\cdot n=0. \tag{3}$$

- The friction on $\Lambda\times]0,T[$ is modeled by the nonlinear Tresca law (see [8])
$$\begin{cases} |\chi_\tau^\varepsilon|<\varkappa^\varepsilon\Rightarrow\left(\frac{\partial u^\varepsilon}{\partial t}\right)_\tau=0,\\ |\chi_\tau^\varepsilon|=\varkappa^\varepsilon\Rightarrow\exists\beta>0 \text{ such that } \left(\frac{\partial u^\varepsilon}{\partial t}\right)_\tau=-\beta\chi_\tau^\varepsilon,\end{cases} \tag{4}$$

  where $|.|$ denotes the Euclidean norm in $\mathbb{R}^3$ and $\varkappa^\varepsilon$ is a given function.

Problem (1)–(4) is supplemented by the initial conditions

$$u^\varepsilon\,(x,0)=u_0\,(x)\,,\ \ \frac{\partial u^\varepsilon}{\partial t}\,(x,0)=u_1\,(x)\,,\quad \forall x\in\Sigma^\varepsilon. \tag{5}$$

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Existence of spiral strategies for blocking fire spreading

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In this paper we address the problem for blocking fire by constructing a wall ζ whose shape is spiral-like. This is supposed to be the best strategy when a single firefighter is constructing the wall with a finite construction speed σ: the barriers which satisfy this bound on the construction speed are called admissible.

We prove a sharp version of Bressan's Fire Conjecture [2] in this case, i.e. when admissible barriers are spiral-like curves: namely, there exists a spiral-like barrier confining the fire in a bounded region of $\mathbb{R}^2$ if and only if the speed of construction of the barrier σ is strictly larger than the critical speed $\bar{\sigma} = 2.614\ldots$

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The existence of confining spiral barriers for $\sigma > \bar{\sigma}$ is already known [4,13], while we concentrate on the negative side, i.e. if $\sigma \leqslant \bar{\sigma}$, then no admissible spiral blocks the fire.

The proof of these results relies on:

1. the precise definition of spiral barrier and its representation;
2. the analysis of saturated spiral barriers as a Retarded Differential Equation (RDE) in the spirit of [13];
3. the equivalent reformulation of the conjecture as a minimum problem of a functional *for a prescribed functional*;
4. the construction of the optimal closing spiral;
5. the analysis of a differentiable path of admissible spirals along which the functional is differentiable, and in particular increasing when moving from the optimal spiral to any other one (*homotopy argument*).

Due to the complexity of the solution, the evaluation of the quantities needed to prove that the functional is increasing is performed numerically.

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The dynamical systems approach to the Hamilton–Jacobi–Chandrashekar equations

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We introduce several dynamical systems to study local and global properties of solution of $-u_{rr} - \frac{N-1}{r} u_r - e^u + m|u_r|^q = 0$ in $(0, 1]$, in $[1, \infty)$ and in $(0, \infty)$, with $N \geqslant 2$, $q > 1$ and $m > 0$. Several types of singular behaviour occur, depending in particular of the value of q with respect to 2. The singular solutions could be of eikonal type i.e. $u(r) \simeq -q \ln r + \ln mq^q$ if $q > 2$, of Emden–Chandrashekar type, i.e. $u(r) \simeq -2 \ln r + c_N$ if $1 < q < 2$, of Hamilton-Jacobi type i.e. $u(r) \sim \xi_{N,m,q} r^{-\frac{2-q}{q-1}}$ and of potential type, i.e. $u(r) \sim \gamma r^{2-N}$ if $1 < q < \frac{N}{N-1}$. We emphasise on the construction of singular solutions using either fixed point methods or analysis of hyperbolic stationary points of the different associated dynamical systems of order 2 and 3 that we introduce.

Joint work with Laurent Véron.

Local entropies in ergodic theory and their applications

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The classical Kolmogorov–Sinai metric entropy and the Adler–Konheim–McAndrew topological entropy measure the complexity of a dynamical system but do not provide any information about the complexity of individual trajectories, let alone overall sequences of points. To answer the last question, Kolmogorov proposed to consider the "simplest" dynamical system realizing this sequence as a trajectory, and implemented this idea in terms of the shortest program of a universal Turing machine realizing this sequence. Unfortunately, this approach turned out to be not very fruitful: firstly, its practical implementation is extremely difficult, and secondly, the complexity of two trajectories of the same system can differ greatly. To overcome these difficulties, we propose local dynamical entropies measuring the complexity of individual trajectories, and compare them to known approaches.

Let $\Delta := \{\Delta_i\}$ be a finite measurable partition of a Lebesgue space (X, Σ). We refer to the indices of Δ_i as an alphabet $\mathbf{A}$. We say that on a starting segment of length N of a given sequence $\overline{x} := (x_1, x_2, \dots)$ there is a word $\overline{w} := (w_1, \dots, w_n)$ composed of the letters $w_i \in \mathbf{A}$, if there is i such that

$$x_{i+j} \in \Delta_{w_j} \quad \forall j < n+1, i+j \leqslant N.$$

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Denote by $L(\overline{x}, \overline{w}, N)$ the number of occurrences of the word $\overline{w}$ in this starting segment, and let $\overline{p}(\overline{x}, n, N) := \{p_i\}$ be the distribution (frequency) of all such words of length n.

Definition. By the conditional *local entropy* of the trajectory $\overline{x}$ we mean

$$h^{\pm}_{\mathrm{loc}}(\overline{x}|\Delta) := \lim{}^{\pm}_{n\to\infty} \lim{}^{\pm}_{N\to\infty} \frac{1}{n} H(\overline{p}(\overline{x}, n, N)).$$

Here $\pm$ refers to the upper and lower limits, and

$$H(\overline{p}(\overline{x}, n, N)) := -\sum_{i=1} p_i \log p_i$$

is the entropy of the distribution $\overline{p}(\overline{x}, n, N) := \{p_i\}$.

Denote by $L(\overline{x}, n, N)$ the number of different words of length n in the starting piece of length N of the trajectory $\overline{x}$.

Definition. By the conditional *information entropy* of the trajectory $\overline{x}$ we mean

$$h^{\pm}_{\mathrm{info}}(\overline{x}|\Delta) := \lim{}^{\pm}_{n\to\infty} \lim{}^{\pm}_{N\to\infty} \frac{1}{n} \log L(\overline{x}, n, N).$$

Finally, we define the unconditional versions of the *local and information entropies* as follows:

$$h^{\pm}_{\mathrm{loc}}(\overline{x}) := \sup_{\Delta} h^{\pm}_{\mathrm{loc}}(\overline{x}|\Delta), \quad h^{\pm}_{\mathrm{info}}(\overline{x}) := \sup_{\Delta} h^{\pm}_{\mathrm{info}}(\overline{x}|\Delta). \tag{1}$$

Our main results may be formulated as follows.

Theorem.

1. $h_{\mathrm{loc}}(\overline{x}) \leqslant h_{\mathrm{info}}(\overline{x}) \quad \forall \overline{x}$.
2. *Kolmogorov–Sinai metric entropy* $h_\mu(\Delta) = h_{\mathrm{loc}}(\overline{x}|\Delta)$ *for each* μ*-typical trajectory* $\overline{x}$ *of an ergodic dynamical system.*
3. $h_{\mathrm{loc}}(\overline{x}) = h_{\mathrm{info}}(\overline{x}) = 0$ *for each periodic sequence* $\overline{x}$.

We will discuss these results and their applications to various dynamical systems and specific sequences, in particular the ones related to number theory, such as prime numbers, quadratic residues, etc., which were published in [1].

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On ellipticity of operators with shear mappings

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We consider nonlocal boundary value problems where the main operator and the operators in the boundary conditions include differential operators and twisting operators.

Fix a number $\alpha > 0$ non-commensurable with π. Consider the infinite cylinder $Y = \mathbb{S}^1 \times \mathbb{R}$, on which the group $\Gamma = \mathbb{Z}$ acts by twistings perpendicularly to the generatrix of the cylinder

$$(x, t) \mapsto (x + k\alpha t, t), k \in \mathbb{Z}.$$

We define the shift operator corresponding to the twistings by the formula

$$(Tu)(x, t) = u(x - \alpha t, t).$$

On the finite cylinder $M = \mathbb{S}^1 \times [0, 1] \subset Y$, consider the nonlocal boundary value problem

$$\mathcal{D} = \begin{pmatrix} D \\ i^*B \end{pmatrix} : H^s(M) \longrightarrow \begin{array}{c} H^{s-m}(M) \\ \oplus \\ H^{s-b-1/2}(\partial M, \mathbb{C}^N). \end{array} \qquad (1)$$

Here D is a differential operator with shifts on M, the operator $B = (B_0, B_1)$ is a pair of differential operators with shifts on the left/right bases of the cylinder M.

The definition of the trajectory symbols for this class of problems is given. We show that the elliptic problems define the Fredholm operators in the corresponding Sobolev spaces. The ellipticity condition of such nonlocal boundary value problems is given.

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Spectral properties of the Schrödinger operator with translation

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We investigate spectral properties of the Schrödinger operator with translation in the free term. In the space $L^2(0,1)$, we consider the self-adjoint operator $\mathcal{A}y = -y''$ with the domain $\mathfrak{D}(\mathcal{A}) = \{y \in W_2^2(0,1)\}$ subject to the Neumann or the Dirichlet boundary conditions. By $\mathcal{L}$ we denote the operator of extension of a function by zero outside $(0,1)$, while $\mathcal{R}$ stands for the operator of restriction of a function to $(0,1)$. The operator $\mathcal{L}$ acts from $L^2(0,1)$ to $L^2(\mathbb{R})$, while the operator $\mathcal{R}$ acts from $L^2(\mathbb{R})$ to $L^2(0,1)$. We define the translation operator $\mathcal{T}$ in $L^2(\mathbb{R})$ by $(\mathcal{T}(\alpha)y)(x) = y(x+\alpha)$, $x \in \mathbb{R}$, where $\alpha \in [0,1]$ is a parameter.

Now we introduce the operator $\mathcal{B}^\alpha$ acting in $L^2(0,1)$ by

$$\mathcal{B}^\alpha y = V\mathcal{R}\mathcal{T}(\alpha)\mathcal{L}y,$$

where V is a complex-valued function and $V \in L_\infty(0,1)$.

The main object is the nonlocal operator $\mathcal{H}^\alpha = \mathcal{A} + \mathcal{B}^\alpha$ in $L^2(0,1)$ with the domain $\mathfrak{D}(\mathcal{H}^\alpha) = \mathfrak{D}(\mathcal{A})$. Our main goal is to study the behaviour of the spectrum of this operator. First, we proved that the operator $\mathcal{H}^\alpha$ has a compact resolvent and its spectrum consists of a countably many eigenvalues λ_n, $n \in \mathbb{N}$, with the only accumulation point at infinity.

Now we formulate our first main result.

Theorem 1. *Let V be a complex-valued function and $V \in L_\infty(0,1)$. Then for sufficiently large n the following representation holds:*

$$\sqrt{\lambda_n} = \pi n + \frac{1}{\pi n}\sum_{i=0}^{\infty}\frac{C_i(n)}{(\pi n)^i}.$$

This series converges uniformly in α and n.

Note that the coefficients $C_i(n)$ are bounded by some constant independent of n. We establish the explicit recurrent formulas for all coefficients $C_i(n)$. Based on these results, we find the explicit form for the coefficients C_0, C_1, and C_2, and we obtain the eigenvalue asymptotics of the operator $\mathcal{H}^\alpha$.

The second main result is devoted to the Bari basis property of the eigenfunctions and the generalized eigenfunctions of the operator $\mathcal{H}^\alpha$ in the space $L^2(0,1)$. We recall that the Bari basis is one generated by systems of projections quadratically close to complete and minimal systems of orthogonal projections.

Theorem 2. *The system of eigenfunctions and generalized eigenfunctions of the operator $\mathcal{H}^\alpha$ forms the Bari basis in the space $L^2(0,1)$.*

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The Kadomtsev–Petviashvili equation and hyperelliptic sigma functions

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The Kadomtsev–Petviashvili equation (KP-equation) of 1970 is one of the most famous (2+1)-equations in the theory of nonlinear waves. It is a natural generalization of the Korteweg–de Vries $(1+1)$-equation (KdV-equation), 1895.

V. E. Zakharov and A. B. Shabat (1974–1979) described solutions to the KP-equation with the condition that they decrease rapidly at infinity. S. P. Novikov developed the theory of finite-zone integration of equations of mathematical physics. In his seminal paper (1974) he wrote: "*Our work is based on certain simple but fundamental algebraic properties of equations admitting the Lax representation which are strongly degenerate in the case of rapidly decreasing functions at infinity, and have therefore not been noted. Finally, it is essential to note that in the periodic case the nonlinear "superposition law for waves" for the KdV-equation has an interesting algebraic-geometric interpretation.*" At the end of the seventies, analyzing the Its–Matveev and Krichever formulas, S. P. Novikov puts forward the conjecture: Jacobi varieties of non-singular algebraic curves are exactly the principally polarized Abelian varieties in whose theta functions the KP-equation is integrated. It was the approach to solve the classical Riemann–Schottky problem. This conjecture was solved by T. Shiota (1986).

Since the mid-90s, V. M. Buchstaber, V. Z. Enol'skii, and D. V. Leikin have developed the theory of multidimensional sigma functions on Jacobians of algebraic curves in the direction of applications to problems of mathematical physics. In 1997, we gave an explicit construction of a basis for the field of meromorphic functions on hyperelliptic Jacobians and explicitly described the generators of the ideal of relations between these basis hyperelliptic functions. It is remarkable that these ideal generators, as differential equations, lead explicitly to the KdV-equation. In this ideal, special cases of the KP-equation were also found.

The focus of the talk will be on the results of article [1]. In this paper, explicit formulas for hyperelliptic solutions to the general KP-equation were obtained, and the long-standing problem of describing the dependence of these solutions on the variation of the coefficients of the hyperelliptic curve equation, which are integrals of the KP-equations, was solved.

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Ergodic properties of the sine-process

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Dyson's sine-process, the scaling limit of radial parts of Haar measures on unitary groups of growing dimension, is the most classical point process of random matrix theory. In the survey talk, we shall consider the ergodic properties of the sine-process, including the speed of convergence in the Soshnikov Central Limit Theorem and the convergence of its stochastic Euler products to the Gaussian Multiplicative Chaos.

Sensitivity of the description of motion of a mechanical system to the method of implementing a constraint with a singularity

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Mechanical systems subjected to unilateral holonomic constraints are considered. It is assumed that the boundary of the area of unconstrained motion possesses a singularity. Possible ways of resolving the singularity based on knowledge related to the mechanical origin of the constraints are indicated.

As is known, the Routh method [1] and its modifications (see, e.g., [2,3]) make it possible not only to effectively solve the problem of the existence of steady motions, in particular, the equilibria of mechanical systems subjected to constraints, but also to investigate sufficient conditions for their stability and instability. However, in a number of cases, such as, for example, the case of continuity, but only piece-wise differentiability of functions defining the imposed constraints, some difficulties can be found, when applying the Routh method (see, e.g., [4]). It is shown that in the case when the boundary of the area of possible motion contains a singularity, knowledge of the mechanical origin of the imposed constraints is required for the correct formulation and solution of the problem of mechanics. In other words, the description of motion of a system subjected to holonomic unilateral constraint is sensitive to the way in which this constraint is implemented.

There is a great deal of modern papers [6–16] and monographs [1,17–21], devoted to various issues of mechanics of systems subjected to constraints. Among them, special mention should be made of the paper [22], in which the need to take into account the forces implementing the constraint for its correct description is emphasized, as well as the paper [4] presenting results which prompted the author's interest in this

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topic. Among the proposed methods, it is necessary to highlight the analytical approach to the study of critical and related mechanisms, developed in the works by V. A. Samsonov [23] and his school [24, 25]. In particular, they studied the restructuring of the configuration manifold of mechanisms with one degree of freedom as the parameter passes through the critical value, and the problems involved in correctly determining the trajectories of the critical mechanism using the limiting transition in the parameter. They described a typical bifurcation of equilibrium positions in the vicinity of a singular point. Also of note are the recent publications [26, 27] close in subject to the listed works. Among the numerous papers on the mechanics of systems subjected to constraints, it is necessary to mention the publications [28, 29], in which, in particular, a two-folded covering of the area of unconstrained motion is used to substantiate the mechanics of such systems. Finally, we note that the use of close methods in dry friction systems makes it possible to detect very subtle dynamic effects relateg to the so called Painlevé paradoxes (cf. [30]).

The results are partially exposed in [31].

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On homogenization of Lavrentiev–Bitsadze equations in partially perforated domains (random and deterministic)

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In the paper we study the Lavrentiev–Bitsadze equation of mixed type in inhomogeneous partially perforated domain. Theorem of unique solvability of such problems in perforated domain can be found in [1] (see also [2]).

We consider a perforated domain $D_\varepsilon \subseteq \mathbb{R}^2$ assuming that the perforation is located in the half-plane $\{x_2 > 0\}$ and has a random structure as long as the "lower" part $D_\varepsilon \cap \{x_2 < 0\}$ does not possess any inhomogeneous micro structure.

To define the perforated part of the domain, we first introduce a random set $U \subseteq \mathbb{R}^2$ which is statistically homogeneous with respect to the group of translations. Then we transform it into the small scaled domain $U_\varepsilon = \{x = (x_1, x_2) :\ \frac{x}{\varepsilon} \in U,\ x_2 > 0\}$, where $\varepsilon > 0$ is a small parameter. The random porous medium occupies the domain $D^1 = \{(x_1 - 1)^2 + x_2^2 < 1, x_2 > 0\}$.

Now the elliptic part of the Lavrentiev–Bitsadze equation is to be considered in the random perforated domain $D^1_\varepsilon = D^1 \cap U_\varepsilon$. The "hyperbolic part" of the equation is considered in the domain $D^2 = \{(x_1, x_2) :\ x_2 < 0,\ x_2 > -x_1,\ x_2 > x_1 - 2\}$.

Let us denote the parts of the boundary $\Gamma_0 := \{(x_1, x_2) :\ x_2 \geqslant 0,\ (x_1-1)^2 + x_2^2 = 1\}$, $\Gamma := \{(x_1, x_2) :\ x_2 = 0,\ x_1 \in [0; 2]\}$, $\Gamma_1 := \{(x_1, x_2) :\ x_2 = -x_1,\ x_1 \in [0; 1]\}$, $\Gamma_2 := \{(x_1, x_2) :\ x_2 = x_1 - 2,\ x_1 \in [1; 2]\}$, $S_\varepsilon = D^1 \cap \partial U_\varepsilon$. Thus, the domain $D_\varepsilon = D^1_\varepsilon \cup (int\ \Gamma) \cup D^2$ is constructed.

We study the asymptotic behavior as $\varepsilon \to 0$ of the solution $u^\varepsilon(x_1, x_2)$ to the following problem:

$$\left\{\begin{array}{ccc} -u^\varepsilon_{x_2x_2} - (\operatorname{sign} x_2) u^\varepsilon_{x_1x_1} = f(x_1, x_2) & \text{in} & D_\varepsilon, \\ u^\varepsilon = 0 & \text{on} & \Gamma_0 \bigcup \Gamma_1, \\ \frac{\partial u^\varepsilon}{\partial n_\varepsilon} = 0 & \text{on} & S_\varepsilon, \end{array}\right. \tag{1}$$

where $n_\varepsilon(x_1, x_2, \frac{x_1}{\varepsilon}, \frac{x_2}{\varepsilon})$ is the unit normal vector to S_ε internal for the pores. We also assume that $f(x_1, x_2) \equiv 0$ for $x_1 \leqslant 0$.

We prove the convergence of the solution of problem (1) to the solution of the limit problem

$$\left\{\begin{array}{ccc} \operatorname{div}\,(K \nabla u^0) = -\theta f(x_1, x_2) & \text{in} & D^1, \\ \frac{\partial^2 u^0}{\partial x_2^2} - \frac{\partial^2 u^0}{\partial x_1^2} = 0 & \text{in} & D^2, \\ \frac{\partial (u^0)^-}{\partial x_1} - \frac{\partial (u^0)^-}{\partial x_2} = \frac{\partial (u^0)^+}{\partial x_1} - \frac{\partial (u^0)^+}{\partial \gamma}, \qquad [u^0] = 0 & \text{on} & \Gamma, \\ u^0 = 0 & \text{on} & \Gamma_0 \bigcup \Gamma_1, \end{array}\right.$$

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where $[u^0] := (u^0)^+ - (u^0)^-$. Let us remind that $\frac{\partial}{\partial\gamma}$ is the conormal derivative, i.e. $\frac{\partial g}{\partial\gamma} := K\nabla g\overrightarrow{\nu}$ with $\overrightarrow{\nu} = (0,-1)^\top$. Here K is a constant deterministic matrix.

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Stochastic Keller–Segel model

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This talk concerns the solvability of the stochastic hyperbolic Keller–Segel model on $\mathbb{R}^2$ with an infinite dimensional multiplicative noise and integrable initial data. The model is composed of a stochastic nonlinear hyperbolic conservation law and an elliptic equation.

We present a method based on the kinetic theory [1] and the stochastic compactness arguments [2,3]. We apply the stochastic Jakubowski–Skorokhod representation theorem to show the existence of a stochastic weak entropy solution.

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On some properties of singular systems of linear integro-differential Volterra equations with a convolution-type kernel

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We consider systems of linear integro-differential Volterra equations with convolution-type kernels and matrices of constant coefficients,

$$\sum_{i=0}^{k} A_i x^{(i)}(t) + \int_{\alpha}^{t} K(t-s)x(s)ds = f(t),\ t \in T = [\alpha, \beta] \subset \mathbb{R}, \tag{1}$$

where $k = 1, 2, \cdots,$ A_i are constant $(n \times n)$-matrices, $K(t-s)$ is an $(n \times n)$-matrix, and $x(t)$ and $f(t)$ are the desirable and the given functions, respectively. It is assumed that the matrix that multiplies the higher derivative of the unknown vector function is degenerate:

$$\det A_k = 0. \tag{2}$$

The initial value problem for such systems differs from the standard Cauchy problem. To identify a specific solution of systems of the form (1), it is assumed that at the initial point the solutions of the system satisfy the conditions

$$\mathrm{x}(\alpha) = \mathrm{a},\ \mathrm{a} = \begin{pmatrix} a_0^\top & a_1^\top & \dots & a_{k-1}^\top \end{pmatrix}^\top,\ \mathrm{x} = \begin{pmatrix} x^\top & \dot{x}^\top & \dots & (x^{(k-1)})^\top \end{pmatrix}^\top, \tag{3}$$

where $a_0,\ a_1,\ \dots,\ a_{k-1}$ are given vectors from $\mathbb{R}^n$, $\top$ denotes transposition. It is clear that for the existence of solutions to problem (1), (3), it is necessary (but not always sufficient) to fulfill the Kronecker–Capelli criterion at the point $t = \alpha$ for vectors $x^{(k)}(\alpha)$. In other words, for the Cauchy problem to be solvable, it is necessary that the initial vectors a_i in formulas (3) belong to some linear manifold $\mathcal{R} \subset \mathbb{R}^{kn},\ m \leqslant kn$. Therefore, the initial value problem for systems (1) is set in the form of relations

$$P\mathrm{x}(\alpha) = \mathrm{a}, \tag{4}$$

where P is an $(m \times kn)$ full-rank matrix, a is a given vector. For $P = I_{kn}$, problem (1) and (4) coincides with the classic Cauchy problem, where I_{kn} is the identity of dimension kn. The matrix P projects arbitrary initial vectors a onto a valid manifold.

In this talk, we establish conditions for the solvability of such systems, as well as for the existence and uniqueness of solutions to the initial value problems. We also provide examples to illustrate the theoretical results.

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On some dynamical systems on manifolds arising in fluid and gas mechanics

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The report will discuss the reduction of fluid and gas mechanics equations for certain exact solutions to dynamical systems on manifolds. The Lie–Ovsiannikov group-theoretical approach allows one to construct broad classes of exact solutions generated by the invariance of the original equations under the Lie group of continuous transformations. For the so-called partially invariant solutions, the system of differential equations can be reduced to a dynamical system on a manifold. The properties of solutions of such systems are described in detail in the case where the reduced system is an implicit differential equation having a set of folded singular points on the manifold, the type of which can change depending on the physical parameters of the system. The possibility of switching between different solution regimes by a shock-type discontinuity is proven. For the shallow water model on a rotating attracting sphere, various types of stationary solutions are investigated, and the existence of solutions with a bore-type discontinuity is proven. The geometric properties of such solutions are also investigated. The presented results were obtained in collaboration with A. A. Cherevko and E. S. Stetsyak.

Differential models for growing sandpiles

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The equilibrium configurations of certain evolutionary models for the dynamics of granular materials are described by a system of Monge–Kantorovich type PDEs.

I will specifically discuss the case of a sandpile on a flat support with a prescribed boundary, generated by a vertical source.

In the mathematical model, the possible profiles of the pile correspond to 1-Lipschitz extensions of the boundary data, while the density of sand transported along a profile depends on the geometry of the segments along which the slope is maximal.

I will describe the possible solutions to the problem, characterizing the cases in which the equilibrium configuration is unique.

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Global dynamics of a mathematical model on smoking

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Smoking is a leading cause of preventable death in many nations due to its adverse effects on many body organs, resulting in strokes, heart disease, and other respiratory disorders. Smoking can raise the risk of lung cancer in both men and women. Smoking increases the risk of getting cardiovascular diseases. Media campaigns, commercials, and quitting programs can significantly reduce smoking rates. This article develops a non-linear compartmental model to examine how media knowledge affects the spread of smoking among smokers and non-smokers. With this motivation, a discrete mathematical model is framed and developed in this research work. In addition to this, the relapse of population who temporarily quit smoking back to becoming smokers and also from smokers and temporary smokers to permanently quit smokers are analysed. The necessary conditions for the stability of equilibria in the discrete model are defined. Numerical simulations are performed to highlight model's complex dynamics.

Birth of fixed points in families of compositions of pairs of involutions and the emergence of oscillations in the Wilander model

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Analysis of thermohaline circulations is one of the important tasks in the study of the dynamics of ocean currents (and, as a consequence, in the study of the stability of a number of real processes), including the use of block models for such currents [1,2]. We

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study the Wilander parametric model and give an exhaustive description of the typical emergence of such oscillations in it. In this model, the salinity and the temperature of the surface water layer are affected by the salinity and the temperature of the atmosphere above it as well as the constant temperature and salinity of the underlying deep water layer [3, 4]. Their interaction is described by Newton's law, where the exchange coefficients q_S and q_T for salinity and temperature are constant for the atmosphere and satisfy the relation $0 < q_S < q_T$, and there is turbulent exchange between the two layers of water, where the coefficient is *the transfer function* of the difference in the densities of these layers. This function is nonnegative, does not decrease, takes values close to zero for significantly negative values of this difference, and grows rapidly near zero.

Under natural restrictions on the parameters of the model for a wide class of finite-parameter families of transfer functions with a jump at zero, we reduce the analysis of the occurrence of thermohaline oscillations in the model under consideration to the study of the occurrence of nontrivial fixed points of the composition of involutions of the real line with a common fixed point in typical finite-parameter smooth families of pairs of such involutions [5]. We obtain normal forms of such families [6], which ultimately gives a clear description of the typical occurrence of such oscillations for the type of transfer functions under consideration.

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Epidemic fractional mathematical model with vaccination and quarantine compartments

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We present a mathematical model with vaccination and quarantine compartments. We use a system of fractional differential equations (SIQVR) to track the dynamics of model variables. We examine the well-posedness and boundedness of solutions for the model. The basic reproduction number (BRN) and equilibrium points of the model

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are analyzed. In order to effectively manage the transmission of infection within the outlined model, we employ the strategy of optimal control. This approach involves implementing control measures and interventions, guided by mathematical optimization techniques, to minimize the spread of disease. These control strategies may encompass vaccination campaigns, quarantine protocols, social distancing measures, and other preventive actions. Furthermore, to evaluate the effectiveness of proposed model and the applied optimal control strategy, we conduct a series of numerical simulations. Computational results involve running the model under different scenarios, considering a range of parameters, and meticulously analyzing the resulting outcomes.

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Compact equilibria in the liquid drop model

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This work addresses the liquid drop model introduced by Gamow in 1930 and Bohr and Wheeler in 1939 to describe the structure of atomic nuclei in nuclear physics. The problem involves finding a surface in the three-dimensional space that is critical for a specific energy functional balancing surface tension and nonlocal repulsion, subject to a volume constraint. Spherical solutions always exist and minimize the energy for sufficiently small volumes. However, for larger volumes, constructing non-minimizing critical points becomes more challenging. In this study, we present a new class of large-volume solutions resembling "pearl collars" arranged along an axis in the shape of a large circle, with geometry close to Delaunay's unduloids surfaces of constant mean curvature. We also construct non-minimizing solutions with small mass that resemble two nearly identical spheres connected by a narrow neck. This is joint work with Monica Musso, Andrés Zúñiga, and Rupert Frank.

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Asymptotic properties of solutions to the characteristic problem for the ultrahyperbolic equation

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We deal with the following problem:

$$\left(\partial_{ts}^2 + \partial_{x_1}^2 + \ldots + \partial_{x_d}^2 - \partial_{y_1}^2 - \ldots - \partial_{y_n}^2\right) U = 0, \qquad (1)$$
$$U|_{t=0} = U_0,$$

where U is a function of the variables

$$(t, s, x_1, \ldots, x_d, y_1, \ldots, y_n) \in \mathbb{R}^{N+2}, \quad N := d + n \geqslant 1.$$

The set $\{t = 0\}$ where the initial data U_0 are given is a characteristic hyperplane for ultrahyperbolic equation (1). The well-posedness of this problem in a certain set of functions was established by A. S. Blagoveshchensky [1]. In particular, the following conservation law was obtained:

$$\|U(t)\|_{L_2} = \|U_0\|_{L_2}, \quad t \in \mathbb{R}.$$

These two features make it reasonable to study the corresponding nonstationary scattering problem for equation (1), considering t as the time variable. One of the crucial issues in such an investigation would be the asymptotic behavior of solutions for large t, which is the subject of the present talk.

As in the case of *hyperbolic* equations, the asymptotics of solutions at infinity is given along null geodesics. It is convenient to formulate the result in terms of a function $u(x, y)$ of the variables

$$x = (x_0, x_1, \ldots, x_d) \in \mathbb{R}^{d+1}, \quad y = (y_0, y_1, \ldots, y_n) \in \mathbb{R}^{n+1},$$

related to U as follows:

$$u(x, y) = U((x_0 - y_0)/2, (x_0 + y_0)/2, x_1, \ldots, x_d, y_1, \ldots, y_n).$$

Let

$$x = \tau\theta, \quad y = (\tau + p)\omega, \quad (\theta, \omega, p) \in S^d \times S^n \times \mathbb{R}$$

(S^m is the unit sphere in $\mathbb{R}^{m+1}$), where τ is a large positive parameter. Assume also that $\theta_0 \neq \omega_0$, which means that the characteristic direction (θ, ω) is transversal to the hyperplane where the initial data are given. Then, for U_0 from the Schwartz class, the solution U is a smooth function outside the hyperplane $\{t = 0\}$, and the following asymptotic relation is valid:

$$u(x, y) = \frac{1}{\tau^{N/2}} F(\theta, \omega, p)(1 + o(1)), \quad \tau \to +\infty.$$

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Non-polynomial invariants and the integrability of two-dimensional differential systems

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Studying the integrability and solvability of differential systems is a classical problem of analysis. This talk is devoted to the integrability of two-dimensional differential systems

$$x_t = P(x, y), \quad y_t = Q(x, y), \tag{1}$$

where $P(x, y)$ and $Q(x, y)$ are elements of the field of two-variable rational functions $\mathbb{C}(x, y)$. We say that system (1) is integrable if there exists a continuously differentiable non-constant function $I(x, y)$: $D = D_x \times D_y \subseteq \mathbb{C}^2 \to \mathbb{C}$ called a first integral such that this function is constant on all integral curves $(x(t), y(t))$ contained in D. A continuously differentiable non-constant function $F(x, y)$: $D = D_x \times D_y \subseteq \mathbb{C}^2 \to \mathbb{C}$ is called an invariant of system (1) if it satisfies the linear partial differential equation $\mathcal{X}F = \lambda(x, y)F$, where $\mathcal{X}$ is the vector field associated to system (1). The function $\lambda(x, y)$ is commonly refereed to as the cofactor of the invariant $F(x, y)$. Obviously, a first integral $I(x, y)$ is an invariant with zero cofactor.

System (1) is Liouvillian integrable, whenever it has a first integral that belongs to a Liouvillian extension of the field $\mathbb{C}(x, y)$. Singer's work [1] gives rise to a powerful method for solving the Liouvillian integrability problem. A sufficient number of multi-parameter systems (1) such that their Liouvillian first integrals are completely classified is now available, see, e.g., [2]. Less is known about systems (1) with non-Liouvillian first integrals. The aim of this talk is to present a method of finding first integrals that are expressible via special functions and belong to some Picard–Vessiot extension of the field $\mathbb{C}(x, y)$. The method is based on the existence of two independent invariants that satisfy a linear second-order ordinary differential equation with respect to one of the variables. In addition, these invariants have the same cofactor. We study the existence of non-Liouvillian first integrals for the generalized Liénard differential systems

$$x_t = y, \quad y_t = -h(x)y^2 - f(x)y - g(x), \tag{2}$$

where $h(x)$, $f(x)$, and $g(x)$ are elements of the field of one-variable rational functions $\mathbb{C}(x)$. We present novel families of systems (2) without polynomial invariants.

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Asymptotic analysis of solutions for a nonlinear viscoelastic model

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The asymptotic behavior of solutions to partial differential equations modeling the deformation of elastic and viscous materials in thin domains has been extensively studied in the literature. For instance, in [1], the authors analyzed a mathematical model describing the quasi-static frictional contact between a piezoelectric body and a deformable foundation under the normal compliance condition and Tresca's friction law. They derived the limit problem as one dimension of the domain tends to zero. The asymptotic study of a dynamical problem in isothermal elasticity involving a nonlinear dissipation term and the nonlinear Tresca friction is discussed in [3].

The aim of this paper is to investigate the asymptotic behavior of a nonlinear viscoelastic contact problem in a thin domain, $\mathcal{O}^\varepsilon \subset \mathbb{R}^3$, characterised by a logarithmic source term with the Dirichlet condition on one part of its boundary and the non-linear slip conditions governed by the Tresca friction law on the other parts. As the domain thickness ε tends to zero, we demonstrate that the solution of the three-dimensional problem converges to a two-dimensional limit problem with the Tresca free boundary conditions. Furthermore, the uniqueness of the solution to the limit problem is established.

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On the Ω-blow-up phenomenon in simplest skew products on cells

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In this work, we consider the following types of the C^0- Ω-blow-up in skew products on nD-cells ($n \geqslant 2$): (i) the C^0- Ω-blow-up in the family of fiber maps of continuous skew products with a closed set of periodic points (see [1], [2]); (ii) the C^0- Ω-blow-up in C^1-smooth skew products with a bounded set of (least) periods of periodic points (see [3]).

Here we show that the Ω-blow-up of type (i) defines the structure of the nonwandering set and prove the theorem on the coincidence of the nonwandering set with the set of periodic points in the case of a closed set of periodic points of a continuous skew product on a multidimensional cell.

The structure of the nonwandering set defines the dynamics of maps under consideration (in particular, we describe here some aspects of the ω-limit behaviour of trajectories).

The results obtained in the framework of item (i) also make it possible to study the Ω-blow-up of type (ii). At first, we prove different criteria of such Ω-blow-up. Secondly, we prove that we get the new scenario of the transition to the entropy chaos, starting from maps with zero topological entropy.

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Problems of distributed control of emulsion oscillations of weakly viscous compressible liquids and the existence of a wavefront

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The problem of distributed control of oscillations of an effective (averaged) medium corresponding to a two-phase medium of slightly viscous liquids is considered. The averaged model is described by a boundary value problem for an integro-differential equation. It is shown that for this model, it is impossible to bring vibrations to a state of rest in a finite time by force action on the entire region.

The question of the existence of a wavefront for the averaged model is also considered. It is shown that with sufficient smoothness of the density of liquids and initial conditions, the solution of the problem will be a finite function of a spatial variable for each fixed finite moment in time.

Relatively uniformly continuous semigroups on ordered vector spaces

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Relatively uniformly continuous semigroups on vector lattices were introduced and studied in [4–6] to extend the theory of C_0-semigroups to the vector lattice setting. It is quite natural and flexible to consider an abstract Cauchy problem on a dense subspace of a Banach space. In many cases, the vector lattice structure may be absent. However, in various applications, underling PDEs describe processes in a compact area, and then the space still possesses a structure of an ordered vector space with a generating cone.

In this talk, relatively uniformly continuous operator semigroups on ordered vector spaces are introduced and several well known results are extended in the vector lattice setting to ordered vector spaces with generating cones.

It is joint work with E. Emelyanov and S. Gorokhova, see [1]. The research is carried out in the framework of the Hacettepe University (project FHD-2024-21076).

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Contracting and expanding σ-demi ab-continuous operators

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The concepts of compact and continuous operators are of great importance in Banach lattice theory. In 1966, the notions of demicontinuous and demicompact operators were introduced based on the norm convergence, to construct and analyze the structure of fixed point sets of nonlinear operators in Hilbert and Banach spaces. However, in vector lattices, there are various types of convergence in addition to the norm convergence. Alongside classical concepts of the order and the norm convergence, unbounded convergences such as the unbounded order, the unbounded norm, and the unbounded absolute weak convergence defined for vector lattices, form an integral part of this research. In 2024, a class of demi ab-continuous operators was studied based on various types of a and b convergences in Banach lattices, see [2].

Inspired by Knuth's foundational work, we introduce several asymptotic notions, including $O_a^b((y_n))$ and $\Omega_a^b((y_n))$, which are defined with respect to arbitrary sequential convergences. In the present talk, we focus on contracting and expanding σ-demi ab-continuous operators, which represent specific subclasses of the general σ-demi ab-continuous operators, by employing $O_a^b((y_n))$ and $\Omega_a^b((y_n))$. Moreover, we explore fundamental properties of contracting and expanding σ-demi ab-continuous operators.

This work is based on the manuscript, see [1].

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The routes of trajectories of the Lotka–Volterra operators, bigraphs and their application to description of the evolution of several viruses interaction

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Let the Lotka–Volterra mapping $V_1 : S^4 \to S^4$ corresponding to a block skew-symmetric degenerate matrix have the following form:

$$V_1 : \begin{cases} x_1^{'} = x_1(1 + a_1x_3 - b_1x_4 + c_1x_5), \\ x_2^{'} = x_2(1 - a_2x_3 + b_2x_4 - c_2x_5), \\ x_3^{'} = x_3(1 - a_1x_1 + a_2x_2), \\ x_4^{'} = x_4(1 + b_1x_1 - b_2x_2), \\ x_5^{'} = x_5(1 - c_1x_1 + c_2x_2). \end{cases} \tag{1}$$

Some cases of operators of this type were considered in [1]. This operator has an infinite number of fixed points. In addition, we find the following fixed points on the edges of the S^4 simplex:

$$M_1\left(\frac{a_2}{a_1+a_2},\ \frac{a_1}{a_1+a_2},\ 0,\ 0,\ 0\right),\ M_2\left(\frac{b_2}{b_1+b_2},\ \frac{b_1}{b_1+b_2},\ 0,\ 0,\ 0\right),$$

$$M_3\left(\frac{c_2}{c_1+c_2},\ \frac{c_1}{c_1+c_2},\ 0,\ 0,\ 0\right)$$

on the edge $\Gamma_{12} = \mathrm{co}\,\{e_1,\ e_2\}$, the points

$$N_1\left(0,0,\frac{b_1}{a_1+b_1},\frac{a_1}{a_1+b_1},0\right),\quad N_2\left(0,0,\frac{b_2}{a_2+b_2},\frac{a_2}{a_2+b_2},0\right)$$

on the edge $\Gamma_{34} = \mathrm{co}\,\{e_3, e_4\}$, and the points

$$K_1\left(0,0,0,\frac{c_1}{b_1+c_1},\frac{b_1}{b_1+c_1}\right),\quad K_2\left(0,0,0,\frac{c_2}{b_2+c_2},\frac{b_2}{b_2+c_2}\right)$$

on the edge $\Gamma_{45} = \mathrm{co}\,\{e_4, e_5\}$. The order of these points determines the dynamic properties of the mapping V_1.

