(Adv) Introduction to Computational Fluid Dynamics

Semester Project Overview

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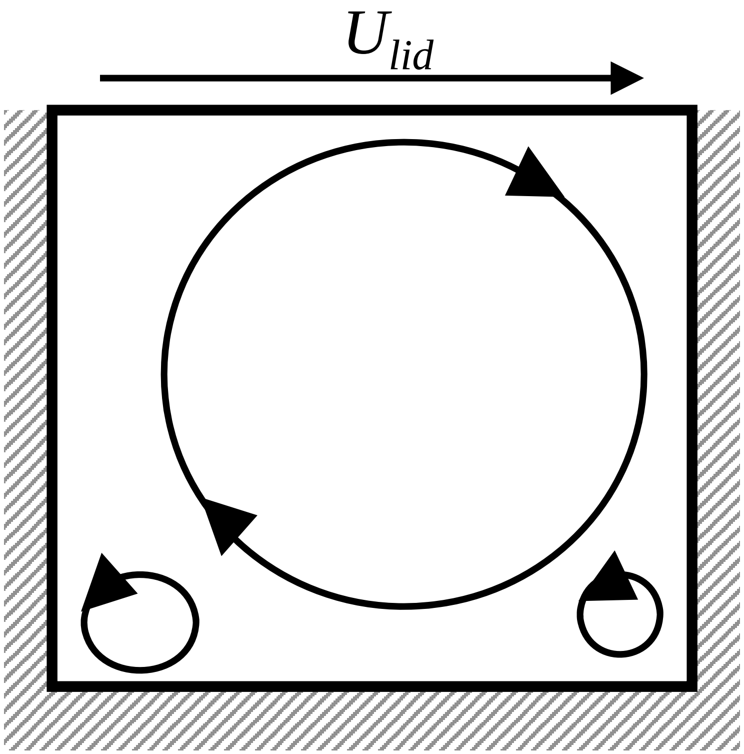
Project: Incompressible, Lid-Driven Cavity

- **For the project, solve the steady, incompressible N-S equations (w/ time derivative preconditioning) for the viscous flow in a square, 2D, lid-driven cavity**
- **The conditions are:**
  - $0 \leq x \leq 0.05$ m
  - $0 \leq y \leq 0.05$ m
  - $Re = 100 =$
  - $L = 0.05$ m
  - $U_{lid} = 1.0$ m/s
  - $\rho = 1.0$ kg/m^3



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Project: Incompressible, Lid-Driven Cavity

- **Assume the following:**
  - **Constant density**
  - **Constant viscosity**
  - **Constant temperature**
- **The energy equation is thus decoupled from the mass and momentum equations and doesn't need to be solved**
- **We will be interested in steady-state solutions only, so the temporal accuracy will not be important**
- **We will use time-derivative preconditioning to help solve these equations (OK for steady-state cases)**
- **Manufactured solutions will be used for code order of accuracy verification**

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The governing equations will include:

- **Time derivative preconditioning**
- **Artificial viscosity**
- **Manufactured solution source terms (supplied for the code verification case)**

$$\frac{1}{\beta^2}\frac{\partial p}{\partial t}+\rho\frac{\partial u}{\partial x}+\rho\frac{\partial v}{\partial y}-S=f_{mass}(x,y)$$

$$\rho\frac{\partial u}{\partial t}+\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}+\frac{\partial p}{\partial x}-\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=f_{xmtm}(x,y)$$

$$\rho\frac{\partial v}{\partial t}+\rho u\frac{\partial v}{\partial x}+\rho v\frac{\partial v}{\partial y}+\frac{\partial p}{\partial y}-\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)=f_{ymtm}(x,y)$$

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- The baseline discretization approach will be the simple explicit method
- Replace the temporal derivative with a forward difference in time
- Replace spatial derivatives with centered differences
- For the continuity equation, this results in:

$$\frac{1}{\beta_{i,j}^2}\frac{p_{i,j}^{n+1}-p_{i,j}^{n}}{\Delta t}+\rho\frac{u_{i+1,j}^{n}-u_{i-1,j}^{n}}{2\Delta x}+\rho\frac{v_{i,j+1}^{n}-v_{i,j-1}^{n}}{2\Delta y}-S_{i,j}=f_{mass}(x,y)$$

or

$$p_{i,j}^{n+1}=p_{i,j}^{n}-\beta_{i,j}^2\Delta t\left[\rho\frac{u_{i+1,j}^{n}-u_{i-1,j}^{n}}{2\Delta x}+\rho\frac{v_{i,j+1}^{n}-v_{i,j-1}^{n}}{2\Delta y}-S_{i,j}-f_{mass}(x,y)\right]$$

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The baseline discretization approach for the x-mtm eqn. is:

$$\rho\frac{u_{i,j}^{n+1}-u_{i,j}^{n}}{\Delta t}+\rho u_{i,j}^{n}\frac{u_{i+1,j}^{n}-u_{i-1,j}^{n}}{2\Delta x}+\rho v_{i,j}^{n}\frac{u_{i,j+1}^{n}-u_{i,j-1}^{n}}{2\Delta y}+\frac{p_{i+1,j}^{n}-p_{i-1,j}^{n}}{2\Delta x}$$

$$-\mu\frac{u_{i+1,j}^{n}-2u_{i,j}^{n}+u_{i-1,j}^{n}}{\Delta x^{2}}-\mu\frac{u_{i,j+1}^{n}-2u_{i,j}^{n}+u_{i,j-1}^{n}}{\Delta y^{2}}=f_{xmtm}(x,y)$$

or

$$u_{i,j}^{n+1}=u_{i,j}^{n}-\frac{\Delta t}{\rho}\left[\rho u_{i,j}^{n}\frac{u_{i+1,j}^{n}-u_{i-1,j}^{n}}{2\Delta x}+\rho v_{i,j}^{n}\frac{u_{i,j+1}^{n}-u_{i,j-1}^{n}}{2\Delta y}+\frac{p_{i+1,j}^{n}-p_{i-1,j}^{n}}{2\Delta x}\right.$$

$$\left.-\mu\frac{u_{i+1,j}^{n}-2u_{i,j}^{n}+u_{i-1,j}^{n}}{\Delta x^{2}}-\mu\frac{u_{i,j+1}^{n}-2u_{i,j}^{n}+u_{i,j-1}^{n}}{\Delta y^{2}}-f_{xmtm}(x,y)\right]$$

