# Assignment – 3

1.

Those configuration for which the rank of J decreases are of special configuration. Such configuration are called singularity or singular configurations.

a. To know whether the given configuration is singular or not- we need know that 'J(Jacobian)' is the function of joint variable , it is not constant .

Rank of J(q) < min[6,n]

OR det[J(q)]= 0 ; then we can find the singular configuration.

b. $$J_{11} = \begin{bmatrix} -a2s1c2 - a3s1c23 & -a2s2c1 - a3c1s23 & -a3c1s23 \\ a2c1c2 + a3c1c23 & -a2s1s2 - a3s1s23 & -a3s1s23 \\ 0 & a2c2 + a3c23 & a3c23 \end{bmatrix}$$

Det $J_{11}$ = a2a3s3(a2c2+a3c23)

For sin(theta 3)= 0 , i.e., theta3 = 0 or 180

The situation in figure arises when elow is fully extended or fully retracted – singularity arises.

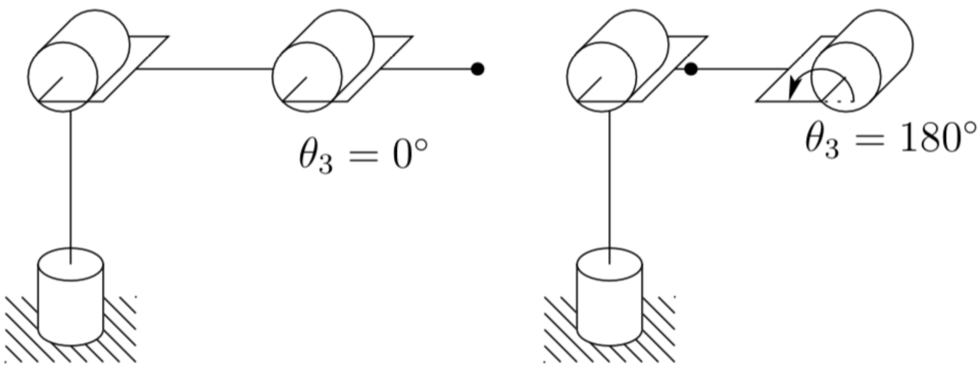



Fig. elbow manipulator in singular configuration.

12. Universal robot 5 is the most convenient type of robot . It comes along with the easy programmable software with 3d visualisation and not only that it comes with analog touchscreen display that allows to move robot arm to desired waypoint in given workspace.

