# Mathematical methods of managing a retail store as a dynamic system: the promotional pricing aspect

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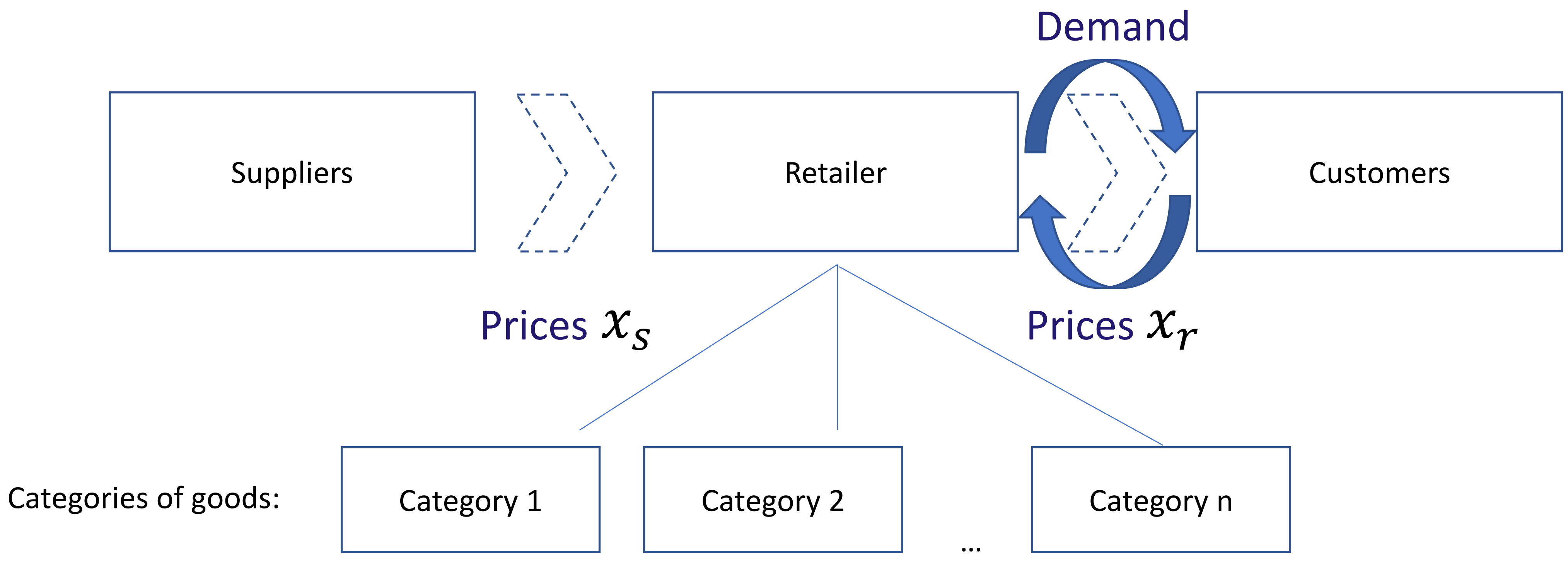
*Dedicated to the memory of academician S.P. Novikov*

The study proposes an innovative approach to managing a retail store as a multi-level stochastic system. We address the negative impact of uncoordinated pricing decisions by category managers through iterative corrections of the loyalty program conditions. The model combines: 1) the formalism of a stochastic game between the Center and category agents; 2) the representation of the loyalty system as a managed queue with financially dependent capacity.

# Subject area

Our subject area is retail, which can be modeled as follows. Goods flow from suppliers (at price Xs) to the retailer, who categorizes them (1…n) into discrete separately managed units* and sells to customers (at price Xr). Customer purchases—varying in timing, frequency, and product combinations—act as disturbances, destabilizing the system. The retailer adjusts category-level prices Xr to steer demand and restore stability. However, managing prices per category creates inefficient control policies across the system.




*AC Nielsen's definition: "Category management is the process of managing product categories as **separate** business units..."