The y-momentum equation is discretized similarly

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Stability for this method comes from a combination of the convective stability limit and the diffusive limit

The convective stability limit for the time step is:

$$\Delta t_c \leq \frac{\min(\Delta x, \Delta y)}{|\lambda|_{\max}}$$

Where $|\lambda|_{\max} = \max(\lambda_x, \lambda_y)$ **and**

$$\lambda_x = \frac{1}{2}\left(|u| + \sqrt{u^2 + 4\beta^2}\right)$$

$$\lambda_y = \frac{1}{2}\left(|v| + \sqrt{v^2 + 4\beta^2}\right)$$

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The diffusive stability limit for the time step is:

$$\Delta t_d \leq \frac{\Delta x \Delta y}{4\nu}, \quad \nu = \mu / \rho$$

We can combine the convective and diffusive time step limits and add the CFL number:

$$\Delta t \leq \min(\Delta t_c, \Delta t_d) = CFL \cdot \min(\Delta t_c, \Delta t_d)$$

For the explicit methods, the CFL generally can vary between zero and one

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The boundary conditions for velocity are:

- **Bottom and side walls:** $u = v = 0$
- **Top wall:** $u = U_{lid}$, $v = 0$

For the pressure at the wall, use linear extrapolation from the interior:

- **For example, at the bottom wall:** $p_{i,1}^{n} = 2p_{i,2}^{n} - p_{i,3}^{n}$
- **At the right wall:** $p_{\mathrm{imax},j}^{n} = 2p_{\mathrm{imax}-1,j}^{n} - p_{\mathrm{imax}-2,j}^{n}$
- **The other two walls are handled similarly**

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Notice that the pressure only shows up as a derivative in the governing equations

- **This means that its value is not important**
- **What is important is its gradient**
- **As you solve this problem, the value of the pressure may tend to drift due to small numerical errors**
- **After each time step, simply rescale the pressure by adding or subtracting an amount from the pressure at each point to enforce a pressure at some reference location in the domain**
- **This reference point should be set to some value (e.g., p_{inf} = 1 N/m^2) somewhere within the cavity, e.g., at the cavity center (note: for the MMS case, it should be consistent with your chosen manufactured solution)**

Local versus Global Time Stepping

Your routine to set the time step should look like:

```
delta_t_max = 1.0e99
Loop i = 2 to imax-1
  Loop j = 2 to jmax-1
    ! Local Time Step (max allowable at i,j)
    delta_t(i,j) = _________   ! Array
    ! Global Time Step (max allowable in domain)
    delta_t_max = min(delta_t(i,j),delta_t_max)
  End Loop over j
End loop over i
! Use delta_t(i,j) for local time stepping
! Use delta_t_max for global time stepping
```

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Recommendations:

- **You may be able to use CFL numbers larger than one**
- **You probably want to use *at least* 65 points in each coordinate direction for the actual driven cavity flow**
- **Since the explicit approach is not very efficient, it may take more than 100,000 iterations to converge your iterative residuals eight orders of magnitude on finer meshes**
- **Output the iterative residuals every 100 iterations or so**
- **Output your solution every 5,000 iterations or so**
- **The templates you will receive will have a "restart" capability due to the long run times**
  - **Code will output the file 'restart.out' when it outputs the solution**
  - **Copy 'restart.out' to 'restart.in' and change *irstr* to 1 in input deck**