Proposition 1. *The following statements are true:*

1) *The segments N_1K_1 and N_2K_2 intersect (internally) only if $M_2 \in riM_1M_3$.*

2) *If $M_1 = M_2$, then $N_1 = N_2$, and vice versa.*

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3) $M_2 = M_3$ *only with* $K_1 = K_2$.

4) *If* $M_1 = M_2 = M_3$, *then the segments* N_1K_1 *and* N_2K_2 *coincide, and vice versa.*

Proposition 2. *The following statements are true:*

1) *If* $M_1 \neq M_2$ *and* $M_2 \neq M_3$, *then* $Fix\,(V) = \Gamma_{12} \cup \Gamma_{345}$.

2) *If* $M_1 = M_2 \neq M_3$, *then* $Fix\,(V) = \Gamma_{12} \cup \Gamma_{345} \cup \mathrm{co}\,\{M_1N_1\}$.

3) *If* $M_2 = M_3 \neq M_1$, *then* $Fix\,(V) = \Gamma_{12} \cup \Gamma_{345} \cup \mathrm{co}\,\{M_2K_1\}$.

4) *If* $M_1 = M_2 = M_3$, *then* $Fix\,(V) = \Gamma_{12} \cup \Gamma_{345} \cup \mathrm{co}\,\{M_1N_1K_1\}$.

The Lotka–Volterra operator V_1 defined by system (1) can be interpreted in an epidemiological context as a model of interaction between two competing viruses (or strains of infection) spreading in a population. In a detailed paper, we will present the epidemiological interpretation of each variable and each term of the equations, as well as the meaning of the geometric structure of the points M_i, N_i, K_i.

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Analysis of gauged Witten equations

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Gauged linear sigma model was considered by Witten as a uniform model to explain the phenomena of mirror symmetry and Landau–Ginzburg/Calabi–Yau correspondence conjecture. The aim of its A-model theory is to build a virtual cycle theory over the moduli space of curves. There are some ways to build analogous algebraic objects. In this talk, I will report our approach to consider this model via analysis. We will introduce the basic setting of the energy functional, its minimizer equation-gauged witten equation (GWE). Some analytic properties of GWE, and a compactness theorem. This is the ongoing joint work with Y. Ruan and T. Jarvis.

Cone search functions with operator coefficients on a normed-space-valued cone metric space

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In 1964, A. I.Perov proved a generalization of the known Banach contraction mapping principle in metric spaces with a metric taking values in a cone of the vector space $\mathbb{R}^n$. A positive linear operator in $\mathbb{R}^n$ with spectral radius less than 1 was used as a contraction coefficient. This idea by Perov received a great response among mathematicians.

Earlier, in 2009–2013, the author introduced the concept of (α, β)-search functional and proved the zero existence theorem for such functionals in a metric space. There were obtained many fixed point and coincidence consequences for mappings of metric spaces, generalizing a number of known results of various authors.

Let X be a nonempty set and $(S, \|\cdot\|)$ be a Banach space. Let K be a positive convex closed acute cone. The cone K in the set S defines a partial order $\leqslant_K$ on S, we say $x \leqslant_K y$ iff $y - x \in K$. Thus, $(S, \leqslant_K)$ is a partially ordered set. The norm in the space S is assumed to be monotone with respect to the order $\leqslant_K$. That is the cone K is normal with the normality coefficient 1.

The cone metric on X associated with the cone K is a mapping $d : X \times X \to K$ satisfying the axioms of a usual metric. The space (X, d_K) with the cone metric is called a *cone metric space*.

Using natural Perov's idea above, the author introduced a concept of (A, B)-search cone functions on a cone metric space as analogs of (α, β)-search functionals. Here A, B are linear operators with special properties in a normed space, which are used instead of coefficients α, β. A zero existence theorem is proved for such functions. It implies several results concerning fixed point and coincidence existence for multivalued mappings on cone metric spaces, which generalize some known theorems of other authors.

The talk is mainly based on shot communication [1].

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Global solvability of free boundary magnetohydrodynamics problem for two fluids

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We consider a free boundary problem of magnetohydrodynamics in a bounded domain in $\mathbb{R}^3$. It describes the motion of viscous incompressible electrically conducting capillary fluid inside the other viscous incompressible fluid under the action of magnetic field. The interface between the fluids is unknown. We assume that the initial position of the free boundary can be considered as a small normal perturbation of the sphere. Under smallness assumptions on the initial data, we prove solvability of the free boundary problem of magnetohydrodynamics in an infinite time interval. The scheme of the proof is similar to that used in [1], where the similar result was

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proved for the case where a finite isolated mass of a viscous incompressible electrically conducting fluid was surrounded by a vacuum region.

The solution is obtained in the Sobolev–Slobodetskii spaces $W_2^{2+l,1+l/2}$, $l \in (\frac{1}{2}, 1)$. To reduce the free boundary problem to a problem in a fixed domain, we use the Hanzawa coordinate transform. The linearized problem can be decomposed into two parts: the hydrodynamic part and the magnetic one. The unique solvability of the linear conjugation problem for the magnetic field is established in [2].

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Hamilton's equations and neural networks in dynamic system modeling

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The study investigates the dynamics of industry development through the lens of sustainable growth. Sustainable growth is understood as the transition of the economic sector towards "green finance." The objective of this transition is to shift the consumer model from a linear to a circular one. This task is approached using financial and economic indicators of the oil and gas industry over the period T.

Mathematical formulation of the problem. Let $Z(t)$ denote the state of the dynamic system at time t, where $\bar{Z} \in \mathbb{R}^n$ is defined as:

$$\bar{Z} = (z_1, \ldots, z_{n_1}, z_{1+n_1}, \ldots, z_{n_1+n_2}, z_{n_1+n_2+1}, \ldots, z_{n_1+n_2+n_3}),$$

the first n_1 coordinates correspond to indicators of environmental protection funding; the next n_2 coordinates represent indicators of funding for social needs; and the subsequent n_3 coordinates indicate funding for production needs.

We have a set of N sequential, equally spaced observations of the state of the system, denoted as: $\left\{\bar{Z}\ (t)\right\}_{t=1}^{N}$. To adequately assess the development rates of the industry in the context of sustainable growth, it is essential to formulate clear functional criteria and develop corresponding algorithms that allow for data analysis and result interpretation.

To solve the posed problem, we propose to use methods from mechanics, specifically Hamilton's equations. In the first stage of the research, financial and economic indicators were processed using Principal Component Analysis (PCA). This method allowed for the transformation of the original data into new variables (principal components) that better reflect the structure of the system in terms of the implementation of green technologies. Then, in the space of new coordinates obtained using PCA, Hamilton's equations were reconstructed using neural networks. The next step of

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the research was to study the stability of the resulting Hamiltonian system. For this purpose, the Jacobian was calculated, which allows for assessing the local stability of the system based on the analysis of the eigenvalues of the linear operator.

Thus, the proposed approach combines methods from mechanics and modern machine learning tools for indepth analysis of financial and economic systems within the framework of sustainable development. The results of this research may be useful for formulating strategies for transitioning to green technologies and assessing their impact on economic sustainability.

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On the energy constrained optimal mixing problem

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The optimal mixing problem addresses the question of how fast a passive scalar can be mixed under the influence of an incompressible velocity field. The talk focuses on the case where the energy of the allowed fields, i.e. their L^2 norm, is uniformly bounded in time. In that setting perfect mixing in finite time is permitted and indeed realized in some examples by Depauw (2003) and Lunasin, Lin, Novikov, Mazzucato, Doering (2012). On the other hand, a lower bound on the time in which perfect mixing can be achieved is known due to Lin, Thiffeault, Doering (2011). In the talk we will show an improvement of the lower bound for the special case of initial data depending only on one spatial coordinate. We will also discuss an example for which the new lower bound is sharp.

Blow-up problem for a parabolic equation with nonlinear memory and absorption under nonlinear nonlocal boundary condition

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We consider the nonlinear nonlocal parabolic equation

$$u_t = \Delta u + a \int_0^t u^q(x,\tau)\, d\tau - bu^m,\ x \in \Omega,\ t > 0, \tag{1}$$

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with the nonlinear nonlocal boundary condition

$$\frac{\partial u(x,t)}{\partial \nu} = \int_\Omega k(x,y,t)u^l(y,t)\,dy,\ x \in \partial\Omega,\ t > 0, \tag{2}$$

and the initial datum

$$u(x,0) = u_0(x),\ x \in \Omega, \tag{3}$$

where a, b, q, m, l are positive numbers, Ω is a bounded domain in $\mathbb{R}^N$ for $N \geqslant 1$ with smooth boundary $\partial\Omega$, ν is the unit outward normal vector on $\partial\Omega$.

Throughout this paper we suppose that the functions $k(x,y,t)$ and $u_0(x)$ satisfy the following conditions:

$$k(x,y,t) \in C(\partial\Omega \times \overline{\Omega} \times [0,+\infty)),\ k(x,y,t) \geqslant 0;$$

$$u_0(x) \in C^1(\overline{\Omega}),\ u_0(x) \geqslant 0 \text{ in } \Omega,\ \frac{\partial u_0(x)}{\partial \nu} = \int_\Omega k(x,y,0)u_0^l(y)\,dy \text{ on } \partial\Omega.$$

The initial boundary value problem (1)–(3) with $a = 0$ was considered in [1, 2]. We prove global existence and finite time blow-up results.

Theorem 1. *Let at least one from the following conditions hold:*

a) $\max(q,l) \leqslant 1$;

b) $l \leqslant 1$, $1 < q \leqslant m$;

c) $1 < l < m$, $q \leqslant m$.

Then every solution of (1)–(3) *is global.*

To formulate finite time blow-up result, we suppose that there are some $k_1 > 0$ and $t_1 > 0$ such that

$$\int_{\partial\Omega} k(x,y,t)\,dS_x \geqslant k_1 \text{ for any } y\ \in \Omega \text{ and } t \geqslant t_1. \tag{4}$$

Theorem 2. *Let* $q > \max(m,1)$ *or* $l > \max(m,1)$ *and* (4) *hold. Then solutions of* (1)–(3) *blow up in finite time for any* $u_0(x) \not\equiv 0$.

The results of the talk were published in [3, 4].

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Multiline solitons of the Kadomtsev–Petviashvili-II equation and divisors on rational curves

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The Kadomtsev–Petviashvili (KP) equation was derived as a universal model for the wave propagation in weakly dispersive weakly nonlinear spatially 2D systems under the assumption that the waves weakly depend on the y-variable. It has two real forms, KP-I and KP-II. In particular, the KP-II equation is used as a model for the surface waves in the shallow water when the surface tension is not too strong. Multiline soliton solutions of the KP-II equation can be constructed either using Darboux transformations or as degenerations of finite-gap solutions.

It is necessary for physical applications to use real regular solutions. S. P. Novikov pointed out that real regular soliton solutions shall be constructed as degenerations of real regular finite-gap solutions. The rational spectral curves generating real regular solution solutions were constructed in a series of works by the authors (see [1–3]).

Using degenerate spectral curves in the finite-gap theory, it is necessary to refine the definition of the divisor, because if a pair of divisor points approach a double point from different directions, then the Abel transform remains finite, and it is necessary to apply resolution of singularities procedure to make the dynamics prescribed by the equation continuous. It is very likely that in generic situation highly complicated singularities may occur.

We show that the situation is essentially simplified in the special important case of real regular mutlisoliton solutions of the KP-II equation. If we consider the rational M-curves corresponding to the totally positive Schubert cells, and the divisors satisfying the Dubrovin-Natanzon conditions, then only two simplest types of blow-ups take place.

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Fractional calculus and universality theorems

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Fractional calculus can be extended to functions in the complex domain. In particular, the generalization of the Grünwald–Letnikov derivative to the complex plane makes sense under weaker conditions than the other definitions.

The class of zeta functions plays a special role. These functions are also relevant in fractional calculus. More precisely, we are talking about the α-order fractional derivative of the Riemann ζ function given by

$$\zeta^{(\alpha)}(s) = e^{i\pi\alpha} \sum_{n=2}^{\infty} \frac{\log^{\alpha} n}{n^s}, \quad s \in \mathbb{C}, \ \alpha \in \mathbb{R} \setminus \mathbb{Z}.$$

One of the most relevant property of the Riemann zeta function called universality was proved by Voronin in 1975. He proved that a wide class of analytic functions can be approximated by shifts of $\zeta(s)$, i.e. $\zeta(s + i\tau)$ with $\tau \in \mathbb{R}$.

In this talk, we deal with the role of universality theorems in fractional calculus. In particular, we discuss the universality property for the fractional derivative $\zeta^{(\alpha)}(s)$ along with the main applications in dynamical systems.

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Impulsive fractional differential inclusions involving Caputo–Hadamard fractional derivative

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In this work, we establish existence results for a class of initial value problems for impulsive fractional differential inclusions involving the Liouville–Caputo–Hadamard fractional derivative of order $0 < \alpha < 2$, when the right hand side is convex, as well as nonconvex, valued. The topological structure of the set of solutions is also considered. This communication deals with the existence of solutions of the initial value problem (IVP for short), for the impulsive fractional order differential inclusion,

$$\begin{gathered} {}^{CH}D^{\alpha}y(t) \in F(t, y(t)), \\ \text{for a.e. } t \in J := [a, T], \quad a > 0, \quad t \neq t_k, \quad k = 1, \dots, m, \quad 0 < \alpha < 1, \end{gathered} \tag{1}$$

$$\Delta y|_{t=t_k} = I_k(y(t_k^-), \quad k = 1, \dots, m, \tag{2}$$

$$y(a) = y_a, \tag{3}$$

where ${}^{CH}D^{\alpha}$ is the Caputo-Hadamard fractional derivative, $F : J \times \mathbb{R} \to \mathcal{P}(\mathbb{R})$ is a multivalued map, $\mathcal{P}(\mathbb{R})$ is the family of all nonempty subsets of $\mathbb{R}$, $I_K : \mathbb{R} \to \mathbb{R}$, $k = 1, \dots, m$ are continuous functions, $a = t_0 < t_1 < \dots < t_m < t_{m+1} = T$, $\Delta y|_{t=t_k} = y(t_k^+) - y(t_k^-)$, $y(t_k^+) = \lim\limits_{\varepsilon \to 0^+} y(t_k + \varepsilon)$ and $y(t_k^-) = \lim\limits_{\varepsilon \to 0^-} y(t_k + h)$ represent the right and left limits of $y(t)$ at $t = t_k$, $k = 1, \dots, m$, and $y_a \in \mathbb{R}$.

Also

$${}^{CH}D^{r}y(t) \in F(t, y(t)), \text{ for a.e. } t \in J = [a, T],\ a > 0,\ t \neq t_k,\ k = 1, \dots, m, 1 < r \leqslant 2, \tag{4}$$

$$\Delta y|_{t=t_k} = I_k(y(t_k^-)), \quad k = 1, \dots, m, \tag{5}$$

$$\Delta y'|_{t=t_k} = \overline{I}_k(y(t_k^-)), \quad k = 1, \dots, m, \tag{6}$$

$$y(a) = y_1, y'(a) = y_2, \tag{7}$$

where ${}^{CH}D^{r}$ is the Caputo–Hadamard fractional derivative, $F : J \times \mathbb{R} \to \mathcal{P}(\mathbb{R})$ is a multivalued map, $\mathcal{P}(\mathbb{R})$ is the family of all nonempty subsets of $\mathbb{R}$, $I_k, \overline{I}_k : \mathbb{R} \to \mathbb{R}$, $k = 1, \dots, m$, are continuous functions, $a = t_0 < t_1 < \dots < t_m < t_{m+1} = T$, $\Delta y|_{t=t_k} = y(t_k^+) - y(t_k^-)$, $\Delta y'|_{t=t_k} = y'(t_k^+) - y'(t_k^-)$, $y(t_k^+) = \lim\limits_{\varepsilon \to 0^+} y(t_k + \varepsilon)$ and $y(t_k^-) = \lim\limits_{\varepsilon \to 0^-} y(t_k + h)$ represent the right and left limits of $y(t)$ at $t = t_k$, $k = 1, \dots, m$, and $y_1, y_2 \in \mathbb{R}$.

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Fractional Caputo–Fabrizio stochastic differential equations

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Fractional derivatives are the basis of fractional calculus, a crucial tool for modeling and analyzing systems with nonlocal properties. Unlike integer-order derivatives, which consider only instantaneous rates of change at a given point, fractional derivatives consider the behavior of the solution over some range of values, including memory effects in the system. There are different types of fractional derivatives, each with its own definition and properties. The choice of a particular type of fractional derivative depends on the problem conditions and the desired mathematical characteristics of the derivative [1]. In recent years, some researchers have had great success using the Caputo–Fabrizio derivative [2,3]. Fractional stochastic differential equations combine stochastic processes with fractional calculus. These equations are used in modeling systems with random behavior, often representing stochastic processes occurring in environments with nonlocal and memory-dependent dynamics. Such equations describe quite accurately the properties of real-world systems such as biological populations, financial markets, and control system technologies, where prehistories, memory effects, and unpredictable events significantly influence the current dynamics. In this paper, we consider a system of fractional stochastic Caputo–Fabricio equations in the n-dimensional Euclidean space $\mathbb{R}^n$ together with the initial condition,

$$\begin{cases} {}^{CF}D_t^{\alpha}x(t) = a(t,x(t)) + \sigma(t,x(t))\frac{dW(t)}{dt}, \ t \in [0,T], \\ x(0) = \xi, \end{cases} \tag{1}$$

where x is the stochastic process, ${}^{CF}D_t^{\alpha}$ is the Caputo–Fabrizio fractional derivative with $\alpha \in (1/2, 1]$, and the functions $a : [0,T] \times \mathbb{R}^n \to \mathbb{R}^n$, $\sigma : [0,T] \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are measurable and continuous. Let $(\Omega, \mathcal{F}, \mathbb{P})$ denote a complete probability space with standard properties and filtering $\{\mathcal{F}_t\}_{t\geqslant 0}$, and $W(t)$ be an m-dimensional Wiener process on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$, adapted to the filter $\{\mathcal{F}_t\}_{t\geqslant 0}$.

We have established the conditions of existence and uniqueness, continuous dependence on initial data, and regularity of solutions to problem (1).

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Differential geometric approach to ellipsoid characterizations

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In 1932, Stephan Banach posed the following problem: Let V be a normed space and assume that for some fixed $1 < k < \dim V$, all k-dimensional linear subspaces of V are isometric. Does this imply that the norm of V is an inner product one? An equivalent geometric formulation reads as follows: if all k-dimensional central cross-sections of a symmetric convex body (the norm's unit ball) are linearly equivalent, is the body necessarily an ellipsoid?

The problem has been solved affirmatively in some dimensions and remains open in others. Known partial solutions are based mainly on algebraic topology. In a recent joint work with D. Mamaev and A. Nordskova the problem was solved in the case $k = 3$ and $\dim V = 4$ where topological methods do not help. In fact, the solution can be localized: it suffices to assume the linear equivalence only for an open set of linear subspaces, and then it follows that the respective part of the norm's unit ball coincides with an ellipsoid or a cylinder.

The solution is based on differential geometric approach which has grown from attempts to generalize Gauss' Theorema Egregium to surfaces in non-Euclidean normed spaces. This problem also remains open in general but the special case of zero curvature at a point has been solved, and it is closely related to the localized Banach's problem.

The key idea of the proof is to reduce the problem to a question about integrability of some special systems of first-order PDEs. These systems form a finite-dimensional family parameterized by certain tensors determined by the convex body in question. The relevant conjecture (confirmed in dimensions 3 and 4) is that most of these systems are not integrable and integrable ones correspond to ellipsoids. Another ingredient of the solution is a localized version of the classical Kakutani characterization of inner product norms: If for every k-dimensional linear subspace from some open set there exist a unit-norm linear projector onto this subspace, then the norm coincides with an inner product one or a cylindrical one on the respective subset.

In the talk I will give an overview of the above mentioned problems and the arising connections between geometry of normed vector spaces, differential geometry of surfaces, and integrability.

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Methods of solving differential-difference equations with incommensurable shifts of arguments

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We consider the boundary value problem

$$A_R u = -\sum_{i,j=1}^{n} (R_{ij} u_{x_j})_{x_i} = f(x) \quad (x \in Q), \tag{1}$$

$$u(x) = 0 \quad (x \notin Q). \tag{2}$$

Here Q is a bounded domain in $\mathbb{R}^n$ with smooth boundary ∂Q, $f \in L_2(Q)$, the difference operators $R_{ij} : L_2(\mathbb{R}^n) \to L_2(\mathbb{R}^n)$ are given by

$$R_{ij} u(x) = \sum_{h \in M_{ij}} a_{ijh}(u(x+h) + u(x-h)) \quad (a_{ijh} \in \mathbb{R}),$$

where $M_{ij} \subseteq M$ are finite sets of vectors with incommensurable coordinates. The generalized solution u of boundary value problem (1), (2) belongs to the Sobolev space $\mathring{H}^1(Q)$.

For elliptic differential-difference equations with integer shifts of the variables, the theory of boundary-value problems was constructed in the works by Skubachevskii (see [1]).

Incommensurable shifts of the arguments greatly complicates the study of such boundary value problems, especially in the case where the orbit of the boundary under the shifts present in the difference operators is infinite (see [5]).

Under the conditions of strong ellipticity [3], we can apply the Ritz method for solving (1)–(2), as well as the method of approximation of the solution to the "irrational" problem by a sequences of solutions to the "rational" problems. The last problems are redusable to the boundary value problem for a differential equation with nonlocal boundary conditions (see [4]).

The proposed approximation algorithms for the one-dimentional case are implemented in the Maple software environment [2].

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New concept of measure in metric spaces and the Riemann Hypothesis

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In [1], a new measure is considered in an infinitedimensional unite cube. Some curves having zero measure in the sense of the product measure become immeasurable in the new meaning. This gives the negative answer to J. P. R. Christensen's question on the Haar measure, firstly found in paper [2] by M. Csornyei. An explanation for this phenomenon is given in [1].

Consider a compact metric space $\langle G, d\rangle$. Let Ω be a family of open balls in G and $\Pi(\Omega)$ denote the algebra induced by subsets of this family. Suppose that a finite additive set function μ is given in $\Pi(\Omega)$, and this set function can be uniquely continued to the σ-algebra Ξ generated by the algebra Π. Denote this measure as μ. We define now a new measure by using a new definition of the zero measure sets.

Definition 1. A subset $A \subset \Omega$ is called a subset of zero measure if for any $\varepsilon > 0$ there exist open balls $B_1, B_2, ...$ such that

1) $A \subset B = B(\varepsilon) = \bigcup\limits_{k=1}^{\infty} B_K$;

2) *none of these balls contains any others*;

3) $\sum_{k=1}^{\infty} \mu(B_k) < \varepsilon$.

The inner and outer measures μ_{*0} and μ_0^* are defined now in a similar way by using coverings with open balls satisfying condition 2) above. Since each open ball can be enclosed in some subset from σ-algebra, we have

$$\mu_{*0}(A) \leqslant \mu_*(A) \leqslant \mu^*(A) \leqslant \mu_0^*(A)$$

for a given subset $A \subset \Omega$.

Definition 2. A subset A is called μ_0-measurable if and only if there exists a subset $D \subset \Xi$ such that the symmetric difference $D \bigtriangleup A$ has zero measure.

Denote by Σ the minimal subalgebra in Ξ containing all unions of open balls satisfying conditions 2) and 3) for any $\varepsilon > 0$. It is obvious that $\Pi(\Omega) \subset \Sigma \subset \Xi$.

Theorem 1. *For any subset $A \in \Xi$, the following is true*:

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1) *for any real* $\varepsilon > 0$, *there exists a subset* $A_\varepsilon \in \Sigma$ *such that* $A \subset A_\varepsilon$ *and* $\mu(A_\varepsilon) \leqslant \mu(A) + \varepsilon$.

2) *there is a sequence of subsets* $A_{\varepsilon_1}, A_{\varepsilon_2}, \ldots$ *with* $\varepsilon_n \to 0$ *such that* $\mu\Big(\bigcap\limits_{n=1}^{\infty} A_{\varepsilon_n} \setminus A\Big) = 0$.

3) *the relation* $\bigcap\limits_{n=1}^{\infty} A_{\varepsilon_n} \in \Sigma$ *holds.*

Theorem 2. *If* Ω *is a proper family, then the Lebesgue extensions of the* σ*-algebra and the* σ*-summable algebra induced by the family* Ω *are coincident.*

This theorem justifies the consideration of the σ-summable algebra instead of the σ-algebra.

Theorem 3. *The family of open balls in the infinite-dimensional unite cube [1] is a proper one.*

The main difference between the product measure and the new one is given by the lemma below.

Lemma. *Let* (λ_n) *be an unbounded sequence of positive real numbers with the property that any finite subsequence of this sequence is linearly independent over the field of rational numbers. Then the curve* $(\{t\lambda_n\})\,, t \in [0,1]$, *is not a* μ_0*-measurable set in* $\omega = [0,1]^\infty$.

Therefore curves negligible with respect to the Lebesgue product measure acquire a property of uniform distributions. As an application of the new concept, we get our main result.

Theorem 4. *Let* $0 < r < 1/4$ *be a real number. Then there exist a sequence* θ_n *in* ω *and a sequence* m_n *of integers such that* $\lim\limits_{n\to\infty} F_n(s+it, \theta_n) = \zeta(s+it)$ *for every real* t uniformly in the disc $|s - 3/4| \leqslant r$, *where*

$$F_n(s+it,\theta_n) = \prod_{p\leqslant m_n} \left(1 - \frac{e^{-2\pi i \vartheta_p^n}}{p^{s+it}}\right)^{-1}; \quad \theta_n = (\theta_p^n),$$

the product is taken over all prime numbers, and the components of θ_n *are indexed by prime numbers.*

Corollary 1. *The Riemann Hypothesis is true, i. e.* $\zeta(s) \neq 0$ *when* $\sigma > 1/2$.

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Operator norms “calculation” in Morrey and dual Morrey spaces

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We consider maximal operators, singular integral operators, and commutators with a BMO function in Morrey spaces and dual Morrey spaces. We give a simple proof of their boundedness. Moreover, we establish that their norms are “almost the same” as the norms in Lebesgue spaces with the corresponding power weight.

Application of guiding functions to the study of sweeping processes with delay

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In this paper a system consisting of a differential inclusion and a sweeping process with delay is considered (see [1]). Conditions have been found under which such a system has periodic solutions. For the proof we use multivalued condensing operators with respect to the vector measure of non-compactness (see [2]), the theory of topological degree for such operators, as well as Krasovsky functionals (see [3]), which are analogs of guiding functions in the case of equations with delays.

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Aggrandization of spaces: classical approach and new approach, and applications

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We present a classical approach to the aggrandization of spaces of functions, and we discuss the case of spaces of analytic functions. We then propose new ideas for aggrandization. We base on recent work on the so-called local aggrandization of Lebesgue spaces and extend this approach to the case of arbitrary Banach spaces of functions on metric spaces. We show that grand spaces of holomorphic functions can be equivalently defined in terms of an aggrandization associated only with the boundary.

Nonlocal dynamics of systems with delay and nonlinearity having compact support

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Consider the system of differential equations with delayed feedback

$$\dot{u} = (A_0 + \varepsilon A_1)u + F\big(u(t-T)\big), \tag{1}$$

where $u = (u_1, u_2) \in \mathbb{R}^2$, A_0 and A_1 are 2×2 matrices, $T > 0$ is the delay, and the parameter ε is positive and sufficiently small ($0 < \varepsilon \ll 1$). We fix $C_{[-T,0]}(\mathbb{R}^2)$ as the phase space of system (1)

The main assumption is that the nonlinear delayed feedback F has a compact support. We consider some particularly interesting classes of systems.

First, we investigate system (1) with the function F depending on the scalar product v of some fixed vector h such that $\|h\| = 1$ by the vector u (that is, $v = (h, u)$). In this case, F is a compactly supported function since F is zero if v is outside of a segment $[-p, p]$:

$$F(u) = \begin{cases} f(v) & \text{if } |v| \leqslant p, \\ 0, & \text{if } |v| > p, \end{cases}$$

where $p > 0$ is some fixed constant. We assume that the function $f(v)$ is smooth.

Consider another function having compact support,

$$F(u) = \begin{cases} f(u_1)\Phi(u_2) & \text{if } |u_1| \leqslant p, \\ \quad 0 & \text{if } |u_1| > p. \end{cases}$$

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Such nonlinear vector-functions with delay occur in many fields of science: in various problems of radiophysics, biology, and physiology, when describing some relay systems, etc.

As for the matrix A_0, we assume that it has eigenvalues with the zero real part.

We consider the questions of existence, stability, and asymptotics of periodic solutions to system (1) for sufficiently small values of ε.

We show that there can simultaneously exist local cycles in a neighborhood of the equilibrium state (with an asymptotically small amplitude) and nonlocal cycles with an amplitude of order unity or with an asymptotically large amplitude. The asymptotics of these cycles are constructed.

It is shown that the original system can have rather different dynamical properties for different definitions of compactly supported nonlinear functions.

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Local analysis of nonlinear systems with a large delay

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A system of two equations with delay is considered. The purpose of this work is to study the local dynamics of this system under the assumption that the delay parameter is sufficiently large. Critical cases in the problem of stability of an equilibrium state are identified and it is shown that they have infinite dimension. The research is based on the use of special methods of infinite-dimensional normalization. Classical methods based on the application of the theory of invariant integral manifolds and normal forms turn out to be directly inapplicable. As the main results, special nonlinear boundary value problems are constructed, which play the role of normal forms. Their nonlocal dynamics determine the behavior of all solutions of the original system in the vicinity of the equilibrium state.

This work was carried out within the framework of a development programme for the Regional Scientific and Educational Mathematical Center of the Yaroslavl State University with financial support from the Ministry of Science and Higher Education of the Russian Federation (Agreement on provision of subsidy from the federal budget No. 075-02-2025-1636).

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On a method for solving the Riccati equation

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The article explores issues related to constructing particular solutions of the Riccati equation. Developing an algorithm for finding particular solution to the Riccati equation is one of the pressing problems in the theory of differential equations. In the course of the research, it was established that there exists a class of coefficient-conjugate Riccati equations for which a particular solution can be obtained by solving a linear nonhomogeneous differential equation or a Bernoulli differential equation. New properties of the Riccati equation were also identified. A procedure for transforming a Riccati equation of the general type into a coefficient-conjugate Riccati equation is presented, and an algorithm for constructing a particular solution to an arbitrary Riccati equation is developed.

Since the Riccati equation is closely related to second-order linear differential equations, linear systems of differential equations, and linear integro-differential equations with variable coefficients, the article presents algorithms for finding their particular solutions through the solution of the Riccati equation. Theoretical conclusions are supported by examples.

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Evolution equations and stochastic processes in models of anomalous diffusion

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Experimentally well-established, anomalous diffusion (AD) is a phenomenon observed in many different natural systems belonging to different research fields. In particular, AD has become foundational in living systems after a large use of single-particle tracking techniques in the recent years. Generally speaking, AD labels all those diffusive processes that are governed by laws that differ from that of classical diffusion, namely, all that cases when particles' displacements do not accomodate to

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the Gaussian density function and/or the variance of such displacements does not grow linearly in time.

We establish the physical origin of AD within the classical picture of a test-particle kicked by infinitely many surrounding particles. We consider a stochastic dynamical system where the microscopic thermal bath is the source for the mesoscopic Brownian motion of a bunch of N particles that express the environment of a single test-particle. Physical conservation principles, namely the conservation of momentum and the conservation of energy, are met in the considered particle system in the form of the fluctuation-dissipation theorem for the motion of the surround-particles.

The key feature of the considered particle system is the extra-randomness reflecting the inhomogeniety of the environment and individuality on the test-particle. The inhomogeniety of the environment is introduced via the distribution of the masses of the surround-particles. When the number of mesoscopic Brownian particles N is large enough for providing a crowded environment, then the test-particle displays AD characterised by the distribution of the masses of the surround-particles. More precise, we prove that, in the limit $N \to \infty$, the test-particle diffuses according to a quite general (non-Markovian) Gaussian process $(Z_t)_{t\geqslant 0}$ characterised by the covariance function

$$\mathrm{Cov}(Z_t, Z_s) = C(v(t) + v(s) - v(|t-s|)), \tag{1}$$

where $v(\cdot)$ is determined by the distribution of the masses of the surround-particles. With a particular choice of distribution of surround-particles, we obtain fractional Brownian motion (fBm) with Hurst parameter $H \in (1/2, 1)$ as a special case. In this respect, we remind that the fBm experimentally turned out to be the underlying stochastic motion in many living systems. We present also some distributions of masses of the surround-particles which lead to a mixture of independent fractional Brownian motions with diferent Hurst parameters or to a classical Wiener process as a limiting process $(Z_t)_{t\geqslant 0}$. Moreover, we present some distributions of masses of the surround-particles leading to the limiting processes which perform a transition from ballistic diffusion to superdiffusion, or from ballistic diffusion to classical diffusion.

Next, the individuality of the test-particle is introduced via making its coupling parameter random. This allows to establish the physical basis for the model of the randomly scaled (or, superstatistical) fBm and other related (non-Gaussian) processes. The randomly scaled (or, superstatistical) fBm is widely discussed in the physical and mathematical literature (keywords: “generalized grey Brownian motion,” “gamma-grey Brownian motion,” etc.).

We discuss the Kolmogorov–Fokker–Planck equations for the above stochastic processes. These are integro-differential evolution equations containing operators of fractional calculus (e.g., generalized time-fractional derivatives, as a special case).

This is a joint work with Christian Bender, Mirko D’Ovidio, Gianni Pagnini and Merten Mlinarzik. The research has been fulfilled during my work at Kassel University, Germany.

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On spectral properties of the difference operator with incommensurable shifts

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The talk addresses the problem of eigenfunctions and eigenvalues for a difference operator with incommensurable shifts. The relationship between this problem and the boundary-value problem for a differential-difference operator with incommensurable shifts is considered. An estimate on the location of the spectrum is discussed as well.

The original problem arises in the study of the smoothness of generalized solutions to the problem

$$-(v - \varepsilon Rv)''(t) = f_0(t) \quad (t \in Q), \tag{1}$$

$$v(t) = 0 \quad (t \in [-1,\, 0] \cup [\pi,\, \pi + 1]), \tag{2}$$

where $(Rv)(t) = v(t-1) + v(t+\tau)$, $\tau \in \mathbb{R} \setminus \mathbb{Q}$ is such that $0.9 < \tau < 1$ and $p - q\tau \neq \pi$ for all $p,\, q \in \mathbb{Z}$, $\varepsilon \neq 0$, $Q = (0,\, \pi)$, and $f_0 \in L_2(Q)$.

In order to study the properties of boundary value problem (1), (2), we consider the following eigenvalue problem for the operator R, $u \in L_2(\mathbb{R})$:

$$(Ru)(t) = \lambda u(t) \quad (\text{a.e. on } Q), \tag{3}$$

$$u(t) = 0 \quad (\text{a.e. on } \mathbb{R} \setminus Q). \tag{4}$$

It is proved that all eigenvalues of problem (3), (4) belong to the set

$$\left\{ \lambda \in \mathbb{C} \;\middle|\; |\lambda| \in \{0\} \cup \left(1,\, \frac{1+\sqrt{5}}{2}\right) \right\}.$$

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On blow-up conditions for solutions of systems of second-order quasilinear elliptic inequalities

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We study the system

$$\begin{cases} -\operatorname{div} A_1(x, \nabla u_1) \geqslant F_1(x, u_2) & \text{in } \mathbb{R}^n, \\ -\operatorname{div} A_2(x, \nabla u_2) \geqslant F_2(x, u_1) & \text{in } \mathbb{R}^n, \end{cases} \tag{1}$$

of differential inequalities, where $n \geqslant 2$ and A_i are Caratheodory functions such that

$$C_1|\xi|^{p_i} \leqslant \xi A_i(x, \xi), \quad |A_i(x, \xi)| \leqslant C_2|\xi|^{p_i - 1}, \quad i = 1, 2,$$

with some constants $C_1, C_2 > 0$ and $p_1, p_2 > 1$ for almost all $x \in \mathbb{R}^n$ and for all $\xi \in \mathbb{R}^n$

An ordered pair $(u_1, u_2) \in W^1_{p_1,loc}(\mathbb{R}^n) \times W^1_{p_2,loc}(\mathbb{R}^n)$ is called a solution of (1) if $F_1(x, u_2), F_2(x, u_1) \in L_{1,loc}(\mathbb{R}^n)$ and, moreover,

$$\int_{\mathbb{R}^n} A_1(x, \nabla u_1)\nabla\varphi\, dx \geqslant \int_{\mathbb{R}^n} F_1(x, u_2)\varphi\, dx$$

and

$$\int_{\mathbb{R}^n} A_2(x, \nabla u_2)\nabla\varphi\, dx \geqslant \int_{\mathbb{R}^n} F_2(x, u_1)\varphi\, dx$$

for any non-negative function $\varphi \in C_0^\infty(\mathbb{R}^n)$.

For solutions of system (1) such that

$$\operatorname*{ess\,inf}_{\mathbb{R}^n} u_i = 0, \quad i = 1, 2,$$

we obtain exact blow-up conditions, i.e. conditions guaranteeing that these solutions are equal to zero almost everywhere in $\mathbb{R}^n$. In particular, we strengthen results given in [1]. All formulations and detailed proof can be found in [2].

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On the method of guiding functions in the problem of the existence of bounded solutions for differential inclusions

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At the end of the 20th — beginning of the 21st centuries, in connection with the new opportunity of applications to current problems of mathematics, mechanics, control theory, physics and other sciences, the need arose for a significant expansion of the classes of guiding functions under consideration, first introduced by M. A. Krasnosel'skii and A. I. Perov (see [3]). In particular, for differential equations, a class of guiding functions on a given set (see [4]) and a class of multivalent vector guiding functions (see [7]) were introduced, which were later generalized to the case of differential inclusions (see [1, 2, 5, 6]). In the present talk, along with the classical method of guiding functions, the method of guiding functions on a given set and the method of multivalent vector guiding functions are applied to the problem of the existence of bounded solutions in nonlinear objects governed by differential inclusions, the right-hand side of which has convex compact values, satisfies the upper Carathéodory conditions and the sublinear growth condition.

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Classical solution of the first mixed problem for the semilinear hyperbolic equation in curvilinear half strip

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Consider a domain $Q \subset \mathbb{R}^2$ bounded by the following curves: $x_0 = s(x_1),\ s : \mathbb{R} \supset [x_1^{(0)}; x_1^{(1)}] \ni x_1 \to s(x_1) \in \mathbb{R}$ (lower base), $x_1 = \gamma^{(0)}(x_0),\ \gamma^{(0)} : \mathbb{R} \supset [x_0^{(0)}; +\infty) \ni x_0 \to \gamma^{(0)}(x_0) \in \mathbb{R}$ (left side border), and $x_0 = \gamma^{(1)}(x_1),\ \gamma^{(1)} : \mathbb{R} \supset [x_0^{(1)}; +\infty) \ni x_0 \to \gamma^{(1)}(x_0) \in \mathbb{R}$ (right side border). The boundary satisfies the conditions mentioned in [1].