Number of link = 6

Number of joints= 5

Nature of joint = revolute

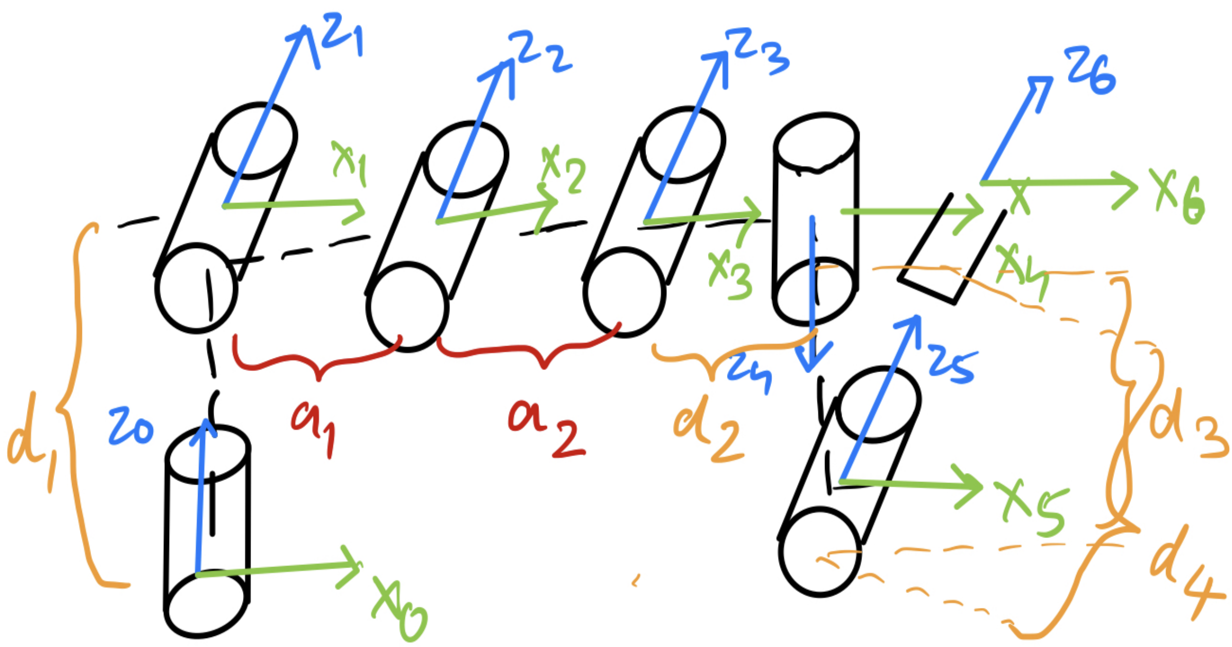
Degree of freedom = 6

Workspace= 850 mm

Payload = 5kg

Footprint = 149mm diameter

Length of robot arm <=6m (as cable length is 6m)




| **UR3e** | | | | |
|---|---|---|---|---|
| **Kinematics** | theta [rad] | a [m] | d [m] | alpha [rad] |
| Joint 1 | 0 | 0 | $d_1$ | π/2 |
| Joint 2 | 0 | $a_1$ | 0 | 0 |
| Joint 3 | 0 | $a_2$ | 0 | 0 |
| Joint 4 | 0 | 0 | $d_2$ | π/2 |
| Joint 5 | 0 | 0 | $d_3$ | -π/2 |
| Joint 6 | 0 | 0 | $d_4$ | 0 |

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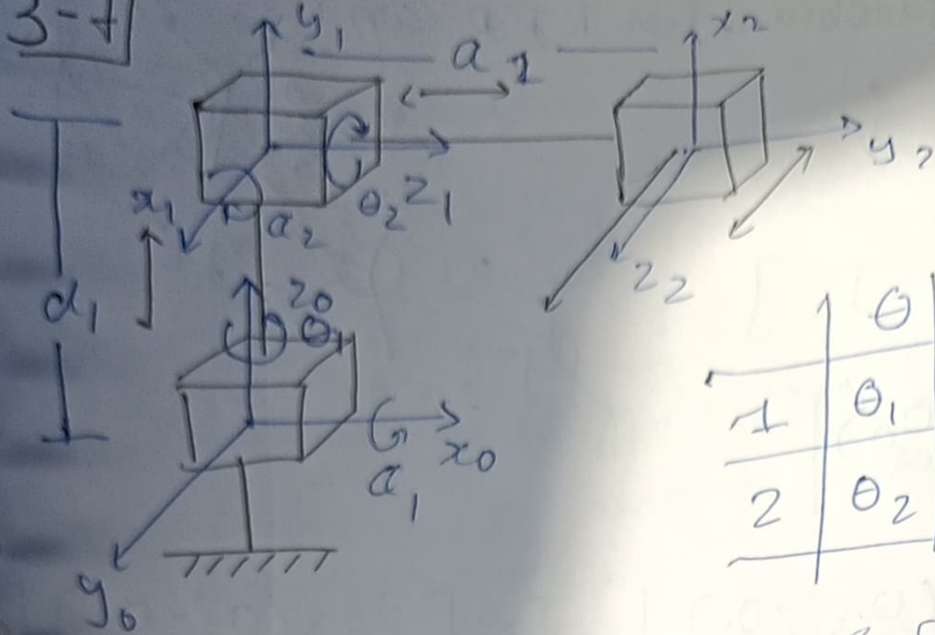


| | $\theta$ | $\alpha$ | $r$ | $d$ |
|---|---|---|---|---|
| 1 | $\theta_1$ | $\alpha_1$ | 0 | $d_1$ |
| 2 | $\theta_2$ | $\alpha_2$ | $a_1$ | 0 |

