BAYESIAN ESTIMATION OF RISK-NEUTRAL PROBABILITY

Alexander Vlasov (avlasov@nes.ru)

New Economic School – 2025-06-20

CONTENTS

IMPLIED PROBABILITY DENSITY

$$S = \sum_{s=1}^{K} \phi(s) q(s)$$

$$\phi^* \stackrel{d}{=} \frac{\phi(s)}{\sum_{s=1}^{K} \phi(s)} = \frac{\phi(s)}{1/(1+r)} = (1+r)\phi(s)$$

$$S = \frac{1}{1+r} \sum_{s=1}^{K} \phi^*(s) q(s) = \frac{1}{1+r} \mathbb{E}^*[q(s)].$$

For TAS utility, the implied risk-neutral probability is

$$\phi^*(s) = \pi \frac{\beta u'[c(s)]}{u'[c(0)]}(1+r) = \pi \frac{u'[c(s)]}{u'[c(0)]} \approx \pi,$$

for $c(s) \approx c(0)$ or u' is constant (risk-neutrality).

BREEDEN-LITZENBERGER (1978) FORMULA

For short-term, $r \approx 0$.

$$\begin{aligned} C(S,t) &= \mathbb{E}^* \left[[S(T) - \mathcal{K}]^+ \mid S(t) = S \right] \\ &= \int_0^{+\infty} (x - \mathcal{K})^+ \, dP^*(x) \\ &= \int_{\mathcal{K}}^{+\infty} x \, dP^*(x) - \mathcal{K} \int_0^{+\infty} \, dP^*(x) \\ &= \int_{\mathcal{K}}^{+\infty} x \, dP^*(x) - \mathcal{K}(1 - P^*(\mathcal{K})) \end{aligned}$$

$$\begin{aligned} \frac{\partial C}{\partial \mathcal{K}} &= -\mathcal{K}p^*(\mathcal{K}) - 1 + P^*(\mathcal{K}) + \mathcal{K}p^*(\mathcal{K}) = P^*(\mathcal{K}) - 1 \\ \frac{\partial^2 C}{\partial \mathcal{K}^2} &= p^*(\mathcal{K}). \end{aligned}$$

REFERENCES [1]

Breeden, Douglas T. and Robert H. Litzenberger (1978). "Prices of State-Contingent Claims Implicit in Option Prices". In: *The Journal of Business* 51(4), pp. 621–651. ISSN: 0021-9398. JSTOR: 2352653.