The semilinear hyperbolic equation

$$\partial_{x_0}^2 w(\mathbf{x}) - a^2(\mathbf{x})\partial_{x_1}^2 w(\mathbf{x}) + L^{(0)}(\partial_{x_0} w(\mathbf{x}), \partial_{x_1} w(\mathbf{x}), w(\mathbf{x}), \mathbf{x}) = 0 \tag{1}$$

with the initial conditions

$$w(s(x_1), x_1) = \varphi(x_1),\ \partial_{x_0} w(s(x_1), x_1) = \psi(x_1), x_1 \in [x_1^{(0)}; x_1^{(1)}] \tag{2}$$

and the boundary conditions

$$w(x_0, \gamma^{(j)}(x_0)) = \mu^{(j)}(x_0), x_0 \in [x_0^{(j)}; +\infty), j \in \{0, 1\} \tag{3}$$

is considered in $\overline{Q}$.

Theorem 1. *Let the boundary satisfy conditions* (2.1) *and* (2.2) *from* [1] *and the operator $L^{(0)}$ satisfy the generalized Lipschitz condition with respect to the first three variables with the Lipschitz function $K(\boldsymbol{x}) \in C^2$. Suppose also that $\varphi \in C^2([x_1^{(0)}; x_1^{(1)}])$, $\psi \in C^1([x_1^{(0)}; x_1^{(1)}])$, and $\mu^{(i)} \in C^2([x_0^{(i)}; +\infty)), i \in \{0, 1\}$. Then the classical solution to problem* (1)–(3) *exists and is unique in $C^2(\overline{Q})$ if and only if the matching conditions on the given functions of problem* (1)–(3) *are met.*

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Classical solutions of hyperbolic equations with discontinuous conditions

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In this report, we consider the following mixed problem:

$$\rho\partial_t^2 u(t,x) = \partial_x \sigma(t,x) + f(t,x), \;\; t \in (0,\infty), \;\; x \in (0,\infty), \tag{1}$$

$$E\partial_t\partial_x u = \partial_t \sigma(t,x) + \tau^{-1}\sigma(t,x), \;\; t \in (0,\infty), \;\; x \in (0,\infty), \tag{2}$$

$$\sigma(0,x) = \sigma_0(x) + v\sqrt{E\rho}\chi_{\{0\}}(x), \;\; x \in [0,\infty), \tag{3}$$

$$u(0,x) = u_0(x), \;\; \partial_t u(0,x) = u_1(x) + v\chi_{\{0\}}(x), \;\; x \in [0,\infty), \tag{4}$$

$$\partial_t^2 u(t,0) - h\sigma(t,0) = \mu(t), \;\; t \in [0,\infty), \tag{5}$$

where χ_A is the indicator function of a set A. We assume that system (1)-(2) is strictly hyperbolic, i.e., the inequality $E\rho > 0$ holds [1]. Problem (1)–(5) models the longitudinal vibrations of a Maxwell-type viscoelastic rod after a massive load hits its free end $x = 0$.

Due to discontinuous initial conditions, problem (1)–(5) has no global classical solution defined on the set $[0,\infty) \times [0,\infty)$. However, it is possible to define a classical solution on a smaller set $([0,\infty) \times [0,\infty)) \setminus \Gamma$ that will satisfy Eqs. (1) and (2) on the set $([0,\infty) \times [0,\infty)) \setminus \Gamma$ in the standard sense and some additional conjugation conditions on the set Γ.

Definition 1. A pair of functions u and σ is a classical solution of problem (1)–(5) if it can be written in the form $u = u^{(1)} + u^{(2)}$ and $\sigma = \sigma^{(1)} + \sigma^{(2)}$, where $(u^{(1)}, \sigma^{(1)})$ is a classical solution of the problem (1)–(5) with $v = 0$ and $(u^{(2)}, \sigma^{(2)})$ satisfies Eqs. (1) and (2) with $f \equiv 0$, the initial conditions

$$\sigma^{(2)}(0,x) = u^{(2)}(0,x) = \partial_t u^{(2)}(0,x) = 0, \;\; x \in [0,\infty),$$

the boundary condition

$$\partial_t^2 u^{(2)}(t,0) - h\sigma^{(2)}(t,0) = 0, \;\; t \in [0,\infty),$$

and the following matching condition:

$$\left[(\sigma^{(2)})^+ - (\sigma^{(2)})^-\right](t, x = at) = \sqrt{E\rho}\,v\gamma(t), \;\; t \in [0,\infty), \tag{6}$$

where $\gamma(0) = 1$ and $a = \sqrt{E/\rho}$.

Remark 1. The function γ should be determined from physical considerations. Following [2], we can show that $\gamma(t) = \exp(-t/(2\tau))$.

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Remark 2. It is easy to see that $[(u)^+ - (u)^-](0,0) = -v$ follows from system (1)–(2) and (6). Therefore, conjugation condition (6) properly handles the discontinuous initial condition $\partial_t u(0,x) = u_1(x) + v\chi_{\{0\}}(x)$.

Theorem 1. *Let the conditions*

$$f \in C^{1,2}([0,\infty) \times [0,\infty)), \;\; \mu \in C^1([0,\infty),$$
$$w_0 \in C^2([0,\infty), \;\; u_0 \in C^2([0,\infty), \;\; u_1 \in C^2([0,\infty)$$

be satisfied. Problem (1)–(5) *has a unique solution in the sense of Definition 1 if the matching conditions*

$$\rho\sigma_0'(0) - h\sigma_0(0) = \mu(0), \quad \mu'(0) = \frac{h\sigma_0(0)}{\tau} - Ehu_1'(0) - \frac{\rho\sigma_0'(0)}{\tau} + E\rho u_1''(0)$$

are satisfied.

The *proof* is carried out by the method of characteristics, see, e.g., [2–4].

The obtained results agree with previously known [5, 6]. This method can also be used to solve problems of oscillations of finite rods and beams.

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Solving the Dirichlet problem for a fractional order equation

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In the domain $\Omega = \{(x,t) \mid 0 < x < +\infty,\ t > 0\}$, find a solution to the problem

$$D_{0t}^{\alpha} u(x,t) - u_{xx}(x,t) = f(x,t),$$

$$D_{0t}^{\alpha-1} u\big|_{t=0} = \varphi(x), \quad D_{0t}^{\alpha-2} u\big|_{t=0} = \psi(x),$$
$$u(0,t) = \nu(t), \quad \lim_{x\to+\infty} u(x,t) = 0,$$

where D_{0t}^{α} is the Riemann–Liouville fractional derivative of order $\alpha \in (1,2)$.

The complete solution is expressed as

$$\begin{aligned} u(x,t) = &\int_0^t \nu(t-\tau) G_0(x,\tau)\, d\tau \\ &+ \int_0^t \int_0^\infty G_1(x,\xi,t-\tau) f(\xi,\tau)\, d\xi d\tau \\ &+ \int_0^\infty G_2(x,\xi,t)\varphi(\xi)\, d\xi + \int_0^\infty G_3(x,\xi,t)\psi(\xi)\, d\xi, \end{aligned}$$

where

$$\begin{aligned} G_0(x,t) &= t^{\alpha/2-1}\phi\left(-\frac{\alpha}{2}, \frac{\alpha}{2}; -x t^{-\alpha/2}\right), \\ G_1(x,\xi,t) &= \frac{1}{2}\int_0^t \tau^{\alpha/2-1}\left[\phi\left(-\frac{\alpha}{2}, \frac{\alpha}{2}; -|\xi-x|\tau^{-\alpha/2}\right) - \phi\left(-\frac{\alpha}{2}, \frac{\alpha}{2}; -(\xi+x)\tau^{-\alpha/2}\right)\right] d\tau, \\ G_2(x,\xi,t) &= \frac{1}{2} t^{\alpha/2-1}\left[\phi\left(-\frac{\alpha}{2}, \frac{\alpha}{2}; -|x-\xi| t^{-\alpha/2}\right) + \phi\left(-\frac{\alpha}{2}, \frac{\alpha}{2}; -(x+\xi) t^{-\alpha/2}\right)\right], \\ G_3(x,\xi,t) &= \frac{1}{2} t^{\alpha/2-2}\left[\phi\left(-\frac{\alpha}{2}, \frac{\alpha}{2}-1; -|x-\xi| t^{-\alpha/2}\right) + \phi\left(-\frac{\alpha}{2}, \frac{\alpha}{2}-1; -(x+\xi) t^{-\alpha/2}\right)\right], \end{aligned}$$

and

$$\phi(\lambda,\mu;z) = \sum_{n=0}^{\infty} \frac{z^n}{n!\Gamma(\lambda n + \mu)}, \quad \lambda > -1, \quad \mu \in C,$$

is the Wright function.

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Approximation in strong norms of solutions of variational problems with bilateral constraints in variable domains

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We consider a sequence of convex integral functionals $F_s\colon W^{1,p}(\Omega_s) \to \mathbb{R}$ and a sequence of weakly lower semicontinuous and generally non-integral functionals $G_s\colon W^{1,p}(\Omega_s) \to \mathbb{R}$, where $\{\Omega_s\}$ is a sequence of domains in $\mathbb{R}^n$ contained in a bounded domain $\Omega \subset \mathbb{R}^n$ ($n \geqslant 2$) and $p > 1$. Along with this, we consider the sequence of sets $V_s = \{v \in W^{1,p}(\Omega_s)\colon \varphi_s \leqslant v \leqslant \psi_s \text{ a.e. in } \Omega_s\}$, where φ_s and ψ_s are functions in $W^{1,p}(\Omega_s)$ such that $\varphi_s \leqslant \psi_s$ a.e. in Ω_s.

The involved domains, functionals, and constraints are subject to a number of conditions.

In particular, it is assumed that the sequence of domains Ω_s exhausts the domain Ω, the sequence of spaces $W^{1,p}(\Omega_s)$ is strongly connected with the space $W^{1,p}(\Omega)$, the sequence $\{F_s\}$ Γ-converges to a functional $F\colon W^{1,p}(\Omega) \to \mathbb{R}$, the sequence $\{G_s\}$ converges in a strong sense to a functional $G\colon W^{1,p}(\Omega) \to \mathbb{R}$, the sequence $\{F_s + G_s\}$ satisfies the uniform convexity condition, and $\operatorname{meas}\{\psi_s - \varphi_s < \alpha\} \to 0$ for a positive measurable function $\alpha\colon \Omega \to \mathbb{R}$.

The main result we present is that the sequence of minimizers u_s of the functionals $F_s + G_s$ on the sets V_s is approximated in $W^{1,p}$-norms by a special Γ-realizing sequence $\{w_s\}$ for the minimizer u of the functional $F + G$ on a set in $W^{1,p}(\Omega)$ defined by bilateral constraints. This Γ-realizing sequence is constructed based on an arbitrary Γ-realizing sequence for u and the functionals F_s, it depends on the given constraints φ_s and ψ_s, and each its element w_s belongs to the corresponding set V_s.

For details concerning the above conditions and the proof of the mentioned result, see [3]. The proof itself is based on the constructions and results given in [1, 2].

The importance of the Γ-realizing sequence $\{w_s\}$ is that using it, we establish the convergence of the minimizers u_s to the minimizer u in L^p-norms and the convergence of the values $F_s(u_s)$ to $F(u)$, i.e., we have that $\{u_s\}$ is also a Γ-realizing sequence for u. Together with the fact that the sequence $\{F_s + G_s\}$ satisfies the uniform convexity condition, this implies that the sequence $\{u_s\}$ is approximated in $W^{1,p}$-norms by any Γ-realizing sequence for the minimizer u.

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Direct and inverse scattering: spectral parameter power series approach

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We study the direct and inverse scattering problems for the Schrödinger equation and the Zakharov–Shabat system. Representations for the Jost solutions are obtained in the form of power series in the transformed spectral parameter. In terms of this parameter, the Jost solutions are convergent power series in the unit disk. The coefficients of the series are computed following a simple recurrent integration procedure. This essentially reduces the solution of the direct scattering problem to the computation of the coefficients and location of zeros of an analytic function inside the unit disk.

The inverse scattering problem is reduced to a system of linear algebraic equations for the power series coefficients, while the potential is recovered from the first coefficient. The overall approach leads to a simple and efficient method for the numerical solution of both direct and inverse scattering problems, which is illustrated by numerical examples.

Two approaches to average a stochastic perturbation of an integrable system

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The two approaches mentioned in the title are: write the perturbed equation in the action-angle variables and then average the obtained fast/slow system or, alternatively, inspired by the Krylov–Bogolyubov work, reduce the problem to averaging a suitable auxiliary system with fast rotating coefficients in the original space. I will explain that the second approach has significant advantages over the first one. The talk is based on a joint work with Huang Guan and Andrey Piatnitski.

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Analyzing hepatitis B dynamics through fractional modeling and uncertainty quantification using a homotopy method

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Mathematical models play a crucial role in understanding the dynamics of viral infections. In this study, we present a fuzzy fractional-order model for the hepatitis B virus (HBV) to address the uncertainty inherent in biological systems. The model incorporates a double parametric-based homotopy approach to analyze the dynamics of infection and the removal of infected cells. We also investigate the existence and uniqueness of the proposed model. Our results demonstrate that the inclusion of a fractional-order derivative, combined with fuzzy logic, leads to a reduction in peak viral load and minimizes cellular damage, particularly when early treatment is initiated. However, complete eradication of the virus may require a longer duration. These findings enhance our understanding of HBV progression through various infection stages, offering valuable insights into disease prediction and improving clinical intervention strategies.

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Approximate controllability of semilinear delay heat equation by Tikhonov regularization

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In this work, approximate controllability of semilinear heat equation with delay is studied using the Tikhonov regularization. Consider the semilinear delay equation of

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heat propagation

$$\begin{cases} \dfrac{\partial z(t,x)}{\partial t} = \Delta z(t,x) + \chi_V u(t,x) + f(t, z(t-\tau,x), u(t,x)), \quad (t,x) \in [0,T] \times \Omega, \\ z(t,x) = 0 \quad \text{on } (0,T) \times \partial\Omega, \\ z(s,x) = \eta(s,x), \quad (s,x) \in [-\tau, 0] \times \Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$, $N \geqslant 1$, is a bounded domain, $z : [0,T] \times \Omega \to \mathbb{R}$ is the heat distribution, $u : [0,T] \times \Omega \to \mathbb{R}$ is the heat source, $V \subset \Omega$, χ_V is the characteristic function of V, $\eta : [-\tau, 0] \times \Omega \to \mathbb{R}$ is a continuous function, $\partial\Omega$ denotes the boundary of Ω and $f : [0,T] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a nonlinear function.

Equation (1) preserves unique mild solution $z \in H^1([-\tau, T]; H^2(\Omega; \mathbb{R}))$ when f is measurable in $[0,T]$ and satisfies Lipschitz condition in $\mathbb{R}^2$ for some constant $L_f > 0$. In this discussion, problem (1) for a partial differential equation is transformed into an equivalent operator equation. The operator equation would be analyzed for its well-posedness and controllability using the Tikhonov reqularization. Previous works [1, 2] have applied the Thikhonov regularization for abstract semilinear delay control systems. In this work, our idea is to construct a sequence of regularized controls that steers the operator equation from a given initial state to an ε-neighborhood of the desired state in finite time. Then, we get a sequence of weak solutions of heat equation (1) corresponding to this sequence of controls. Sufficient conditions have been explored to guarantee the convergence of both the sequences. The main result proves the approximate controllability of heat equation (1) under the assumption that the associated linear equation is approximately controllable.

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An invariant manifold bounded by a periodic solution in differential delay equations with cyclic monotone negative feedback

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It is shown that N-dimensional systems of differential delay equations with cyclic monotone negative feedback possess a two-dimensional invariant manifold, on which phase curves spiral outward towards a bounding periodic orbit. Statements on the attractor location and on parameter thresholds concerning stability and oscillation are included. The results apply to models for gene regulatory systems, e.g. the "repressilator" system.

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Spectral decomposition and skew product for group actions

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In this talk, we present various types of group action versions of the classical spectral decomposition theorem by S. Smale which generalize the previous works. In particular, we use the skew-product map associated with a group action to derive the spectral decomposition of the nonwandering set in a given direction. This talk is based on the joint work with C. Morales and Y. Tang.

Quenching phenomenon in the time-fractional Kawarada problem: analysis and computation

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This is a joint work with PhD student Cao D. D. In this talk, a time-space fractional Kawarada problem is considered, where the time fractional derivative is in the Caputo–Hamard sense, while the spatial derivative is in the Riesz sense. The existence, uniqueness, and quenching behavior of a solution to the considered model are investigated. Then, the finite difference scheme is established for solving the quenching solution to the equation model. The numerical simulations show the effectiveness and feasibility of the theoretical analysis.

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Central discontinuous Galerkin finite element method for the time-fractional convection equation

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This study presents an efficient numerical method for solving the time-fractional convection equation, where the solution may exhibit weak regularity at the initial time. To address this, the Caputo fractional derivative is discretized by the L1 formula on nonuniform time meshes. For the spatial discretization, the central discontinuous Galerkin (CDG) method on staggered grids is employed to handle the spatial derivative. Fully discrete schemes are constructed for one- and two-dimensional time-fractional convection equations. Numerical stability and optimal error estimates of the proposed schemes are rigorously proved. Finally, numerical experiments are conducted to verify the theoretical results.

Investigation of a system of hyperbolic equations with integral and two-point boundary conditions

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We consider the following nonlocal boundary value problem with two-point and integral boundary conditions for a system of hyperbolic equations in the domain $Q = [0, T] \times [0, l]$:

$$z_{tx} = f\left(t, x, z\left(t, x\right)\right), \tag{1}$$

$$A_1 z\left(t, 0\right) + A_2 z\left(t, l\right) = \varphi\left(t\right), \;\; t \in [0, T], \tag{2}$$

$$\int_0^T n\left(t\right) z\left(t, x\right) = \psi\left(x\right), \;\; x \in [0, l], \tag{3}$$

where $z\left(t, x\right) = z_1\left(t, x\right), z_2\left(t, x\right), \ldots, z_n\left(t, x\right)$ is the unknown n-dimensional vector-function, $f : Q \times \mathbb{R}^n \to \mathbb{R}^n$, $\varphi\left(t\right)$ and $\psi\left(x\right)$ are differentiable n-dimensional vector functions defined on the segments $[0, T]$ and $[0, l]$, and A_1, $A_2 \in \mathbb{R}^{n \times n}$ are given matrices with $A = A_1 + A_2$ and $det A \neq 0$. The function $n\left(t\right)$ is an $n \times n$ matrix-valued function with $det \int_0^T n\left(t\right) dt \neq 0$ and $A_i n\left(t\right) = n\left(t\right) A_i, \; i = 1, 2$. The fulfillment of the compatibility condition $\int_0^T n\left(t\right) \varphi\left(t\right) dt = A_1 \psi\left(0\right) + A_2 \psi\left(l\right)$ is necessary for the solvability of problem (1)–(3).

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Theorem. *Problem* (1)–(3) *is equivalent to the integral equation*

$$z(t,x) = A^{-1}\varphi(t) + n^{-1}(T)\psi(t) - A^{-1}A_1 n^{-1}(T)\psi(0) -$$
$$-A^{-1}A_2 n^{-1}(T)\psi(l) + \int\limits_0^T \int\limits_0^l G(t,x,\tau,s) f(\tau,s,z)\, d\tau\, ds,$$

where

$$G(t,x,\tau,s) = A^{-1}n^{-1}(T) \begin{cases} A_1 \int\limits_0^t n(\tau)\, d\tau, \ \ 0 \leqslant \tau \leqslant t, \ 0 \leqslant s \leqslant x, \\ -A_2 \int\limits_0^t n(\tau)\, d\tau, \ \ 0 \leqslant \tau \leqslant t, \ x < s \leqslant l, \\ -A_1 \int\limits_t^T n(\tau)\, d\tau, \ \ t < \tau \leqslant T, \ 0 \leqslant s \leqslant x, \\ A_2 \int\limits_t^T n(\tau)\, d\tau, \ \ t < \tau \leqslant T, \ x < s \leqslant l. \end{cases}$$

By applying the Banach contraction mapping principle, sufficient conditions for the existence and uniqueness of a solution to the problem have been established. Similar problems have been considered in [1, 2].

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Commutative Poisson algebras from deformations of noncommutative algebras and non-Abelian Hamiltonian systems

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By a well-known procedure — usually referred to as taking the classical limit — quantum systems reduce to classical systems equipped with a Hamiltonian structure, either symplectic or Poisson. According to deformation quantisation theory, a formal deformation of a commutative algebra $\mathcal{A}$ yields a Poisson bracket on $\mathcal{A}$, and the classical limit of a derivation on the deformation becomes a Hamiltonian derivation on $\mathcal{A}$, defined by this bracket.

In this talk, I will present a generalisation of this framework to formal deformations of arbitrary noncommutative associative algebras $\mathcal{A}$ [1]. I will show that such a deformation gives rise to a commutative Poisson algebra structure on

$$\Pi(\mathcal{A}) := Z(\mathcal{A}) \times (\mathcal{A}/Z(\mathcal{A}))$$

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where $Z(\mathcal{A})$ denotes the centre of $\mathcal{A}$, and equips $\mathcal{A}$ with the structure of a $\Pi(\mathcal{A})$-Poisson module. In this setting, the limiting Heisenberg derivations become Hamiltonian derivations of $\mathcal{A}$, but with Hamiltonians taking values in $\Pi(\mathcal{A})$ rather than in $\mathcal{A}$ itself. This framework enables the definition of non-Abelian Hamiltonian systems within the setting of commutative Poisson algebras.

I will illustrate the construction using the simplest example: a non-Abelian Hamiltonian system defined on the quantum plane $\mathbb{C}[q]\langle x, y\rangle / \langle yx - qxy\rangle$ at $q = -1$. The method has been successfully applied to several deformations of noncommutative algebras, including quantum algebras associated with the Kontsevich integrable map, the $M_q(2)$ algebra, the quantised Grassmann algebra, and quantisations of the Volterra hierarchy [1–4].

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On the sell growth equation

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The talk is based on article [1].

A process of growth and division of cells is modeled by the following first-order linear functional partial differential equation, the so-called sell growth equation:

$$\frac{\partial n(x,t)}{\partial t} + g\frac{\partial n(x,t)}{\partial x} = b\alpha^2 n(\alpha x, t) - (b+\mu)n(x,t),$$

where $n(x,t)$ denotes the density distribution of cells structured by size x at time t (i.e., $\int_{x_1}^{x_2} n(x,t)dx$ is the number of cells that at time t have a size between x_1 and x_2), $g > 0$ is the rate of growth, $\mu > 0$ is the rate of death, and $b > 0$ is the rate at which cells divide into $\alpha > 1$ equally sized daughter cells.

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The above equation is supplemented by a given initial distribution,

$$n(x,0) = n_0(x),$$

where n_0 is a probability distribution function, and the boundary condition

$$n(0,t) = 0.$$

The analytical solution to this problem was given by A. A. Zaidi, B. Van Brunt, and G. C. Wake in paper [2]. In the talk we discuss how to simplify the arguments given in the above mentioned paper by using the theory of operator semigroups. This theory enables us to prove the existence and uniqueness of the solution in various spaces and to express this solution in terms of Dyson-Phillips series. The asymptotics of the solution is also discussed from the viewpoint of the theory of operator semigroups.

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Investigation of infection progression with resident macrophages

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The spread of a viral infection through a cell or tissue culture can be described by a system of reaction-diffusion equations that can have a traveling wave solution. The main characteristics of such systems, which have biological significance, are the virus replication number, wave speed, and viral load.

The virus replication number is an analog of the basic reproduction number from SIR-type epidemiological models. It indicates whether the infection will develop. It comes from the condition of the loss of stability for the “healthy” stationary point.

In the spatially heterogeneous system, if the system allows a wave solution, then the wave propagation speed can be estimated. This rate correlates with the severity of the infectious disease. Also in this case, the integral of the concentration of virus particles along the spatial axis is convergent, and its value can be found from the corresponding algebraic equation. This integral is called the viral load and characterizes the infectivity of the disease for respiratory viral infections.

When the virus enters the body, the mechanisms of the immune response are activated. An important component of the immune response is inflammation. Recently, the role of resident macrophages, which are involved in inflammation, has been actively studied in the biological literature.

In this paper, we formulate a model based on reaction-diffusion equations to describe the process of viral infection spreading in tissues. The model takes into account

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inflammation in the form of pro-inflammatory cytokines and resident macrophages. In this model, the virus replication number was found, and a system of algebraic equations for determining the wave speed and the total viral load was obtained. Also, the number which characterizes the strength of the immune response was obtained. It is called the immunity effectiveness number. It has been shown that in the case of inflammation, the development of infection in a spatially heterogeneous case is determined not only by the viral replication number, but also by the immunity effectiveness number.

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Initial-value problems for hyperbolic equations with nonlocal potentials: series expansions of solutions

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Initial-value problems for equations of the kind

$$\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{\partial^2 u}{\partial x^2}(x,t) + au(x+h,t),$$

where a and h are real parameters, are considered.

Representations of classical solutions by function series are constructed.

The said function series consist of iterated means of translated solutions of the same initial-value problem for the wave equation.

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Inverse problem for evolution equation with regular integro-differential operator

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Let $\mathcal{Z}, \mathcal{U}$ be Banach spaces, $\mathcal{L}(\mathcal{Z})$ denote the Banach algebra of all bounded linear operators in $\mathcal{Z}$, $\rho(A)$ be the resolvent set of an operator A, and $\sigma(A) = \mathbb{C} \setminus \rho(A)$ its spectrum, where $\mathbb{R}_+ = \{a \in \mathbb{R} : a > 0\}$, $\overline{\mathbb{R}}_+ = \mathbb{R}_+ \cup \{0\}$, $K \in C(\overline{\mathbb{R}}_+; \mathcal{L}(\mathcal{Z}))$. Define the convolution operator

$$(J^K z)(t) := \int_0^t K(t-s)z(s)ds := (K * z)(t), \quad t > 0,$$

and the integro-differential operator of the Riemann–Liouville type

$$(D^{1,K} z)(t) := D^1 (J^K z)(t) := D^1 \int_0^t K(t-s)z(s)ds, \quad t > 0.$$

Denote by $\widehat{h}$ the Laplace transform of a function $h : \mathbb{R}_+ \to \mathcal{Z}$.

Suppose that $K \in C^1(\overline{\mathbb{R}}_+; \mathcal{L}(\mathcal{Z}))$, then there exists the Laplace transform $\widehat{K}(\lambda)$ of K, which is a single-valued analytic function in $\Omega_{r_0} := \{\lambda \in \mathbb{C} : |\lambda| > r_0\} \cup \infty$. Consider the inverse problem

$$(D^{1,K} z)(t) = Az(t) + B(t)u + g(t), \quad t \in (0, T], \tag{1}$$

where $K \in C^1([0,T]; \mathcal{L}(\mathcal{Z}))$, $A \in \mathcal{L}(\mathcal{Z})$, $B \in C([0,T]; \mathcal{L}(\mathcal{U}; \mathcal{Z}))$, $g \in C([0,T]; \mathcal{Z})$, with the initial condition

$$(J^K z)(0) = 0 \tag{2}$$

and with the overdetermination condition

$$\int_0^T z(t)d\nu(t) = z_T \in \mathcal{Z}, \tag{3}$$

where the function $\nu : (0,T] \to \mathbb{R}$ has a bounded variation, briefly, $\nu \in BV((0,T]; \mathbb{C})$. In this case, the additional unknown element $u \in \mathcal{U}$ in equation (1) must be found using additional condition (3).

An element $u \in \mathcal{U}$ is called a solution to problem (1)–(3) if the corresponding solution to Cauchy-type problem (1), (2) satisfies condition (3). Problem (1)–(3) is called well-posed if for all $z_T \in \mathcal{Z}$, $g \in C([0,T]; \mathcal{Z})$ there exists a unique solution $u \in \mathcal{U}$ of the problem satisfying at the same the inequality

$$\|u\|_{\mathcal{U}} \leqslant C \left(\|z_T\|_{\mathcal{Z}} + \|g\|_{C([0,T];\mathcal{Z})} \right), \tag{4}$$

where $C > 0$ does not depend on z_T, g.

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An element u is a solution of problem (1)–(3) if and only if it satisfies the equation

$$\chi u = \psi, \tag{5}$$

where χ and ψ are defined by the formulas

$$\chi := \int\limits_0^T (K(0)-A)^{-1}B(t)d\nu(t) + \int\limits_0^T\int\limits_0^t Z(t-s)B(s)dsd\nu(t) \in \mathcal{L}(\mathcal{U};\mathcal{Z}),$$

$$\psi := z_T - \int\limits_0^T (K(0)-A)^{-1}g(t)d\nu(t) - \int\limits_0^T\int\limits_0^t Z(t-s)g(s)dsd\nu(t) \in \mathcal{Z}.$$

Theorem 1 (see [1]). *Let $A \in \mathcal{L}(\mathcal{Z})$, $K \in C^1(\overline{\mathbb{R}}_+;\mathcal{L}(\mathcal{Z}))$, $(K(0)-A)^{-1} \in \mathcal{L}(\mathcal{Z})$, there exist the Laplace transform $\widehat{K}(\lambda)$ being a single-valued analytic function in Ω_{r_0}, $g \in C([0,T];\mathcal{Z})$, $B \in C([0,T];\mathcal{L}(\mathcal{U};\mathcal{Z}))$, $\nu \in BV((0,T];\mathbb{C})$, and $z_T \in \mathcal{Z}$. Then problem* (1)–(3) *is well-posed if and only if there exists an inverse operator $\chi^{-1} \in \mathcal{L}(\mathcal{Z};\mathcal{U})$. In this case, the solution has the form $u = \chi^{-1}\psi$.*

The work is supported by the grant of Russian Science Foundation and the Government of the Chelyabinsk region, project No. 24-21-20015, `https://rscf.ru/project/24-21-20015/`.

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The method of transferring from the left to the right (on the regularity of solutions to the Venttsel problem)

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We discuss a priori estimates, Fredholm-type theorems and the improving of integrability property for solutions of the Venttsel problem to elliptic and parabolic equations.

The talk is based on the joint papers with D. E. Apushkinskaya, D. K. Palagachev, and L. G. Softova.

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Investigation of a multi-group epidemiological model

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We study a model of the spread of an epidemic in a population, based on delay differential equations. As in the classical SIR model, we are interested in the dynamics of the number of susceptible, infected, and recovered individuals in the population. We consider two classes of susceptible individuals (e.g., age groups or different immunity statuses) denoted as S_1 and S_2 . They become infected I_1 and I_2 (with cross-infection possible) and then recover as R_1 and R_2 respectively. Recovered individuals become susceptible again once they lose immunity.

The mathematical model is as follows:

$$\begin{cases} \frac{dS_1(t)}{dt} = -J_1(t) + J_1(t-\tau_1-\tau_2), \\ \frac{dS_2(t)}{dt} = -J_2(t) + J_2(t-\tau_1-\tau_2), \\ \frac{dI_1(t)}{dt} = J_1(t) - J_1(t-\tau_1), \\ \frac{dI_2(t)}{dt} = J_2(t) - J_2(t-\tau_1), \\ \frac{dR_1(t)}{dt} = J_1(t-\tau_1) - J_1(t-\tau_1-\tau_2), \\ \frac{dR_2(t)}{dt} = J_2(t-\tau_1) - J_2(t-\tau_1-\tau_2), \end{cases}$$

where τ_1 is the disease duration (also called the infectious period), τ_2 is the duration of immunity, J_1 and J_2 are the numbers of the newly infected at time t in each class,

$$J_1(t) = \frac{S_1(t)}{N}(\beta_{11}I_1(t) + \beta_{12}I_2(t)), \quad J_2(t) = \frac{S_2(t)}{N}(\beta_{21}I_1(t) + \beta_{22}I_2(t)),$$

where $N = N_1 + N_2$, $N_1 = S_1(t) + I_1(t) + R_1(t)$, and $N_2 = S_2(t) + I_2(t) + R_2(t)$ is the total population of the first and the second class respectively.

We have shown a connection between the two-class model and the single-class model (see [1–3]). By reducing the equations to the integral form, we found non-negative stationary solutions and studied their stability. Numerical modeling was performed and a comparison with theoretical results was made.

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2D rotating Bose–Einstein condensation at the critical rotation speed

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We study the minimizers of a magnetic 2D non-linear Schrödinger energy functional in a harmonic trapping potential, describing a rotating Bose–Einstein condensate. In the case of a repulsive interaction potential, we derive an effective Thomas–Fermi-like model in the rapidly rotating limit where the centrifugal force compensates the confinement. The available states are restricted to the lowest Landau level. The coupling constant of the Thomas–Fermi functional is to link the emergence of vortex lattices (the Abrikosov problem). When turning from repulsive to attractive interactions, the system is unstable since there is a balance between kinetic and interaction energies. In the regime where the strength of the interaction approaches a critical value from below, the system collapses to a profile obtained from the (unique) optimizer of a Gagliardo–Nirenberg interpolation inequality. This was established before in the case of fixed rotation frequency. We extend the result to rotation frequencies approaching, or even equal to, the critical frequency at which the centrifugal force compensates the trap. We prove that the blow-up scenario is to leading order unaffected by such a strong deconfinement mechanism. In particular, the blow-up profile remains independent of the rotation frequency.

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Analysis of the parameters of a mathematical model of plasma transport in a helical magnetic field

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The paper presents a mathematical model of plasma confinement in a helical magnetic field based on mathematical modeling [1]. Plasma confinement is carried out by

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transferring a pulse from a magnetic field with helical symmetry to a rotating plasma. Plasma transport in a spiral magnetic field is described by a stationary equation of the second order in an axially symmetric formulation [2]. Setting the parameters of the installation allows us to calculate the diffusion coefficient, evaluate the effect of plasma confinement, and select optimal parameters (the depth of magnetic field corrugation, plasma potential, etc.). The distribution of the concentration of the substance obtained using numerical modeling confirmed the confinement effect obtained in the experiment. The dependences of the integral characteristics of the substance on are obtained. The model was calibrated using new experimental data obtained at the SMOLA facility created at the Budker Institute of Nuclear Physics SB RAS [3].

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Asymptotic solution of a Schrödinger-type equation with both fast and slow time dependencies of the Hamiltonian

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We study asymptotic solutions of the equation

$$\mathcal{K}(\varepsilon t, \frac{t}{\varepsilon})\mathbf{\Psi} = -i\Gamma\frac{\partial \mathbf{\Psi}}{\partial t}, \quad \varepsilon \ll 1, \tag{1}$$

in a Hilbert space. The operators $\mathcal{K}(\xi, \tau)$ for real ξ and τ, as well as the operator Γ, are self-adjoint. The operators Γ and Γ^{-1} are bounded and

$$\mathcal{K}(\xi, \tau+1) = \mathcal{K}(\xi, \tau), \quad \tau = t/\varepsilon, \quad \xi = \varepsilon t.$$

Equation (1) can be regarded as the Schrödinger equation with the non-selfadjoint Hamiltonian $\Gamma^{-1}\mathcal{K}$. Asymptotic solutions of equation (1) without dependence on τ were constructed in [2]. The stationary Dirac equations, as well as the equations describing the propagation of monochromatic waves in irregular waveguides, encounted in mechanics and electrodynamics, can be written in form (1), where the variable t is the distance along the waveguide axis; see, e.g., [2] for waves in an irregular elastic layer.

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We construct the full asymptotic expansion of the solution of (1) in the form

$$\Psi = e^{\frac{i}{\varepsilon}\int\limits_{\xi_*}^{\xi}\beta(\xi')d\xi'}\Big(\Phi^{(0)}(\xi,\tau)+\varepsilon\Phi^{(1)}(\xi,\tau)+\varepsilon^2\Phi^{(2)}(\xi,\tau)+\dots\Big),$$
$$\Phi^{(j)}(\xi,\tau+1)=\Phi^{(j)}(\xi,\tau),\quad j=1,2,\dots, \tag{2}$$

found through the eigenvalues and eigenvectors of the homogenized problem

$$\langle\mathcal{K}\rangle(\xi)\varphi(\xi)=\beta(\xi)\varphi(\xi),\quad \langle\mathcal{K}\rangle(\xi)=\int\limits_0^1\mathcal{K}(\xi,\tau)\,d\tau. \tag{3}$$

Let problem (3) have an eigenvalue separated from the rest of the spectrum by a gap independent of $\hbar$. Corrections are found for the Berry phase and for all approximations in (2) caused by fast oscillations. Estimates for all the terms are given under the proper assumption about the smoothness of the operator family $\mathcal{K}$ for particular cases.

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Bernoulli problem. Hessians and gradient flows

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The Bernoulli problem is formulated as follows. Let $\Omega\subset\mathbb{R}^2$ be a fixed bounded simply connected domain with Jordan boundary $\partial\Omega\in C^\infty$. Within this domain, we study smooth Jordan curves $\Gamma\Subset\Omega$ admitting a parametrization $\Gamma: x=f(\theta)$, $\theta\in\mathbb{S}^1$. Each such curve Γ divides Ω into two parts: a simply connected subdomain $\Omega^+\Subset\Omega$ and the complementary curvilinear annulus $\Omega^-=\Omega\setminus\overline{\Omega^+}$. The curves Γ are unknown and must be identified as part of the solution to the problem. Let $h:\partial\Omega\to\mathbb{R}$ and $Q:\Omega\to\mathbb{R}$ be given positive smooth functions. The problem is to find a curve Γ and a potential $u:\Omega^-\to\mathbb{R}$ such that

$$\Delta u=0\quad\text{in }\Omega^-,\qquad u=h\quad\text{on }\partial\Omega,\qquad u=0,\ |\nabla u|^2=Q^2\quad\text{on }\Gamma.$$

The Bernoulli problem is a prototypical free boundary problem with broad applications. It arises naturally in fluid mechanics, optimal thermal and electrical insulation design, electromechanical processes and galvanization, plasma equilibrium configurations, and tumor growth modeling. One of the most widely used mathematical

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approaches reformulates the Bernoulli problem as a variational problem for an integral functional defined over a variable domain. In our geometrically simpler setting, the Bernoulli functional takes the form

$$\mathcal{J}_b(f) = \int_{\Omega^-} \left(|\nabla u|^2 + Q^2\right)\, dx,$$

Most shape optimization problems when stated without geometric constraints are ill-posed. A standard and effective remedy is to penalize the objective functional by adding appropriate regularization terms. These terms are interpreted as geometric energies of the curve Γ. The most common are the capillary energy $\mathcal{E}_p$ (lengths of Γ) and the elastic energy $\mathcal{E}_e$ ($1D$-Willmore functional).

Another geometric energy is the Möbius–O'Hara energy $\mathcal{E}_m$ arising in knot theory. If all three energies are finite, then Γ is $C^{1+\alpha}$-curve without double points. Therefore, the most suitable choice for regularizing the Bernoulli functional is the combined functional

$$\mathbb{J} = \varepsilon_e\mathcal{E}_e + \varepsilon_m\mathcal{E}_m + \varepsilon_p\mathcal{E}_p + \mathcal{J}_b, \quad \varepsilon_\alpha > 0, \quad \alpha = e, m, p.$$

One of the key issues in the theory is the development of efficient and robust algorithms for numerical implementation. The most classical approach is the steepest descent method, which can be interpreted as a time discretization of the gradient flow problem

$$\partial_t f + d\mathbb{J}[f] = 0 \text{ in } \mathbb{S}^1 \times [0, \infty), \qquad f\big|_{t=0} = f_0, \tag{1}$$

where $d\mathbb{J}$ is the Hadamard gradient of $\mathbb{J}$. Let us introduce the space $\mathfrak{M}$ of all Jordan curves $\Gamma = f(\mathbb{S}^1) \subset \Omega$ with $f \in W^{2,2}(\mathbb{S}^1)$. The space $\mathfrak{M}$ can be endowed with a Riemannian metric. The boundary of $\mathfrak{M}$ includes singular curves (with cusps, self-intersections, or contact with $\partial\Omega$). The key observation is that $\mathbb{J}(f) \to \infty$ as $\Gamma = f(\mathbb{S}^1) \to \partial\mathfrak{M}$. We prove that for any smooth immersion f_0 such that $\Gamma_0 = f_0(\mathbb{S}^1) \in \mathfrak{M}$, problem (1) admits a unique smooth solution such that $\Gamma_t = f(t, \mathbb{S}^1) \to \Gamma_\infty \in \mathfrak{M}$ in the uniform metric as $t \to \infty$.

