

# BAYESIAN ESTIMATION OF RISK-NEUTRAL PROBABILITY

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## IMPLIED RISK-NEUTRAL PROBABILITY DENSITY

$$S = \sum_{s=1}^K \phi(s)q(s)$$

$$\phi^* \stackrel{d}{=} \frac{\phi(s)}{\sum_{s=1}^K \phi(s)} = \frac{\phi(s)}{1/(1+r)} = (1+r)\phi(s)$$

$$S = \frac{1}{1+r} \sum_{s=1}^K \phi^*(s)q(s) = \frac{1}{1+r} \mathbb{E}^*[q(s)].$$

## IMPLIED RISK-NEUTRAL PROBABILITY DENSITY

For TAS utility, the state-prices are

$$\phi(s) = \pi \frac{\beta u'[c(s)]}{u'[c(0)]},$$

↪ implied risk-neutral probability is

$$\phi^*(s) = \pi \frac{\beta u'[c(s)]}{u'[c(0)]} (1 + r) = \pi \frac{u'[c(s)]}{u'[c(0)]} \approx \pi,$$

for  $u'$  is constant (risk-neutrality) or  $c(s) \approx c(0)$ .

## BREEDEN-LITZENBERGER (1978) FORMULA

For short-term,  $r \approx 0$ .

$$\begin{aligned}C(S, t) &= \mathbb{E}^* [[S(T) - \mathcal{K}]^+ \mid S(t) = S] \\&= \int_0^{+\infty} (x - \mathcal{K})^+ dP^*(x) \\&= \int_{\mathcal{K}}^{+\infty} x dP^*(x) - \mathcal{K} \int_0^{+\infty} dP^*(x) \\&= \int_{\mathcal{K}}^{+\infty} x dP^*(x) - \mathcal{K}(1 - P^*(\mathcal{K}))\end{aligned}$$

$$\begin{aligned}\frac{\partial C}{\partial \mathcal{K}} &= -\mathcal{K}p^*(\mathcal{K}) - 1 + P^*(\mathcal{K}) + \mathcal{K}p^*(\mathcal{K}) = P^*(\mathcal{K}) - 1 \\ \frac{\partial^2 C}{\partial \mathcal{K}^2} &= p^*(\mathcal{K}).\end{aligned}$$

Ait-Sahalia and Duarte (2003)

## BAYESIAN FORMULATION

$$y_i = \lambda \sum_{j=1}^K X_{ij} \beta_j + \varepsilon_i,$$

where  $\varepsilon_i \sim N(0, \sigma^2)$ , and  $\beta = (\beta_1, \dots, \beta_K) \in \Delta^{K-1}$ , where  $\Delta^{K-1}$  denotes the simplex of dimension  $(K - 1)$ .

In vector form,

$$y = \lambda X \beta + \varepsilon.$$

In order to insure  $\beta \in \Delta^{K-1}$ , we select the Dirichlet prior:

$$p(\beta \mid \alpha) = \text{Dirichlet}(\beta \mid \alpha) \propto \prod_{i=1}^K x_i^{\alpha-1}.$$

## PRIOR FOR CONCENTRATION COEFFICIENT

$\alpha$  is a vector of concentration parameters for Dirichlet distribution.

It is reasonable to select a distribution with positive support that is relatively flat on  $[0, 1]$ .

## PRIORS AND POSTERIOR

$$p(\beta \mid \alpha) = \text{Dirichlet}(\beta \mid \alpha) \propto \prod_{i=1}^K x_i^{\alpha-1}.$$

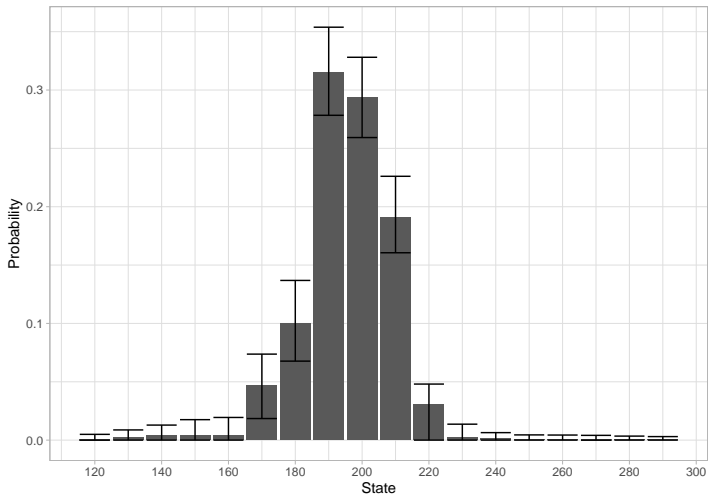
$$p(\alpha) = \text{Weibull}(1.1, 1)$$

$$p(\sigma^2) \propto \frac{1}{\sigma^2}$$

$$p(\alpha, \lambda, \sigma, \beta \mid y) \propto N(y \mid \lambda X \beta, \sigma) \times \text{Dirichlet}(\beta \mid \alpha) \times \frac{1}{\sigma^2}$$

