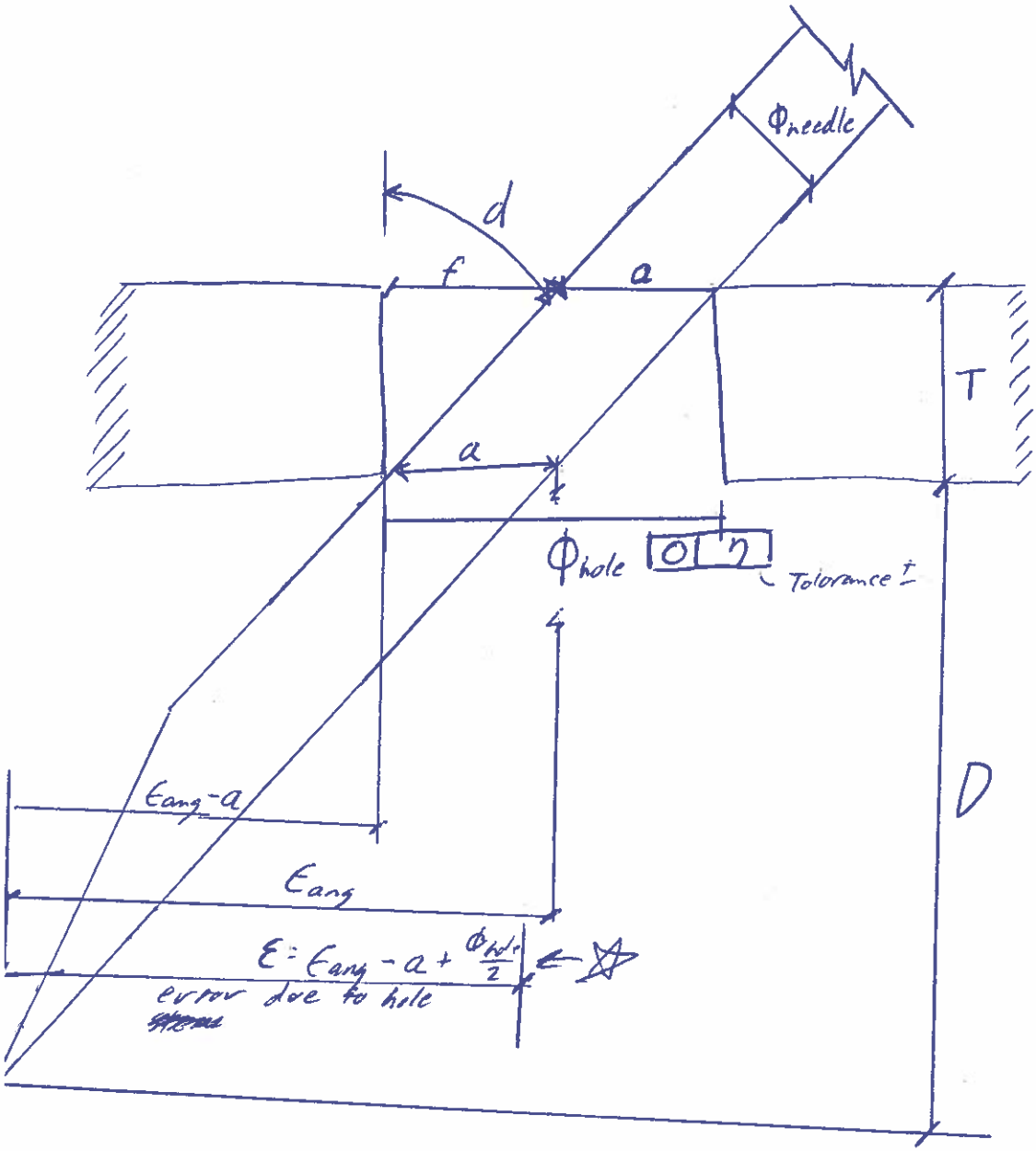


Find d

$$f = \Phi_{hole} \pm \eta - a$$

$$f = \Phi_{hole} \pm \eta - \frac{\Phi_{needle}}{\sin(90-d)}$$

} from Φ_{hole} ①

$$\tan(d) = \frac{f}{T} \leftarrow 2^{nd} \text{ eqn for } "f" \text{ ②}$$

Combine ① & ②

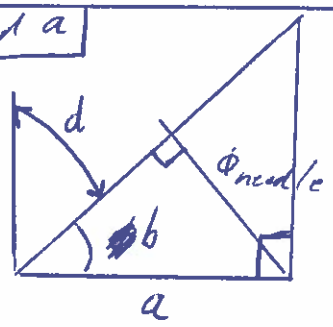
$$T\tan(d) = \Phi_{hole} \pm \eta - \frac{\Phi_{needle}}{\sin(90-d)}$$ → Solved on next page

$$\alpha = \Phi_{hole} \pm \eta$$

$$d = \sin^{-1}\left(\frac{-\Phi_{needle}}{\sqrt{T^2+\alpha^2}}\right) + \tan^{-1}\left(\frac{\alpha}{T}\right)$$