Jumps in Besov spaces and fine properties of Besov and fractional Sobolev functions

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We analyze functions in the Besov spaces $B^{1/q}_{q,\infty}(\mathbb{R}^N, \mathbb{R}^d), q \in (1, \infty)$, and functions in the fractional Sobolev spaces $W^{r,q}(\mathbb{R}^N, \mathbb{R}^d), r \in (0, 1), q \in [1, \infty)$. We prove for functions $u \in B^{1/q}_{q,\infty}(\mathbb{R}^N, \mathbb{R}^d)$ the summability of the difference between one-sided approximate limits in power q, $|u^+ - u^-|^q$, along the jump set $\mathcal{J}_u$ of u with respect to the Hausdorff measure $\mathcal{H}^{N-1}$, and establish the best bound from above for the integral $\int_{\mathcal{J}_u} |u^+ - u^-|^q d\mathcal{H}^{N-1}$ in terms of Besov constants. We show for functions $u \in B^{1/q}_{q,\infty}(\mathbb{R}^N, \mathbb{R}^d), q \in (1, \infty)$, that

$$\liminf_{\varepsilon \to 0^+} \frac{1}{\varepsilon^N} \int_{B_\varepsilon(x)} |u(z) - u_{B_\varepsilon(x)}|^q dz = 0 \tag{1}$$

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for every x outside of an $\mathcal{H}^{N-1}$-sigma finite set. For functions $u \in W^{r,q}(\mathbb{R}^N, \mathbb{R}^d)$, we prove that

$$\lim_{\varepsilon \to 0^+} \frac{1}{\varepsilon^N} \int_{B_\varepsilon(x)} |u(z) - u_{B_\varepsilon(x)}|^q dz = 0 \tag{2}$$

for $\mathcal{H}^{N-rq}$ a.a. x, where $q \in [1, \infty)$, $r \in (0, 1]$, and $rq \leqslant N$.

In addition, we prove Lusin-type approximation for fractional Sobolev functions $u \in W^{r,q}(\mathbb{R}^N, \mathbb{R}^d)$ by Hölder continuous functions in $C^{0,r}(\mathbb{R}^N, \mathbb{R}^d)$.

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On a coefficient inverse problem for a quasilinear parabolic equation

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In $G_{[0,T]} = \{(t, x, z) |\, 0 \leqslant t \leqslant T, x \in E_1, z \in E_1\}$, consider the Cauchy problem

$$u_t = \alpha_1(t, x) u_{xx} + \alpha_2(t, x) u u_{zz} +$$

$$+\alpha_3(t, x)(u_z)^2 + \alpha_4(t, x) u^2 + \alpha_5(t, x, z) f(t, x, z), \tag{1}$$

$$u(0, x, z) = u_0(x, z),\ x \in E_1, z \in E_1. \tag{2}$$

The functions $f(t, x, z)$ and $u_0(x, z)$ are defined in $G_{[0,T]}$ and E_2 respectively, the coefficients $\alpha_i(t, x)$, $i = \overline{1, 3}$, are continuously differentiable real-valued functions of t, x, $0 \leqslant t \leqslant\ T$, $T > 0$, $T = const$, $\alpha_1(t, x) \geqslant a_1 > 0$, $a_1 = const$.

The overdetermination conditions

$$u(t, x, b_1(t)) = \varphi_1(t, x),\ u(t, x, b_2(t)) = \varphi_2(t, x),\ u(t, d(t), z) = \psi(t, z) \tag{3}$$

are imposed with $b_1(t) \neq b_2(t)$, $b_j(t) \in C^1[0, T]$, $j = 1, 2$. The functions $\varphi_1(t, x)$, $\varphi_2(t, x)$, and $\psi(t, z)$ are assumed to satisfy the matching conditions

$$\varphi_1(0, x) = u_0(x, b_1(0)), \quad \varphi_2(0, x) = u_0(x, b_2(0)), \quad \psi(0, z) = u_0(d(0), z),$$

$$\varphi_1(t, d(t)) = \psi(t, b_1(t)), \varphi_2(t, d(t)) = \psi(t, b_2(t)).$$

The solution to inverse problem (1)–(3) in $G_{[0,t^*]}, 0 < t^* \leqslant T$, is a triple of functions $\alpha_4(t, x)$, $\alpha_5(t, x, z) = \beta_1(t, x) + \beta_2(t, z)$, and $u(t, x, z)$, satisfying equations (1)–(3).

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The existence and uniqueness of the classical solution to inverse problem (1)–(3) in the class of smooth bounded functions is proved in this paper. The proof of the theorem is by passing from inverse problem (1)–(3) to the direct auxiliary Cauchy problem for the loaded equation. The solvability of the direct problem is proved by the weak approximation method [1, 2].

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Differential and functional differential equations involved in Morrey type classes

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Is showed a problem studied in cooperation with Professor Atsushi Tachikawa. We treat the regularity problem for minimizers $u(x)$ of quadratic and nonquadratic growth functional having integrand $A(x, u, Du)$. We point out that concerning the dependence on the variable x is assumed only that $A(x, u, p)$ is in the class of Vanishing Mean Oscillation class, as a function of x. Namely, is not assumed the continuity of $A(x, u, p)$ with respect to x. Are treated partial regularity and global regularity of the minimizer u.

Generalized nonlinear Schrödinger equations describing femtosecond pulse propagation, and their conservation laws

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Current progress in laser physics makes it possible to generate an optical pulse containing a few cycles. Such ultrashort pulses are used in nonlinear optics, medical diagnostics and imaging, for controlling chemical reaction and other applications. In the framework of slowly evolving wave approximation (SEWA), the femtosecond pulse propagation in a medium with cubic nonlinear response is governed by the dimensionless equation [1] for the slowly varying envelope of the electric field strength

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$A(z,x,t)$ with taking into account the pulse self-steepening and mixed derivatives:

$$\mathcal{D}_t \frac{\partial A}{\partial z} + iD_2 \mathcal{D}_t \frac{\partial^2 A}{\partial t^2} + iD \frac{\partial^2 A}{\partial x^2} + i\alpha \mathcal{D}_t^2(|A|^2 A) = 0, \quad \mathcal{D}_t = 1 + i\gamma \frac{\partial}{\partial t}, \quad (1)$$

where $(z,x,t) \in \Omega = (0,L_z] \times (0,L_x) \times (0,L_t)$, $D > 0$, $\gamma > 0$, $D_2, \alpha \in \mathbb{R}$, $D_2 \neq 0$. Equation (1) is called the generalized nonlinear Schrödinger equation (GNLSE).

We propose a novel transform of Eq. (1) to a form containing neither the derivative of the term describing the nonlinear response nor mixed derivatives. In new variables the process is described by three equations containing linear differential operators only. Under corresponding boundary conditions, we derive the conservation laws [2] for (1): energy, Hamiltonian, spectral invariant, and some other invariants.

Invariants under discussion are very useful in mathematical modeling of very short optical pulse propagation both in theoretical analysis of laser pulse propagation and for developing the conservative finite-difference schemes for computer simulation of the problem. Results of computer simulations are presented.

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A posteriori error identities for evolutionary problems with monotone spatial operators

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We discuss quantitative relations (error identities) that characterise distances between exact solutions of initial boundary value problems with monotone spatial operators and functions considered as approximations. Restrictions imposed on such functions are minimal and actually come down to the condition that they must belong to the same functional class as the generalized solution of the problem under consideration. Error identities hold for special measures of deviations from exact solutions whose structure is dictated by the spatial differential operator. They reflect the most general relations between deviations and those values that can be observed in a numerical experiment. The identities serve as basic tools for different purposes such as obtaining fully computable and guaranteed a posteriori estimates, comparison of solutions generated by different data, and for analysis of modeling errors. For elliptic boundary value problems these questions are studied in [1]. Examples related to parabolic and parabolo-hyperbolic evolutionary problems are presented in [2,3].

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The homogenized dynamical model of thermoelastic composite reinforced by thin inclusions

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Composite materials are widely used in civil engineering, mechanical engineering, medicine, and other industries. In this regard, the construction of new mathematical models that most accurately describe the behavior of structures build of such materials is crucial. To find effective models suitable for use in engineering practice, it is necessary to take into account the heterogeneity of the medium and physical effects occurring at the micro- and mesoscale levels. In particular, this is relevant for the description of media reinforced with thin fibers. Thus, in this paper, we consider a model of fibrous composites, which are elastic bodies reinforced by filaments. We assume that the threads behave like rod-type inclusions.

Namely, our study is devoted to the problem of description of dynamical behavior of thermoelastic composite body containing thin deformable rod-type inclusions. We use the two-dimensional model of dynamics of a linear thermoelastic body reinforced by thin thermoelastic filaments. The filaments with zero thickness are rectilinear and parallel to each other. The distance between two neighboring filaments is proportional to a small positive dimensionless parameter ε. This model is formulated in a weak sense (Problem A_ε).

We state well-posedness of Problem A_ε. Then we justify the passage to the limit as $\varepsilon \to 0+$ for Problem A_ε by the homogenization method. In our consideration, we use the two-scale convergence approach (see, e.g., [1, 2]).

This study is a continuation of our study started for a simplified model problem and then extended to the stationary thermoelastic vibration problem in [3] and the quasi-static model of thermoelasticity in [4]. In the present investigation, the method of two-scale convergence on thin manifolds is applied for the first time to the thermoelastic problem describing fully dynamical behaviour of the composite body reinforced by thin filaments.

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Delay epidemic models determined by latency, infection, and immunity duration

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Novel single and two-strain epidemic models are introduced, formulated through systems of delay differential equations (DDEs), in which the temporal dynamics of newly exposed individuals are explicitly incorporated. Transitions between epidemiological compartments — exposed, infectious, recovered, and susceptible — are governed by time delays corresponding to the durations of each stage. The existence and positivity of solutions for these models are rigorously established. By reducing the DDEs to integral equations, stationary solutions and their stability are analyzed.

For the two-strain model, it is revealed through competition between strains that the variant with a higher strain-specific basic reproduction number ($\mathcal{R}_0$) typically outcompetes the other. However, when $\mathcal{R}_0$ surpasses critical thresholds, stability of stationary solutions is lost, giving rise to periodic oscillations. Under these conditions, coexistence of both strains is observed, and their dynamics are dependent not only on $\mathcal{R}_0$ values but also on additional parameters such as delay durations and interaction rates.

Finally, the theoretical framework is validated through comparison of model predictions with empirical data from seasonal influenza, demonstrating its applicability to real-world epidemiological patterns. Understanding of strain competition and oscillatory behavior in infectious disease dynamics is advanced by this work.

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On isospectral potentials on periodic discrete graphs

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We consider discrete Schrödinger operators $H = \Delta + Q$ with periodic (possibly complex-valued) potentials Q on periodic graphs $\mathcal{G}$, where Δ is the discrete Laplacian. The spectrum of the operator H is the union of spectra of *Floquet* operators (matrices) $H(k)$ acting on the finite quotient graph $\mathcal{G}_*$ and depending on the parameter $k \in \mathbb{T}^d := \mathbb{R}^d/(2\pi\mathbb{Z})^d$ called *quasimomentum*, where d is the dimension of the periodic graph (i.e., the rank of its period lattice). The family of spectra $\sigma(H(k))$ of $H(k)$, $k \in \mathbb{T}^d$, is called the *Floquet spectrum* of the Schrödinger operator H. The spectrum of $H(0)$ is the *periodic spectrum* of H. Two potentials Q_1 and Q_2 are *Floquet isospectral* (respectively, just *isospectral*), if $\Delta + Q_1$ and $\Delta + Q_2$ have the same Floquet (respectively, periodic) spectrum.

We are interested in the following problem. To what extent is the potential Q determined by the periodic spectrum or by the Floquet spectrum of the Schrödinger operator H on an arbitrary (but fixed) periodic graph? The following partial answers are obtained.

- For a given potential Q there are generically $n!$ complex-valued potentials isospectral to Q, where n is the number of the vertices of the quotient graph. Moreover, for sufficiently large real potentials Q with pairwise distinct values at the vertices of the quotient graph, there are $n!$ *real* potentials isospectral to Q.

- If a real potential Q is isospectral to the zero potential, then $Q = 0$. If a real potential Q is isospectral to the "*degree*" potential æ (where æ(v) is the degree of the vertex v) and the quotient graph has no loops, then $Q =$ æ.

- We provide examples of periodic graphs for which the Floquet spectrum of the Schrödinger operator $H = \Delta + Q$ generically determines the potential Q uniquely up to symmetries in the quotient graph.

The proof of the obtained results is based on the spectral invariants of the Schrödinger operator on periodic graphs from [1]. Our results extend the results of Kappeler obtained for the lattice $\mathbb{Z}^d$ [2,3] to the case of arbitrary periodic graphs.

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Continuation of solutions of differential equations through the formation of singularities

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Continuations of solutions of evolutionary ODEs and PDEs, allowing the formation of singularities in a finite time, are considered. A unified scheme for defining of such continuations is proposed. This scheme based on an extension of the phase space and continuation of trajectories into the extended space. The Koopman representation of the extended phase flow is studied. The dynamics should be defined for observables that depend on the variables of the phase space of the original system only. The additional parameters of the extended phase space take a role of probabilistic parameters. Thus, the restriction to the subalgebra of observables of the original system presents the extension of the one-parameter family of dynamical mappings through the moment of singularity formation. The extended one-parameter family of dynamical mappings is the random process due to its dependence on the probabilistic parameter. The mean value of this process defines the averaged continuation of trajectories through the boundary of the existence domain of the initial system. The scheme of continuation of trajectories is applied to the finite dimensional Hamiltonian system, the focusing nonlinear Schrödinger equation, the linear Schrödinger equation with a degenerated Hamiltonian.

Dirac's geometric structures

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The main geometric objects of Lagrangian and Hamiltonian mechanics are the tangent TQ and cotangent T^*Q bundles of smooth configuration manifolds Q. In Dirac dynamics, constraints of a mechanical system are considered as *integrable differential distributions* in these spaces. The variational approach to describing the dynamics of such systems, in particular, allows one to consider problems with Lagrangians degenerate in velocities, which are important in relativistic mechanics. In [1], Dirac dynamics is extended to Hamiltonian systems on symplectic manifolds, a special case of which is the cotangent bundle of a smooth manifold with the natural symplectic structure. The constraints of mechanical systems are described in the general case as *non-integrable differential distributions* in these spaces. The operation of symplectic projection of a Hamiltonian vector field is introduced, which allows one to construct equations of motion taking the constraints into account. In the case of integrability of differential distributions describing the constraints, we obtain Dirac Hamiltonian dynamics. To formulate equations describing the dynamics of such systems, it is necessary to project Hamiltonian and Lagrangian vector fields describing the dynamics

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of a constraint-free system onto these distributions describing the constraints. For this purpose, the various projection operators are considered (see [1]). In particular, a method of symplectic projection of Hamiltonian flows to preserve the Hamiltonian structure is proposed. In the non-degenerate case, the uniqueness of such a projection is proved.

In our study, we consider the possibility of implementing symplectic projection in some degenerate cases. As an important application, this allows us to study systems with *one non-integrable constraint*. For this purpose, it is proposed to extend the symplectic projection method to affine differential constraints.

Next, the symplectic projection is extended to Dirac dynamics, which is described within the framework of the so-called generalized geometry [2–4]. The main geometric object of such a consideration is the double bundle $TT^*Q \oplus T^*T^*Q$. The geometric structures based on this object properly describe mechanical systems.

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Hardy inequalities in grand Lebesgue spaces $L_{p)}$, $0 < p \leqslant 1$, for quasi-monotone functions

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In this work, we establish the boundedness of the Hardy operator for quasi-monotone functions in the grand Lebesgue spaces $L_{p)}(0,1)$, $0 < p \leqslant 1$. All inequalities are proved with sharp constants. Some results of Rovshan A. Bandaliyev et al. are deduced as particular cases.

For $0 < p < \infty$, we denote by $L_{p,w}(0,1)$ the set of all Lebesgue measurable functions such that

$$\|f\|_{L_{p,w}(0,1)} = \|f\|_{p,w} = \left(\int_0^1 |f(x)|^p \, w(x)\, dx\right)^{\frac{1}{p}} < \infty, \tag{1}$$

where $w \in L_1^{loc}(0,1)$ and $w(x) > 0$ a.e.

In 1992, T. Iwainiec and C. Sbordone [5] introduced a new type of function spaces $L_{p)}(\Omega)$, $1 < p < \infty$, where Ω is a bounded open set $\Omega \subset \mathbb{R}^n$, called grand Lebesgue spaces. Namely, the grand Lebesgue space is defined as the space of the Lebesgue

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mesurable functions f on Ω such that

$$\|f\|_{p)} = \sup_{0<\varepsilon<p-1} \left(\frac{\varepsilon}{|\Omega|} \int_\Omega |f(x)|^{p-\varepsilon}\, dx \right)^{\frac{1}{p-\varepsilon}} < \infty,$$

where $|\Omega|$ is the Lebesgue measure of Ω.

These spaces were intensively studied during the last years due to different applications (see [3,4]) and continue to attract attention of researchers (see [6–8]).

We state the following definitions and propositions useful in the proofs of main results.

Definition 1 (see [1]). Let $0 < p \leqslant 1$. We say that a function f belongs to the grand Lebesgue space $L_{p)}(0,1)$ if f is non-negative and Lebesgue measurable on $(0,1)$ and

$$\|f\|_{L_{p)}(0,1)} = \sup_{0<\varepsilon<\frac{p}{2}} \left(\varepsilon \int_0^1 |f(x)|^{p-\varepsilon}\, dx \right)^{\frac{1}{p-\varepsilon}} < \infty.$$

Definition 2 (see [1]). Let $0 < p \leqslant 1$. We denote by $\mathcal{A}_p$ the class of all measurable functions $f \in L_{p)}(0,1)$ such that

$$\|f\|_{\mathcal{A}_p} = \sup_{0<\varepsilon\leqslant\frac{p}{2}} \left(\varepsilon \int_0^1 \left(x^{p-\varepsilon-1} - 1\right) f^{p-\varepsilon}(x) dx \right)^{\frac{1}{p-\varepsilon}} < \infty.$$

Remark 1. It was proved in [1] that for $0 < p < 1$, the space $L_{p)}(0,1)$ is a quasi-Banach function space over $(0,1)$. In this case, if $w \equiv 1$, then quantity (1) becomes a quasi-norm in the usual Lebesgue space $L_p(0,1)$.

The following definition is well-known (see [2]).

Definition 3. We say that a function f is *quasimonotone* on $]0,\infty[$ if for some real number α the function $x^\alpha f(x)$ is a decreasing or an increasing function of x. More precisely, given $\beta \in \mathbb{R}$, we say that $f \in Q_\beta$ if $x^{-\beta} f(x)$ is non-increasing and $f \in Q^\beta$ if $x^{-\beta} f(x)$ is non-decreasing.

Proposition 1. *Let* $0 < p < 1$, $0 < \varepsilon < \frac{p}{2}$.

(*a*) *If* $\beta > -1,\ f \in Q_\beta$ *and* $0 \leqslant a < b \leqslant \infty$, *then*

$$\left(\int_0^b f(y) dy \right)^{p-\varepsilon} \leqslant (p-\varepsilon)\,(\beta+1)^{1-p+\varepsilon} \left(\int_0^b y^{p-\varepsilon-1} f^{p-\varepsilon}(y) dy \right). \tag{2}$$

(*b*) *If* $\beta > -1,\ f \in Q^\beta$, *then*

$$\left(\int_0^x f(y) dy \right)^{p-\varepsilon} \leqslant (p-\varepsilon)\,(\beta+1)^{1-p+\varepsilon} \int_0^x \left[y^{-\beta}\left(x^{\beta+1} - y^{\beta+1}\right) \right]^{p-\varepsilon-1} f^{p-\varepsilon}(y) dy. \tag{3}$$

The constants in these inequalities are the best possible.

Theorem 1. *Let $0 < p < 1$, $0 < \varepsilon < \frac{p}{2}$, $f \in \mathcal{A}_p$ and $f \in Q_\beta$, $\beta \geqslant 0$. Then*

$$\|H_1 f\|_{L_{p)}(0,1)} \leqslant C \|f\|_{\mathcal{A}_p}. \tag{4}$$

If $C > 0$ is the sharp constant in (4)*, then*

$$\left(\frac{p}{2-p}\right)^{\frac{2}{p}} \leqslant C \leqslant (\beta+1)^{\frac{2}{p}-1} \left(\frac{p}{1-p}\right)^{\frac{1}{p}}. \tag{5}$$

Remark 2. If $\beta = 0$ in (5), then we have Theorem 3 of [1].

Theorem 2. *Let $0 < p < 1$, $0 < \varepsilon < \frac{p}{2}$, $w(t) = \int_t^1 \frac{(1-y^{\beta+1})^{p-\varepsilon-1}}{y^{\beta(p-\varepsilon-1)+1}} dy$, $0 < y < 1$, and $f \in Q^\beta$, $\beta \geqslant 0$. Then the inequality*

$$\|H_1 f\|_{L_{p)}(0,1)} \leqslant C \|f\|_{L_{p),w}(0,1)} \tag{6}$$

holds, where

$$\|f\|_{L_{p),w}(0,1)} = \sup_{0<\varepsilon<\frac{p}{2}} \left(\varepsilon \int_0^1 \left(\int_t^1 \frac{(1-y^{\beta+1})^{p-\varepsilon-1}}{y^{\beta(p-\varepsilon-1)+1}} dy\right) f^{p-\varepsilon}(t) dt\right)^{\frac{1}{p-\varepsilon}}.$$

If $C > 0$ is the best constant in (6)*, then*

$$\left(\frac{p}{2}\right)^{\frac{2}{p}} \leqslant C \leqslant \left((\beta+1)^{1-\frac{p}{2}} p\right)^{\frac{1}{p}}. \tag{7}$$

Remark 3. If $\beta = 0$ in (7), then we have Theorem 4 of [1].

Similar results hold for $p = 1$.

Corollary 1. *Let $f \in L_{1)}(0,1)$, $f \in Q^\beta$, $\beta \geqslant 0$. Then there exists a constant $C > 0$ such that*

$$\|H_1 f\|_{L_{1)}(0,1)} \leqslant C \sup_{0<\varepsilon<\frac{p}{2}} \left(\varepsilon \int_0^1 \left(\int_t^1 \frac{(1-y^{\beta+1})^{-\varepsilon}}{y^{-\beta\varepsilon+1}} dy\right) f^{1-\varepsilon}(t) dt\right)^{\frac{1}{1-\varepsilon}}. \tag{8}$$

If C is the best constant in (8)*, then*

$$\frac{1}{4} \leqslant C \leqslant (\beta+1)^{\frac{1}{2}}. \tag{9}$$

Remark 4. If $\beta = 0$ in (9), then we obtain Theorem 6 of [1].

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Petrovsky surfaces and short-wave solutions for strictly hyperbolic systems with abruptly varying coefficients

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It is well known that short-wave asymptotic solutions of strictly hyperbolic linear systems in the sense of Petrovskii, whose coefficients do not depend on a small parameter (or depend on it regularly), are described in terms of the Maslov canonical operator on a set of Lagrangian surfaces. These surfaces are invariant with respect to Hamiltonian fields whose Hamiltonians satisfy the characteristic equation for the leading symbol of the hyperbolic system.

If the coefficients are discontinuous or depend singularly on the small parameter (i.e., their weak limits are not smooth), then the solution has a more complicated form near the support of the singularity; in general case, the corresponding theory has not been developed. We describe the asymptotics of the solution of the Cauchy problem in the case where the coefficients change abruptly, i.e., they or their weak limits are discontinuous on some hypersurface in the space of independent variables. In this situation, the Lagrangian surfaces undergo reconstruction at the points corresponding to the specified surface, and the reconstruction is controlled by the geometry of the projective hypersurface in the dual space, determined by the leading symbol of the system (the Petrovskii surface). It is proved that the solution can be expanded into an asymptotic series whose terms are expressed in terms of the Maslov canonical operator on the reconstructed Lagrangian surfaces; the functions to which these operators are applied satisfy an auxiliary scattering problem for a linear system of ordinary differential equations, and the coefficients of the monodromy operator of such a problem determine the coefficients of reflection and transmission of waves through the surface of the jump in coefficients.

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Another look at boojums in a liquid crystal model

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Several authors (Alama, Bronsard, Golovaty, Mironescu) have recently studied the minimizers $\{u_\varepsilon\}$ of the energy

$$E_\varepsilon(u) = E_\varepsilon^{g,\alpha}(u) = \frac{1}{2}\int_\Omega \left(|\nabla u|^2 + \frac{1}{2\varepsilon^2}(1-|u|^2)^2 \right) dx + \frac{1}{2\varepsilon^s}\int_{\partial\Omega} W(u,g)\, ds \qquad (1)$$

over $H^1(\Omega,\mathbb{C})$, where

$$W(u,g) = \frac{1}{2}(|u|^2-1)^2 + [(u,g)-\cos\alpha]^2 \qquad (2)$$

with $\alpha \in (0,\pi/2)$ and $s \in (0,1)$, and where $g : \partial\Omega \to S^1$ is a smooth function of degree $D > 0$. The motivation comes from the thin-film limit of the Landau–de Gennes energy for nematic liquid crystals. It turns out for $s < \frac{1}{2}$ that boundary defects (called "boojums") appear in the limit as $\varepsilon \to 0$. However, the first term in (2) is not part of the physical model but rather added artificially to the energy for "technical reasons".

There are two main goals of this work:

1. To show that the results obtained for energy (1) remain valid for the energy derived from the physical model, namely with $W(u,g) = [(u,g)-\cos\alpha]^2$, rather than $W(u,g)$ as defined in (2).
2. To obtain more precise information on the minimizers $\{u_\varepsilon\}$ as $\varepsilon \to 0$; for example, to show that $|u_\varepsilon| \to 0$ uniformly on $\bar{\Omega}$.

On solutions of Hamilton–Jacobi equations with exponential dependence on momentum

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Let $f(\cdot) : \mathbb{R} \to \mathbb{R}$ and $g(\cdot) : \mathbb{R} \to \mathbb{R}$ be continuously differentiable functions such that $f(\cdot)$ is monotonically increasing and $g(\cdot)$ is monotonically decreasing. We assume that there are points x_* and x^* at which these functions vanish, $f(x_*) = 0$, $g(x^*) = 0$, and $x_* < x^*$. Let time moment $T > 0$ be given. In the rectangle $\Pi = [0,T]\times[x_*,x^*]$, we consider the Hamilton–Jacobi equation

$$\partial u/\partial t + H\,(x, \partial u/\partial x) = 0 \qquad (1)$$

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together with the initial condition

$$u(0,x) = u_0(x), \qquad x \in [x_*, x^*], \tag{2}$$

where $u_0(\cdot)$ is a given continuously differentiable function, and the Hamiltonian has one of the following two forms:

$$\mathbf{A}. \quad H(x,p) = h(x) + f(x)e^p + g(x)e^{-p},$$

$$\mathbf{B}. \quad H(x,p) = h(x) - f(x)e^p - g(x)e^{-p}.$$

Here $h(\cdot)$ is a given continuously differentiable function.

Problems with an exponential dependence of the Hamiltonian on the impulse variable are not typical for the theory of Hamilton–Jacobi equations. At the same time, such problems arise in applied research (see, e.g., [1]) and require study.

It is proved that for Cauchy problem (1), (2) with the Hamiltonian of form **A** there exists a unique continuous viscosity [2] solution, and a scheme for its construction is proposed. It is also shown that for the problem with the Hamiltonian of form **B** there is no continuous viscosity solution, and to find the generalized solution, additional conditions need to be specified. The constructions are based on the method of characteristics [3] and on solving variational problems.

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From a small reactive part of the boundary to the whole interior non-reactive domain

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We prove the approximate controllability, for the interior final observation, of the solution of a linear heat equation in $\Omega \times (0,T)$, when the control is placed on a small part of the boundary l_ε, which is heterogeneous and where we assume a Robin-type boundary condition. We apply the homogenization process proving that the solution of the microscopic problem converges as $\varepsilon \to 0$, to a function $u_0(x,t)$ that is the unique solution to a suitable global state parabolic problem with a Robin-type boundary

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condition on a part of the boundary. Next we consider a microscopic optimal control problem and prove the convergence of the state and the optimal control. At last we prove the approximate controllability by passing to the limit in a penalty parameter of the cost functional.

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On exceptional sets in weighted harmonic analysis

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The formulations of many theorems in mathematical analysis, differential equations, probability theory, and other fields involve sets that are, in one sense or another, considered negligible. For example, sets of Lebesgue measure zero are well known. Statements are often described as being true "almost everywhere," "almost certainly," and so on. Additionally, there are widely used theorems that guarantee the convergence of Fourier series at all points that do not belong to some "exceptional" set [1].

An abstract set $\mathcal{E}$, on which functions of a linear functional class $\mathcal{F}$ are defined, is referred to as the *basic set* of $\mathcal{F}$. A given function f from $\mathcal{F}$ is not necessarily defined on the entire basic set $\mathcal{E}$. A subset of $\mathcal{E}$ where f is not defined is called the *exceptional set* of f.

Functions f and g from $\mathcal{F}$ are considered equal only if they are identical. Identity implies that they coincide and that their domains are the same. Specifically, f and g are distinct if their exceptional sets are different. For this reason, a linear functional class is not necessarily a vector space in the conventional sense.

In the talk, we study exceptional sets adapted to problems with the Laplace–Bessel operator $\Delta_\gamma = (\triangle_\gamma)_x = \sum\limits_{k=1}^{n} (B_{\gamma_k})_{x_k}$, where $(B_\gamma)_t = \frac{\partial^2}{\partial t^2} + \frac{\gamma}{t}\frac{\partial}{\partial t}$, $t > 0$, $\gamma > 0$.

The work was carried out within the framework of the state assignment of the Ministry of Education and Science No. FEGS-2023-0003.

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Invariant subspace problem in Krein space and associated problems

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We will discuss the problem of the existence of maximal sign-definite invariant subspaces in a Hilbert space with an indefinite metric (Krein space). In general, this is an open difficult problem, the prospects for solving which are absolutely unclear. The talk will deliver the results on the classes of operators for which it has a positive solution. The connection of this problem with the factorization theory of operator pencils, with the so-called "half-range problem," with the theory of diagonalization of operator matrices will be traced. Some applications of the results to concrete problems will be demonstrated.

Special attention will be paid to the existence of the factorization

$$L(\lambda) = \lambda^2 + \lambda B + C = (\lambda - Z_2)(\lambda - Z_1),$$

with special properties of the divisor Z_1, provided that $L(\lambda)$ is an elliptic pencil.

On some properties of solutions to mixed problems for Vlasov–Poisson system with external magnetic field

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The Vlasov–Poisson system describes the kinetics of high temperature plasma in a fusion reactor. If sufficiently large number of particles hit the reactor wall, then the wall be destroyed. Therefore, one of the most important problems in plasma theory is plasma confinement [1]. We present the sufficient conditions for external magnetic field providing the existence of a global classical solution to the mixed problem for the Vlasov–Poisson system in the half-space with the density distribution functions supported at a given distance from the boundary [2, 3].

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On statement of boundary value problems for linear and nonlinear elliptic differential-difference equations related to variational problems

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The need to study elliptic differential-difference equations is due to their important and numerous applications. Firstly, they are related to variational problems arising in the theory of multilayer plates and shells, as well as in the theory of control systems with aftereffect, etc. Secondly, elliptic differential-difference equations appear in connection with elliptic problems with nonlocal boundary conditions arising in plasma theory (Bitsadze–Samarskii problem), in the theory of diffusion processes, etc. In the linear setting, the equation

$$A\widetilde{R}_Q u = f, \quad x \in Q, \tag{1}$$

is considered in a bounded domain $Q \subset \mathbb{R}^n$ with boundary $\partial Q \in C^\infty$, where A is a linear strongly elliptic differential operator, $\widetilde{R}_Q$ is a linear difference operator with constant coefficients and shifts in spatial variables. Assuming $w(x) = \widetilde{R}_Q u(x)$, we reduce equation (2) to the form

$$Aw = f, \quad x \in Q, \tag{2}$$

where the new function $w(x)$ must satisfy the nonlocal boundary conditions when equating the traces of the function w on some parts of the boundary to some linear combinations of the traces of w on the shifts of these parts inside the domain.

However, if a differential-difference equation is obtained from the problem of minimization of a functional containing the function and its derivatives with shifts in spatial variables to the power of p, $p \in (1, \infty)$, then we do not come to an equation of form (1). For example, in the simplest case we have the equation

$$-\sum_{1\leqslant i\leqslant n} \partial_i R_Q^* \left(|R_Q \partial_i u|^{p-2} R_Q \partial_i u\right) = f, \qquad x \in Q, \tag{3}$$

where R_Q is a linear difference operator with constant coefficients containing shifts in spatial variables. Note that $R_Q^* R_Q$ is a linear difference operator too. If $p = 2$, then the operator ∂_i and the linear difference operator $R_Q^* R_Q$ commute on $\mathring{W}_p^1(Q)$. Thus,

$$-\sum_{1\leqslant i\leqslant n} \partial_i R_Q^* R_Q \partial_i u = -\sum_{1\leqslant i\leqslant n} \partial_i^2 \widetilde{R}_Q u,$$

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where $\widetilde{R}_Q := R_Q^* R_Q$. In this way we obtain equation (1). Note that ∂_i and a linear difference operator do not commute on $L_q(Q)$, $1/p+1/q=1$. Therefore, if $p \neq 2$, we obtain the new equation

$$-\sum_{1\leqslant i\leqslant n} \partial_i R_Q^* \left(|\partial_i R_Q u|^{p-2} \partial_i R_Q u\right) = f, \quad x \in Q, \tag{4}$$

So, in the nonlinear case, we obtain two different types of differential-difference equations (1) and (4), which are investigated by different methods and have different applications.

Asymptotics of eigenvalues and eigenvectors of special type Toeplitz banded matrices

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Let $a(t)$ be a Lebesgue integrable function defined on the unit circle $T = \{t \in \mathbb{C} : |t| = 1\}$. Denote by $T_n(a)$ the Toeplitz matrix $T_n(a) = (a_{j-k})_{j,k=1}^{n-1}$, where n is a natural number and a_l is the l-th coefficient of the Fourier series of $a(t)$. Note that a Toeplitz matrix can be viewed as an operator acting in a finite-dimensional vector space. The function $a(t)$ is called the symbol of the Toeplitz matrix (Toeplitz operator) $T_n(a)$. We consider banded Toeplitz matrices generated by the symbol $a(z) = (z + z^{-1} - 2)^3$. This symbol is a simple example of a symbol vanishing together with its derivatives up to the second order inclusive at the points ± 1. It should also be noted that the Toeplitz matrices defined by this symbol are self-adjoint, and the symbol itself is real. Asymptotic formulas for the eigenvalues of such Toeplitz matrices were found in [1]. We discuss these formulas here. The problem of finding the eigenvalues and eigenvectors of very large Toeplitz matrices of great importance, in particular in the theory of integrable models od statistical mechanics.

Note that for symmetric banded Toeplitz matrices, asymptotic formulas for the components of the eigenvectors were obtained in [2] for the case of a non-degenerate symbol.

We present also formulas for the eigenvectors, similar to the formulas for the eigenvalues found in [1], using the aforementioned formulas for the eigenvalues. The proof of these formulas for eigenvectors is based on a combination of the result on the expression of the components of eigenvectors in the form of determinants of some matrices expressing the components of eigenvectors through the roots of the symbol, allowing one to find the asymptotic formulas themselves, and the method of successive approximations, allowing one to find the values of the components approximately, but with arbitrarily high accuracy and making it possible to prove the found asymptotic formulas.

The formulas themselves, despite their cumbersome form, provide an effective method for finding the components of eigenvectors for Toeplitz matrices of very large sizes, in particular for sizes for which standard methods for finding the eigenvectors of matrices do not work.

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Resonance in nonlinear systems with damped oscillatory perturbations

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We consider time-decaying perturbations of non-isochronous oscillatory systems in the plane. It is assumed that the perturbations oscillate with an asymptotically constant frequency with a limit value satisfying the resonance condition. The occurrence of attractive solutions with a resonant amplitude is investigated. Such effects in the corresponding problems with a small parameter are usually associated with a nonlinear resonance. Using a combination of averaging techniques and the Lyapunov function method, model equations are derived whose solutions describe the perturbed dynamics. It is shown that resonant solutions occur in the phase-locking regime. The conditions for the existence and stability of such a mode are found, and the corresponding threshold values are described.

The report is based on papers [1, 2].

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Homogenization of a parabolic convolution-type equation with periodic coefficients

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Let $u_\varepsilon(\mathbf{x},t)$, $\mathbf{x}\in\mathbb{R}^d$, $t\in[0,\infty)$, $\varepsilon>0$, be the solution of the Cauchy problem for a parabolic equation with a nonlocal convolution-type operator,

$$\begin{aligned}\frac{\partial u_\varepsilon(\mathbf{x},t)}{\partial t} &= \varepsilon^{-d-2}\int_{\mathbb{R}^d} a((\mathbf{x}-\mathbf{y})/\varepsilon)\mu(\mathbf{x}/\varepsilon,\mathbf{y}/\varepsilon,t/\varepsilon^2)(u_\varepsilon(\mathbf{y},t)-u_\varepsilon(\mathbf{x},t))\,d\mathbf{y},\\ u_\varepsilon(\mathbf{x},0)&=\varphi(\mathbf{x}).\end{aligned} \tag{1}$$

Here $\varphi\in L_2(\mathbb{R}^d)$. It is assumed that $a\in L_1(\mathbb{R}^d)$, $\mu\in L_\infty(\mathbb{R}^{2d}\times\mathbb{R}_+)$, and

$$a(\mathbf{x})=a(-\mathbf{x})\geqslant 0,\quad \mathbf{x}\in\mathbb{R}^d;\quad 0<\int_{\mathbb{R}^d}(1+|\mathbf{x}|^3)a(\mathbf{x})\,d\mathbf{x}<\infty;$$

$$0<\mu_-\leqslant\mu(\mathbf{x},\mathbf{y},t)\leqslant\mu_+<\infty,\quad \mu(\mathbf{x},\mathbf{y},t)=\mu(\mathbf{y},\mathbf{x},t),\quad \mathbf{x},\mathbf{y}\in\mathbb{R}^d,\ t\in\mathbb{R}_+;$$

$$\mu(\mathbf{x}+\mathbf{m},\mathbf{y}+\mathbf{n},t+j)=\mu(\mathbf{x},\mathbf{y},t),\quad \mathbf{x},\mathbf{y}\in\mathbb{R}^d,\ t\in\mathbb{R}_+,\ \mathbf{m},\mathbf{n}\in\mathbb{Z}^d,\ j\in\mathbb{Z}_+.$$

We show that, as $\varepsilon\to0$, u_ε converges to the solution u_0 of the homogenized problem

$$\frac{\partial u_0(\mathbf{x},t)}{\partial t}=-\operatorname{div} g^0\nabla u_0(\mathbf{x},t),\quad u_0(\mathbf{x},0)=\varphi(\mathbf{x}). \tag{2}$$

Here $g^0=\{g^0_{kl}\}$ is a positive definite effective matrix defined in terms of the solutions of some auxiliary problems. Let $v_k(\mathbf{x},t)$ be a $\mathbb{Z}^d$-periodic (in $\mathbf{x}$) solution of the problem

$$\frac{\partial v_k(\mathbf{x},t)}{\partial t}=\int_{\mathbb{R}^d}a(\mathbf{x}-\mathbf{y})\mu(\mathbf{x},\mathbf{y},t)(v_k(\mathbf{y},t)-v_k(\mathbf{x},t)+y_k-x_k)\,d\mathbf{y},$$

$$v_k(\mathbf{x},t)=v_k(\mathbf{x},t+1),\quad \mathbf{x}\in\Omega,\ t\in\mathbb{R}_+;\quad \int_\Omega v_k(\mathbf{x},t)\,d\mathbf{x}=0.$$

Here $\Omega=[0,1)^d$. The entries of the effective matrix g^0 are given by

$$\begin{aligned}g^0_{kl}&=\frac12\int_0^1 dt\int_\Omega d\mathbf{x}\int_{\mathbb{R}^d}d\mathbf{y}\,a(\mathbf{x}-\mathbf{y})\mu(\mathbf{x},\mathbf{y},t)\\&\quad\times\left((x_k-y_k)(x_l-y_l)-(x_k-y_k)v_l(\mathbf{y},t)-(x_l-y_l)v_k(\mathbf{y},t)\right),\quad k,l=1,\dots,d.\end{aligned}$$

Theorem 1. *Under the above assumptions, let u_ε be the solution of problem* (1)*. Let u_0 be the solution of homogenized problem* (2)*. Then for $\varepsilon>0$ we have*

$$\left\|u_\varepsilon(\cdot,t)-u_0(\cdot,t)\right\|_{L_2(\mathbb{R}^d)}\leqslant\frac{C\varepsilon}{(t+\varepsilon^2)^{1/2}}\|\varphi\|_{L_2(\mathbb{R}^d)},\quad t>0,$$

with some constant C.