## EXAMPLE. AAPL

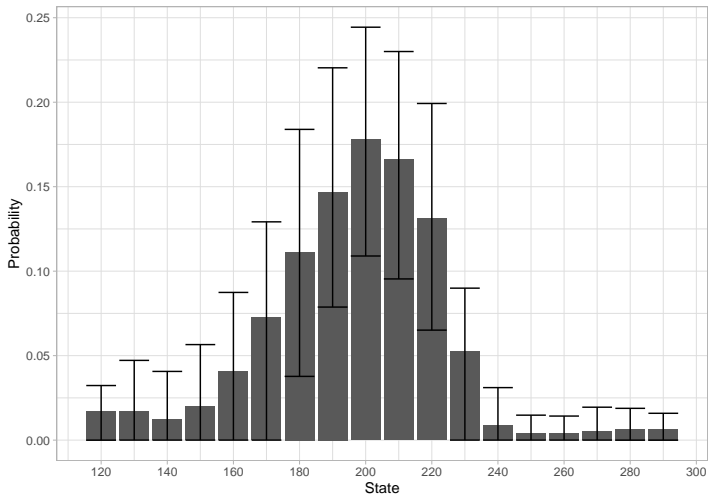
Calculated in 2025-05-23 with expiration date 2025-06-13



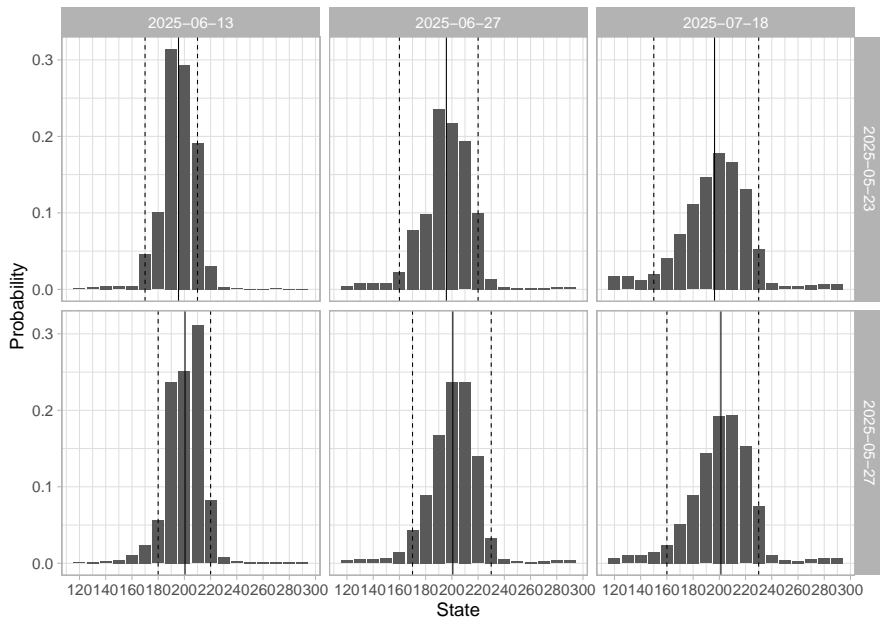


## EXAMPLE. AAPL

Calculated in 2025-05-23 (same) with expiration date 2025-07-18 (+35 days).

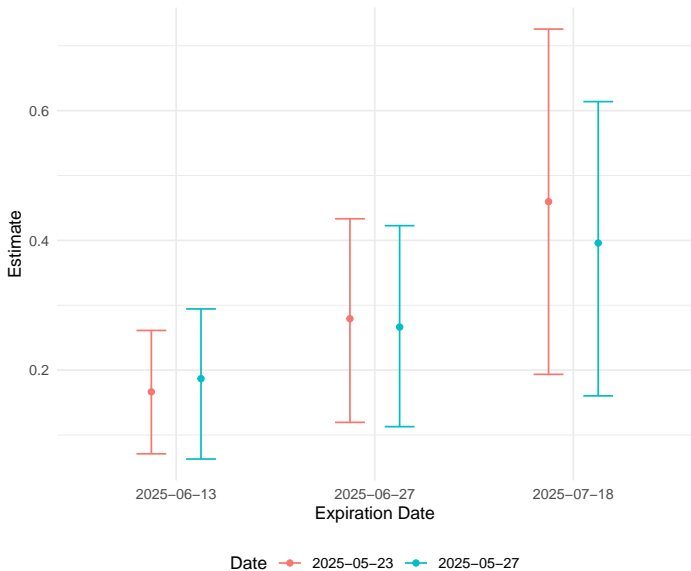


# BETA ESTIMATES





# ALPHA ESTIMATES





# REFERENCES [1]



Ait-Sahalia, Yacine and Jefferson Duarte (Sept. 2003). “Nonparametric Option Pricing under Shape Restrictions”. In: *Journal of Econometrics* 116(1), pp. 9–47. ISSN: 0304-4076. DOI: 10.1016/S0304-4076(03)00102-7. (Visited on 05/31/2025).



Breeden, Douglas T. and Robert H. Litzenberger (1978). “Prices of State-Contingent Claims Implicit in Option Prices”. In: *The Journal of Business* 51(4), pp. 621–651. ISSN: 0021-9398. JSTOR: 2352653. (Visited on 05/31/2025).