# Problem setup

The agent will maximize its utility and make a choice from a set of choices that represents the set of maximums of its objective function:

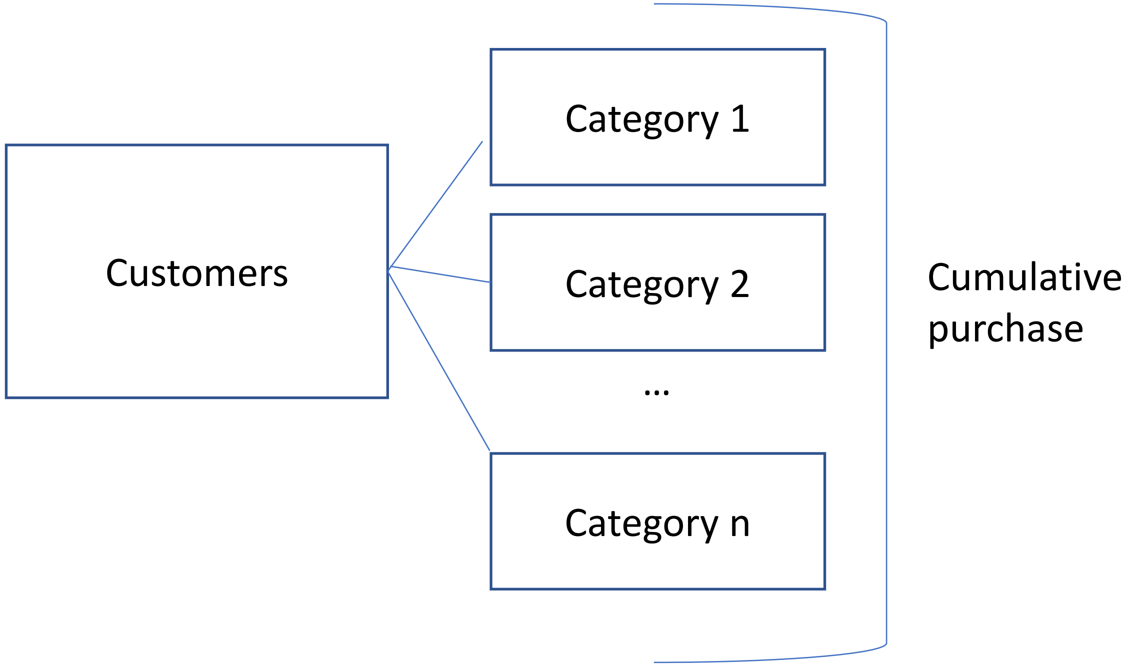
$$P(f(\cdot), A) = \operatorname{Arg}\max_{y \in A} f(y)$$

Therefore, the agent's choice set depends on its preferences f(×) and the set A from which it makes its choice.