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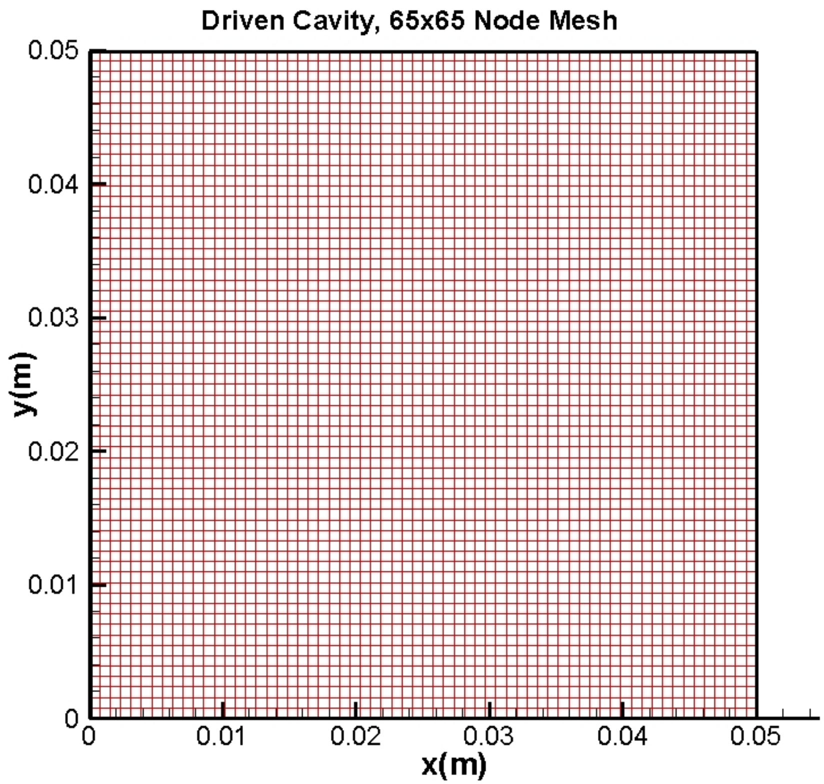
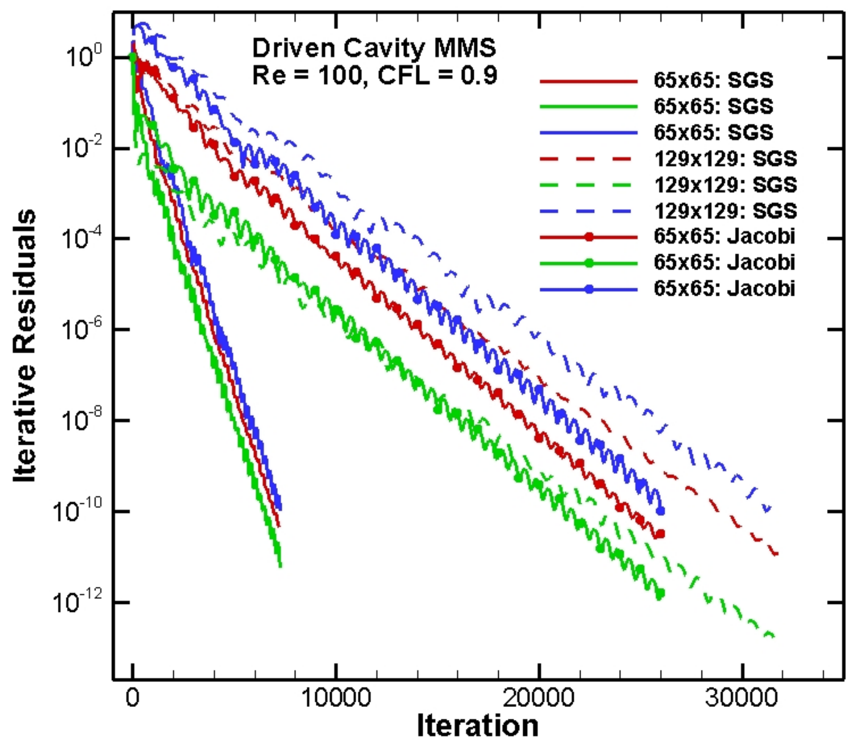
- **All Students:**
  - Artificial viscosity should be used
  - Baseline values of constants are: $k = 0.1$ and $C^{(4)} = 0.01$
  - Compare your results to the results you get from running FLUENT (or some other CFD code)
  - Compare results to benchmark numerical solutions and/or experimental data (to be provided)
  - Formal project report required
- **Those taking Intro CFD as a graduate course:**
  - Also implement the explicit point symmetric Gauss-Seidel method
  - Also run Reynolds numbers of 500 and 1,000