↑ find $0 \le d \le 90$

Find a



$$b = 90° - d$$

$$\sin(b) = \frac{\Phi_{needle}}{a}$$

$$a = \frac{\Phi_{needle}}{\sin(90-d)}$$

$$T\tan(d) = \overbrace{\phi_{hole} \pm \gamma}^{\dot{}} - \frac{\phi_{needle}}{\sin(90-d)} \rightarrow \sin(90-d) = \sin\left(\frac{\pi}{2}-d\right)$$

~~sin(90-d)~~

$$\sin\left(\frac{\pi}{2}\right)\cos(d) \leftarrow \cos\left(\frac{\pi}{2}\right)\sin(d)$$

$\sin\left(\frac{\pi}{2}\right) \rightarrow 1$; $\cos\left(\frac{\pi}{2}\right) \rightarrow \emptyset$

$1 \rightarrow \cos d$

$$T\tan(d) = \alpha - \frac{\phi_{needle}}{\cos(d)}$$

$$T\tan(d) + \frac{\phi_{needle}}{\cos(d)} = \alpha$$

↳ trig identity $\tan(d) = \frac{\sin(d)}{\cos(d)}$

$$T\frac{\sin(d)}{\cos(d)} + \frac{\phi_{needle}}{\cos(d)} = \alpha$$

$$T\sin(d) - \alpha\cos(d) = -\phi_{needle}$$

~~$T\tan(d) - \alpha\cos(d)$~~

~~$T\sin(d) - \alpha\sin\left(\frac{\pi}{2}-d\right) = \phi_{needle}$~~

$A\sin\alpha - B\cos\alpha = R\sin(\alpha - a)$

$R = \sqrt{A^2+B^2}$

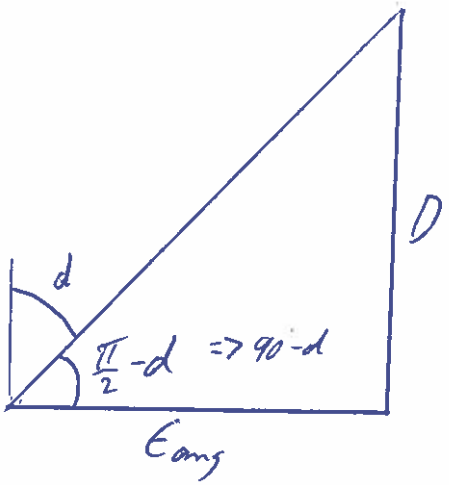
$\alpha = \tan^{-1}\left(\frac{B}{A}\right)$

$$T\sin(d) - \alpha\cos(d) = \sqrt{T^2+\alpha^2}\,\sin\left(d - \tan^{-1}\left(\frac{\alpha}{T}\right)\right)$$

$$\sqrt{T^2+\alpha^2}\,\sin\left(d - \tan^{-1}\left(\frac{\alpha}{T}\right)\right) = -\phi_{needle}$$

$$d - \tan^{-1}\left(\frac{\alpha}{T}\right) = \sin^{-1}\left(\frac{-\phi_{needle}}{\sqrt{T^2+\alpha^2}}\right)$$

$$d = \sin^{-1}\left(\frac{-\phi_{needle}}{\sqrt{T^2+\alpha^2}}\right) + \tan^{-1}\left(\frac{\alpha}{T}\right)$$



$\epsilon_{ang} =$

$\tan\left(\frac{\pi}{2} - d\right) = \frac{D}{\epsilon_{ang}}$

$\epsilon_{ang} = \frac{D}{\tan\left(\frac{\pi}{2} - d\right)}$

Calculate $\epsilon = \epsilon_{ang} - a + \frac{\phi_{hole}}{2}$

- hole diam
- tolerance hole η
- D
- ϕ_{needle}

plot this for 110 ϕ_{needle}