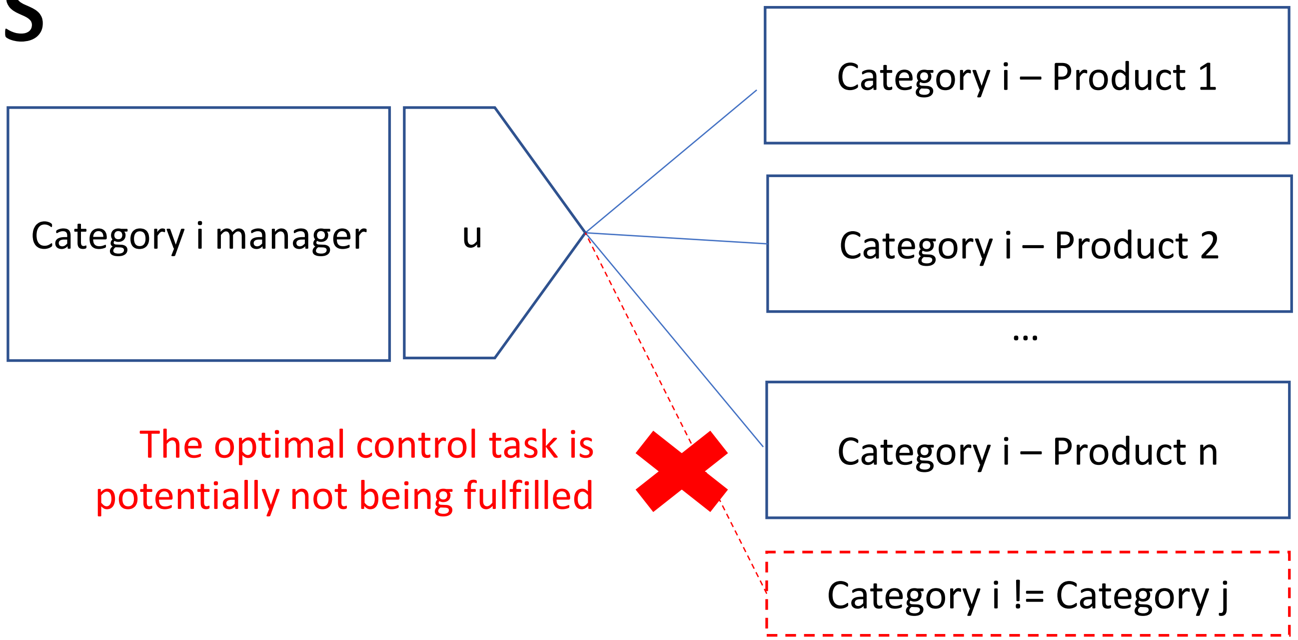
The problem of optimal control, that is, of admissible control that is maximally efficient:

$$\Phi(u) \to \max_{u \in U} .$$

Where u is some control




**VS**



# Example of constrained optimization problem formulation using the example of promotional pricing for a consumer with a known final budget

The total revenue function of a store can be expressed as the sum of revenue across all categories:

$$R_{global} = \sum_{i=1}^{n} R_i$$

Objective function:

$$\max_{\mathrm{P_k}} \mathrm{R_{global}} \text{ to} \sum_{i=1}^{n} S_i \leq B, S_j = f(S_k, P_k) \quad \forall j \neq k$$

aims to find such a state where all price would yield maximal spending by customer across multiple categories of goods.

**Constraints**

1) Budget constraint $$\sum_{i=1}^{n} S_i <= B$$

Where

$S_i$ - Spending in category i;

B – total budget.

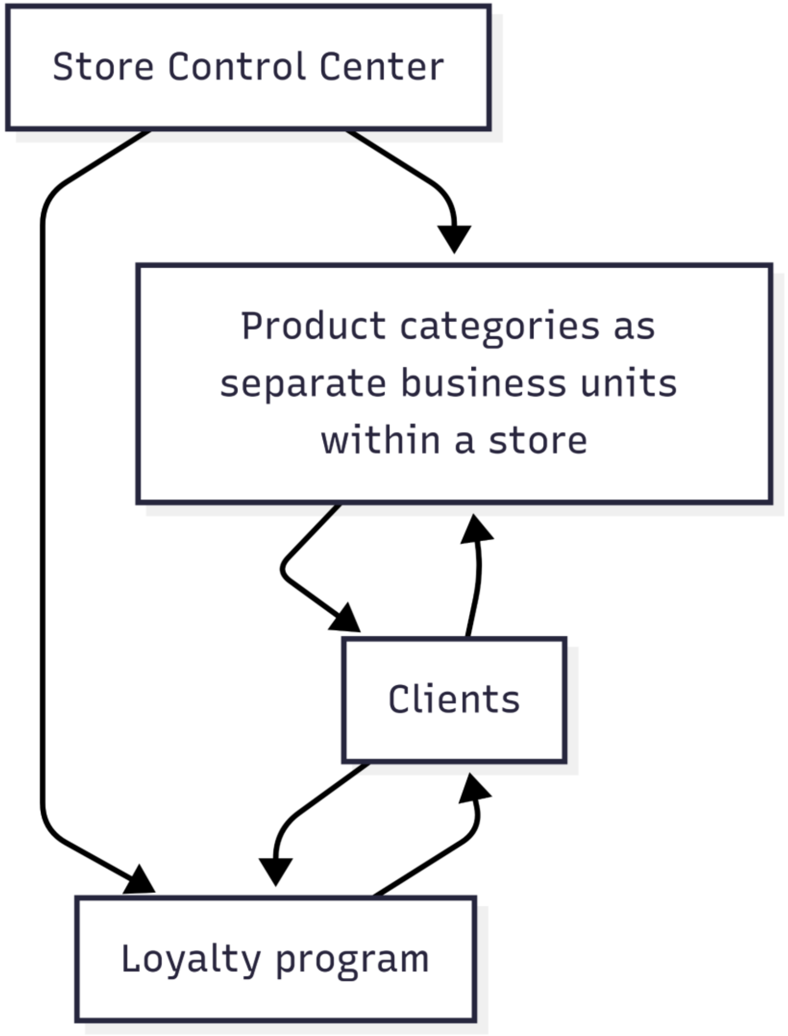
2) Switching constraint $$S_j = f(S_k, P_k) \quad \forall j \neq k$$