Lid Driven Cavity: Re = 100

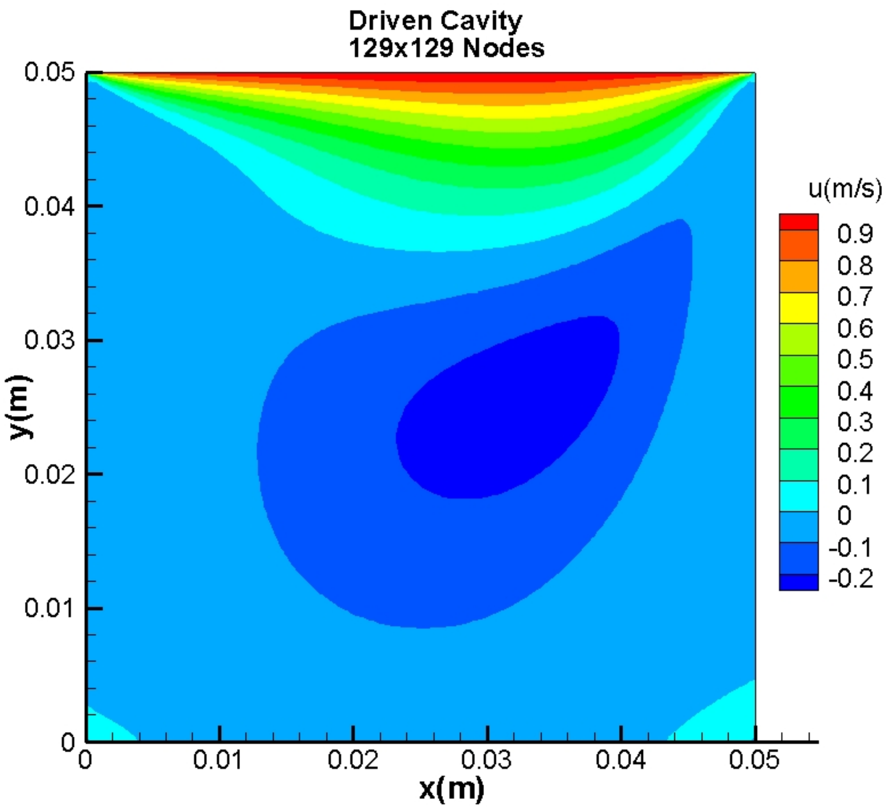


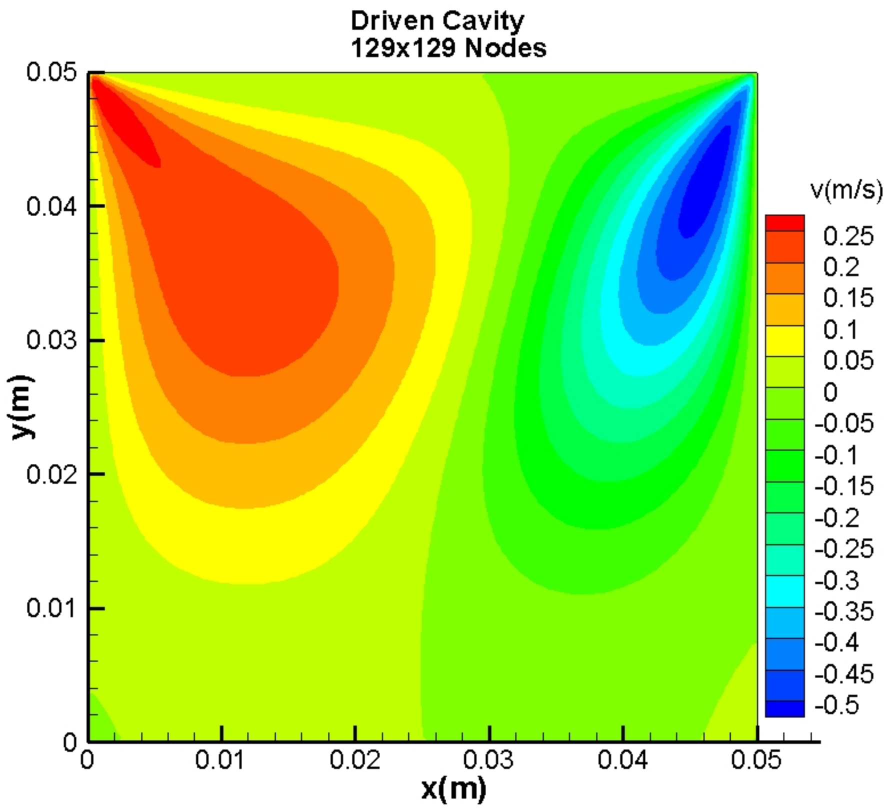
Grid



Iterative Residual History

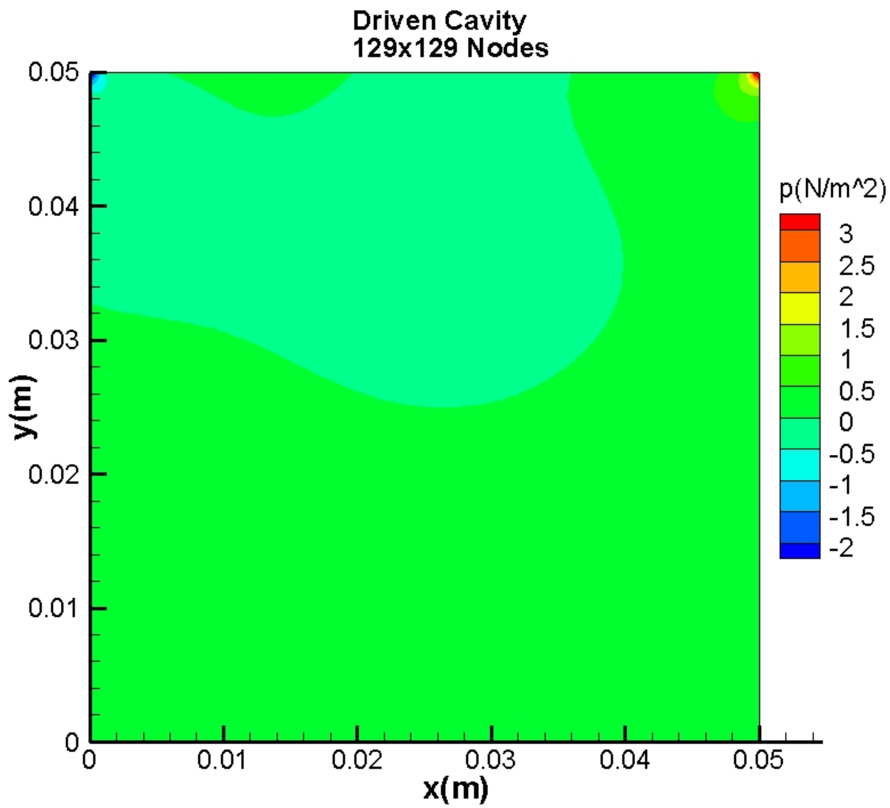
Lid Driven Cavity: Re = 100