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On singular solutions of the Davey–Stewartson II equation

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We expose the examples of blow-ups of solutions of the Davey–Stewartson II equation obtained in [1] by means of surface theory and demonstrate how these tools can be used to explain the Ozawa solution to this equation [2].

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On coercivity problem for functional-differential operators with orthotropic contractions in the three-dimensional case

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Let us consider the functional-differential equation

$$A_R u(x) = -\sum_{i,j=1}^{3} \left(R_{ij} u_{x_i}\right)_{x_j} = f(x), \quad x \in Q,$$

where $Q \subset \mathbb{R}^3$ is a bounded domain containing the origin and

$$R_{ij} v(x) = a_{ij0} v(x) + a_{ij1} v(q^{-1} x_1, p x_2, r x_3) + a_{ij,-1} v(q x_1, p^{-1} x_2, r^{-1} x_3).$$

Here $a_{ijk} \in \mathbb{C}$, $i, j = 1, 2, 3$; $k = 0, \pm 1$, and $p, q, r > 1$, which means that one coordinate is contracted while the other two are expanded, and vise versa.

The conditions for strongly ellipticity (fulfillment of the Gårding-type inequality) are obtained.

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The magnon bound states in the Heisenberg model

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In the work [1], S. Tashpulatov has considered the three-magnon system in an isotropic non-Heisenberg ferromagnet model with spin values $s = 1$ with the nearest neighbors interactions. We consider the energy operator of three-magnon systems in the Heisenberg model and investigated the structure of essential spectra and discrete spectrum of the system in the two- and three-dimensional lattice $Z^{\nu}, \nu = 1, 2, 3$. Hamiltonian of the system has the form

$$H = \sum_{m,\tau} (\overrightarrow{S}_m \overrightarrow{S}_{m+\tau}), \tag{1}$$

where $J < 0$ is the parameter of the bilinear exchange interaction between atoms, $\overrightarrow{S}_m = (S^x_m, S^y_m, S^z_m)$ is the operator of the atomic spin $\frac{1}{2}$ at the site m, and summation over τ ranges the nearest neighbors. Hamiltonian H acts in a symmetric complex Fock space $(\mathcal{H}, (\cdot,\cdot)_{\mathcal{H}})$. We let φ_0 denote the vector, called the vacuum, uniquely defined by the conditions $S^+_m \varphi_0 = 0$ and $S^z_m \varphi_0 = \frac{1}{2}\varphi_0$, where $||\varphi_0|| = 1$. We set $S^{\pm}_m = S^x_m \pm i S^y_m$, here S^-_m and S^+_m are the magnon creation and annihilation operators at the site m. The vector $S^-_p S^-_q S^-_r \varphi_0$ describes the state of the system of three magnons at the sites p, q and r with spin $s = \frac{1}{2}$. The vectors $\psi = \sum_{p,q,r} f(p, q, r) S^-_p S^-_q S^-_r \varphi_0$ constitute an orthonormal system. We let $\mathcal{H}_3$ denote the closure of this space of three-magnon states of operator H. We denote by H_3 the restriction of the operator H to the space $\mathcal{H}_3$.

Theorem 1.

a). *If $\nu = 1$ and the total quasimomentum of the has the form $\Lambda = \pi$, then the essential spectrum of the operator $\widetilde{H}_{3\Lambda}$ consists of a three values $\sigma_{ess}(\widetilde{H}_{3\Lambda}) = \{-12J, -8J, -10J\}$, and the relation $1 \leqslant N \leqslant 10$ holds for the number of three-magnon bound states N of operator $\widetilde{H}_{3\Lambda}$.*

b). *If $\nu = 1$ and the total quasimomentum of the has the form $\Lambda = 0$, then the essential spectrum of the operator $\widetilde{H}_{3\Lambda}$ consists of a unique segment: $\sigma_{ess}(\widetilde{H}_{3\Lambda}) = [0, -24J]$, and the relation $0 \leqslant N \leqslant 9$ holds for the number of three-magnon bound states N of operator $\widetilde{H}_{3\Lambda}$.*

c). *If $\nu = 1$ and the total quasimomentum of the has the form $\Lambda \neq \pi$, and $\Lambda \neq 0$, then the essential spectrum of the operator $\widetilde{H}_{3\Lambda}$ consists of union of three segments: $\sigma_{ess}(\widetilde{H}_{3\Lambda}) = [-4J \sum_{i=1}^{3}(1 - \cos\frac{\Lambda_i}{2}), -4J \sum_{i=1}^{3}(1 + \cos\frac{\Lambda_i}{2})] \cup [-8J +$*

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$2J(2\cos\frac{\Lambda_1}{2}+\sum_{i=2}^{3}(cos^2\frac{\Lambda_i}{2}), -8J-4J\cos\frac{\Lambda_1}{2}+2J\sum_{i=2}^{3}(cos^2\frac{\Lambda_i}{2})] \cup [-10J+4J\sum_{i=2}^{3}(\cos\frac{\Lambda_i}{2})+2Jcos^2\frac{\Lambda_1}{2}, -10J-4J\sum_{i=2}^{3}(\cos\frac{\Lambda_i}{2})+2Jcos^2\frac{\Lambda_1}{2}]$ *and the relation* $1\leqslant N\leqslant 10$ *holds for the number of three-magnon bound states* N *of operator* $\widetilde{H}_{3\Lambda}$.

Theorem 2. *If* $\nu=2$ *and the total quasimomentum of the system has the form* $\Lambda=(\Lambda_1,\Lambda_2)=(\pi,\pi)$, *then the essential spectrum of the operator* $\widetilde{H}_{3\Lambda}$ *consists of a three values* $\{-24J,-16J,-20J\}$, *i.e.* $\sigma_{ess}(\widetilde{H}_{3\Lambda})=\{-24J,-16J,-20J\}$ *and the relation* $1\leqslant N\leqslant 18$ *holds for the number of three-magnon bound states* N *of operator* $\widetilde{H}_{3\Lambda}$.

Theorem 3. *If* $\nu=2$ *and the total quasimomentum of the system has the form* $\Lambda=(\Lambda_1,\Lambda_2)=(\pi,0)$, *and* $\Lambda=(0,\pi)$, *then the essential spectrum of the operator* $\widetilde{H}_{3\Lambda}$ *consists of the union of three intervals:* $\sigma_{ess}(\widetilde{H}_{3\Lambda})=[-12J,-36J]\cup[4(-5+\sqrt{5})J,4(-7+\sqrt{5})J]\cup[2(-8+\sqrt{5})J,2(-16+\sqrt{5})J]$, *and the relation* $1\leqslant N\leqslant 19$ *holds for the number of three-magnon bound states* N *of operator* $\widetilde{H}_{3\Lambda}$.

Theorem 4. *If* $\nu=2$ *and the total quasimomentum of the system has the form* $\Lambda=(\Lambda_1,\Lambda_2)=(0,0)$, *then the essential spectrum of the operator* $\widetilde{H}_{3\Lambda}$ *consists of the union of three intervals:* $\sigma_{ess}(\widetilde{H}_{3\Lambda})=[0,-48J]\cup[2z_1,-16J+2z_1]\cup[z_1,-32J+z_1]$, *and the relation* $1\leqslant N\leqslant 19$ *holds for the number of three-magnon bound states* N *of operator* $\widetilde{H}_{3\Lambda}$.

Theorem 5. *If* $\nu=2$ *and the total quasimomentum of the system has the form* $\Lambda=(\Lambda_0,\Lambda_0)$, *then the essential spectrum of the operator* $\widetilde{H}_{3\Lambda}$ *consists of the union of five intervals:* $\sigma_{ess}(\widetilde{H}_{3\Lambda})=[-24J(1-cos\frac{\Lambda_o}{2}),-24J(1+cos\frac{\Lambda_o}{2})]\cup[-8J(1-cos\frac{\Lambda_o}{2})+2z_1,-8J(1+cos\frac{\Lambda_o}{2})+2z_1]\cup[-8J(1-cos\frac{\Lambda_o}{2})+2z_2,-8J(1+cos\frac{\Lambda_o}{2})+2z_2]\cup[-16J(1-cos\frac{\Lambda_o}{2})+z_1,-16J(1+cos\frac{\Lambda_o}{2})+z_1]\cup[-16J(1-cos\frac{\Lambda_o}{2})+z_2,-16J(1+cos\frac{\Lambda_o}{2})+z_2]$, *and the relation* $4\leqslant N\leqslant 22$ *holds for the number of three-magnon bound states* N *of operator* $\widetilde{H}_{3\Lambda}$.

Theorem 6. *If* $\nu=3$ *and the total quasimomentum of the system has the form* $\Lambda=(\pi,\pi,\pi)$, *then the essential spectrum of the operator* $\widetilde{H}_{3\Lambda}$ *consists of the four values:* $\sigma_{ess}(\widetilde{H}_{3\Lambda})=\{-36J,-32J,-24J,-34J\}$ *and the relation* $4\leqslant N\leqslant 30$ *holds for the number of three-magnon bound states* N *of operator* $\widetilde{H}_{3\Lambda}$.

Theorem 7. *If* $\nu=3$ *and the total quasimomentum of the system has the form* $\Lambda=(0,0,0)$, *then the essential spectrum of the operator* $\widetilde{H}_{3\Lambda}$ *consists of the union of three segments:* $\sigma_{ess}(\widetilde{H}_{3\Lambda})=[0,-72J]\cup[2z_1,-24J+2z_1]\cup[z_1,-48J+z_1]$, *and the relation* $1\leqslant N\leqslant 27$ *holds for the number of three-magnon bound states* N *of operator* $\widetilde{H}_{3\Lambda}$.

Theorem 8. *If* $\nu=3$ *and the total quasimomentum of the system has the form* $\Lambda=(\pi,0,0)$, *then the essential spectrum of the operator* $\widetilde{H}_{3\Lambda}$ *consists of the union of six segments:* $\sigma_{ess}(\widetilde{H}_{3\Lambda})=[-12J,-60J]\cup[-4J+2z^\star,-20J+2z^\star]\cup[-4J+2\widetilde{z},-20J+2\widetilde{z}]\cup[-4J+z^\star+\widetilde{z},-20J+z^\star+\widetilde{z}]\cup[-8J+z^\star,-40J+z^\star]\cup[-8J+\widetilde{z},-40J+\widetilde{z}]$, *and the relation* $4\leqslant N\leqslant 30$ *holds for the number of three-magnon bound states* N *of operator* $\widetilde{H}_{3\Lambda}$.

Theorem 9. *If* $\nu=3$ *and the total quasimomentum of the system has the form* $\Lambda=(\pi,\pi,0)$, *then the essential spectrum of the operator* $\widetilde{H}_{3\Lambda}$ *consists of the union* *of five segments:* $\sigma_{ess}(\widetilde{H}_{3\Lambda}) = [-24J, -48J] \cup [-8J, -16J] \cup [-4(5+\sqrt{5})J, -4(10+\sqrt{5})J] \cup [-16J, -32J] \cup [-2(14+\sqrt{5})J, -2(24+\sqrt{5})J]$, *and the relation* $3 \leqslant N \leqslant 29$ *holds for the number of three-magnon bound states* N *of operator* $\widetilde{H}_{3\Lambda}$.

Theorem 10. *If* $\nu = 3$ *and the total quasimomentum of the system has the form* $\Lambda = (\Lambda_0, \Lambda_0, \Lambda_0)$, *then the essential spectrum of the operator* $\widetilde{H}_{3\Lambda}$ *consists of the five segments:* $\sigma_{ess}(\widetilde{H}_{3\Lambda}) = [-36J(1-cos\frac{\Lambda_0}{2}), -36J(1+cos\frac{\Lambda_0}{2})] \cup [-12J(1-cos\frac{\Lambda_0}{2}) + 2z_1, -12J(1+cos\frac{\Lambda_0}{2})+2z_1] \cup [-12J(1-cos\frac{\Lambda_0}{2})+2z_2, -12J(1+cos\frac{\Lambda_0}{2})+2z_2] \cup [-12J(1-cos\frac{\Lambda_0}{2}) + 2z_1, -12J(1+cos\frac{\Lambda_0}{2}) + 2z_1] \cup [-24J(1-cos\frac{\Lambda_0}{2}) + z_1, -24J(1+cos\frac{\Lambda_0}{2}) + z_1] \cup [-24J(1-cos\frac{\Lambda_0}{2}) + z_2, -24J(1+cos\frac{\Lambda_0}{2}) + z_2]$ *and the relation* $4 \leqslant N \leqslant 30$ *holds for the number of three-magnon bound states* N *of operator* $\widetilde{H}_{3\Lambda}$.

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On the global regularity of the Navier–Stokes equations

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In this presentation, we give a sufficient condition to guarantee the existence of a smooth solution of the d-dimensional Navier–Stokes equation with nice decreasing properties at infinity for $d \geqslant 3$. In this way, we prove the existence of smooth physically reasonable solutions to the Navier–Stokes problem. Additionally, we show the existence of a smooth curve of entire vector fields of order 2 that extends the solution to the complex domain for positive time.

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Local properties of solutions of viscous Hamilton–Jacobi–Chandrashekar equations

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We study the local properties of solutions to the equation

$$-\Delta u = e^u - m|\nabla u|^q$$

in a punctured domain $\Omega \setminus \{0\}$ of $\mathbb{R}^N$ or in an exterior domain $\mathbb{R}^N \setminus B_{r_0}$ in the range $q > 1$ and $m > 0$. We prove a series of a priori estimates depending on q and of the sign of $q-2$, and give a precise description of behaviour of solutions near an isolated singularity or at infinity in $\mathbb{R}^N$. This extends a similar study concerning the Chipot–Weissler equation $-\Delta u = u^p - m|\nabla u|^q$.

This is a joint work with M.-F. Bidaut-Véron.

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Nonlocal reaction-diffusion equations in biomedical applications

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Reaction–diffusion equations have long served as a foundational tool for modeling the dynamics of spatially distributed systems in biology and medicine, capturing processes such as pattern formation, tissue growth, disease spread, and chemical signaling. However, classical models based on local interactions often fall short in describing complex biological phenomena where the spatial interactions are intrinsically nonlocal. Nonlocal reaction–diffusion equations extend the traditional framework by incorporating integral terms that account for long-range spatial effects, delayed interactions, or heterogeneous connectivity, providing a more accurate and versatile approach to modeling biomedical systems.

This lecture presents an introduction to nonlocal reaction–diffusion equations and explores their applications in biomedical contexts. We begin by discussing the mathematical formulation of nonlocal models, highlighting how they differ from local counterparts in terms of structure, analysis, and solution behavior. Key theoretical aspects related to spectral properties and stability of solutions [1] will be briefly reviewed, with an emphasis on how nonlocality alters the qualitative dynamics of the system.

The main focus of the lecture is on biomedical applications where nonlocal effects are essential. We will discuss two main types of applications: emergence and evolution of biological entities (species, cell clones, virus quasi-species) and the interaction of local and systemic effects in biomedical processes [2].

Through these applications, we demonstrate how nonlocal terms can capture spatial memory, synchronization, and collective behavior more faithfully than local models. We also discuss some challenges in in the analysis of such equations, and present recent results and open questions in the field.

The lecture aims to bridge the gap between mathematical theory and biomedical modeling, offering insights into how nonlocal reaction–diffusion equations can serve as powerful tools in understanding and predicting the dynamics of complex biological systems.

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On the regularity of generalized eigenfunctions of differential-difference operators with the Dirichlet conditions

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We consider the spectral problem for a differential-difference operator with the Dirichlet boundary conditions. Unlike ordinary differential operators, solutions of boundary value problems for such operators can be non-smooth [1]. We have shown that the smoothness of generalized eigenfunctions of differential-difference operators can be violated in the interior of the interval [2]. In this talk, we discuss the necessary and sufficient conditions for violation of smoothness of generalized eigenfunctions. We will provide an example of a differential-difference operator for which all eigenfunctions are non-smooth.

It is a joint work with A. L. Skubachevskii.

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Generalized Saigo fractional operators involving product of hypergeometric functions of two variables and polynomials

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This paper presents a detailed investigation into the generalized Saigo fractional operators acting on the product of hypergeometric functions of two variables and a general class of q-polynomials. The study emphasizes the role of q-analogues of Fox's H-function and Meijer's G-function, and explores their behavior under fractional q-calculus. New theorems are established for the q-analogues of Saigo-type fractional integrals and derivatives, particularly in combination with special functions of two variables. The work also derives several special cases, including reductions to the basic analogues of Meijer's G-function and MacRobert's E-function. The results

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contribute significantly to the theory of q-series and can be applied to solving certain q-difference and q-integral equations involving special functions.

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Nonlinear conjugate gradient scheme for penalty finite element approximations of the Carreau–Yasuda Stokes problem

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The steady-state Carreau–Yasuda Stokes problem for incompressible fluid flows is approximated by a penalty Stokes problem in which the incompressibility constraint is compensated by a penalty term. The penalty Stokes problem is formulated as an unconstrained nonlinear minimization problem. The solution of the minimization problem over a finite element space is considered a semi-discrete approximation of the solution. A nonlinear conjugate gradient scheme (NCGS) is introduced for a fully discrete approximation of the solution of the penalty Stokes problem. Convergence of the NCGS and the corresponding error estimates are demonstrated by establishing upper bounds on the 1st and 2nd-order Gateaux derivatives of the associated penalty objective functional. Numerical solutions of a cavity flow problem are presented as a case study to demonstrate the successful convergence of the NCGS.

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On Brezis's first open problem: a complete solution

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In 2023, H. Brezis [1] published a list of his "favorite open problems," which he described as challenges he had "raised throughout his career and has resisted so far." We provide a complete resolution to the first one — Open Problem 1.1 — in Brezis's favorite open problems list: the existence of solutions to the long-standing Brezis–Nirenberg problem on a three-dimensional ball. More precisely, let Ω be the unit ball B_1 in $\mathbb{R}^3$. Consider the following Brezis–Nirenberg [2] problem:

$$\begin{cases} \Delta u + \lambda u + u^5 = 0 & \text{in } B_1, \\ u = 0 & \text{on } \partial B_1. \end{cases} \tag{1}$$

H. Brezis' first open problem, implicitly raised by Brezis and Nirenberg [2, Remark (6)(d)], is as follows:

Open Problem 1.1 (Implicit in [2]). *Assume that*

$$0 < \lambda < \frac{\lambda_1}{4}. \tag{2}$$

Does there exist a non-trivial solution $u \not\equiv 0$ to (1)?

As remarked by H. Brezis, under condition (2), any solution to (1), if exists, must be non-radial and sign-changing. When $\lambda > \lambda_1$, there are sign-changing solutions but there is no positive solution to (1), see [2]. They are obtained by bifurcation from non-radial sign-changing eigenfunctions.

Our main result provides a complete resolution to Brezis' Open Problem 1.1 and, in fact, establishes even more.

Theorem 1 (Sun-Wei-Yang [3])**.** *Assume that*

$$0 < \lambda < +\infty. \tag{3}$$

Then there are infinitely many (sign-changing) solutions to (1).

Our building block is the Del Pino–Musso–Pacard–Pistoia sign-changing solutions to the Yamabe problem and non-degeneracy.

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On new method for solving the problem of anomalous diffusion control based on perturbation theory

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Let's consider the first boundary value problem for non-homogeneous equation for fractional diffusion

$$\begin{cases} \frac{du(x,t)}{dt} = D\frac{d^\alpha u(x,t)}{\partial x^\alpha}, \\ u(1,t) = u(0,t) = 0, \\ u(x,0) = u(x). \end{cases} \tag{1}$$

where

$$\frac{d^\alpha u(x,t)}{\partial x^\alpha} = \frac{1}{\Gamma(2-\alpha)} \frac{\partial^2}{\partial x^2} \int\limits_0^x \frac{u(x,\tau)}{(x-\tau)^{\alpha-1}} d\tau$$

is the fractional derivative in Riemann–Liouville sense [1] and $1 < \alpha < 2$.

Theorem. *The function*

$$u(x,t) = \sum_{n=1}^{\infty} e^{\lambda Dt} [\int\limits_0^t f_t e^{-\lambda_n Dt} dt + \varphi_n] x^{\alpha-1} E_{\alpha,\alpha}(\lambda_n x^\alpha) \tag{2}$$

is the solution of boundary value problem (1)*. Here*

$$E_{\alpha,\alpha}(\lambda_n x^\alpha) = \sum_{k=0}^{\infty} \frac{(\lambda_n x^\alpha)^k}{\Gamma(\alpha - \alpha k)}$$

is the Mittag-Leffler function [2]*, φ_n is the coefficient of the decomposition for the function $u(x,t)$ and $f(\lambda,t)$ by functional feature,*

$$\omega_n(\lambda_n, x) = x^{\alpha-1} E_{\alpha,\alpha}(\lambda_n x^\alpha).$$

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Weak solution of fourth order of accuracy difference scheme for the nonlinear system of coupled sine-Gordon equations

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The sine-Gordon equation, a prominent nonlinear hyperbolic partial differential equation, plays a significant role in modeling various physical phenomena, including nonlinear optics, quantum field theory, and especially biological systems such as DNA dynamics. In this study, we investigate a coupled nonlinear sine-Gordon system that models the rotational motion of base pairs in the DNA double helix. We develop an unconditionally stable finite difference scheme of fourth-order accuracy within the framework of Sobolev spaces to approximate the solution of this system. The primary goal is to establish the existence and uniqueness of weak solutions by employing variational methods. To this end, we construct the necessary functional framework and derive a priori estimates ensuring the well-posedness of the numerical model. Additionally, a comprehensive numerical analysis is performed to examine the stability and convergence properties of the proposed scheme. The results not only contribute to the theoretical understanding of high-order numerical methods for nonlinear wave equations but also offer valuable insights into the mathematical modeling of DNA dynamics using coupled sine-Gordon systems.

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Local well posedness of the Korteweg–de Vries equation in weighted Sobolev space

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We study the initial value problem for the Korteweg–de Vries equation

$$\begin{cases} \partial_t u + \partial_x^3 u + \partial_x(u^2) = 0, & x \in \mathbb{R},\, t > 0, \\ u(x,0) = u_0(x), & x \in \mathbb{R}, \end{cases} \tag{1}$$

in the weighted Sobolev space $H^{3/4}(\mathbb{R}) \cap L^2(|x|^{2r}dx)$, where $r \in (0, 3/8)$. The Korteweg–de Vries equation (1) proposed as a model to resonant interactions of long waves with different dispersion relations.

We apply the Banach fixed point theorem to the integral equation version of the initial value problem (1), i.e.

$$u(x,t) = e^{it\partial_x^3}u_0 - \int_0^t e^{i(t-t')\partial_x^3} u u_x dt', \tag{2}$$

where $e^{it\partial_x^3}$ is the Airy semigroup. We are interested in local well-posedness results for (2) in weighted Sobolev spaces. To control nonlinearities in the integral part of the Duhamel formulation of the solution we will develop the methods to be adapted to Bourgain spaces $X_{s,b}$. In the past years, new techniques based on Bourgain spaces, have been used to address the IVP associated to many dispersive equations in low regularity Sobolev spaces [1–3].

Theorem. *Let $u_0 \in H^{3/4} \cap L^2(|x|^{2r}dx)$ for $r \in (0, 3/8)$. Then, there exist $T > 0$ and a unique solution u to the system of the integral equation* (2) *such that*

$$u(\cdot, t) \in H^{3/4} \cap L^2(|x|^{2r}dx), \quad t \in [0, T].$$

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Grazing diffraction of short waves by contours with nonsmooth curvature

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We consider 2D problems of diffraction by a contour whose curvature suffers a jump at a point, and an incident wave arrives at the jump point tangentially. With the aim of constructing short-wavelength asymptotic formulas for the wavefields, we apply a systematic boundary-layer technique which goes back to the research of Fock [1] and was further developed by Babich and Kirpichnikova [2].

Problems of grazing diffraction by contours with jumping curvature are numerous. Some of them were investigated in our joint works with A. P. Kiselev and in papers of other researchers (see, e.g., [3, 4] and literature therein). In the talk, prominence is given to the diffraction of a whispering gallery wave running along the concave curve towards the point where the contour passes, with a jump in curvature, to the convex curve. Our concern is not only the influence of curvature jump, but also the effects of change of curvature sign — for half a century, steady interest has been seen in such phenomena. Asymptotic formulas are obtained for waves arising in the vicinity of the jump point, both inside and outside emerging boundary layers.

This work was performed at the Saint Petersburg Leonhard Euler International Mathematical Institute and supported by the Ministry of Science and Higher Education of the Russian Federation.

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On variational inequalities with non-smooth solutions

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We consider deformation models of elastic systems with nonlinear conditions. This kind of condition arises due to an obstacle to the displacement from the equilibrium

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position of one of the ends of the investigated elastic system. Depending on the applied external force, the corresponding end either remains free or reaches the boundary point of the obstacle. The deviation of the studied physical system from the equilibrium position is a solution to a variational inequality with Stieltjes integrals. This allows us to take into account singularities localized at separate points, such as concentrated forces and elastic supports, and analyse both solutions and relations at each point. The corresponding variational inequalities are obtained from the problems of minimizing the potential energy functionals for the investigated physical systems.

The necessary and sufficient conditions for the extremum of the energy functional are established. Theorems of existence and uniqueness of solutions are proved; formulas for exact solutions of the corresponding variational inequalities are obtained in the explicit form; the dependence of solutions on the size of the obstacle is studied; algorithms for finding approximate solutions are developed.

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Well-posedness of one viscoelastic model

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In a bounded domain $\Omega \in \mathbb{R}^n, n = 2, 3$, on the time interval $[0, T], T > 0$, we consider the following initial-boundary value problem (see [1, 2]):

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \mu_0 \Delta v - \mu_1 \frac{\partial \Delta v}{\partial t} - 2\mu_1 \mathrm{Div}\Big(\sum_{i=1}^{n} v_i \frac{\partial \mathcal{E}(v)}{\partial x_i}\Big) -$$

$$-2\mu_1 \mathrm{Div}\Big(\mathcal{E}(v) W_\rho(v) - W_\rho \mathcal{E}(v)\Big) -$$

$$-\frac{\mu_2}{\Gamma(1-\lambda)} \mathrm{Div} \int_0^t (t-s)^{-\lambda} \mathcal{E}(v)\big(s, z(s; t, x)\big)\, ds + \nabla p = f,$$

$$z(\tau; t, x) = x + \int_t^\tau v\big(s, z(s; t, x)\big)\, ds, \quad t, \tau \in [0, T], \quad x \in \Omega,$$

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$$v|_{t=0} = v_0, \qquad v|_{[0,T]\times\partial\Omega} = 0.$$

Here $v(x,t)$ is the vector function of the fluid particle velocity, $p(x,t)$ is the pressure function, $f(x,t)$ is the vector function of the density of external forces, $z(\tau,t,x)$ is the trajectory of a particle of the fluid indicating at time τ the location of a particle of the fluid located at time t at point x, $\mu_0, \mu_1, \alpha > 0$, $\mu_2 \geqslant 0$ are some constants, $\mathcal{E}(v) = (\mathcal{E}_{ij}(v))_{j=1,\ldots,n}^{i=1,\ldots,n}$, $\mathcal{E}_{ij}(v) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$ is the strain rate tensor, $W(v) = (W_{ij}(v))_{j=1,\ldots,n}^{i=1,\ldots,n}$, $W_{ij}(v) = \frac{1}{2}(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i})$ is the vorticity tensor, $W_\rho(v) = \int_{\mathbb{R}^n} \rho(x-y)W(y)dy$, $\rho : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth compactly supported function such that $\int_{\mathbb{R}^n} \rho(y)dy = 1$ and $\rho(x) = \rho(y)$ for x and y with the same Euclidean norms, $\Gamma(1-\lambda)$ is the Euler gamma function.

The weak solvability of the initial-boundary value problem described above is considered at the report.

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Интегрирование системы уравнений в частных производных для σ-функции Вейерштрасса

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Доклад посвящён поиску аналитических решений системы уравнений (см. [1]) в частных производных

$$z\frac{\partial u}{\partial z} - 4g_2\frac{\partial u}{\partial g_2} - 6g_3\frac{\partial u}{\partial g_3} - u = 0, \tag{1}$$

$$\frac{\partial^2 u}{\partial z^2} - 12g_3\frac{\partial u}{\partial g_2} - \frac{2}{3}g_2^2\frac{\partial u}{\partial g_3} + \frac{z^2}{12}g_2\, u = 0, \tag{2}$$

которой при начальных условиях $u(0; g_2, g_3) = 0$, $u'_z(0; g_2, g_3) = 1$ удовлетворяет σ-функция Вейерштрасса (см. [2]). Если рассматривать g_2, g_3 как параметры, то $\sigma(z; g_2, g_3)$ является целой функций переменного z и допускает запись в виде степенного ряда

$$\sigma(z; g_2, g_3) = \sum_{k=0}^{\infty} a_k(g_2, g_3)\, z^{2k+1}, \tag{3}$$

сходящегося во всей комплексной плоскости.

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С использованием представления (3) для решений уравнений (1), (2) при начальных условиях вида $u(0;g_2,g_3)=0, u'_z(0;g_2,g_3)=v(g_2;g_3)$ получены дифференциально-рекуррентные формулы и представления для коэффициентов степенного ряда. В частности, рассмотрена счётная система обыкновенных дифференциальных уравнений на функции $c_k(g_2^3/g_3^2)$, через которые выражаются $a_k(g_2,g_3)$.

Особое внимание уделяется частному решению $\sigma(z;g_2,g_3)$ и найденным представлениям для коэффициентов ряда Тейлора этой специальной функции [3]. Так, приводится несколько новых замкнутых нерекуррентных формул для коэффициентов степенного ряда σ-функции и обсуждается подход к численной эффективной аппроксимации значений $\sigma(z;g_2,g_3)$, основанный на полученном двумерном рекуррентном соотношении.

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Об одном методе решения краевой задачи для неоднородного уравнения третьего порядка в трехмерном пространстве

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В данной работе рассматривается краевая задача для неоднородного уравнения третьего порядка с кратными характеристиками в трехмерном пространстве. Единственность решения задачи доказана методом интегралов энергии, а существование — методом разделения переменных. Решение представлено в виде построенной функции Грина. При обосновании равномерной сходимости установлено отличие от нуля «малого знаменателя».

В области $D=\{(x,y,z):0<x<p,\ 0<y<q,\ 0<z<r\}$ рассмотрим уравнение третьего порядка вида

$$L[u]\equiv u_{xxx}-u_{yy}-u_{zz}=f(x,y,z), \tag{1}$$

где $p,q,r\in R$, и для него исследуем следующую задачу.

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Задача 1. Найти решение уравнения (1) в области D из класса $C^{3,2,2}_{x,y,z}(D) \cap C^{2,1,1}_{x,y,z}(\overline{D})$, удовлетворяющее краевым условиям:

$$\begin{cases} \alpha u(x,0,z) + \beta u_y(x,0,z) = 0, \\ \gamma u(x,q,z) + \delta u_y(x,q,z) = 0, \\ u(x,y,0) = u(x,y,r) = 0, \end{cases} \tag{2}$$

$$\begin{cases} au(0,y,z) + bu_{xx}(0,y,z) = \psi_1(y,z), \\ cu(p,y,z) + du_{xx}(p,y,z) = \psi_2(y,z), \\ u_x(p,y,z) = \psi_3(y,z), \end{cases} \tag{3}$$

где $a,b,c,d,\alpha,\beta,\gamma,\delta \in R\backslash\{0\}$, а $f(x,y,z), \psi_i(y,z)$, $i=\overline{1,3}$ — заданные достаточно гладкие функции.

Отметим, что для уравнения (1) на плоскости, краевые задачи исследованы при $b=d=0$ в работах [1, 2], а при $a=c=0$ и $\beta=\delta=0$ в работе [3].

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О задаче преследования управляемым БПЛА самолетного типа движущегося объекта

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Важные прикладные значения имеют задачи автономного управления беспилотного летательного аппарата (БПЛА) и обеспечения автономного преследования–сближения БПЛА с надземной целью.

В докладе рассматривается БПЛА, оснащенный видеокамерой и органами управления: тягой двигателя и аэродинамическими рулями (углы отклонения руля высоты, руля направления). БПЛА, как объект управления, представляет собой динамическую систему. Математическая модель движения БПЛА представляется нелинейной системой из шести дифференциальных уравнений. Фазовыми координатами считаем высоту, продольную и боковую дальность, углы курса, наклона траектории и путевую скорость.

Требуется разработать такой алгоритм управления движением БПЛА, который обеспечивает гарантированное сближение с целью не хуже, чем на заданные величины по расстоянию и по углу.

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Вообще обеспечение гарантированного сближения с целевым объектом в пределах определенного допустимого отклонения особенно важно для тех нелинейных систем, для которых классические методы преследования (или сближения) не всегда применимы.

Для построения алгоритма управления движением БПЛА предполагается, что надземная цель находится в поле зрения видеокамеры. Будем рассматривать движение БПЛА в вертикальной плоскости. Линеаризируем систему уравнений БПЛА относительно установившегося режима полета. На основе полученных данных с видеокамеры и с учетом закона движения БПЛА получена координатно-временная зависимость БПЛА и цели. Далее построен алгоритм управления таким образом, чтобы преследование (сближение) БПЛА цели осуществлялось методом наведения по линии погони. Для линеаризованной системы определяются управляющие воздействия, переводящие движение БПЛА из начального состояния в конечное состояние (цели), которые подставляются в исходную систему. Далее численно интегрируется эта нелинейная система дифференциальных уравнений и в процессе этого в каждый момент времени проверяются условия отклонения соответствующих величин, вычисленных для линейной и нелинейной систем. Если нарушается хотя бы одно из условий, то для фиксированного момента отклонения вычисляется состояние системы и относительно этого состояния линеаризуется нелинейная система. Для этого состояния находим управляющие воздействия, переводящие движение системы к цели, подставляем их в исходную нелинейную систему дифференциальных уравнений и численно интегрируем в каждый момент времени, проверяя условия отклонения и так далее. Это приводит к динамическому обновлению траектории БПЛА, обеспечивая гибкость и адаптивность управления, обеспечивая отклонение от цели не больше, чем на заданную величину. Для иллюстрации конструктивности алгоритма приведены конкретные примеры. Работа выполнена при поддержке Комитета по высшему образованию и науке РА, в рамках исследовательского проекта № 23-2DP-1B001.

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Аналитическое решение линеаризованной системы Больцмана—Максвелла

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Рассматривается линеаризованная система уравнений Больцмана—Максвелла, описывающая поведение плазмы под действием внешнего возмущающего электромагнитного поля [1]. В работе получено аналитическое решение для указанной системы с помощью представления самосогласованного электрического поля в виде суперпозиции продольной и поперечной волн. Каждой волне соответствует своя система интегро-дифференциальных уравнений, сумма их решений является решением исходной системы.

Система уравнений для продольной волны имеет вид:

$$v\frac{\partial \mathcal{F}_{\|}(x,v)}{\partial x} + \mathrm{A}\mathcal{F}_{\|}(x,v) = v\mathcal{E}_{\|}(x) + \int_{-\infty}^{+\infty} \mathcal{F}_{\|}(x,s)k(s)ds, \tag{1}$$

$$\frac{d\mathcal{E}_{\|}(x)}{dx} = \mathrm{B}_1 \int_{-\infty}^{+\infty} \mathcal{F}_{\|}(x,s)k(s)ds, \tag{2}$$

а для поперечной волны — следующий вид:

$$v\frac{\partial \mathcal{F}_{\perp}(x,v)}{\partial x} + \mathrm{A}\mathcal{F}_{\perp}(x,v) = v\mathrm{B}_2 \int_{-\infty}^{+\infty} \mathcal{F}_{\perp}(x,s)sk(s)ds, \tag{3}$$

$$\mathcal{E}_{\perp}(x) = \int_{-\infty}^{+\infty} \mathcal{F}_{\perp}(x,s)sk(s)ds; \tag{4}$$

здесь параметры B_1 и B_2 принимают вещественные значения, а параметр A — комплексные значения, зависящие от свойств плазмы. Величины $\mathcal{F}_{\|}(x,v)$, $\mathcal{F}_{\perp}(x,v)$ и $\mathcal{E}_{\|}(x)$, $\mathcal{E}_{\perp}(x)$ представляют собой возмущения функции распределения электронов и напряженности электрического поля, индексы $\|$ и $\perp$ соответствуют продольной и поперечной волне, а переменные (x,v) имеют смысл координаты и скорости. Функция $k(s)$ представляет собой обезразмеренную невозмущенную функцию распределения электронов, в качестве которой в работе рассмотрены функция Ферми—Дирака и Максвелла.

Общие решения систем (1), (2) и (3), (4) построены в аналитическом виде с помощью сочетания метода расширения области (до всей плоскости (x,v)) и метода преобразования Фурье в пространствах обобщенных функций D' и Z'; о таких пространствах см. [2]. Искомые функции найдены в интегральном виде с явно выписанным ядром и свободной плотностью.

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О спектрах операторов свертки с потенциалами

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В докладе будут представлены недавние результаты о спектрах операторов, представляющих собой сумму многомерного оператора свертки и потенциала. В явном виде описано положение существенного спектра. Основные результаты касаются точечного спектра. Получены достаточные условия, гарантирующие существование собственных значений в лакунах существенного спектра. Отдельно обсуждается вопрос о наличии бесконечных серий собственных значений, накапливающихся к краям существенного спектра. Выписывается несколько достаточных условий, гарантирующих существование таких серий. Данные достаточные условия покрывают широкий класс ядер свертки и потенциалов, включая негладкие случаи либо случаи сложных осцилляций. Рассмотрен также случай несамосопряженного оператора с малым потенциалом, здесь получены предварительные результаты о собственных значениях, возникающих из краев существенного спектра.

Модели и методы математической иммунологии

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В докладе представлены математические модели динамики иммунных процессов при инфекционных заболеваниях и вопросы структурной и практической идентификации моделей по реальным данным [1]. Модели формулируются на основе систем дифференциальных уравнений с запаздывающим аргументом и их расширения путем включения пространственной динамики компонент иммунных реакций и интеграции с блоками процессов клеточного и системного уровня в рамках мультифизического подхода [2]:

$$\Sigma_{t,x} = \begin{cases} \partial_t \boldsymbol{y}(t,x) & = \boldsymbol{f}(t, \boldsymbol{y}(t-\tau, x), \boldsymbol{p}) + D\Delta \boldsymbol{y}(t,x), \ t \in [0, t_{mod}], \ x \in [0, L], \\ \boldsymbol{y}_{obs}(t) & = \boldsymbol{h}(\boldsymbol{y}(t,x), \boldsymbol{p}), \\ \boldsymbol{y}(0,x) & = \boldsymbol{y}_{0,x}(\boldsymbol{p}), \\ \boldsymbol{y}(t,0) & = \boldsymbol{y}_{t,0}(\boldsymbol{p}), \quad \boldsymbol{y}(t,L) = \boldsymbol{y}_{t,L}(\boldsymbol{p}). \end{cases} \tag{1}$$

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Рассмотрены особенности моделирования динамики и исходов вирусных инфекций, представляющих значительную проблему для населения: ВИЧ и SARS-CoV-2. Разработана модель, описывающая размножение ВИЧ на клеточном уровне с учетом конкуренции с дефектными интерферирующими вирионами (ДИВ), см. [3]. С помощью моделей исследованы перспективные подходы к лечению ВИЧ инфекции на основе комбинированной терапии, и с использованием ДИВ. Для этого используются результаты анализа свойств мультистабильности и гистерезиса калиброванных моделей [4]. Предложен подход к определению топологической структуры кооперативных клеточных сетей в иммунной системе человека по данным иммунного ответа на антигенные возмущения различной природы.