Where

- $S_j$ is the customer's spending in category j, where j is any category other than the promoted category.
- $S_k$ is the customer's spending in the promoted category.
- $P_k$ is a discount depth
- $f(S_k, P_k)$ is a function that expresses how spending in a non-promoted category depends on spending in a promoted category and on the intensity of promotion in the corresponding category.

# Problem solution framing

## Stochastic game between three levels




The problem of managing the disbalance between category-level and store-level objectives can be formally defined as a stochastic setting where the Center does not control agent actions directly, so the Center chooses control actions to respond to agents actions in multi-stage game (Bellman) in order to optimize its objectives.

$$\max_{u_t \in \mathcal{U}} \mathbb{E}_{a_t \sim \mathcal{P}_t} \left[ R_{\text{global}}(a_t, u_{t-1}) + \gamma \cdot V_{t+1}(s_{t+1}) \right]$$

**where:**

$a_t$ - agents' (categories) actions at time $t$, drawn from $\mathcal{P}_t$,

$u_{t-1}$ - Center's response chosen at previous step $t-1$,

$R_{\text{global}}(a_t, u_{t-1})$ - revenue at time $t$, depending on $a_t$ and prior Center response,

$s_{t+1}$ - system state after actions $(a_t, u_t)$,

$V_{t+1}(s_{t+1})$ - expected future revenue starting from next state,

$\gamma$ - discount factor (optional).