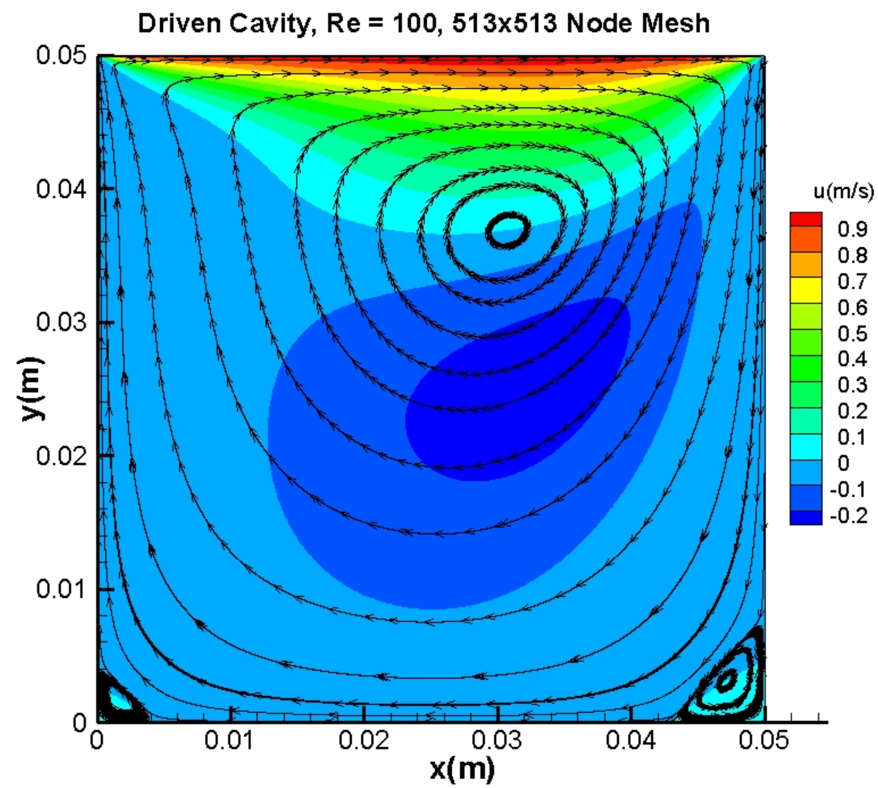




u-Velocity **v-Velocity**

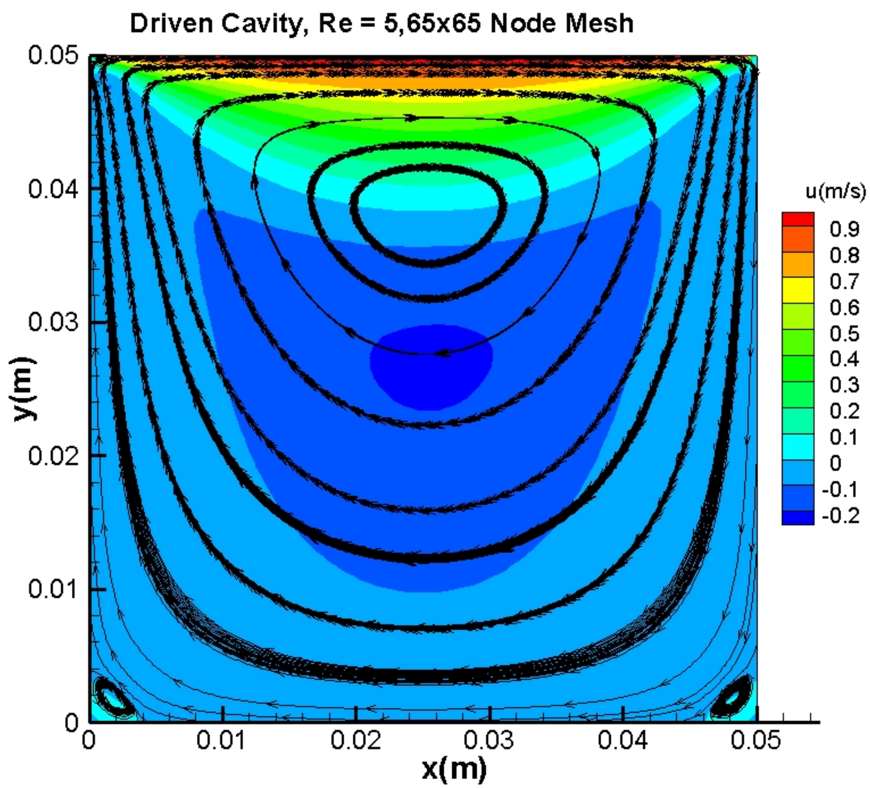
Lid Driven Cavity: Re = 100



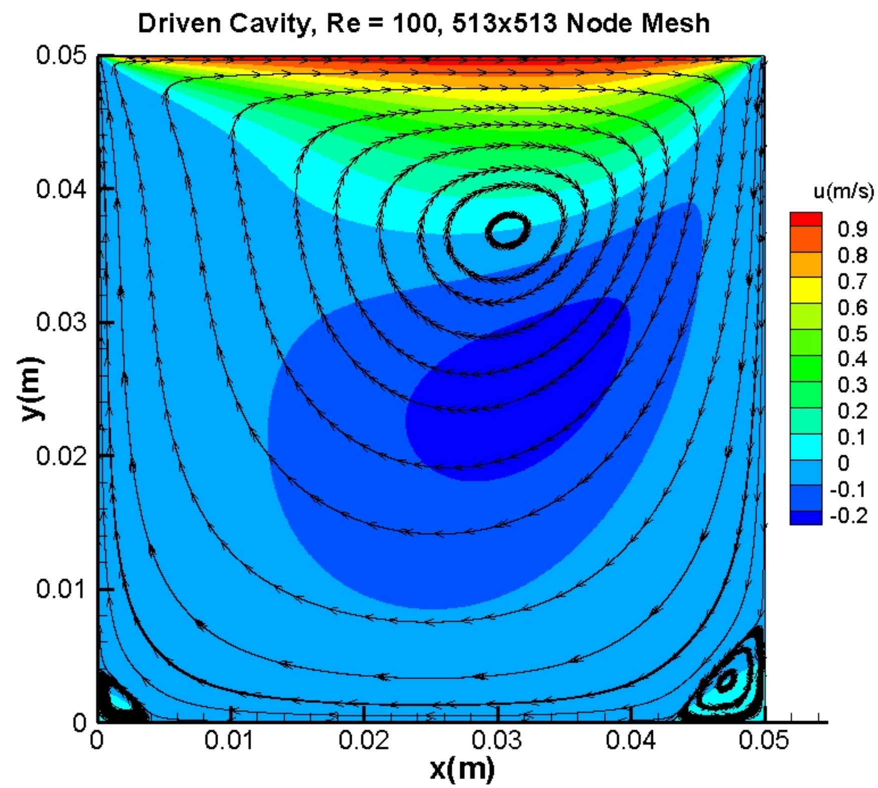


Pressure **Streamlines**