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Об эквивариантных краевых задачах и некоторых приложениях

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Пусть Ω — произвольная ограниченная область в пространстве $\mathbb{R}^n$ с границей $\partial\Omega$ и $\mathcal{L} = \sum_{|\alpha|\leqslant m} a_\alpha(x)D^\alpha, D^\alpha = (-i\partial)^{|\alpha|}/\partial x_1^{\alpha_1}...\partial x_n^{\alpha_n}$, $\alpha \in \mathbf{Z}_+^n, |\alpha| = \sum_k \alpha_k$ — некоторая формально самосопряженная дифференциальная операция с гладкими ($C^\infty(\overline{\Omega})$) комплексными матричными коэффициентами $a_\alpha(x)$. Пусть L_0 — минимальный оператор с областью определения $D(L_0)$, а $L = (L_0)^*$ — максимальный оператор $\mathcal{L}$, $C(L) = D(L)/D(L_0)$ — граничное пространство, $\Gamma : D(L) \to C(L)$ — фактор-отображение. Краевая задача $Lu = f, \Gamma u \in B \subset C(L)$ называется *корректной*, если соответствующее расширение $L_B = L|_{D(L_B)}$, $D(L_B) = \Gamma^{-1}B$ имеет непрерывный двусторонний обратный оператор.

Пусть G — группа Ли, гладко действующая в замкнутой области $\overline{\Omega}$ и на границе $\partial\Omega$, и пусть это действие сохраняет объем области. Пусть дифференциальная операция $\mathcal{L}$ инвариантна относительно действия группы, то есть $g(\mathcal{L}u) = \mathcal{L}(gu)$. Тогда пространства $D(L)$, $D(L_0)$, $C(L)$ инвариантны относительно действия группы G. Краевую задачу $Lu = f, \quad \Gamma u \in B$, будем называть *$G$-инвариантной*, если пространство B инвариантно относительно указанного действия группы G. Если группа G компактна, то, как известно, гильбертово пространство представления разлагается в прямую сумму конечномерных инвариантных подпространств неприводимых представлений. А если группа еще и коммутативна, то такие представления одномерны. Пусть пространство представления компактной группы G будет граничным пространством $C(L)$. Тогда мы имеем разложения

$$C(L) = \sum_{k=0}^{\infty} \oplus \tilde{C}^k, \; C(\mathrm{ker}L) = \sum_{k=0}^{\infty} \oplus C^k(\mathrm{ker}L), \; B = \sum_{k=0}^{\infty} \oplus B^k.$$

Если наша G-инвариантная краевая задача корректно поставлена, разложения в прямую сумму $C(L) = C(\mathrm{ker}L) \oplus B$ индуцируют разложения в прямую сумму $C^k := C^k(\mathrm{ker}L) \oplus B^k = \sum_l \tilde{C}^{k_l}$ с конечномерными проекторами $\Pi^k : C^k \to C^k(\mathrm{ker}L)$ вдоль B^k и теперь проверка *корректности G-инвариантной краевой краевой задачи* может быть показана *проверкой двух свойств*:

$$1) \; C^k(\mathrm{ker}L) \cap B^k = 0; \quad 2) \; \exists \varkappa > 0, \; \forall k, \; \|\Pi^k\|_{C^k} < \varkappa.$$

О приложениях. Мы рассматриваем постановку смешанной задачи в шаре с наиболее общим линейным граничным условием для волнового уравнения и доказываем существование обобщенного решения такой задачи. Также исследуется спектр оператора общей корректной SO-эквивариантной краевой задачи для уравнения Пуассона в круге и шаре, выделяя случаи нарушения корректности этой задачи для уравнения Гельмгольца как нарушение свойства 1). Выясняется, что выполнение свойства 2) является следствием свойства корректности задачи для уравнения Пуассона. Еще одно приложение связано с квантовой механикой. Рассматривается уравнение Шредингера для водородоподобного атома с кулоновским потенциалом и неточечным шаровым ядром. Найдены собственные значения и собственные функции оператора, заданного произвольной инвариантной относительно вращения краевой задачей на сферической границе ядра, доказано, что собственные значения оператора эквивариантной граничной задачи не зависят от выбора этой граничной задачи.

Системы управления с последействием на временны́х графах

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Концепция временно́го графа была предложена в [1,2]. В отличие от пространственной сети, ребра такого графа отождествляются с промежутками времени, а каждая внутренняя вершина является точкой разветвления процесса, дающей несколько различных сценариев дальнейшего его протекания по числу выходящих из нее ребер. Как и в пространственных сетях, здесь также могут возникать

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условия типа Кирхгофа. Им будет удовлетворять такая траектория течения процесса, которая является оптимальной с учетом сразу всех сценариев.

В работах [1, 2] с помощью идеи глобального запаздывания [3] на графы перенесена задача Н. Н. Красовского об успокоении системы управления с последействием [4, 5], до этого рассматривавшаяся на временно́м интервале. В [2, 6] дана стохастическая интерпретация системы управления на временно́м дереве. А именно, к ней приведет, в частности, замена коэффициентов в уравнении на интервале дискретными случайными процессами с дискретным временем.

В докладе обсуждаются новые постановки задач оптимального управления с последействием на временны́х графах. В частности, будет рассмотрена конструкция динамического временно́го графа, когда длины ребер не фиксированы и также могут выбираться из соответствующих условий оптимальности.

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Об эллиптических уравнениях в пространствах со смешанной нормой

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Исследуется разрешимость псевдодифференциального уравнения [1]

$$(Au)(x) = v(x), \quad x \in C, \tag{1}$$

где $C \subset \mathbb{R}^m$ — выпуклый острый конус, A — эллиптический псевдодифференциальный оператор с символом $A(\xi)$ порядка α_j по j-ой группе переменных, $j = 1, 2, \ldots, n$, $\alpha = (\alpha_1, \ldots, \alpha_n)$, решение ищется в пространстве Соболева—Слободецкого $H^s(C)$, $s = (s_1, \ldots, s_n)$, $v \in H^{s-\alpha}(C)$ [2].

С помощью специальной факторизации эллиптического символа [1, 2] описывается картина разрешимости такого модельного эллиптического псевдодифференциального уравнения в конусе. Найдена структура общего решения уравнения

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при дополнительных условиях на индекс $æ = (æ_1, \dots, æ_n)$ волновой факторизации символа.

В трехмерном случае рассмотрена следующая краевая задача. Пусть $C = C_2 \times C_1$, $C_2 = \{x \in \mathbb{R}^3 : x = (x_1, x_2, x_3), x_2 > a|x_1|,\ a > 0,\ x_3 = 0\}$, $C_1 = \{x \in \mathbb{R}^3 : x = (x_1, x_2, x_3), x_1 = 0, x_2 = 0, x_3 > 0\}$. В пространстве $H^s(C)$, $s = (s_1, s_2)$, показатель s_1 соответствует двумерному подпространству, а показатель s_2 — одномерному. Зададим граничные условия Дирихле на двух гранях

$$u|_{ax_1 - x_2 = 0} = g_1(x_1 + ax_2, x_3), \quad u|_{ax_1 + x_2 = 0} = g_1(x_1 - ax_2, x_3) \tag{2}$$

и рассмотрим краевую задачу (1), (2).

Теорема 1. *Пусть $s_1 > 1/2$ и символ $A(\xi)$ допускает волновую факторизацию относительно C с индексом $æ = (æ_1, æ_2)$ таким, что $1/2 < æ_1 - s_1 < 3/2$, $-1/2 < æ_2 - s_2 < 1/2$. Пусть, кроме того, $g_{1,2} \in H^{s'}(\mathbb{R}_+), s' = (s_1 - 1/2, s_2), v \in H^{s-\alpha}$. Тогда однозначная разрешимость краевой задачи (1), (2) в пространстве $H^S(C)$ эквивалентна однозначной разрешимости системы линейных интегральных уравнений (относительно двух неизвестных функций), которая строится по данным исходной задачи.*

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ОТО и уравнения Власова: космология и пространство Лобачевского

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Рассмотрены вывод и свойства уравнений Власова—Эйнштейна и Власова—Пуассона и космологические решения.

В классических работах (см. [1–4]) уравнения для полей предлагаются без вывода правых частей. Здесь мы даем вывод правых частей уравнений Максвелла и Эйнштейна в рамках уравнений Власова—Максвелла—Эйнштейна из классического, но немного более общего принципа наименьшего действия [5–11]. Получающийся вывод уравнений типа Власова даёт уравнения Власова—Эйнштейна отличные от тех, что предлагались ранее [12–15]. Предлагается способ перехода от кинетических уравнений к гидродинамическим следствиям [5–8], как это делалось раньше уже самим А. А. Власовым [4]. В случае гамильтоновой механики от гидродинамических следствий уравнения Лиувилля возможен переход

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к уравнению Гамильтона—Якоби, как это делалось уже в квантовой механике Е. Маделунгом [16], а в более общем виде В. В. Козловым [17, 18]. Таким образом получаются в нерелятивистском случае решения Милна—Маккри, а также нерелятивистский и релятивистский анализ решений типа Фридмана нестационарной эволюции Вселенной. Это позволяет определить константу Хаббла не на основе метрики, как это делалось ранее [1–3], а как положено, на основе наблюдаемой материи, написать уравнения для неё на основе движения материи в заданной метрике, проанализировать Лямбду Эйнштейна и причину ускоренного расширения Вселенной как релятивистский эффект [19–21]. Факт ускоренного расширения позволяет также определить знак кривизны: она отрицательна, и мы живем в пространстве Лобачевского.

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Система дискриминантов, структура кратностей корней и решение дифференциальных уравнений

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Алгебраические уравнения и системы уравнений играют фундаментальную роль во многих разделах математики. Знание кратностной структуры корней алгебраического уравнения позволяет решать задачи классификации и типизации решений в теории дифференциальных уравнений и в качественной теории динамических систем.

Общая задача выяснения картины кратностей корней алгебраического многочлена по его коэффициентам была поставлена ещё Артуром Кэли. С тех пор этой задаче было посвящено множество работ. Но в основном полученные результаты относятся к анализу кратностей корней общих уравнений с комплексными коэффициентами. Полученные при этом системы дискриминантов не позволяют выяснять вещественно-комплексный состав кратных корней в случае уравнений с вещественными коэффициентами.

В докладе представлены результаты автора по классификации вещественных алгебраических уравнений, то есть по выяснению, сколько вещественных и сколько комплексно-сопряжйных пар корней и каких кратностей имеет заданный многочлен n-ой степени с вещественными коэффициентами. Предложена система дискриминантов, по вектору знаков которых определяется картина кратностей корней, как вещественных, так и комплексно сопряжённых. Правда, в некоторых случаях ответ оказывается двузначным, в связи с чем появляется задача пополнения системы дискриминантов новыми функциями, решение которой входит в план дальнейшей работы.

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О решении уравнения равновесия тонкой нанопластины в рамках теории градиента микроструктурной деформации

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Представленная работа посвящена построению решения уравнения равновесия тонкой изотропной прямоугольной нанопластины постоянной толщины в рамках теории градиента микроструктурной деформации и моментной теории [1]. В предположении, что процесс деформирования является изотермическим и смещения точек пластины малы по сравнению с ее толщиной, с применением многочленов Чебышева первого рода в качестве базиса в гильбертовом пространстве функций получено решение дифференциального уравнения изгибающей поверхности рассматриваемой нанопластины в виде частичной суммы двойного ряда по этим многочленам. Неизвестные коэффициенты в представлении решения найдены методом коллокации. При этом в качестве точек коллокации использованы корни многочленов Чебышева первого рода. Рассмотрены граничные условия: свободное опирание, защемление, свободный край и их комбинации [1–3]. В результате выполненных матричных преобразований [4] краевая задача сведена к решению системы линейных уравнений. Получена оценка отклонения построенного решения по норме в банаховом пространстве существенно ограниченных функций. В зависимости от значений нелокальных параметров масштаба длины представлены результаты вычисления изгиба пластины, обусловленного действием постоянной и распределенной синусоидальной нагрузки, для которой возможно аналитическое решение в случае простого опирания ее сторон, и проведен их анализ. Проведено сравнение с результатами [1, 3, 5]. Показано, что построенное решение сходится по норме в банаховом пространстве существенно ограниченных функций к точному достаточно гладкому решению. Полученные результаты моделирования изгиба срединной плоскости изотропной нанопластины обобщают результаты исследования тонких изотропных нанопластин при использовании теории Кирхгофа [2].

Работа выполнена при поддержке Российского Фонда Фундаментальных Исследований (проект 24-21-00381).

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О фундаментальных соленоидальных системах в кубической области

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В ряде работ академика Ладыженской О. А. (например, в [1]) указывалось на важность построения фундаментальной системы в пространстве соленоидальных функций для простейших областей типа куба, шара и др. (см. также [2]).

Теоретически, существование такой системы не требует доказательства, это общеизвестно. Последний факт активно используется специалистами при доказательстве теорем существования для $2D$ и $3D$ систем Навье–Стокса и для дальнейшего анализа качественных свойств решения, доказанного на существование. Однако, для численного решения граничных задач как для системы уравнений Стокса, так и системы Навье-Стокса, возникает необходимость в построении вышеуказанной фундаментальной системы [3, 4].

В связи со сказанным, главная цель представленного доклада—это построение фундаментальных систем в пространстве соленоидальных функций для областей, являющихся прямоугольными параллелепипедами.

Новым в нашем подходе является введение понятия функции тока и активное использование понятия ротор, когда размерность независимых переменных $d \geqslant 2$.

В результате, нами дан ответ на вопрос О.А. Ладыженской для областей, представляющих собой прямоугольные параллелепипеды, в частности, для кубической области.

Работа выполнена при поддержке Комитета науки Министерства науки и высшего образования Республики Казахстан (грант BR20281002).

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Метод осреднения в задаче о набеге на берег волн, описываемых линеаризованными уравнениями мелкой воды

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Применение коротковолновых асимптотик к практическим задачам о распространении в водоеме коротких (по сравнению с горизонтальными размерами водоема) длинных (по сравнению с его глубиной) гравитационных волн, таких, как волны цунами, наталкивается на трудность, связанную с тем, что глубина водоема, которая входит в коэффициенты уравнений, может сильно меняться на расстояниях, значительно меньших длины волны. В такой ситуации естественно использовать осреднение, однако классические методы осреднения, в которых предполагается периодичность коэффициентов уравнения по "быстрым" переменным, здесь не работают, и авторами был разработан новый метод, свободный от предположения о периодичности [1–3]. Этот метод применим к невырожденным уравнениям (т.е., в случае уравнений мелкой воды, к распространению волн вдали от берега). В докладе метод распространяется на случай вырождения, что позволяет рассмотреть набег волн на берег.

Работа выполнена в Институте проблем механики им. А. Ю. Ишлинского РАН по теме государственного задания (№ госрегистрации 124012500442-3).

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Системные неравенства Пуанкаре и Фридрихса. Полевые формы задачи Неймана

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I (*неравенства Пуанкаре*). Пусть $G \subset \mathbb{R}^3$ — ограниченная область с липшицевой границей. Пусть далее $\vec{\Phi}(x) = (\Phi_1(x), \Phi_2(x), \Phi_3(x))$, $x \in G$ — нормированное векторное поле в области G.

Имеют место следующие утверждения.

1. Допустим, что система функций $\Phi_1(x)$, $\Phi_2(x)$, $\Phi_3(x)$ линейно независима в $L_2(G)$. Тогда для любой вектор-функции $\vec{u} = (u_1(x), u_2(x), u_3(x)) \in \vec{W}_2^1(G)$ справедливо неравенство
$$\int_G |\vec{u}|^2\,dx \leqslant M\left(\int_G |\nabla\vec{u}|^2\,dx + \left|\int_G \left(\vec{u},\vec{\Phi}\right)\vec{\Phi}dx\right|^2\right),$$
где $M > 0$ — постоянная, не зависящая от $\vec{u}$.

2. Допустим, что система $\Phi_1(x)$, $\Phi_2(x)$, $\Phi_3(x)$ линейно независима в $L_2(G)$ и при этом матрица Грама этой системы не содержит в своём спектре число $\lambda = \mathrm{mes}\, G$. Тогда для любой функции $\vec{u} \in \vec{W}_2^1(G)$ справедливо неравенство
$$\int_G |\vec{u}|^2\,dx \leqslant M\left(\int_G |\nabla\vec{u}|^2\,dx + \left|\int_G \left[\vec{\Phi},\left[\vec{u},\vec{\Phi}\right]\right]dx\right|^2\right),$$
где $M > 0$ — постоянная, не зависящая от $\vec{u}$.

II (*неравенства Фридрихса*). Пусть $G \subset \mathbb{R}^3$, $\Gamma = \partial G$ и $\vec{\Phi} = (\Phi_1(\gamma), \Phi_2(\gamma), \Phi_3(\gamma))$, $\gamma \in \Gamma$ — нормированная векторная система на Γ.

Имеют место следующие неравенства.

1. Пусть вектор-функция $\vec{\Phi}$ такова, что её компоненты образуют в пространстве $L_2(\Gamma)$ линейно независимую систему элеметов. Тогда для любой вектор-функции $\vec{u} \in \vec{W}_2^1(G)$ справедливо неравенство
$$\int_G |\vec{u}|^2\,dx \leqslant M\left(\int_G |\nabla\vec{u}|^2\,dx + \left|\int_\Gamma \left(\vec{u},\vec{\Phi}\right)\vec{\Phi}d\gamma\right|^2\right),$$
где $M > 0$ — постоянная, не зависящая от $\vec{u}$.

2. Пусть вектор-функция $\vec{\Phi} = (\Phi_1(\gamma), \Phi_2(\gamma), \Phi_3(\gamma))$ такова, что матрица Грама её компонент не содержит в своём спектре число $\lambda = \mathrm{mes}\ \Gamma$. Тогда для любой функции $\vec{u} \in \vec{W}_2^1(G)$ справедливо неравенство
$$\int_G |\vec{u}|^2\,dx \leqslant M\left(\int_G |\nabla\vec{u}|^2\,dx + \left|\int_\Gamma \left[\vec{\Phi},\left[\vec{u},\vec{\Phi}\right]\right]d\gamma\right|^2\right),$$

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где $M > 0$ — постоянная, не зависящая от $\vec{u}$.

Пример. В случае $\vec{\Phi} = \vec{n} = (n_1, n_2, n_3)$, где $\vec{n}$ – вектор нормали на границе Γ, получаем неравенства

$$\int_G |\vec{u}|^2 \, dx \leqslant M \left(\int_G |\nabla \vec{u}|^2 \, dx + \left| \int_\Gamma (\vec{u}, \vec{n}) \, \vec{n} d\gamma \right|^2 \right)$$

и

$$\int_G |\vec{u}|^2 \, dx \leqslant M \left(\int_G |\nabla \vec{u}|^2 \, dx + \left| \int_\Gamma [\vec{n}, [\vec{u}, \vec{n}]] \, d\gamma \right|^2 \right).$$

III (*задача Неймана*). Установленные неравенства позволяют рассмотреть задачу Неймана

$$-\Delta \vec{u}(x) = \vec{h}(x), \quad x \in G, \tag{1}$$

$$\left. \frac{\partial \vec{u}}{\partial \vec{n}} \right|_\Gamma = 0 \tag{2}$$

в соответствующих полевых формах.

Приведём одну из таких форм.

Теорема 1. *Для любой вектор-функции $\vec{\Phi} = (\Phi_1(x), \Phi_2(x), \Phi_3(x))$, компоненты которой линейно независимы в $L_2(G)$, и любого функционала $\vec{h} \in \left(\vec{W}_2^1(G)\right)^*$, удовлетворяющего условию $\left(\vec{h}, \vec{c}\right) = 0$, где $\vec{c} = (c_1, c_2, c_3)$ — постоянная вектор-функция, существует единственное слабое решение $\vec{u}_0 \in \vec{W}_2^1(G)$, такое, что*

$$\int_G \left(\vec{u}_0, \vec{\Phi}\right) \vec{\Phi} dx = \vec{0}.$$

При этом справедлива оценка

$$\int_G |\vec{u}_0|^2 \, dx + \int_G |\nabla \vec{u}_0|^2 \, dx \leqslant M \left\| \vec{h} \right\|^2_{\left(\vec{W}_2^1(G)\right)^*}, \quad M > 0.$$

Любое другое решение задачи (1), (2) *отличается от $\vec{u}_0$ на аддитивный постоянный вектор $\vec{c} = (c_1, c_2, c_3)$.*

Аналогично формулируются результаты для ядер «вектор-функционалов»

$$\int_G \left[\vec{\Phi}, \left[\vec{u}, \vec{\Phi}\right]\right] dx, \quad \int_\Gamma \left(\vec{u}, \vec{\Phi}\right) \vec{\Phi} d\gamma, \quad \int_\Gamma \left[\vec{\Phi}, \left[\vec{u}, \vec{\Phi}\right]\right] d\gamma.$$

Результаты работы были получены в рамках выполнения государственного задания Минобрнауки России (проект FSWF-2023-0012).

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Построение обобщенных операторов Бушмана–Эрдейи с использованием композиционного метода

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Теория операторов преобразования является хорошо разработанным самостоятельным разделом математики, находящимся на стыке дифференциальных, интегральных и интегро-дифференциальных уравнений, функционального анализа, теории функций, комплексного анализа, теории специальных функций и дробного интегро-дифференцирования, теории обратных задач и задач рассеяния. Значительный вклад в эту теорию и её приложения к дифференциальным уравнениям с частными производными внесли работы воронежского математика Валерия Вячеславовича Катрахова [1].

В докладе будет рассказан композиционный метод построения операторов преобразования [2–4]. Этим методом получаются по единой схеме все известные ранее в явном виде операторы преобразования, а также получены многочисленные новые классы операторов преобразования обобщенных операторов Бушмана–Эрдейи.

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Многообразия, порожденные семейством спектральных краевых задач

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Мы рассмотрим линейное многообразие стационарных операторов Шредингера на отрезке. По фиксированному оператору и краевым условиям определяются собственные значения и собственные функции. Оказывается, корректно определена «обратная задача» — восстановление оператора по собственной функции. Это возможно по той причине, что класс функций, которые могут быть собственными для некоторого оператора Шредингера, настолько узок, что допускает полное описание. В докладе будет дано аналитическое и топологическое описание многообразия собственных функций и связанных с ним подмногообразий операторов Шредингера.

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О периодическом η-инварианте

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Эта-инвариант впервые появился в работе Атьи, Патоди и Зингера [1] как регуляризация типа дзета-функции сигнатуры квадратичной формы, отвечающей самосопряженному оператору. По определению он является спектральным инвариантом и рассматривался как показатель спектральной асимметрии оператора. В теории индекса эллиптических операторов на многообразии с цилиндрическими концами и/или коническими особенностями эта-инвариант выражает вклад бесконечности/конической точки в формулу индекса. Также эта-инвариант име-

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ет приложения в геометрии, топологии, анализе, теории чисел, математической физике.

Позже Мельроуз [2] обобщил эта-инвариант на случай семейств операторов с параметром, эллиптических в смысле Аграновича—Вишика [3], построив специальную регуляризацию числа вращения определителя обратимой матричнозначной функции. Используя подход Мельроуза, в работе [4] авторы построили эта-инвариант эллиптических псевдодифференциальных операторов на прямой с периодическими коэффициентами и предъявили формулу индекса для операторов на прямой с коэффициентами, периодическими на бесконечности.

Доклад посвящен исследованию периодического эта-инварианта. Рассматривается случай периодических операторов на прямой, и далее строится обобщение на случай бесконечного двумерного цилиндра. Важной особенностью периодических операторов в многомерном случае является отсутствие следового свойства функционалов, которые ввел в своей работе Мельроуз, определенных на алгебре таких операторов. В связи с этим эта-инвариант теряет стандартное логарифмическое свойство, и интерес представляют формулы «дефекта» данных функционалов при действии на коммутаторах. В качестве примера вычисляется эта-инвариант для оператора Шрёдингера.

Исследование выполнено за счет гранта Российского научного фонда № 24-21-00336.

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Начальная задача для одного двумерного гиперболического дифференциально-разностного уравнения

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Теория дифференциально-разностных уравнений — уравнений, содержащих дифференциальные операторы и операторы сдвига, — представляет собой один из важных разделов современной теории дифференциальных уравнений.

Интерес к таким уравнениям обусловлен их многочисленными приложениями в теории управления, в механике деформируемого твёрдого тела, в релятивистской электродинамике, в моделировании колебаний кристаллической решётки, в

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исследованиях нейронных сетей и во многих других областях.

Существенные результаты в исследовании задач для дифференциально-разностных уравнений с частными производными были получены А. Л. Скубачевским, В. В. Власовым, А. Б. Муравником, В. Ж. Сакбаевым и их учениками.

Доклад посвящён исследованию вопроса разрешимости начальной задачи в полуплоскости $D = \{(x,t) : x \in \mathbb{R}, t > 0\}$ с классическими начальными условиями Коши для гиперболического уравнения со сдвигом по пространственной переменной в свободном члене:

$$u_{tt}(x,t) - a^2 u_{xx}(x,t) + b\, u(x-h,t) = 0, \quad (x,t) \in D,$$

где $a > 0$, b, $h \neq 0$ — заданные действительные числа.

С подробными результатами можно ознакомиться в работе [1].

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Асимптотика решений вольтерровых интегро-дифференциальных уравнений

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Пусть H, H_l $(l = 1, 2)$ — гильбертовы пространства, $\mathcal{L}(H, H_l)$ — банахово пространство линейных ограниченных операторов, действующих из H в H_l, $\mathcal{L}(H) := \mathcal{L}(H,H)$. Пусть $G_1 \in \mathcal{L}(H_1)$, A, G_2, C_l — плотно определённые замкнутые операторы, $A : \mathcal{D}(A) \subset H \to H$, $G_2 : \mathcal{D}(G_2) \subset H_2 \to H_2$, $C_l : \mathcal{D}(C_l) \subset H \to H_l$, причём A, G_l — самосопряжённые положительно определённые операторы.

Будем считать, что оператор-функции $C_l^* \exp(-tG_l) C_l A^{-1}$ $(l = 1, 2)$ сильно непрерывны на $[0, +\infty)$.

Отсюда следует, что $\mathcal{D}(A^{1/2}) \subset \mathcal{D}(C_l)$. Обозначим через $\omega_{G_l} := \inf\{\lambda \in \sigma(G_l)\}$, где $\sigma(G_l)$ — спектр оператора G_l, нижнюю грань оператора G_l.

Будем считать, что существуют $a_1 \in (0,1)$, $a_2 > 0$ такие, что

$$\sum_{l=1}^{2} \frac{1}{\omega_{G_l}} \|C_l u\|^2 \leqslant a_1 \|A^{1/2} u\|^2, \quad \|C_1 u\|^2 \geqslant a_2 \|A^{1/2} u\|^2 \quad \forall\, u \in \mathcal{D}(A^{1/2}) = \mathcal{D}(C_1).$$

В гильбертовом пространстве H рассмотрим задачу Коши для неполного интегро-дифференциального операторного уравнения второго порядка:

$$\frac{d^2 u}{dt^2} = -Au + \int\limits_0^t \sum_{l=1}^{2} C_l^* \exp\big(-G_l(t-s)\big) C_l u(s)\, ds + f(t), \qquad u(0) = u^0, \quad u'(0) = u^1. \tag{1}$$

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Доклад посвящён исследованию асимптотического поведения при $t \longrightarrow +\infty$ решений интегро-дифференциального уравнения (1) в случае правой части вида

$$f(t) = g(t) + \sum_{k=0}^{n} e^{-i\sigma_k t} f_k(t), \quad \sigma_0 = 0, \quad 0 \neq \sigma_k \in \mathbb{R} \quad (k = \overline{1,n}),$$

где $\|g(t)\| = o(1)$, $\|f_k'(t)\| = o(1)$ $(k = \overline{0,n})$ при $t \longrightarrow +\infty$.

Для полного интегро-дифференциального операторного уравнения вопрос асимптотического поведения решений исследован в [1].

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Оптимизация траекторий космических аппаратов с малой тягой на основе квадратичного сглаживания функционала и метода продолжения

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Доклад посвящен развитию подхода на основе метода продолжения решения основной задачи механики космического полёта с малой тягой: оптимизации управления в задачи с ограниченной тягой (ОТ-задача). Для этого используется набор необходимых условий в форме принципа максимума Понтрягина. Решение соответствующей краевой задачи ищется с помощью метода продолжения по параметру. Использование метода продолжения для решения ОТ-задачи является одним из лучших численных подходов, однако для его успешной реализации требуется хорошее начальное приближение [1].

В работе предложена новая постановка метода продолжения для ОТ-задачи, использующая её связь с модельной задачей, имеющей квадратичный функционал (типа «энергия») для получения непрерывного управления, зависящего от параметра. Введением в функционал типа «топливо» регулирующего слагаемого типа «энергия» достигается непрерывная аппроксимация релейного управления в ОТ-задаче [2, 3].

Рассмотрена модельная задача без расхода массы, имеющая такие же ограничения на управление, как и ОТ-задача, это гарантирует существование решения при любых начальных и конечных условиях со свободным временем перелёта. Полученная параметризованная задача всегда имеет решение и позволяет идентифицировать отсутствие искомого решения в ОТ-задачи во время процедуры

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продолжения. При достижении заданного значения параметра аппроксимирующее управление соответствует оптимальному релейному управлению для ОТ-задачи с наперёд заданной точностью.

В качестве примера приводятся тестовые решения модельных задач.

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Граничные условия потенциала Гельмгольца

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В конечной области $\Omega \subset \mathbb{R}^n$ с гладкой границей $\partial\Omega$ рассмотрим уравнение Гельмгольца

$$-\Delta_x u - k^2 u = f, \;\; x \in \Omega, \tag{1}$$

где k — произвольное действительное число.

Фундаментальным решением уравнения (1) назовём функцию $\varepsilon(x, ik)$, удовлетворяющую уравнению

$$-\Delta_x \varepsilon(x, ik) - k^2 \varepsilon(x, ik) = \delta(x), \quad x \in \mathbb{R}^n. \tag{2}$$

Функция $\varepsilon(x, ik)$ строится методом спуска из фундаментального решения уравнения теплопроводности со спектральным параметром

$$\frac{\partial \varepsilon(x,t)}{\partial t} - \Delta_x \varepsilon(x,t) - \lambda \varepsilon(x,t) = \delta(x,t), \quad x \in \mathbb{R}^n, \; t > 0$$

и задаётся в виде

$$\varepsilon_n(x, ik) = \varepsilon_\Delta(x) \cdot \frac{n-2}{\Gamma\left(\frac{n}{2}\right)} \left(\frac{ik}{2|x|}\right)^{\frac{n-2}{2}} K_{-\frac{n-2}{2}}\left(ik|x|\right), \tag{3}$$

где $K_{-\frac{n-2}{2}}(z)$ — функция Макдональда, а

$$\varepsilon_\Delta(x) = \frac{1}{(n-2)\,\omega_n\,|x|^{n-2}}, \quad n \geqslant 3 \tag{4}$$

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— фундаментальное решение уравнения Лапласа.

Потенциалом Гельмгольца уравнения (1) назовём следующий интегральный оператор:

$$u(x,\lambda) = \int\limits_{\Omega} \varepsilon_n(x-y, ik)\, f(y)\, dy. \tag{5}$$

Имеет место

Теорема 1. *Для любой функции* $f \in L_2(\Omega)$ *потенциал Гельмгольца* $u \in W_2^2(\Omega)$ *удовлетворяет уравнению*

$$-\Delta_x u(x) - iku(x) = f(x) \tag{6}$$

и граничному условию

$$N[u] = -\frac{u(x)}{2} + \int\limits_{\partial\Omega} \left(\frac{\partial \varepsilon_n}{\partial n_y}(x-y, ik)\, u(y) - \varepsilon_n(x-y, ik) \frac{\partial u}{\partial n_y}(y) \right) dS_y = 0, \quad x \in \partial\Omega. \tag{7}$$

Обратно, если функция $u \in W_2^2(\Omega)$ *удовлетворяет уравнению* (6) *и условию* (7), *то она совпадает с потенциалом Гельмгольца* (5).

Отметим, что при $k = 0$ теорема 1 доказана в работе Т. Ш. Кальменова, Д. Сурагана [1].

Задача (6)–(7) используется при сведении задачи Зоммерфельда излучения электромагнитических волн в $\mathbb{R}^n$ к конечной области.

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Асимптотика выхода на бегущую волну на траектории седло—узел

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Рассматривается полулинейное уравнение гиперболического либо параболического типа

$$\delta \frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial \varphi}{\partial t} + f(\varphi) = 0, \quad x \in \mathbb{R},\ t > 0; \quad \delta = \mathrm{const} \geqslant 0,$$

допускающее пару равновесных решений: $\varphi \equiv 0$ и $\varphi \equiv 1$. Строится асимптотическое решение, которое на далеких временах $t \to \infty$ выходит на бегущую волну $\varphi(x,t) \approx \Phi(x - S(t)) + \mathcal{O}(t^{-1})$ с условиями стабилизации к равновесиям

$$\Phi(s) \to 0 \ \text{ при } s \to -\infty, \quad \Phi(s) \to 1 \text{ при } s \to +\infty.$$

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Эта волна описывает динамический переход от одного равновесия к другому. Функция $\Phi(s)$ является решением обыкновенного дифференциального уравнения. Скорость такой волны $S'(t)$ зависит от времени и для нее строится асимптотика при $t \to \infty$ [1]. Выяснено, что универсальная часть асимптотики (не зависящая от начальных данных) содержит логарифмы и не может быть построена в виде степенного ряда. Этот результат принципиально отличается от известных утверждений [2].

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Задача Коши для неоднородного итерированного уравнения гиперболического типа с оператором Бесселя

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Доклад посвящен исследованию задачи Коши — нахождению решения $u(x,t) \in C^{2m-1}(\bar{\Omega}) \cap C^{2m}(\Omega)$ неоднородного уравнения высокого порядка вида

$$G_\beta^m(u) \equiv \left(\frac{\partial^2}{\partial t^2} + \frac{2\beta}{t}\frac{\partial}{\partial t} - L\right)^m u(x,t) = f(x,t), \quad (x,t) \in \Omega, \tag{1}$$

удовлетворяющего однородным начальным условиям

$$\left.\frac{\partial^k u}{\partial t^k}\right|_{t=0} = 0, \quad k = \overline{0, 2m-1}, \tag{2}$$

где $\beta \in R$, $\beta \geqslant 0$, $G_\beta^m = G_\beta^1(G_\beta^{m-1})$, $f(x,t)$ — заданная функция, L — не зависящий от t линейный эллиптический дифференциальный оператор конечного порядка, действующий по переменной $x = (x_1, x_2, ..., x_n)$, а Ω — область определения решения задачи (1), (2), зависящая от вида оператора L.

При $\beta \neq 0$ для решения задачи (1), (2) применим оператор преобразования Эрдейи—Кобера дробного порядка [1]:

$$I_{\eta,\beta} f(x) = \frac{2x^{-2(\eta+\beta)}}{\Gamma(\beta)} \int_0^x (x^2 - t^2)^{\beta-1} t^{2\eta+1} f(t)dt, \tag{3}$$

где η, $\beta \in R$, причем $\eta \geqslant -1/2$, $\beta > 0$, $\Gamma(\beta)$ — гамма-функция Эйлера.

Для оператора (3) справедлива следующая теорема [2].

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Теорема 1. *Пусть* $\beta > 0$, $\eta \geqslant -1/2$, *функции* $t^{2\eta+1}[B_\eta^t]^k u(x,t)$ *интегрируемы в окрестности точки* $t = 0$ *и* $\lim\limits_{t\to 0} t^{2\eta+1}\dfrac{\partial}{\partial t}[B_\eta^t]^k u(x,t) = 0$, $k = \overline{0, m-1}$. *Тогда имеет место равенство*

$$(B_{\eta+\beta}^t - L)^m I_{\eta,\beta}^{(t)} u(x,t) = I_{\eta,\beta}^{(t)} (B_\eta^t - L)^m u(x,t),$$

где $B_\eta^t \equiv \partial^2/\partial t^2 + [(2\eta+1)/t](\partial/\partial t)$ *— оператор Бесселя, верхний индекс* t *в операторах означает переменную, по которой действуют эти операторы.*

С применением теоремы 1 найдены точные решения задачи (1), (2) при различных значениях параметров β, n и операторах L.

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Условно-корректные граничные интегральные уравнения Фредгольма второго рода для трехмерной задачи дифракции акустических волн

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Рассматривается задача дифракции стационарных акустических волн на трехмерном однородном включении. Такая задача решается, в основном, численно, поскольку ее точное решение может быть найдено лишь в исключительных случаях.

Исходную задачу можно различными способами свести к одному граничному интегральному уравнению Фредгольма первого или второго рода с одной неизвестной функцией [1]. При этом вместо двух искомых функций в трехмерных ограниченной и неограниченной областях требуется найти вспомогательную функцию на двумерной компактной границе включения, что, очевидно, менее затратно с вычислительной точки зрения. После этого приближенное решение исходной задачи может быть найдено в любой точке пространства.

Следует учесть, что полученные интегральные уравнения могут быть некорректно разрешимы на собственных частотах, которые заранее неизвестны. Численное решение на собственных частотах обычно приводит к получению недостоверных результатов. Поэтому требуется изучить свойства интегральных уравнений для этих случаев.

Исследована пара граничных слабо сингулярных интегральных уравнений Фредгольма второго рода с сопряженными интегральными операторами. Установлено, что одно из них на собственных частотах может не иметь решения, а другое разрешимо неединственным образом. При этом существует единственное

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решение второго уравнения, которое дает решение задачи дифракции. Этот факт позволяет не находить собственные частоты, а искать приближенные решения уравнения на всех частотах методом интерполяции.

Ранее такой подход применялся при исследовании и численном решении других условно-корректных граничных интегральных уравнений для задачи дифракции. Подробно он изложен в работах [2, 3].

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Динамика дифференциального уравнения второго порядка с запаздывающей обратной связью импульсного типа

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Доклад посвящен динамике дифференциального уравнения второго порядка с импульсной запаздывающей обратной связью

$$\ddot{x} + \sigma\dot{x} + x = f(x(t-h)),$$

где $\sigma > 0$ и $h > 0$. Относительно нелинейной функции $f(x)$ предполагаем, что она имеет импульсный тип, т. е. $f(x) = 0$ при $x \neq 0$ и $\int\limits_{-a}^{b} f(x)dx = f_0$ при любых $a, b > 0$.

Сначала мы определим класс начальных условий $S(A)$, зависящий от вещественного параметра A и состоящий из функций $\varphi(t) \in \tilde{C}^1$, заданных на отрезке $[-h, 0]$, для которых $\varphi(t) \neq 0$ при $t < 0$, $\varphi(0) = 0$ и $\dot{\varphi}(0) = A$. Для каждой начальной функции из $S(A)$ методом шагов построим решение $x_A(t)$ и найдем первый его положительный корень $t_* = t_*(A) : x_A(t_*(A)) = 0$. При выполнении условия $t_*(A) > h$ можно утверждать, что $x_A(t_*(A)+t) \in S(\overline{A})$, где $\overline{A} = p(A) = \dot{x}_A(t_*(A))$. Таким образом определяется отображение $A_{n+1} = p(A_n)$, динамика которого описывает поведение решений исходного дифференциального уравнения с запаздыванием. Конкретный вид отображения и его свойства зависят от параметров σ, f_0, h.