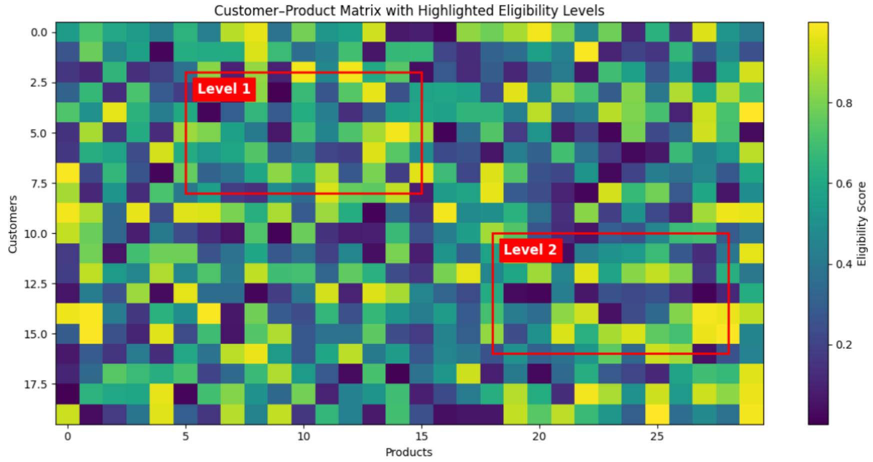
# Problem solution framing

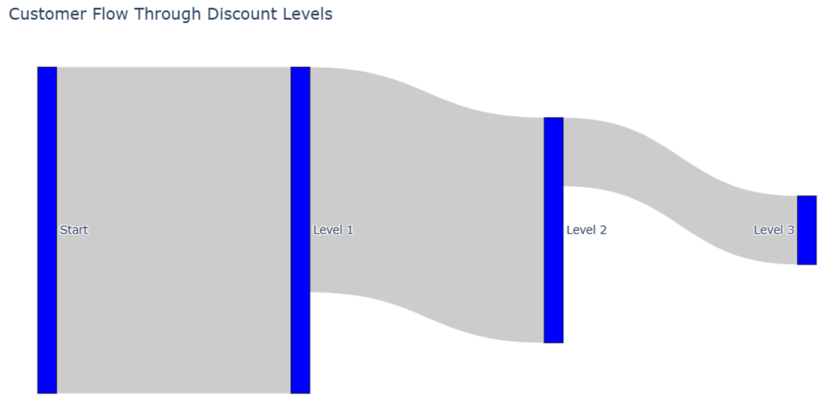
$$x_{i+1}(t+1) = x_{i+1}(t) + u_i(t) \cdot \mu_i \, x_i(t) - D_{i+1}(t), \tag{1}$$

where $x_i(t)$ denotes the number of customers at level $A_i$; $u_i(t) \in \{0, 1\}$ is a control variable indicating admission to the next level; $\mu_i$ is the progression rate (the scaling factor determining the size of the delta from $x_i(t)$); $D_{i+1}(t)$ captures departures from level $A_{i+1}$. The model enforces capacity constraints:

$$x_i(t) \le C_i, \quad \forall\, i,\, t, \tag{2}$$

which effectively block further transitions when a level is full. This blocking mechanism accounts for additional economic information about the system state and serves as the control lever to regulate customer flow and prevent over-discounting.



# Model Predictive Control approach to solution

**Algorithm 1** Price Damage Compensation

**Initialization:** Load current state (customers, finances, price history)
**for every day** $t = 0, 1, 2, \ldots, T$ **do**
    Step 1: Managers set prices $a_t$
    Step 2: Estimate damage: $\Delta R = f(a_t, \text{history})$ ▷ Predict revenue loss
    **if** $\Delta R < -\epsilon$ **then** ▷ If damage is significant
        Step 3: Optimize discounts:

$$\min_{u_t} \left| R^{\text{target}} - R(a_t, u_t) \right| + \lambda \sum u_i x_i$$

Discounts should not cost more than they compensate

        Step 4: Apply optimal $u_t^*$
    **else**
        Step 3-4: Use standard discounts
    **end if**
    Step 5: Update loyalty: $x_{t+1} \leftarrow F(x_t, u_t, a_t)$
    Step 6: Record the result: Record the actual revenue $R_t$
    Step 7: Train the model: Update the damage forecast on new data
**end for**

# Step 3 (Optimize discounts) specification

$$\min_{u} \quad \underbrace{\left| R^{\text{target}} - \sum_{k=1}^{K} p_k(a_t) \cdot q_k(u_t) \right|}_{\text{Term 1: Revenue Gap Penalty}} + \lambda \underbrace{\sum_{i=1}^{L} u_i x_i}_{\text{Term 2: Total Discount Cost}} \qquad \text{// Min. cost of intervention}$$

$$\text{s.t.} \quad q_k(u_t) = \underbrace{d_k^0}_{\text{Baseline demand}} \left( 1 + \gamma \underbrace{\sum_{i=1}^{L} u_i \cdot x_i}_{\text{Total Loyalty Stimulus}} \right) \qquad \text{// Demand boost from overall program satisfaction}$$

$$0 \leq u_i \leq 1, \qquad \text{// Discounts are rates between 0\% and 100\%}$$

$$\sum u_i x_i \leq \text{Budget}_t \qquad \text{// Hard constraint: Do not exceed the budget}$$

# Further research: practical implementation

Short-horizon solution

Pushing

VS

Sequential User-behavior modelling

Steering

Robust Market Interventions

Andrea Galeotti and Benjamin Golub and Sanjeev Goyal and Eduard Talamàs and Omer Tamuz

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**Influential Recommender System**

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# Q&A


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