Lid Driven Cavity: Re Effects

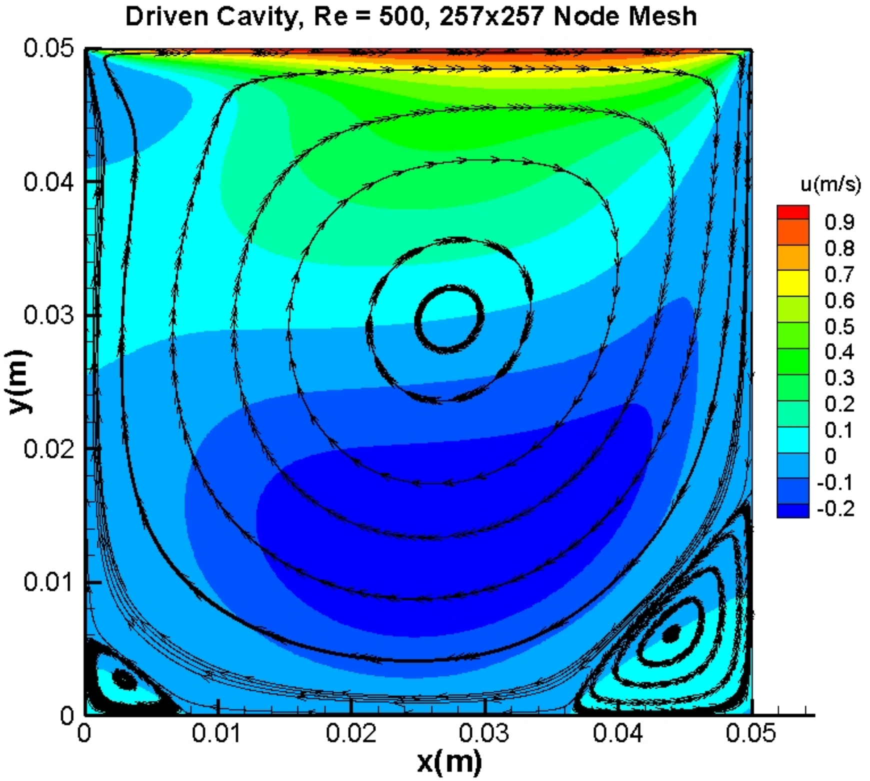


Re = 5

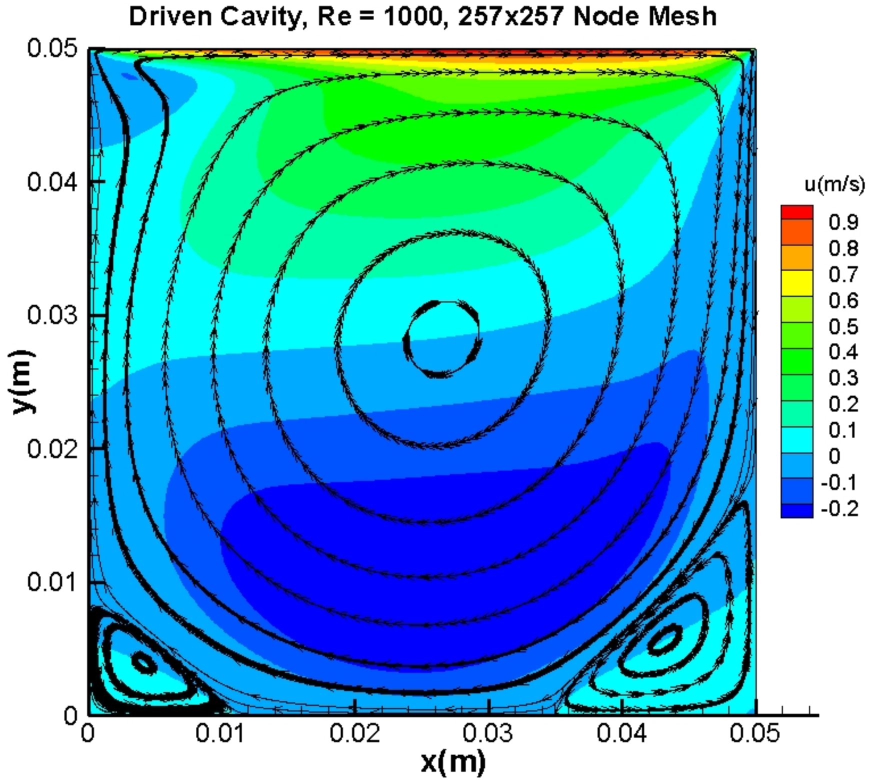


Re = 100

Lid Driven Cavity: Re Effects

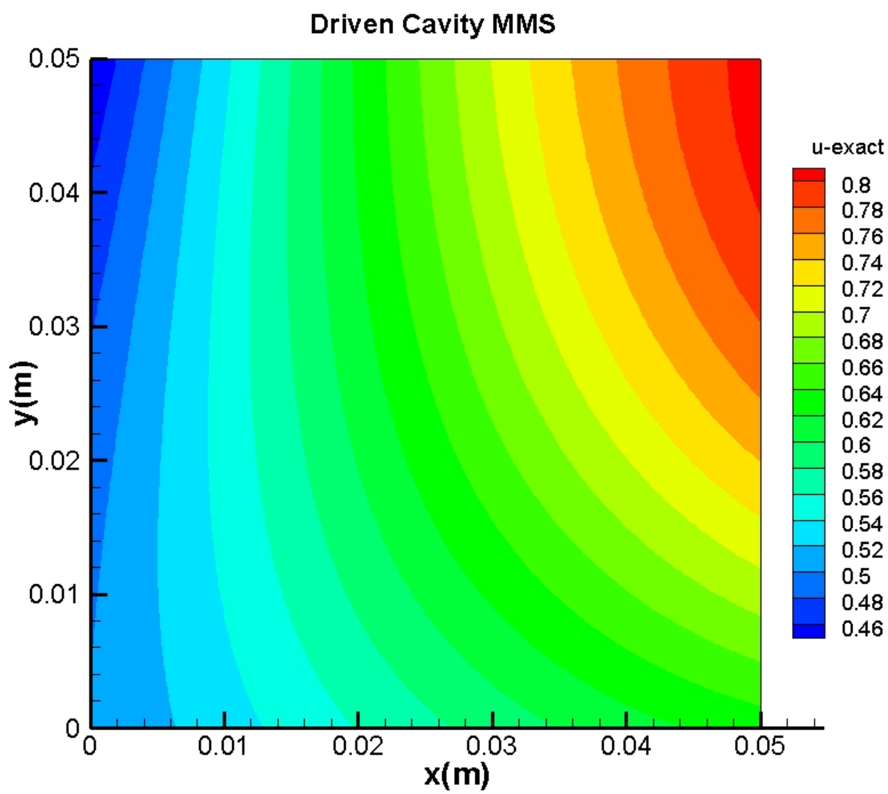