Работа выполнена в рамках программы развития Регионального научно-образовательного математического центра Ярославского государственного универси-

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Однонаправленные импульсы

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В докладе дается обзор результатов по привлекающим в последние годы внимание локализованным решениям волнового уравнения

$$u_{tt} - c^2(u_{xx} + u_{yy} + u_{zz}) = 0, \;\; c = const > 0, \tag{1}$$

обладающим свойством однонаправленности. Оно формулируется как требование, чтобы в разложении решения по плоским волнам присутствовали только однородные плоские волны, бегущие в направлениях, составляющих с некоторым выбранным направлением угол, не превосходящий $\pi/2$.

Первоначально такие решения появились в осесимметрическом случае как интегралы Фурье—Бесселя (например, [1])

$$u = u(\rho, z, t) = \int_0^\infty d\omega e^{i\omega t} \int_0^{\omega/c} dk_z A(k_z, \omega) e^{-ik_z z} J_0(\rho\sqrt{(\omega/c)^2 - k_z^2})\,, \tag{2}$$

где $\rho = \sqrt{x^2 + y^2}$, с произвольным весом A. Простейшее однонаправленное решение было найдено из других, элементарных, соображений и имеет вид [2]

$$u = 1/s(s - z_*), \;\; s = s(t, \rho) = \sqrt{(ct_*)^2 - \rho^2}, \tag{3}$$

$z_* = z + i\zeta, t_* = t + i\tau$, ζ и τ — вещественные свободные параметры, $\zeta < c\tau$, а ветвь корня выбрана так, что $s|_{x=y=0} = ct_*$. Функция (3) нашла применения при моделировании разнообразных малоцикловых оптических импульсов [3].

В [4] отмечена важность класса относительно неискажающихся волн вида

$$u = f(s - z_*)/s, \tag{4}$$

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где форма волны f — произвольная аналитическая функция. Мы установили эквивалентность представлений осесимметрических импульсов вида (2) относительно неискажающихся волн (3) и суперпозиций плоских волн с волновыми векторами, имеющими положительные проекции на ось z, и дали еще несколько интегральных представлений [5]. При этом использовалось представление решений уравнения (1) в терминах их асимптотики в дальней зоне при больших временах. Обнаружены неожиданные примеры конечно-энергетических решений, не имеющих асимптотикой сферическую волну (например, [6]).

Работа выполнена в рамках государственного задания Министерства науки и высшего образования Российской Федерации для Института проблем машиноведения РАН (тема № 124040800009-8).

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Локальные ренормализованные решения анизотропных эллиптических уравнений

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М. Ф. Бидо-Верон [1] ввела понятие локального ренормализованного решения для уравнения с p-лапласианом, поглощением и мерой Радона μ:

$$-\Delta_p u + |u|^{p_0-2}u = \mu, \quad p \in (1, n), \quad 0 < p - 1 < p_0.$$

В монографии [2] Л. Верон обобщил понятие локального ренормализованного решения для уравнения со степенными нелинейностями вида

$$-\operatorname{div} \mathrm{a}(\mathrm{x}, \nabla u) + b(\mathrm{x}, u, \nabla u) = \mu. \tag{1}$$

В настоящей работе это понятие адаптируется к анизотропному уравнению (1)

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с переменными показателями роста и $\mu \in L_{1,\mathrm{loc}}(\Omega)$. В качестве наглядного примера приведем уравнение

$$-\sum_{i=1}^{n}(|u_{x_i}|^{p_i(\mathrm{x})-2}u_{x_i})_{x_i}+|u|^{p_0(\mathrm{x})-2}u=\mu, \quad 0<p_+(\cdot)-1<p_0(\cdot),\ p_+(\cdot)=\max_{i=\overline{1,n}}p_i(\cdot).$$

Пусть Ω — произвольная область пространства $\mathbb{R}^n$, $n \geqslant 2$. В работе найдены условия на структуру анизотропного квазилинейного эллиптического уравнения (1) с переменным ростом и $\mu \in L_{1,\mathrm{loc}}(\Omega)$, достаточные для корректного определения локального ренормализованного решения задачи Дирихле в области Ω. Автором получены локальные оценки, характеризующие регулярность решения, на их основе доказано существование решения без дополнительных ограничений на его рост на бесконечности в неограниченной области Ω. Кроме того, в терминах емкости установлены результаты об устранимой особенности локального ренормализованного решения задачи Дирихле.

В случае изотропного уравнения (1) с переменными показателями нелинейностей аналогичные результаты для задачи Дирихле и во всем пространстве $\mathbb{R}^n$ получены автором в работах [3, 4].

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Функция Грина краевых задач для уравнения теплопроводности в конусе

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Исследуются некоторые свойства функции Грина первой и второй краевых задач для уравнения теплопроводности в конусе. Для параболических уравнений с переменными коэффициентами асимптотика и оценки функции Грина краевых задач в конусе получены в [1, 2], см. также [3] и цитированную там литературу.

Пусть Q — область на единичной сфере $S^{n-1} \subset \mathbb{R}^n$ с непустой границей $\partial Q \subset C^\infty$. Обозначим через

$$K = \{(y,r)\,|\,y \in Q, 0 < r < \infty\}$$

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конус в $\mathbb{R}^n$.

В области $\Omega = K \times (0,T)$, $0 < T < \infty$, с боковой границей $\Sigma = \partial K \times [0,T]$ рассматриваются задачи

$$\begin{cases} \partial_t u - \Delta u &= f \text{ в } \Omega, \\ Bu|_\Sigma &= 0, \\ u|_{t=0} &= 0, \end{cases} \tag{1}$$

где $Bu = u$ или $Bu = \partial u/\partial \bar{n}$.

Для функции Грина задачи (1) выводится некоторое функциональное уравнение. С его помощью получено явное представление функции Грина в виде ряда. Для первой краевой задачи получены оценки функции Грина и установлена асимптотика при больших временах решений с нулевыми правой частью и граничной функцией. Как следствие найдено представление в виде ряда функции Грина задач Дирихле и Неймана в конусе K для уравнения Лапласа.

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Автоколебательные решения начально-краевой задачи для функционально-дифференциального уравнения, возникающей в механике дискретно-распределенных систем

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Для функционально-дифференциального уравнения

$$u_{tt} + (a_0 + a(|u_t|^2))u_t + (\mathrm{I} - \mu\Gamma(i\Omega))(u_{xx} + b(|u_{xx}|^2)u_{xx})_{xx} = 0,$$

где

$$(\mathrm{I} - \mu\Gamma(i\Omega))\varepsilon(t) \equiv \varepsilon(t) - \mu \int\limits_{-\infty}^{0} R(\tau)\mathrm{e}^{-i\Omega\tau}\varepsilon(t+\tau)d\tau, \quad \int\limits_{-\infty}^{0} R(\tau)d\tau = 1, \ 0 < \mu < 1, \tag{1}$$

относительно функции $u(x, t+\tau) = y(x, t+\tau) + iz(x, t+\tau)$, $i = \sqrt{-1}$ в области $\bar{Q}_+ = \{0 \leqslant x \leqslant 1, 0 \leqslant t < \infty\}$ рассматривается трехточечная начально-краевая задача с краевыми

$$u|_{x=0} = u_x|_{x=0} = 0, \ u|_{x=1} = u_x|_{x=1} = 0, u|_{x=l-0} = u|_{x=l+0}, \ u_x|_{x=l-0} = u_x|_{x=l+0},$$

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$$(\mathrm{I}-\mu\Gamma(i\Omega))(u_{xx}+b(|u_{xx}|^2)u_{xx})|_{x=l+0}^{x=l-0}=-Ju_{ttx}|_{x=l}-G_1(a_0+a(|u_{xt}|^2))u_{xt}|_{x=l},$$

$$(\mathrm{I}-\mu\Gamma(i\Omega))(u_{xx}+b(|u_{xx}|^2)u_{xx})_x|_{x=l+0}^{x=l-0}=Mu_{tt}|_{x=l}+G_2(a_0+a(|u_t|^2))u_t|_{x=l}$$

$(0<l<1)$ и начальными условиями

$$u(x,t+\tau)|_{t=0}=g_0(x,\tau)\in D(\bar{Q}_-),\ u_t(x,0)=g_1(x)\in H^2. \tag{2}$$

В формулах (1)–(2) $R(\tau)>0,\infty<\tau<0$, есть непрерывная выпуклая вниз функция (функция релаксации), функции $a(\xi)=a_1\xi+\dots$ и $b(\xi)=b_1\xi+\dots$ предполагаются бесконечно дифференцируемыми, $a_0,\Omega,M,J,G_1,G_2,a_1,b_10$ — положительные параметры, выражение $u|_{x=l+0}^{x=l-0}$ означает $u|_{x=l-0}-u|_{x=l+0}$, а $D(\bar{Q}_-)$, ($\bar{Q}_-=\{0\leqslant x\leqslant 1,\,-\infty<\tau\leqslant 0\}$) и H^2 — функциональные пространства начальных условий. Начально-краевая задача (1)–(2) является приведенной в безразмерных переменных математической моделью динимики вращающегося идеального гибкого вала постоянного круглого сечения из материала с нелинейными наследственными свойствами, концы которого опираются на подшипники, с идеальным твердым круглым диском, насаженным на вал. В (1)–(2) функция $a_0+a(\xi)$ характеризует коэффициент внешнего вязкого трения, функция $b(\xi)$ определяется нелинейной функцией деформации материала вала, M и J соответственно характеризуют массу диска и его момент инерции относительно произвольной оси, перпендикулярной средней линии вала, безразмерные коэффициенты G_1 и G_2 характеризуют геометрические свойства диска, параметр Ω — скорость вращения.

Исследуются условия устойчивости горизонтальной формы вращения, механизмы потери устойчивости, бифурцирующие при этом автоколебательные решения, их характер, устойчивость и зависимость от расположения диска.

О проекторном подходе к асимптотическому решению дискретных задач с малым шагом в критическом случае

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Посвящается светлой памяти
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Рассматриваются задачи для дискретных уравнений вида

$$x(t+\varepsilon^k)=B(t)x(t)+\varepsilon f(x(t),t,\varepsilon),\quad t=0,\varepsilon^k,2\varepsilon^k,...,\quad x(0)=x^0,$$

где $x(t)\in X, dimX<\infty$, $\varepsilon>0$ — малый параметр, $k=1,2$, матрица $A(t)=I-B(t)$ вырожденная.

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При некоторых условиях в [1] для $k = 1$ и в [2] для $k = 2$ найдены первые члены асимптотики погранслойного типа для решений рассматриваемых задач.

Использование ортогональных проекторов пространства X на $KerA(t)$ и на $KerA(t)'$, где штрих означает транспонирование, позволяет хорошо понять алгоритм построения асимптотики и представить в явном виде задачи для нахождения членов асимптотики любого порядка (см. [3] для $k = 1$ и [4] для $k = 2$). Отметим, что при $k = 1$ асимптотика имеет вид $\sum_{j=0}^{\infty} \varepsilon^j (\bar{x}_j(t) + \Pi_{1j}x(\tau_1))$, где $\tau_1 = t/\varepsilon$, $\Pi_{1j}x(\tau_1)$ — пограничные функции. Последовательность определения членов разложения следующая: $(I - P(t))\bar{x}_j$, $(I - P(0))\Pi_{1j}x$, $P(0)\Pi_{1j}x$, $P(t)\bar{x}_j$. Здесь $P(t)$ — ортогональный проектор пространства X на $KerA(t)$. Если $k = 2$, то добавляются еще пограничные функции $\Pi_{2j}x(\tau_2)$, зависящие от $\tau_2 = t/\varepsilon^2$. Последовательность определения членов разложения в этом случае следующая: $(I - P(t))\bar{x}_j$, $(I - P(0))\Pi_{1j}x$, $(I - P(0))\Pi_{2j}x$, $P(0)\Pi_{2j}x$, $P(t)\bar{x}_j(t)$, $P(0)\Pi_{1j}x$.

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Математическая модель динамики полного тороидального тока в расплаве

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На основе анализа результатов математического моделирования и новых экспериментов, проводимых на модифицированной установке [1] ВЕТА (ИЯФ СО РАН, Новосибирск), была сформулирована необходимость в уточнении и расширении существующей модели. Математическая модель для расчета термотоков без учета паров материала [2] давала максимальные амплитуды тока в области расплава, но недостаточные для вращения жидкого металла. В 2024 году был получен результат [3], показавший, что полученные из расчетов в области материала и паров над ним термотоки находятся вне области расплава. Для фокусировки возникающих термотоков потребовался учет магнитного поля, играющего большую роль в области слабо разогретого металла.

В докладе представлена новая математическая модель с учетом магнитного поля. Модель включает расчет полного тока из модифицированных уравнений

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Максвелла по значениям температуры, полученной в образце в результате решения двухфазной задачи Стефана и модельного распределения газа. Использованы выражения для электрического сопротивления и термоэдс через интеграл по энергии электронов для паров материала и экспериментальные зависимости от температуры в образце. Вращение расплава происходит под воздействием термотоков, возникающих в результате мгновенного разогрева образца до температур 6–8 тысяч Кельвинов. В задаче Стефана введен учет джоулева нагрева и эффекта Пельтье на поверхности, как результат расчета термотоков. В результате расчетов определена степень влияния токов на перераспределение тепла.

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Об успокоении системы управления нейтрального типа на временно́м дереве с глобальным сжатием

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Задача об успокоении системы управления с постоянным запаздыванием, описываемой уравнением запаздывающего типа, была поставлена и исследована Н. Н. Красовским [1]. Более трудный случай, когда уравнение содержит также старшие члены с запаздыванием, т.е. имеет нейтральный тип, был рассмотрен А. Л. Скубачевским в [2] (см. также [3]). Аналогичная задача для системы нейтрального типа, но в случае, когда запаздывание не постоянно, а пропорционально времени, была рассмотрена Л. Е. Россовским [4].

С. А. Бутерин [5,6] распространил соответствующую задачу об успокоении системы управления с постоянным запаздыванием на графы, что привело к понятию временно́го графа. В [5] был рассмотрен случай уравнения первого порядка запаздывающего типа, а в [6] — нестационарная управляемая система произвольного порядка нейтрального типа.

В [7] рассмотрена система управления с запаздыванием, пропорциональным времени, на графе-звезде в случае уравнения запаздывающего типа.

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В докладе будет рассмотрена задача об успокоении системы управления нейтрального типа с запаздыванием, пропорциональным времени, на временно́м графе, обладающем произвольной древовидной структурой. Минимизация соответствующего функционала энергии приводит к вариационной задаче. Доказывается ее эквивалентность некоторой краевой задаче для уравнений второго порядка с условиями типа Кирхгофа, а также установлена однозначная разрешимость обеих задач. Как и в работах [2–4, 6], нейтральный тип исходного уравнения приводит к понятию обобщенного решения соответствующей краевой задачи.

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О разрешимости краевой задачи с нелокальными условиями для параболического уравнения с сингулярным коэффициентом

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Рассматривается смешанная задача для параболического уравнения с сингулярным коэффициентом

$$\frac{\partial u(t,x)}{\partial t} = \frac{\partial^2 u(t,x)}{\partial x^2} + \frac{a(x)}{x}\frac{\partial u(t,x)}{\partial x} + f(t,x),\ x \in (0,1),\ t \in (0,T],$$

с краевыми условиями

$$u(t,0) = 0, \quad \int\limits_0^1 u(t,x)\,dx = 0$$

и начальным условием

$$u(0,x) = u_0.$$

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Эта задача изучалась методами теории полугрупп линейных ограниченных операторов. Получены достаточные условия на функции $a(x)$, $f(t,x)$ и начальную функцию $u_0(x)$, при выполнении которых задача имеет единственное решение.

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Представления монодромии систем Жордана–Похгаммера

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Линейные пфаффовы системы на $\mathbb{C}^n$, $n \geqslant 2$ вида

$$\mathrm{d}y(z)_i = \sum_{j=1,\, j\neq i}^{n} (\beta_j y_i(z) - \beta_i y_j(z))\, \frac{\mathrm{d}(z_i - z_j)}{z_i - z_j} \tag{1}$$

называются системами Жордана–Похгаммера с параметрами $\beta_i \in \mathbb{C}$, $i = 1, \ldots, n$. Такие системы вполне интегрируемы в смысле Фробениуса. Фундаментальная матрица решений системы $Y(z)$ имеет порядок n, а ее элементы задаются интегралами гипергеометрического типа

$$y_{ij}(z_1, \ldots, z_n) = \beta_i \int\limits_{\gamma_j} \frac{\Phi(t)}{t - z_i} \mathrm{d}t, \tag{2}$$

где $\Phi(t) = \prod\limits_{j=1}^{n} (t - z_j)^{\beta_j}$, $t \in [0, 1]$, и $\gamma_j(t)$, $j \in \{1, 2, \ldots, n\}$ — так называемые малые петли, представляющие образующие в фундаментальной группе $\pi_1(\mathbb{C} \setminus \{z_1, z_2, \ldots, z_n\})$.

Ветвление фундаментальной матрицы $Y(z)$ определяет представление монодромии системы (1), которое есть некоторое представление ρ_n группы крашеных кос P_n. Если все параметры системы (1) равны между собой, $\beta_1 = \beta = \cdots = \beta_n = \lambda$, то как доказано в работе [1], система Жордана–Похгаммера инвариантна относительно перестановок координат и ее представление монодромии ρ_n эквивалентно представлению Бурау с комплексным параметром $e^{2\pi i\lambda}$ группы кос B_n.

В случае произвольных значений параметров $\beta = (\beta_1, \ldots, \beta_n)$ осутствует инвариантность системы Жордана–Похгаммера относительно перестановок координат, и в этом случае представление монодромии ρ_n системы Жордана–Похгаммера эквивалентно представлению Гасснера с комплексными параметрами $e^{2\pi i\beta_j}$,

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$j = 1, 2, \ldots n$, группы крашеных кос P_n. Вид матриц для значений представления Гасснера на стандартных образующих A_{ij} группы крашеных кос указан в работе [4], а их реализация как матриц монодромии некоторой системы Жордана–Похгаммера с комплексных параметрами $e^{\pm 2\pi i \beta_k}$, $k = 1, \ldots, n$, указана в работе [2].

Системы Жордана–Похгаммера связаны с системами корней A_{n-1}, $n \geqslant 2$, и их деформациями. Мы рассматриваем обобщения систем Жордана–Похгаммера и утверждений об их монодромии на аналоги таких систем, связанные с другими системами корней в стиле работы [3].

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Применение локально регуляризованного экстремального сдвига в задаче реализации эталонного решения системы ОДУ

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Рассматривается управляемая система обыкновенных дифференциальных уравнений, на которую действует неизвестное возмущение. Задача заключается в создании алгоритмов формирования управлений, обеспечивающих реализацию предписанного движения при любом допустимом возмущении, а именно, обеспечивающего близость в метрике пространства непрерывных функций фазовой траектории заданной управляемой системы (а также скорости ее изменения) к эталонной траектории (а также скорости ее изменения) аналогичной системы, функционирующей в условиях отсутствия каких либо внешних воздействий. В качестве допустимых возмущений берется пространство измеримых функций, суммируемых с квадратом евклидовой нормы. Таким образом, задача заключается в отслеживании как траектории эталонной системы, так и скорости ее изменения. Рассматриваются случаи неточных измерений фазовых траекторий обеих систем как во все моменты времени их функционирования, так и в дискретные моменты времени. Указываются ориентированные на компьютерную реализацию алгоритмы решения указанной задачи. Алгоритмы основаны на известном в теории гарантированного управления методе экстремального сдвига. При этом

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осуществляется его локальная (в каждый момент коррекции управления) регуляризация по методу сглаживающего функционала (методу Тихонова). Также приводятся оценки скорости сходимости алгоритмов.

О точных оценках устойчивых решений дифференциальных уравнений с запаздыванием

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Рассмотрим автономное функционально-дифференциальное уравнение

$$\dot{x}(t) - \int_0^h x(t-s)\,dq(s) + \int_h^\tau x(t-s)\,dr(s) = f(t), \quad t \in \mathbb{R}_+, \tag{1}$$

где $0 \leqslant h \leqslant \tau$, функции $q\colon [0,h] \to \mathbb{R}_+$ и $r\colon [h,\tau] \to \mathbb{R}_+$ неубывающие, $q(0) = r(h) = 0$, интеграл понимается в смысле Римана—Стилтьеса, а f — локально суммируемая функция.

Назовем *фундаментальным решением* функцию x_0, являющуюся решением уравнения (1) при $f(t) \equiv 0$ и $x_0(0) = 1$. Без ограничения общности [1, с. 9-10] считаем начальную функцию частью внешнего возмущения f.

Для экспоненциально устойчивых уравнений вида (1) предлагается эффективный метод получения двусторонних оценок фундаментального решения.

Метод позволяет с произвольной точностью найти как показатель, так и коэффициент экспоненциальной оценки решения. Основу метода составляет априорное предположение о положительности фундаментального решения с последующим полным описанием его свойств на полуоси.

Обозначим $F(\lambda) = \lambda + a + \int\limits_0^h e^{-\lambda s}\,dr(s)$, $\lambda \in \mathbb{R}$.

Теорема (см. [2])**.** *Пусть* $(-\omega)$ *— наибольший действительный корень функции* F, $\omega > 0$, *и* $F'(-\omega) \neq 0$. *Тогда фундаментальное решение уравнения* (1) *имеет двустороннюю оценку*

$$e^{-\omega t} \leqslant x_0(t) \leqslant \frac{1}{F'(-\omega)} e^{-\omega t}, \quad t \geqslant 0.$$

Замечание. Поскольку $\lim\limits_{t\to+\infty} x_0(t)e^{\omega t} = \frac{1}{F'(0)}$ и $x_0(0) = 1$, константы 1 и $\frac{1}{F'(-\omega)}$ в оценке точные.

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О единственности решения третьей краевой задачи по времени для неоднородного уравнения четвертого порядка

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Рассмотрим уравнение четвертого порядка вида

$$L(u) \equiv u_{xxxx} + a_1(x)u_x + a_2(x)u - u_{yy} = f(x,y), \tag{1}$$

где $f_{xyy}(x,y) \in C(\bar{\Omega})$.

Для уравнения (1) в области $\Omega = \{(x,y): \ 0 < x < p, \ 0 < y < q, \ p,q \in \mathbb{R}\}$ ставится следующая задача.

Задача 1. Найти функцию $u(x,y)$ из класса $C^{4,2}_{x,y}(\Omega) \cap C^{3,1}_{x,y}(\bar{\Omega})$, удовлетворяющую уравнению (1) в области Ω и следующим краевым условиям:

$$au(x,0) + bu_y(x,0) = 0, \quad cu(x,q) + du_y(x,q) = 0, \quad 0 \leqslant x \leqslant p,$$

$$\left.\frac{\partial^m}{\partial x^m}u\right|_{x=0} = \psi_{m-1}(y), \quad \left.\frac{\partial^m}{\partial x^m}u\right|_{x=p} = \psi_{m+1}(y), \quad m = 2,3, \quad 0 \leqslant y \leqslant q,$$

где $\psi_i(y), \ i = \overline{1,4}$ — заданные достаточно гладкие функции.

Результаты исследования краевых задач для уравнения (1) с постоянными и переменными коэффициентами при наличии второй производной по x в некоторых других частных случаях можно найти в [1–3].

Теорема 1. *Если задача* (1) *имеет решение, то при выполнении условий $ab \geqslant 0$, $cd \leqslant 0$, $a_1(p) \geqslant 0$, $a_1(0) \leqslant 0$ и $2a_2(x) - a_1'(x) \geqslant 0$ оно единственно.*

Доказательство. Предположим обратное: пусть задача (1) имеет два решения $u_1(x,y)$ и $u_2(x,y)$. Тогда функция $u(x,y) = u_1(x,y) - u_2(x,y)$ удовлетворяет однородному уравнению (1) и однородным краевым условиям. Докажем, что $u(x,y) \equiv 0$ в $\bar{\Omega}$. В области Ω справедливо тождество

$$uL(u) \equiv \frac{\partial}{\partial x}\left(uu_{xxx} - u_x u_{xx} + \frac{1}{2}a_1(x)u^2\right) - \frac{\partial}{\partial y}(uu_y) + \\ + u_{xx}^2 + \left(a_2(x) - \frac{1}{2}a_1'(x)\right)u^2 + u_y^2 = 0. \tag{2}$$

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Интегрируя тождество (2) по области Ω и учитывая однородные краевые условия, получим

$$\int\limits_0^p\int\limits_0^q u_{xx}^2(x,y)dxdy+\frac{1}{2}\int\limits_0^p\int\limits_0^q (2a_2(x)-a_1'(x))u^2(x,y)dxdy+$$
$$+\int\limits_0^p\int\limits_0^q u_y^2(x,y)dxdy+\frac{a}{b}\int\limits_0^p u^2(x,0)dx-\frac{c}{d}\int\limits_0^p u^2(x,q)dx+$$
$$+\frac{1}{2}a_1(p)\int\limits_0^q u^2(p,y)dy-\frac{1}{2}a_1(0)\int\limits_0^q u^2(0,y)dy=0.$$

При $2a_2(x)-a_1'(x)>0$ из второго интеграла имеем $u(x,y)\equiv 0$, $(x,y)\in\bar{\Omega}$. Если $2a_2(x)-a_1'(x)=0$, то из третьего интеграла получим $u(x,y)=h(x)$, а из первого интеграла получим $u(x,y)=xb_1(y)+b_2(y)$. Отсюда вывод, что $u=xb_1+b_2$, $b_1,b_2=const$. Из седьмого интеграла имеем $u(0,y)=b_2=0$, а шестой интеграл дает $u(p,y)=pb_1=0$. Отсюда получим $u(x,y)\equiv 0$, $(x,y)\in\bar{\Omega}$. Теорема 1 доказана. □

Замечание 1. Отметим, что при нарушении условий теоремы 1 однородная задача 1 для однородного уравнения (1) может иметь нетривиальные решения. Например, когда $a_1=1$ и $a_2=-\lambda_n<0$, задача

$$\begin{cases} u_{xxxx}+u_x-\lambda_n u-u_{yy}=0\\ au(x,0)+bu_y(x,0)=0, \quad cu(x,q)+du_y(x,q)=0,\\ u_{xx}(0,y)=u_{xx}(p,y)=u_{xxx}(0,y)=u_{xxx}(p,y)=0,\end{cases}$$

имеет нетривиальные решения вида

$$u_n(x,y)=-Y_n(y), \quad n\in N,$$

где λ_n, $Y_n(y)$ соответственно собственные значения и собственные функции задачи

$$\begin{cases} Y''(y)+\lambda Y(y)=0,\\ aY(0)+bY'(0)=0,\\ cY(q)+dY'(q)=0,\end{cases}$$

$$Y_n(y)=a\sin(\sqrt{\lambda_n}y)-b\sqrt{\lambda_n}\cos(\sqrt{\lambda_n}y), \quad \lambda_n=O\left(n^2\right), \quad n\in N.$$

Существование решения задачи 1 доказывается методом Фурье и с использованием функции Грина.

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Оценка сферической производной мероморфных решений алгебраических дифференциальных уравнений

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Объектом нашего внимание будут мероморфные решения алгебраических дифференциальных уравнений N-го порядка в единичном круге $\mathbb{D} = \{|z| < 1\}$. Одной из характеристик, описывающей поведение мероморфной функции f, является рост ее сферической производной $f^{\sharp}(z) = |f'(z)|/(1 + |f(z)|^2)$. В частности, это свойство используется при определении некоторых классов мероморфных функций (нормальные функции в смысле Лехто—Виртанена, классы типа Дирихле, класс Цудзи), в формуле Альфорса—Симидзу для характеристики Неванлинны, и применяется при изучении распределения значений и граничного поведения функций.

Пусть $n, N \in \mathbb{N}$. Рассмотрим уравнение N-го порядка

$$\left(f^{(N)}\right)^n + \sum_{k=1}^{n} P_{k,N}\left(f\right)\left(f^{(N)}\right)^{n-k} = 0, \tag{1}$$

где

$$P_{k,N}(f) = \sum_{j_0=0}^{m_{k,0}} \sum_{j_1=0}^{m_{k,1}} \cdots \sum_{j_{N-1}=0}^{m_{k,N-1}} a_{k,j_0,\ldots,j_{N-1}} \prod_{\ell=0}^{N-1} \left(f^{(\ell)}\right)^{j_\ell}, \quad k = 1, \ldots, n,$$

и $a_{k,j_0,\ldots,j_{N-1}}$ — аналитические функции в $\mathbb{D}$.

Остановимся на случае, когда $N = 1$. Если $\max\limits_{k=1,\ldots,n} \frac{m_k}{k} \leqslant M_0 + 2$, $M_0 \geqslant 0$, то каждое мероморфное решение f уравнения (1) удовлетворяет оценке

$$\left(f^{M_0+1}\right)^{\#} \lesssim \sum_{k=1}^{n} \sum_{j=0}^{m_k} |a_{k,j}|^{\frac{1}{k}}.$$

(Исследования в случае уравнений произвольного порядка N приведены в совместной работе с профессором Jouni Rättyä и доктором Tony Vesikko.)

Зная поточечную оценку роста сферической производной m-ой степени f, можно получить аналогичное свойство самой функции f.

Приведем некоторые из полученных результатов.

Весовой класс Дирихле $\mathcal{D}^{\#}_{\omega^\star}$ (радиальный вес) состоит из мероморфных функций f в $\mathbb{D}$, которые удовлетворяют условию

$$\int_{\mathbb{D}} f^{\#}(z)^2 \omega^\star(z)\, dA(z) < \infty.$$

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При условии $\omega^\star(z) = 1$ получается класс Дирихле.

Класс $\mathcal{N}^\alpha$ известен как класс α-нормальных функций, $0 < \alpha < \infty$. Эти функции удовлетворяют условию

$$\|f\|_{\mathcal{N}^\alpha} = \sup_{z\in\mathbb{D}} f^{\#}(z)(1-|z|^2)^\alpha < \infty.$$

При $\alpha = 1$ получается класс нормальных функций.

Пусть $m \in \mathbb{N} \setminus \{1\}$. Если $f^m \in X$, где X — один из упомянутых классов мероморных функций, то $f \in X$.

Ранее оценка роста сферической производной мероморных решений уравнения (1) получалась автором и другими математиками только при условии быстрого роста сферической производной с применением метода Зальцмана.

Об асимптотике решения задачи Коши для сингулярно возмущенной системы гиперболических уравнений в критическом случае

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Строятся первые члены формального асимптотического разложения (АР) решения задачи Коши для сингулярно возмущенной системы гиперболических уравнений в критическом случае [1]

$$\left\{\begin{array}{lcl} \varepsilon^6(u_{tt} - k_1^2 u_{xx}) & = & -p(au-bv) - \varepsilon^3 q(au-bv)_t - \varepsilon^3 f(u,v), \\ \varepsilon^6(v_{tt} - k_2^2 v_{xx}) & = & p(au-bv) + \varepsilon^3 q(au-bv)_t + \varepsilon^3 f(u,v), \\ |x| < \infty, t > 0 \end{array}\right. \tag{1}$$

$(k_1 \neq k_2)$ с начальными условиями специального вида

$$\left\{\begin{array}{lclllcl} u(x,0) & = & u^0(x/\varepsilon^2), & u_t(x,0) & = & \varphi(x/\varepsilon^2), \\ v(x,0) & = & v^0(x/\varepsilon^2), & v_t(x,0) & = & \psi(x/\varepsilon^2). \end{array}\right. \tag{2}$$

В (1) $0 < \varepsilon \ll 1$ — малый параметр, $a > 0$, $b > 0$, $p > 0$, $q > 0$, $a, b, p, q = const$, $f(u,v)$ определена и достаточно гладкая в области $\Omega = \{|u| < G, |v| < G, G > 0\}$.

Все функции, входящие в правые части условий (2), удовлетворяют ограничениям вида $\left|u^{(k)}(z)\right| \leqslant Ce^{-\sigma z^2}$,$C, \sigma > 0, k = 0, 1, 2, ...$ и некоторым дополнительным условиям. Работа является продолжением работ [2–4], в которых строились АР решений задачи без диссипации [2, 3] или без нелинейности [4]. Построенное АР имеет вид

$$\begin{pmatrix} u(x,t,\varepsilon) \\ v(x,t,\varepsilon) \end{pmatrix} = \begin{pmatrix} S_0^I u(\zeta_1,t) + S_0^{II} u(\zeta_2,t) + \Pi_0 u(\xi,\tau) + Ru \\ S_0^I v(\zeta_1,t) + S_0^{II} v(\zeta_2,t) + \Pi_0 v(\xi,\tau) + Rv \end{pmatrix} \tag{3}$$

В (3) $\zeta_{1,2} = \dfrac{x \mp kt}{\varepsilon^2}, \tau = t/\varepsilon^3, \xi = x/\varepsilon^2$, k выражается через k_1, k_2, a, b, p, q. Главные слагаемые в АР (3) $S_0^J u(\zeta_j,t), S_0^J v(\zeta_j,t)$, $J = I, II$, $j = 1, 2$, имеют вид

$$S_0^J v(\zeta_j,t) = a/b S_0^J u(\zeta_j,t), \quad J = I, II, \quad j = 1, 2, \tag{4}$$

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где $S_0^J u(\zeta_j, t)$ есть решения начальных задач для обобщенных уравнений Кортевега–де Фриза

$$-S_0^J u_t + K S_0^J u_{\zeta\zeta\zeta} + (h(S_0 u))_\zeta = 0, \quad J = I, II, \quad \zeta = \zeta_j, \quad j = 1, 2 \tag{5}$$

с быстро убывающими начальными условиями, исследованных в многочисленных работах, в частности [5]. В (5) $K, h(z)$ выражаются через данные задачи (1). $\Pi_0 u(\xi, \tau), \Pi_0 v(\xi, \tau)$ определяются как решение начальной задачи для системы ОДУ и экспоненциально убывают по переменной τ.

Оценка остаточных членов Ru, Rv в АР (3) дана по невязке.

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Квадратичный и кубический инварианты в волновых дифференциальных уравнениях гидродинамики, электродинамики и теории упругости

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В теории деформаций, являющейся основой для вывода уравнения неразрывности $\operatorname{div} U = 0$, не придано важного значения временно́му закону деформации — лагранжеву закону движения жидкой частицы. При линейном лагранжевом законе движения жидкой частицы в уравнении неразрывности и в соответствующем волновом уравнении возникают квадратичный I_2 и кубичный I_3 инварианты тензора скоростей деформаций или тензора деформаций. При экспоненциальном лагранжевом законе $I_2 = 0$ и $I_3 = 0$. В Санкт-Петербурге произошло неожиданное обрушение спортивно-концертного комплекса после разрезки

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одной из сотни стяжек, удерживающих потолок-крышу, из-за недоучета колебаний. Академик Седов Л. И. в учебнике «Механика сплошной среды» указал, что точное значение коэффициента кубического расширения θ для упругого тела в теории упругости должно считаться по формуле, содержащей три инварианта: $\theta = (1 + 2I_1 + 4I_2 + 8I_3)^{0,5} - 1$, но все учебники используют упрощение $\theta \approx I_1 = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$, где u, v, w — перемещения вдоль осей x, y, z, не учитывающее деформаций сдвига. В точном подходе правая часть равенства с инвариантами поступает в неоднородную правую часть волнового уравнения и дает описание генерации вибраций, звука автоколебаний

$$G\nabla^2 u - \rho\frac{\partial^2 u}{\partial t^2} = -X - (\lambda + G)\frac{\partial}{\partial x}\left[(1 + 2I_1 + 4I_2 + 8I_3)^{0,5} - 1\right].$$

В системе уравнений электродинамики Максвелла, построенной по подобию гидродинамических линий тока, оператор дивергенции в уравнении для напряженности магнитного поля $\operatorname{div}\mathbf{H} = 0$ имеет основания тоже быть дополненным вторым и третьим инвариантами. Тогда магнитное поле будет генерировать дополнительные, не учтенные существующей электродинамикой волны. Если их учесть, то аппарат МРТ будет давать врачу более точные и понятные результаты обследования больного. Волновое уравнение для напряженности магнитного поля для компоненты по оси x будет иметь вид

$$\begin{aligned}\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} - \varepsilon\varepsilon_0\mu\mu_0\frac{\partial^2 H_x}{\partial t^2} = \\ = \frac{\partial J_y}{\partial z} - \frac{\partial J_z}{\partial y} - \frac{(t - t_0)}{\tau q}\frac{\partial}{\partial x}\left[\frac{\partial(H_x, H_y)}{\partial(x, y)} + \frac{\partial(H_y, H_z)}{\partial(y, z)} + \frac{\partial(H_z, H_x)}{\partial(z, x)}\right] - \\ - \left[\frac{(t - t_0)}{\tau q}\right]^2\frac{\partial}{\partial x}\left[\frac{\partial(H_x, H_y, H_z)}{\partial(x, y, z)}\right].\end{aligned}$$

Высшие (квадратичный и кубичный) инварианты тензора скоростей деформаций и тензора деформаций описывают аварийные ситуации, которые возникают из-за генерации периодических волн и уединенной волны типа гидравлического удара Жуковского. Теоретические основы их возникновения достаточно полно проработаны и пора проводить массовые расчеты с использованием высших инвариантов.

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Принцип неопределенности для разных систем координат

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Уравнение аналитической функции $z = f(p)$ рассматривается в новой системе координат с центром в точке $(A, 0)$ для новой комплексной переменной r, если $p = x + iy, A + r = p$, константа $A \neq 0$ действительна, G — область на комплексной плоскости. Второй способ восприятия уравнения $z = f(p + A)$ вытекает из равенства $z = f(r + A)$ при всех $p = r$. При том же z используем замену $r + A = p$ для исходных переменных p, r, в которой $r = p$. Следовательно, это же уравнение совпадает с исходным уравнением $z = f(p)$ с точки зрения одинаковых обратных к $f(p + a)$ и $f(r + A)$ функций $f^{-1}(z) = p = r \in G$ [1, 2]. То же самое получаем в выражении $z = h(s) = h(s - A + A)$ после замены $h(s - A + A) = f(p + A)$ с помощью равенства $h_0(s - A) = f_0(p)$, выполненного при всех $p = s$ для одного многообразия M_0, заданного уравнениями $z = f_0(p)$ и $z = h_0(s)$ в соседних системах координат (при совпадении концов радиус векторов $\overline{p}, \overline{s} - A$); здесь $z = f_0(p) = f(p + A)$ — уравнение сдвинутого влево на $A > 0$ многообразия M. Аналогичный факт основан на следующем рассуждении.

Рассмотрим отображение точек плоскости $(x, y) \to z$, совпадающее всегда с неподвижным M во второй и третьей системе координат, где, по определению, во второй системе координат комплексная ось ix направлена вдоль исходной оси OX, а действительная ось y направлена вдоль исходной оси OY, $p = x + iy \in G$; в третьей системе координат координаты x и y поменялись местами, (ось iy стала осью ix, ось x стала осью y, комплексное i осталось на месте). Два сопоставления неподвижных точек z многообразия M переменной $x + iy$ и переменной $ix + y$ при тех же действительных (x, y) в одной второй системе координат имеют разные аналитические выражения [2], и не могут одновременно совпадать с исходной аналитической функцией $z = f(x + iy) = u(x, y) + v(x, y)i$, (с точки зрения действительных функций u, v второе сопоставление совпадает с исходным $z = u + iv$). Значение новой функции-поля в точках $ix + y$ совпадает со значением исходной функции f в точке $x + iy$ во второй системе координат, так как вторая система координат является результатом поворота третьей системы (с уравнением исходного многообразия $z = f(y + ix)$) на угол $\pi/2$ против часовой стрелки и изменения направления новой оси iOX (направленной вдоль исходной оси OX в отрицательную сторону), на противоположное; в результате этих действий уравнение неподвижного исходного многообразия (графика) во второй системе координат совпадает с исходным уравнением $z = f(p) = u + iv$.

Уравнение $z = f(4A - p)$ отраженного отосительно точки $p = 2A$ многообразия M совпадает также с уравненем $z = h(2A - r) = f(2A - p)$ (как значение в точке).

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Об аппроксимации решений дифференциальных включений дробного порядка

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Доклад посвящен исследованию аппроксимации решений дифференциальных включений в банаховых пространствах следующего вида:

$$ {}^{GC}D_0^{\alpha} x(t) \in Ax(t) + F(t, x(t)), \qquad t \in [0, a], \tag{1} $$

где ${}^{GC}D_0^{\alpha}$ — дробная производная Герасимова–Капуто порядка $\alpha \in (1,2)$, а $A : D(A) \subset E \to E$ — линейный оператор в банаховом пространстве E, порождающий равномерно ограниченное семейство косинус-оператор функций $\{C(t)\}_{t\geqslant 0}$. Предполагается, что $F : [0,a] \times E \multimap E$ — многозначное отображение типа Каратеодори.

Исследование выполнено при финансовой поддержке Российского научного фонда в рамках научного проекта № 22-71-10008.

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Затенение в окрестности гиперболической стационарной точки

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Доклад посвящен поведению траекторий абстрактных параболических задач в случае уравнения с дробной производной по Капуто–Джрбашяну. Специфика состоит в том, что отсутствует экспоненциальное убывание семейств операторов и расположение спектра нельзя корректировать противоположным течением времени; кроме того, спектр линеаризированного оператора выглядит иначе, чем при классической картине. Тем не менее, удается доказать аналоги результатов по затенению [1]. А именно, доказана, например, следующая теорема.

Теорема 1 (О затенении)**.** *Пусть оператор A порождает экспоненциально убывающую аналитическую C_0-полугруппу и $0 \leqslant \beta < \gamma < 1$. Пусть резольвенты операторов A_n, A компактны, сходятся компактно и найдутся такие константы $M \geqslant 1$, $0 < \varphi \leqslant \pi/2$ и $\omega \in \mathbb{R}$, не зависящие от n, что сектор $\omega + \Sigma_{\varphi+\pi/2}$ содержится в $\rho(A_n)$ и $\left\|\lambda^{\alpha-1}\big(\lambda^\alpha I_n - A_n\big)^{-1}\right\|_{B(E_n)} \leqslant \frac{M}{|\lambda-\omega|}$, $\quad \lambda \in \omega + \Sigma_{\psi+\pi/2}$, для любых $n \in \mathbb{N}$ и $0 < \psi < \varphi$. Пусть также для любого $\varepsilon > 0$ найдется $\delta > 0$ такое, что $\|f'(w) - f'(z)\|_{B(E^\beta,E)} \leqslant \varepsilon$ при $\|w - z\|_{E^\beta} \leqslant \delta$ для всех $w, z \in \mathcal{U}_{E^\beta}(u^*;\rho)$, где u^* — гиперболическая стационарная точка задачи*

$$\mathbf{D}_t^\alpha u(t) = Au(t) + f(u(t)),\ t \geqslant 0, u(0) = u^0 \in E^\beta; \tag{1}$$

для любого $\varepsilon > 0$ найдется такое $\delta > 0$, что $\|f_n'(w_n) - f_n'(z_n)\|_{B(E_n^\beta,E_n)} \leqslant \varepsilon$ при $\|w_n - z_n\|_{E_n^\beta} \leqslant \delta$ для всех $w_n, z_n \in \mathcal{U}_{E_n^\beta}(u_n^*;\delta)$, где u_n^* – гиперболические стационарные точки задачи

$$\mathbf{D}_t^\alpha u_n(t) = A_n u_n(t) + f_n(u_n(t)),\ t \geqslant 0, u_n(0) = u_n^0 \in E_n^\gamma; \tag{2}$$

отображения $f_n(\cdot)$ непрерывно дифференцируемы в $\mathcal{U}_{E_n^\beta}(p_n^\beta u^*, \rho)$ и как только $x_n \in \mathcal{U}_{E_n^\beta}(p_n^\beta u^*, \rho)$ и $x_n \xrightarrow{\mathcal{P}^\beta} x$, то $f_n(x_n) \xrightarrow{\mathcal{P}} f(x)$ и $f_n'(x_n) \xrightarrow{\mathcal{P}^\beta\mathcal{P}} f'(x)$. Тогда найдется такое $\rho_0 > 0$, что для любого $\varepsilon_0 > 0$ существует число $n_0 = n_0(\varepsilon_0) \in \mathbb{N}$ такое, что для любых обобщенных решений $u_n(t)$, $n \geqslant n_0$, задачи (2), удовлетворяющих $u_n(t) \in \mathcal{U}_{E_n^\gamma}(u_n^*, \rho_0)$, $0 \leqslant t \leqslant T$ для некоторого $0 < T \leqslant \infty$, существуют такие начальные условия $u^{n,0} \in E^\beta, n \geqslant n_0$, что обобщенные решения $u(t; u^{n,0})$ задачи (1) существуют на $[0, T]$ и удовлетворяют

$$\sup_{0\leqslant t\leqslant T} \|u_n(t) - p_n^\beta u(t; u^{n,0})\|_{E_n^\beta} \leqslant \varepsilon_0 \quad \forall n \geqslant n_0(\varepsilon_0).$$

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О восстановлении решения сингулярного уравнения колебаний

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Рассмотрим начально-краевую задачу

$$B_{t,\vartheta}\, u = B_{x,\gamma}\, u,\ x \in (0,1),\ t > 0, \tag{1}$$

$$\frac{\partial u}{\partial x}(+0,t) = 0,\ u(1,t) = 0, \frac{\partial u}{\partial t}(x,+0) = 0,\ u(x,+0) = \varphi(x), \tag{2}$$

где $B_{x,\gamma} = \partial^2/\partial x^2 + \gamma\, x^{-1}\partial/\partial x$ — оператор Бесселя, $\gamma \geqslant 0$, $\vartheta > 0$.

Следуя [1], рассмотрим оператор F_δ^N, который любой функции φ из пространства Киприянова $W^n_{2,\gamma}$ ставит в соответствие вектор приближенно вычисленных значений первых N коэффициентов в разложении в ряд Фурье–Бесселя (при $\gamma > 0$) или Фурье (при $\gamma = 0$) функции φ и поставим задачу при этих условиях восстановить решение задачи (1),(2) в момент времени $T > 0$ в пространстве $W^n_{2,\gamma}$.

Получен [2] аналог теоремы [1] об оптимальном восстановлении решения задачи (1),(2).

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Оптимальное управление для уравнений дробного порядка, изменяющегося со временем

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Доклад посвящён исследованию возможности применения метода моментов для задачи оптимального управления системами дробного порядка, порядок которых меняется со временем.

В статье [1] были предложены следующие определения производной и интеграла переменного дробного порядка $\alpha(t)$ от некоторой функции $f(t) \in L_1[0, T]$:

$$ {}_0D_t^{\alpha(t)} f(t) = \frac{d}{dt} \int\limits_0^t \phi_\alpha(t-\tau) f(\tau) d\tau - \phi_\alpha(t) f(0), t \in [0, T], \tag{1} $$

$$ {}_0I_t^{\alpha(t)} f(t) = \int\limits_0^t \psi_\alpha(t-\tau) f(\tau) d\tau, t \in [0, T], \tag{2} $$

где $\phi_\alpha(t) = \mathcal{L}^{-1}[s^{sA(s)-1}](t)$, $\psi_\alpha(t) = \mathcal{L}^{-1}[s^{-sA(s)}](t)$, $A(s) = \mathcal{L}[\alpha(t)](s)$; $\mathcal{L}$ и $\mathcal{L}^{-1}$ — операторы прямого и обратного преобразования Лапласа.

В докладе рассматривается линейная стационарная система дробного порядка, которая описывается уравнениями (по повторяющимся индексам подразумевается суммирование)

$$ {}_0D_t^{\alpha_i(t)} q_i(t) = a_{ij} q_j(t) + b_i u_i(t), \quad t \in [0, T], \quad i, j = 1, ..., N, \tag{3} $$

с начальными и конечными условиями $q_i(0) = q_i^0$, $q_i(T) = q_i^T$.

Для системы (3) исследуется задача оптимального управления в форме задачи поиска управления с минимальной нормой при заданном времени управления аналогично работе [2]. Данная задача сведена к l-проблеме моментов, для которой исследована корректность и разрешимость.

Теорема 1. *Пусть задана система* (3) *при* $a_{ij} = \delta_{i+1,j}$, $b_i = \delta_{iN}$ *(цепочка интеграторов) и пусть* $u(\tau) \in L_p[0, T]$, $p > 1$. *Тогда l-проблема моментов для данной системы будет корректна и разрешима, если* $\forall i$ *определена норма функций* $\psi(\alpha_i + \dots + \alpha_N)$ *в пространстве* $L_{p'}[0, T]$, $\frac{1}{p} + \frac{1}{p'} = 1$.

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Математическое моделирование температурных эффектов на распространение вирусной инфекции

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Экспериментально было показано, что распространение респираторной вирусной инфекции можно описать уравнением диффузии [1]. В данной работе представлена математическая модель с учетом роли температуры в процессе распространения инфекции. Подобная задача была рассмотрена в [2], где исследовалась модель без учета иммунного ответа, тогда как в работах [3, 4] были проанализированы модели с включением как врожденного, так и адаптивного иммунного ответа. Описываемая система выглядит следующим образом:

$$\begin{cases} \frac{\partial U}{\partial t} &= \lambda(U_0 - U) - a(T)UV, \\ \frac{\partial I}{\partial t} &= a(T)UV - \beta I, \\ \frac{\partial V}{\partial t} &= D\frac{\partial^2 V}{\partial x^2} + b(T)I(t-\tau) - \sigma(T)V, \\ \frac{dT}{dt} &= \alpha_T(T_s(V) - T), \end{cases} \tag{1}$$

где U — концентрация неинфицированных клеток, I — концентрация инфицированных клеток, V — концентрация вируса, T — температура тела, T_s — значение «нормальной» температуры.

Определяющими характеристиками моделей такого типа, имеющими физиологическое значение, являются скорость распространения волны (вирулентность человека) и полная вирусная нагрузка (тяжесть заболевания). В работе оцениваются эти характеристики и исследуется их зависимость от параметров температуры.

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Принцип излучения для двумерной акустической дифракционной решетки

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Рассматривается двумерная отражательная дифракционная решетка, занимающая «полуплоскость» с гладкой периодической границей и описываемая неоднородной эллиптической краевой задачей для стационарного уравнения акустики с гладкими коэффициентами, периодически меняющимися вдоль границы решетки с тем же периодом. Предполагается, что эта задача самосопряжена относительно формулы Грина, а ее коэффициенты стабилизируются на бесконечном удалении от границы решетки к гладким периодическим функциям.

Принципом излучения мы называем теорему о существовании решения краевой задачи в решетке, главная часть асимптотики которого содержит только приходящие или только уходящие волны. В докладе обсуждается определение приходящих и уходящих волн в решетке, формулируется принцип излучения и вычисляются амплитуды волн в асимптотике решения.

Работа поддержана Российским Научным Фондом, грант 22-21-00136.

Усреднение нелокального несамосопряженного оператора сверточного типа при учете корректора

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Рассмотрим нелокальный оператор сверточного типа

$$(\mathbb{A}_\varepsilon u)(x) := \varepsilon^{-d-2} \int_{\mathbb{R}^d} a((x-y)/\varepsilon)\mu(x/\varepsilon, y/\varepsilon)(u(x)-u(y))\,dy, \quad x \in \mathbb{R}^d, \quad u \in L_2(\mathbb{R}^d).$$

Предполагается, что $a(x)$ — неотрицательная функция класса $L_1(\mathbb{R}^d)$, $\|a\|_{L_1} = 1$; $\mu(x, y)$ — ограниченная отделенная от нуля функция, $\mathbb{Z}^d$-периодическая по каждой переменной; предполагаются конечными моменты $M_k = \int_{\mathbb{R}^d} |x|^k a(x)dx$, $k = 1, 2, 3, 4$. Самосопряженность оператора $\mathbb{A}_\varepsilon$, $\varepsilon > 0$, не предполагается. Оператор $\mathbb{A}_\varepsilon$, $\varepsilon > 0$, (см., например, [1]) ограничен, спектр $\sigma(\mathbb{A}_\varepsilon)$ расположен в правой полуплоскости. В [1] доказана оценка

$$\|(\mathbb{A}_\varepsilon + I)^{-1} - (\mathbb{A}^0 + \varepsilon^{-1}\langle \alpha, \nabla\rangle + I)^{-1}\|_{L_2(\mathbb{R}^d)\to L_2(\mathbb{R}^d)} \leqslant C(a, \mu)\varepsilon, \quad \varepsilon > 0.$$

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Здесь $\mathbb{A}^0 = -\operatorname{div} g^0\nabla$, α — постоянный вектор, g^0 — постоянная положительно определенная матрица, заданные в терминах решений некоторых вспомогательных задач на ячейке $[0,1)^d$.

Наш основной результат представляет следующая теорема.

Теорема. *Справедлива оценка*

$$\|(\mathbb{A}_\varepsilon + I)^{-1} - (\mathbb{A}^0 + \varepsilon^{-1}\langle\alpha,\nabla\rangle + I)^{-1} - \varepsilon\mathbb{K}(\varepsilon)\|_{L_2(\mathbb{R}^d)\to L_2(\mathbb{R}^d)} \leqslant S(a,\mu)\varepsilon^2, \quad \varepsilon > 0.$$

Здесь $\mathbb{K}(\varepsilon)$ — подходящий «корректор», определенный в терминах решений некоторых вспомогательных задач на ячейке.

Метод исследования опирается на теоретико-операторный подход, развитый М. Ш. Бирманом и Т. А. Суслиной и адаптированный к оператору $\mathbb{A}_\varepsilon$ в работе [2].

Исследование В. А. Слоуща и Т. А. Суслиной поддержано РНФ (проект 22-11-00092-П).

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Задача Коши для функциональных включений с каузальными операторами в банаховых пространствах

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Доклад посвящен задаче Коши для функциональных включений, содержащих сумму каузальных однозначных операторов $\mathcal{G}_i, i = 1, ..., n-1$ и композицию многозначного каузального оператора $\mathcal{Q}$ и линейного каузального оператора $\mathcal{S}$ в банаховом пространстве E, следующего вида:

$$x(t) \in \mathcal{G}_0(t)x_0 + \mathcal{G}_1(t)x_1 + \cdots + \mathcal{G}_{n-1}(t)x_{n-1} + \mathcal{S}\circ\mathcal{Q}(x)(t),\ t \in [0,T] \tag{1}$$

$$x(0) = x_0, \quad x'(0) = x_1, \quad \cdots, \quad x^{(n-1)}(0) = x_{n-1}. \tag{2}$$

Здесь $\mathcal{G}_0(\cdot)$ для каждого $t \in [0,T]$ является линейным ограниченным сильно непрерывным оператором, а операторы $\mathcal{G}_i, i = 1, ..., n-1$ удовлетворяют рекуррентной формуле

$$\mathcal{G}_i(t) = \int_0^t \mathcal{G}_{i-1}\,(s)ds.$$

Такие функциональные включения обобщают задачу Коши для полулинейных дифференциальных уравнений и включений произвольного порядка n, а

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также задачу типа Коши для включений и уравнений дробного порядка, не превышающего n.

Для решения задачи (1)–(2) применяется теория топологической степени для многозначных уплотняющих отображений, строится разрешающий многозначный оператор в пространстве непрерывных функций, соответствующих задаче. Опираясь на свойства разрешающего оператора, мы доказываем теорему о существовании решений задачи (1)–(2).

Теорема 1. *При выполнении указанных выше, а также некоторых дополнительных условий, множество решений задачи* (1)–(2) *непусто и компактно.*

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Об устойчивости бегущих волн

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Я собираюсь обсудить один бифуркационный механизм неустойчивости в задаче о динамике бегущих волн в системе УрЧП, полученной как вязкая регуляризация гиперболической системы.

Задача Коши для одномерного уравнения движения в метаматериале

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В пространстве $C[\mathbb{R}]$ непрерывных функций, для которых существуют пределы при $x \to \pm\infty$, представлено исследование математической модели [1] распространения нелинейных продольных волн в метаматериале:

$$u_{tt} - u_{xx} - \alpha u_{ttxx} + \beta u_{xxxx} + \gamma u_{ttxxxx} = \chi \partial_x u_x^2, \tag{1}$$

$$u|_{t=0} = \varphi(x), \ \ u_t|_{t=0} = \psi(x), \quad x \in R, \tag{2}$$

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в которой $u_x = \partial_x u = \partial u/\partial x$; переменные $(t,x) \in \mathbb{R}_+ \times \mathbb{R}$, $\mathbb{R}_+ =]0,+\infty[$, $\mathbb{R} =]-\infty,+\infty[$; коэффициенты $\alpha = \theta^2 l^2/(2\Lambda^2)$, $\beta = (6\theta^2-1)l^2/(12\Lambda^2)$, $\gamma = \theta^2(6\theta^2-1)l^4/(24\Lambda^4)$, $\chi = Ku_0/(\varkappa\Lambda)$ — заданные [1] положительные числовые параметры.

Исследование задачи Коши (1), (2) проведено [2] по следующему плану: прежде всего проверено, что постановка задачи Коши (1), (2) корректна и локальное по времени классическое решение её существует. С этой целью для соответствующего (1) линейного однородного уравнения найдено решение задачи Коши. При этом выяснилось, что для равномерной корректности решения задачи Коши для уравнения движения в метаматериале параметр θ надо рассматривать в промежутке $1/\sqrt{6} \leqslant \theta < 1/\sqrt{3}$. Далее рассмотрена вспомогательная задача Коши

$$v_{tt} - v_{xx} - \alpha v_{ttxx} + \beta v_{xxxx} + \gamma v_{ttxxxx} = \chi \partial_x^2 v^2, \tag{3}$$

$$v|_{t=0} = \varphi'(x), \;\; v_t|_{t=0} = \psi'(x), \quad x \in \mathbb{R}. \tag{4}$$

Для задачи Коши (3), (4) найден временной отрезок $[0,t_1]$ существования и единственности классического решения и оценка нормы в пространстве $C[\mathbb{R}]$ этого локального решения. Затем установлена связь между решениями уравнений (1) и (3). Начиная с этого пункта усиливаются требования к решению уравнения (1): полагаем, что что на временном отрезке $[0,t_1]$ классическое решение $u = u(t,x)$ по переменной x принадлежит пересечению подмножества $C^{(4)}[\mathbb{R}] \subset C[\mathbb{R}]$ с пространством Соболева $W_2^4(\mathbb{R})$. В заключение доклада найдены достаточные условия существования единственного классического глобального ($t \geqslant 0$) решения задачи Коши (1), (2) и условия разрушения решения на конечном временном отрезке.

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Регуляризирующий оператор для решения нелинейного интегрального уравнения первого рода пространстве квадратично-суммируемых функции

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1. Введение. Линейное интегральное уравнение первого рода и его регуляризируемость исследованы в работах Лаврентьева М. М. [1].

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В работе [2] метод Лаврентьева М. М. применена для регуляризации решения широкого класса нелинейного интегрального уравнения первого рода.

В работе [3] исследуемая уравнения изучена с точными данными.

Рассмотрим нелинейное интегральное уравнение первого рода вида

$$\int_0^1 K_h(t,s)M(s,z(s))ds = u(t), \quad t \in [0,1] \tag{1}$$

где $K(t,s)$ — ядро интегрального уравнения, определенное в квадрате $0 \leqslant t, s \leqslant 1$ и непрерывное в этой области, такое, что $|K_h(t,s) - K(t,s)| \leqslant h$; $M(t,s)$ — нелинейная функция, определенная в полосе $-\infty < z \leqslant +\infty$, $0 \leqslant s \leqslant 1$, функция непрерывна в этой полосе и удовлетворяет условию Липшица по z, т. е. $|M(s,z_1(s)) - M(s,z_2(s))| \leqslant N|z_1(s) - z_2(s)|$; $z(s)$ — искомая функция, $u(t)$ — заданная функция, такая, что $||u_0(t) - u_\delta(t)|| \leqslant \delta$.

Допустим, что при $u(t) = u_0(t)$ уравнение имеет единственное решение $z_0(t)$.

2. Регуляризация. Наряду с уравнением (1) введем уравнение второго рода

$$\alpha z(t) + \int_0^1 K_h(t,s)M(s,z(s))d(s) = u_\delta(t). \tag{2}$$

Теорема 1. *Пусть выполнены условия:*

1) *ядро $K_h(t,s)$ симметрично, непрерывно в квадрате $0 \leqslant t, s \leqslant 1$ и положительно определено;*

2) *нелинейная функция $M(s,z)$ определена и непрерывна в полосе $-\infty < z < \infty, 0 \leqslant s \leqslant 1$ и удовлетворяет условию Липшица по z;*

3) *пусть при $u(t) = u_0(t)$ уравнение (1) имеет единственное решение $z_0(t) \in L_2[0,1]$;*

4) *постоянная N_1 удовлетворяет условию $N_1 < 1$.*

Тогда:

а) *при выполнении условий* 1), 2), 4) *уравнение* (2) *при любом $u(t) \in L_2[0,1]$ и любой $\alpha > 0$ имеет единственное решение $z_\alpha(t) \in L_2[0,1]$,*

б) *при выполнении условий* 1), 2), 3), 4) *решение уравнения* (2) $z_\alpha^0(t)$ *при* $u(t) = u_0(t)$ *сходится по норме пространства $L_2[0,1]$ при $h \to 0$ к точному решению уравнения* (1).

Теорема 2. *Пусть:*

1) *выполнены все условия теоремы* 1;

2) *функция $u_\delta(t)$ удовлетворяет неравенству $||u_0(t) - u_\delta(t)|| \leqslant \delta$;*

3) *параметр регуляризации $\alpha(\delta, h)$ выбрана по формуле $\alpha(\delta) = \left(h + \frac{2\delta}{c_1}\right)^{\frac{1}{2}}$.*

Тогда решение уравнения (2) *при $u(t) = u_\delta(t)$ и при $\delta, h \to 0$ сходится по норме пространства $L_2[0,1]$ к точному решению* (1).

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Суммирование в среднем в теории следов

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Рассмотрим класс самосопряженных, ограниченных снизу операторов L_0 с компактной резольвентой, действующих в сепарабельном гильбертовом пространстве H, и соответствующих классов симметрических относительно компактных возмущений V.

По теореме Като–Реллиха оператор $L = L_0 + V$ имеет компактную резольвенту и полуограничен снизу. Пусть $\{\lambda_k\}_{k=1}^{\infty}$, $(\lambda_k < \lambda_{k+1})$ — собственные числа оператора L_0, пронумерованные без учета кратностей, ν_k — кратность λ_k, $\left\{\mu_i^{(k)}\right\}_{i=1}^{\nu_k}$ $\left(\mu_i^{(k)} \leqslant \mu_{i+1}^{(k)}\right)$, $k = 1, 2, \ldots$ — собственные числа оператора L, на которые расщепляют λ_k при возмущении V, P_k — ортогональный проектор на собственное подпространство, соответствующее λ_k. Если $\{f_{k_i}\}_{i=1}^{\nu_k}$ — собственные векторы оператора L_0, образующие ортонормированный базис пространства P_kH, то

$$trP_kV = \sum_{i=1}^{\nu_k} (Vf_{k_i}, f_{k_i})\,.$$

Далее, пусть $r_n = \frac{1}{2} min\,(\lambda_{n+1} - \lambda_n; \lambda_n - \lambda_{n-1})$ и существует последовательность d_n, такая что $0 < d_n \leqslant r_n$, $\inf\limits_{n\geqslant 2} d_n > 0$,

$$\lim_{n\to\infty} \sup_{|z-\lambda_n|\leqslant d_n} \|R_{0n}(z)V\| = 0, \tag{1}$$

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где $R_{0n}(z) = R_0(z) - (\lambda_n - z)^{-1} P_n$, $R_0(z) = (L_0 - z)^{-1}$.

Введем функции

$$\rho(t) = \sum_{\lambda_k < t} \left[\sum_{i=1}^{\nu_k} \left(\lambda_k - \mu_i^{(k)} \right) + tr P_k V \right], \quad K(z) = (R_0(-z)V)^2 R_0(-z).$$

Справедлива

Теорема 1. *Если V — произвольный ограниченный симметрический оператор в H, $trK(z) < \infty$ и выполнено* (1), *то при $t \to +\infty$*

$$\int_0^t \rho(\tau) d\tau = \sum_{\lambda_k < t} tr \left(P_k V^2 - (P_k V)^2 \right) (1 + o(1)) .$$

На основе теоремы 1 для дифференциальных операторов (как обыкновенных, так и в частных производных), возмущенных оператором умножения на ограниченную измеримую функцию $V(x)$, доказывается, что

$$\lim_{t \to +\infty} \frac{1}{t^\alpha} \int_0^t \rho(\tau) d\tau = C(V), \tag{2}$$

где $0 < \alpha \leqslant 1$, а постоянная $C(V)$ есть функционал от возмущения V. А также, в частности, с помощью теоремы тауберова типа для методов Чезаро, показывается, что из соотношения (2) следует

$$\lim_{n \to \infty} \rho(\lambda_n + 0) = C(V)$$

для некоторых двумерных модельных операторов в частных производных, полученные в работах [1–3].

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Об (обобщенных) собственных функциях несамосопряженного квазипериодического оператора

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Рассматривается семейство несамосопряженных разностных операторов Шрёдингера, действующих в ℓ^2 на целочисленной решетке по формуле

$$(A_\theta(\omega,\lambda)\psi)_n = \psi_{n+1} + \psi_{n-1} + \lambda e^{-2\pi i(\theta+\omega n)}\psi_n, \qquad n \in \mathbb{Z},$$

где частота $\omega \in (0,1)$ и константа связи $\lambda > 0$ — заданные числа, а $\theta \in [0,1)$ — параметр, нумерующий операторы. Мы считаем, что $\omega \notin \mathbb{Q}$. При этом функция $n \mapsto e^{-2\pi i(\theta+\omega n)}$ является квазипериодической.

Семейство операторов $\{A_\theta\}$ — популярная модель, которую ввел П. Сарнак в [1]. Справедлива

Теорема. *При $\omega \notin \mathbb{Q}$ спектр A_θ не зависит от θ и описывается формулами*

$$\sigma(A_\theta(\omega,\lambda)) = [-2,2] \quad \text{при} \quad 0 < \lambda \leqslant 1,$$

$$\sigma\big(A_\theta(\omega,\lambda)\big) = \left\{E \in \mathbb{C}: \frac{(Re\,E)^2}{4\operatorname{ch}^2\ln\lambda} + \frac{(Im\,E)^2}{4\operatorname{sh}^2\ln\lambda} = 1\right\} \text{ при } \lambda > 1. \tag{1}$$

В случае, когда частота является диофантовым числом, теорема 1 доказана в [1]. В [2] она была распространена на случай всех $\omega \notin \mathbb{Q}$. В [3] показано, что геометрия спектра очень естественно описывается с помощью метода монодромизации — перенормировочного подхода, который начал развиваться в работах В. С. Буслаева и А. А. Федотова при попытке перенести теорию Блоха–Флоке на разностные операторы на вещественной оси, см. обзор [4].

Для диофантовых ω в [1] изучалась и природа спектра. Показано, что при $\lambda < 1$ нет точечного спектра, а при $\lambda > 1$ спектр является замыканием точечного.

С помощью метода монодромизации мы обсудим наличие точечного спектра для разных значений параметров, а также свойства собственных функций точечного спектра и обобщеннных собственных функций непрерывного спектра. В частности, будут описаны их самоподобные свойства.

Доклад основан на совместных с Д. И. Борисовым (УФИЦ РАН) работах.

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Комбинированный многомасштабный вариационный метод нахождения солитонных решений систем нелинейных уравнений Шрёдингера

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Доклад посвящён нахождению солитонных решений системы уравнений типа Шрёдингера:

$$
\begin{aligned}
&\frac{\partial A_1}{\partial z} + iD_1\frac{\partial^2 A_1}{\partial t^2} + i\gamma\left(A_1^* A_2 e^{-i\Delta_{21}z} + A_2^* A_3 e^{-i(\Delta_{31}-\Delta_{21})z}\right) = 0,\\
&\frac{\partial A_2}{\partial z} + \nu_{21}\frac{\partial A_2}{\partial t} + iD_2\frac{\partial^2 A_2}{\partial t^2} + i\gamma\left(A_1^2 e^{i\Delta_{21}z} + 2A_1^* A_3 e^{-i(\Delta_{31}-\Delta_{21})z}\right) = 0,\\
&\frac{\partial A_3}{\partial z} + \nu_{31}\frac{\partial A_3}{\partial t} + iD_3\frac{\partial^2 A_3}{\partial t^2} + 3i\gamma A_1 A_2 e^{i(\Delta_{31}-\Delta_{21})z} = 0,\ 0 < z \leqslant L_z, -\infty < t < \infty,
\end{aligned} \tag{1}
$$

с финитными начальными условиями. Она описывает взаимодействие трёх лазерных импульсов с кратными частотами в среде с нелинейным квадратичным откликом. Поскольку система (1) является слишком сложной для нахождения её солитонных решений известными ранее методами, был разработан комбинированный подход, включающий в себя метод многих масштабов [1] и вариационный подход [2]. На первом этапе был использован метод многих масштабов для получения упрощённой системы дифференциальных уравнений, решение которой является приближением исходной системы (1). Далее с использованием вариационного подхода были получены солитонные решения упрощённой системы, а анализ её инвариантов [3] позволил определить условия устойчивости.

Компьютерное моделирование исходной системы (1) показало, что найденные решения также являются приближёнными солитонными решениями исходной системы: после незначительных пертурбаций решение выходит на режим устойчивого распространения, соответствующего поведению солитонных решений.

Д. М. Харитонов благодарит Московский центр фундаментальной и прикладной математики (соглашение № 075-15-2025-345) за финансовую поддержку работы.

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Параболические уравнения второго порядка с сингулярными коэффициентами

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Рассматривается задача нахождения неотрицательной функции $u(x,t)$:

$$div(A(x)\nabla u) + (b(x), \nabla u) + V(x)u - u_t = 0,\ x \in \Omega \subset R^n,\ \ t > 0; \tag{1}$$

$$u(x,t) = 0,\ \ x \in \partial\Omega,\ \ t > 0; \tag{2}$$

$$u(x,0) = 0,\ \ x \in \Omega, \tag{3}$$

где $\Omega \subset \mathbb{R}^n$ $(n \geqslant 3)$ — ограниченная липшицева область, содержащая начало координат, $\partial\Omega$ — граница области Ω. Здесь

$$div(A(x)\nabla u) = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_j}\left(a_{ij}(x)\frac{\partial u}{\partial x_i}\right),\ \ (b(x), \nabla u) = \sum_{i=1}^{n} b_i(x)\frac{\partial u}{\partial x_i},\ \ u_t = \frac{\partial u}{\partial t}.$$

Граничное условие (2) выполняется в смысле $u \in L^2([0,T], W_0^{1,2}(\Omega))$. Условие (3) означает, что

$$ess \lim_{t\to 0+} \int\limits_{\Omega} u(x,t)\varphi(x)dx = \int\limits_{\Omega} u_0(x)\varphi(x)dx$$

для любой функции $\phi \in C_0^\infty(\Omega)$.

Предполагается, что выполнены следующие условия.

1) $A(x) = (a_{ij}(x))$ — симметрическая равномерно эллиптическая матрица с ограниченными измеримыми коэффициентами с константой эллиптичности $\mu > 0$;

2) $b(x) = (b_1(x), \cdots, b_n(x))$ — вектор-функция, удовлетворяющая условию $|b| \in L^2_{loc}(\Omega)$ и неравенству $\int\limits_{\Omega} |b|^2\nu^2 dx \leqslant \gamma^2 \int\limits_{\Omega} |\nabla\nu|^2 dx,\ \gamma < \mu$, для любой функции $\nu \in C_0^1(\Omega)$;

3) $0 \leqslant V \in L^1_{loc}(\Omega)$;

4) $0 \leqslant u_0 \in L^2(\Omega),\ T > 0$.

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Определение. Под *решением* уравнения (1) понимается обобщенная функция $u(\cdot,t) \in W_0^{1,2}(\Omega)$ такая, что $u(\cdot,t) \geqslant 0$, $V(\cdot)u(\cdot,t) \in L^1_{loc}(\Omega)$ и выполнено интегральное тождество

$$\int\limits_\Omega (u\psi)\big|_{t_1}^{t_2} dx - \int\limits_{t_1}^{t_2}\int\limits_\Omega u\psi_t dxdt + \int\limits_{t_1}^{t_2}\int\limits_\Omega A\nabla u\nabla\psi dxdt -$$

$$- \int\limits_{t_1}^{t_2}\int\limits_\Omega (b(x),\nabla u)\psi dxdt - \int\limits_{t_1}^{t_2}\int\limits_\Omega V(x)u\psi dxdt = 0$$

для любой функции $\psi \in C^{1,1}(\Omega\times(0,T))$ с компактным носителем, $0 < t_1 < t_2 < T$.

Определим *инфимум спектра* симметрического оператора

$$L \equiv -div(A(x)\nabla) - V(x)$$

следующим образом:

$$\sigma_{\inf}(V;\Omega) = \inf_{0\neq\varphi\in C_0^\infty(\Omega)} \frac{\int\limits_\Omega \big(A(x)\nabla\varphi\nabla\varphi - V(x)\varphi^2\big)dx}{\int\limits_\Omega \varphi^2 dx}.$$

Основным результатом работы является следующая теорема.

Теорема. *Пусть* $\Omega \subset \mathbb{R}^n$ $(n \leqslant 3)$ *— ограниченная липшицева область и* $0 \in \Omega$. *Пусть коэффициенты уравнения* (1) *удовлетворяют условиям* 1)–3) *и, кроме того,* $0 \leqslant u_0 \in L^2(\Omega)$.

1. *Предположим, что для некоторого* $\varepsilon > 0$ *имеем* $\sigma_{\inf}((1+\varepsilon)V;\Omega) > -\infty$. *Тогда задача* (1)–(3) *имеет единственное неотрицательное обобщенное решение при любой неотрицательной начальной функции* $u_0 \in L^2(\Omega)$.

2. *Предположим, что для некоторого* $\varepsilon > 0$ *имеем* $\sigma_{\inf}((1-\varepsilon)V;\Omega) = -\infty$. *Тогда задача* (1)–(3) *не имеет неотрицательных решений, кроме* $u(x,t) \equiv 0$.

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О новом способе решения гомотопии задачи Эйлера продольного изгиба колонны в нелинейной области деформаций

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Задача определения критической продольной нагрузки F_{cr} для однородной колонны AB, скользящий B и неподвижный A концы которой снабжены поворотными пружинами жёсткостей γ_1 и γ_2 Нм, охватывает полную гомотопию всех основных случаев закрепления, кроме, быть может, консоли. Её решение, как правило, осуществлялось для случая закона Гука и в пренебрежении предизгибным укорочением колонны, что фактически означало увеличение её длины при осевом сжатии (!).

Кроме того, вырожденность граничных условий в пренебрежении вышеупомянутого укорочения приводила к вынужденному волюнтаризму исследователей, дополнявших проблему порой безумными условиями, такими, как обращение в бесконечность (!?) упругих моментов пружин в момент начала изгиба [1].

Автор устранил эту вырожденность тем, что *впервые* ввёл условие на *восстановленную* длину и предельным переходом для инфинитезимального изгиба получил систему из алгебраических и трансцендентного уравнений относительно наклона оси $p = \operatorname{tg}\theta$, позволяющую определить величину F_{cr} и профиль самого такого изгиба, в общем случае включающий две точки перегиба (т.п.), с учётом укорочения выше и для произвольной диаграммы сжатия материала, даже без области Гука [2]. При этом сдвиговые деформации в сечении за счёт касательных напряжений τ Па учитывались заменой, предложенной Ф. Енгессером ещё в 19 в.

Получены в компактном виде важные частные случаи гомотопии для: а) одинаковых жёсткостей пружин $\gamma_1 = \gamma_2$; б) идеального шарнира вместо одной пружины $\gamma_1 = 0$; в) идеальной заделки ($\gamma_1 \to \infty$). Для ограниченных жёсткостей получено приближённое решение, практически совпадающее с точным для вышеозначенных случаев $\gamma_1 = \gamma_2$ и $\gamma_1 = 0$.

Что же касается поведения ключевых (концы, т.п.) наклонов оси $\operatorname{tg}\theta_i(F)$, $F > F_{cr}$, проблема сводится к смешанной системе из управляющего обыкновенного дифференциального и сопутствующей группы алгебро-интегральных уравнений [2], метод численного решения которой — задача текущего десятилетия, равно как и учёт сдвиговых деформаций. Однако в предельных случаях $\gamma_{1,2} = 0$ и $\gamma_{1,2} \to \infty$, сводящихся к консоли со свободным концом, возможно численное интегрирование и даже с пертурбативным учётом касательных напряжений. Установлен приближённый *закон секанса* для угла наклона оси на этом конце $\sec\theta_B \approx 1 + k(F - F_{cr})$.

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Работа [1] дает хорошие представления о развитии условий устойчивости линейного неавтономного уравнения с запаздывающим аргументом, идущих от известных теорем А. Д. Мышкиса «о 3/2» [2]. Основные результаты работы [1] применительно к уравнению

$$\dot{x}(t) + a(t)x(h(t)) = 0, \quad t \in [0, +\infty), \tag{1}$$

где $a(t) \geqslant 0$ и $h(t) \leqslant t$, приобретают следующий вид.

Утверждение 1. *Если* $\sup\limits_{t\in[0,+\infty)} \int_{h(t)}^{t} a(s)\,ds \leqslant 3/2$, *то уравнение* (1) *равномерно устойчиво.*

Утверждение 2. *Если* $\int_0^{+\infty} a(s)\,ds = +\infty$ *и* $\varlimsup\limits_{t\to+\infty} \int_{h(t)}^{t} a(s)\,ds < 3/2$, *то уравнение* (1) *асимптотически устойчиво.*

Утверждение 3. *Если* $a(t) \geqslant m > 0$ *и* $\varlimsup\limits_{t\to+\infty} \int_{h(t)}^{t} a(s)\,ds < 3/2$, *то уравнение* (1) *экспоненциально устойчиво.*

Представляем новые условия устойчивости уравнения (1), существенно уточняющие результаты работы [1]. Положим $\mu(\tau) = 0$ для $\tau < 0$, $\mu(\tau) = \tau$ для $\tau \in [0, 1]$ и $\mu(\tau) = 1$ для $\tau > 1$.

Теорема 1. *Если* $\sup\limits_{t\in[0,+\infty)} \int_{t}^{+\infty} \mu\left(\int_{h(s)}^{t} a(\xi)\,d\xi\right) a(s)\,ds \leqslant 1$, *то уравнение* (1) *равномерно устойчиво.*

Теорема 2. *Если* $\int_0^{+\infty} a(s)\,ds = +\infty$ *и* $\varlimsup\limits_{t\in[0,+\infty)} \int_{t}^{+\infty} \mu\left(\int_{h(s)}^{t} a(\xi)\,d\xi\right) a(s)\,ds < 1$, *то уравнение* (1) *асимптотически устойчиво.*

Теорема 3. *Если* $t - h(t) \leqslant R$ *для некоторой константы* $R > 0$, $a(t) \geqslant m$ *для некоторой константы* $m > 0$ *и* $\varlimsup\limits_{t\in[0,+\infty)} \int_{t}^{+\infty} \mu\left(\int_{h(s)}^{t} a(\xi)\,d\xi\right) a(s)\,ds < 1$, *то уравнение* (1) *экспоненциально устойчиво.*

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