Re = 500

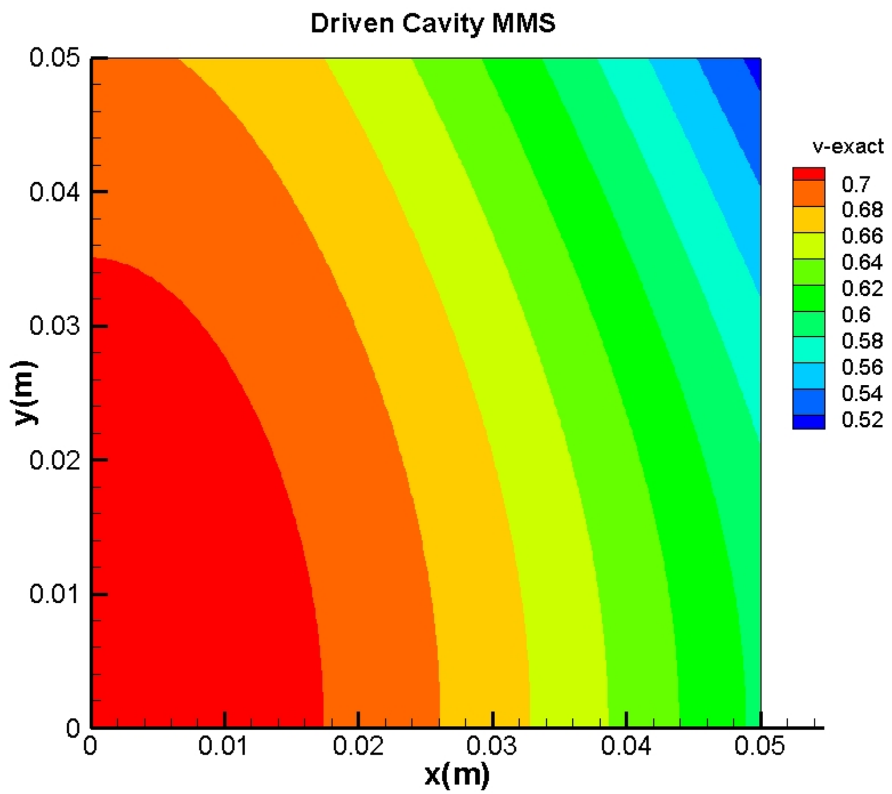


Re = 1000

Manufactured Solution

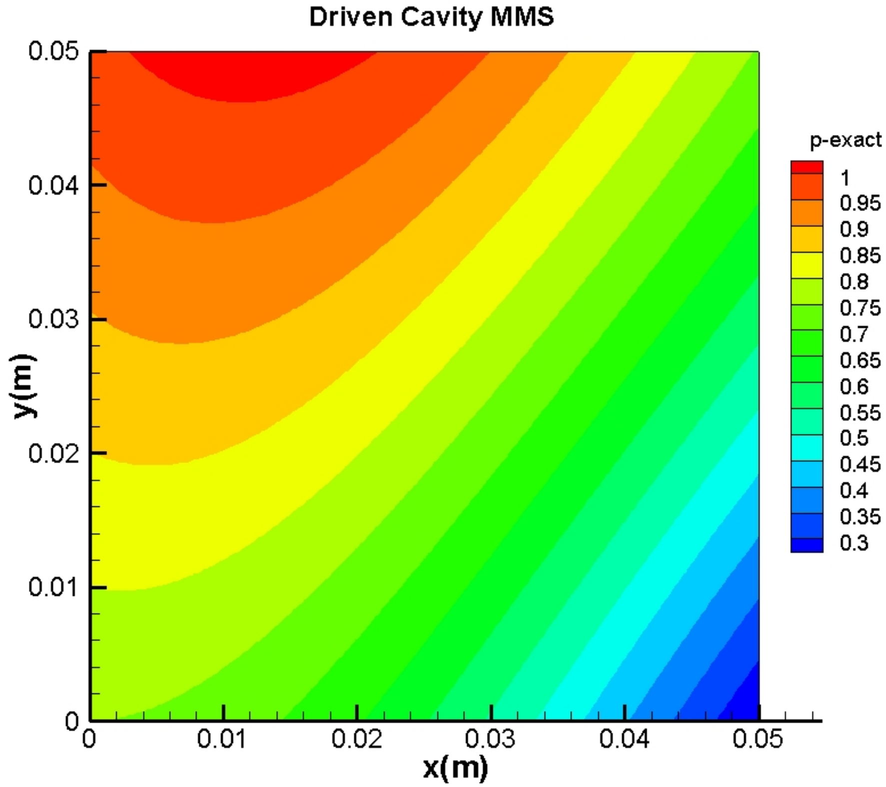


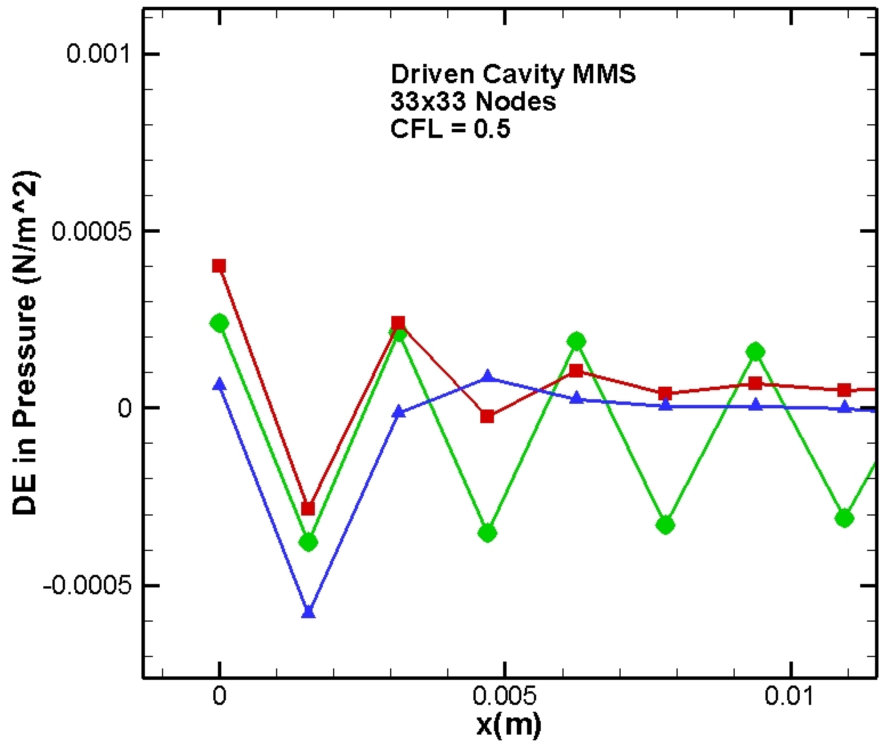
u-Velocity



v-Velocity

Manufactured Solution

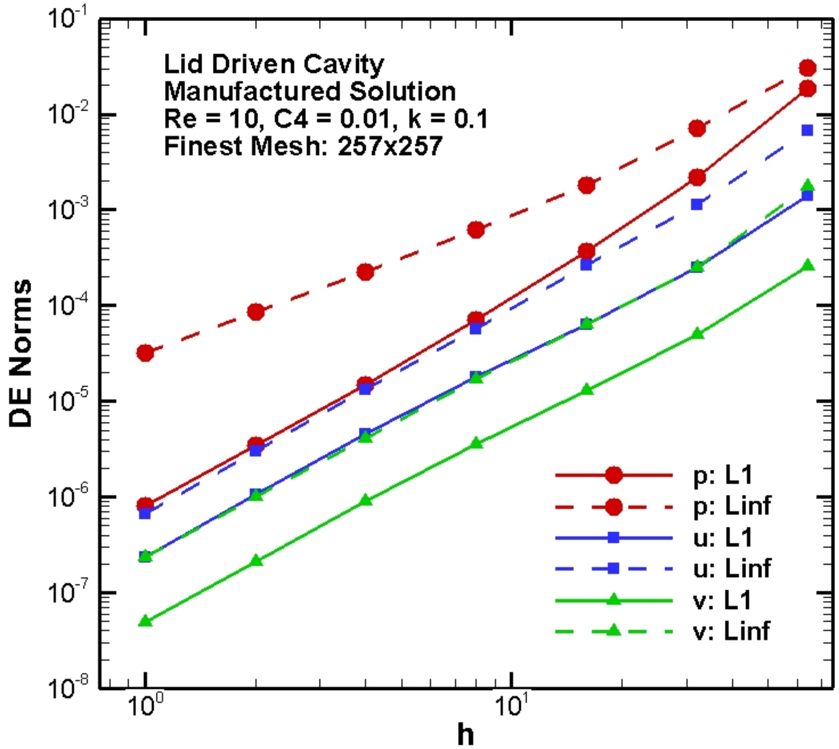


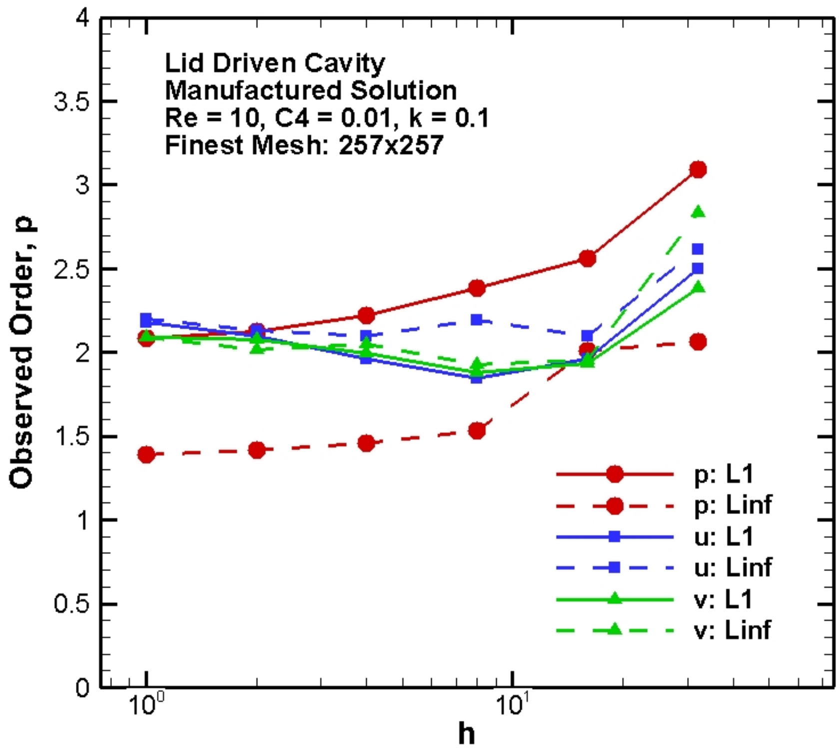


Pressure

Effect of $C^{(4)}$ Constant

Manufactured Solution: Re = 10





DE Norms **Order of Accuracy**