$H_0 = [$ $\quad J_1^0 = \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix}$ $\quad J_1 = \begin{bmatrix} z_0 \\ 0 \end{bmatrix}$

$J = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}$ $\quad z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$H_0^1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\quad z_1 = [\;]$

$H_1^2 = \begin{bmatrix} \; \end{bmatrix}$

$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(3-8)

$$H_0^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_1 C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_1 S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & C\theta_1 & -S\theta_2 & 0 \\ 0 & S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} C\theta_3 & 0 & S\theta_3 & a_3 C\theta_3 \\ 0 & 1 & 0 & a_3 S\theta_3 \\ -S\theta_3 & 0 & C\theta_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & C\theta_4 & -S\theta_4 & 0 \\ 0 & S\theta_4 & C\theta_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^4 = H_0^1 H_1^2 H_2^3 H_3^4$$

$$= \begin{bmatrix} 0 & C\theta_1 S\theta_3 S\theta_4 & C\theta_1 S\theta_3 C\theta_4 & a_3 C\theta_1 C\theta_3 - a_3 S\theta_1 C\theta_1 C\theta_3 + a_2 C\theta_1 \\ 0 & S\theta_1 S\theta_3 S\theta_4 + C\theta_1^2 C\theta_4 - S\theta_2 C\theta_4 C\theta_3 S\theta_4 & S\theta_1 S\theta_3 C\theta_4 - S\theta_3 C\theta_1^2 - S\theta_2 C\theta_1 C\theta_3 C\theta_4 & a_3 C\theta_3 S\theta_1 + a_3 S\theta_3 C\theta_1^2 + a_2 S\theta_1 + a_1 S\theta_1 \\ 0 & C\theta_2 C\theta_3 S\theta_4 & C\theta_2 C\theta_3 C\theta_4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. 2R Manipulator

1) Direct drive

→ Motors are attached direct at the joint of links and base.

2) Remotely driven link

→ Both joints are driven by motors mounted at the base.

→ First is directly connected to motor and second link is connected with belt drive.

→ Angle $P_2$ is determined by driving motor 2 and is independent of angle $P_1$ irrespective in case 1.

→ In this case, $P_1$ & $P_2$ are not joint angles used as in case 1.

3) Five bar linkage.

→ The Eq^n of the manipulator are decoupled, so to control $q_1$ & $q_2$ independently.

→ closed kinematic chain arrangement

→ As $\phi$ (derivative of $V(q)$) is depend on $q_1$ & $q_2$ independently. This is the biggest advantage of five bar linkage.

DisAdvantages:-

→ complex calculation and Individual analysis of each link.

8. $\sum_j d_{kj}\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k$ $\Rightarrow$ Dynamic Eq$^n$ of 2R manipulator

$$\phi_k(q) \Downarrow \frac{\partial V}{\partial q_k}$$

$\therefore$

$$\tau_1 = \frac{1}{3}m_1 l_1^2\ddot{q}_1 + m_2 l_1^2\ddot{q}_1 + m_2\frac{l_1 l_2}{2}\ddot{q}_2\cos(q_2-q_1)$$
$$- m_2\frac{l_1 l_2}{2}\dot{q}_2(\dot{q}_2-\dot{q}_1)\sin(q_2-q_1)$$
$$+ m_1 g\frac{l_1}{2}cq_2 + m_2 g l_1 cq_2$$

$$\tau_2 = \frac{1}{2}m_2 l_2^2\ddot{q}_2 + m_2\frac{l_2^2}{4}\ddot{q}_2 + m_2\frac{l_1 l_2}{2}\ddot{q}_1\cos(q_2-q_1) + m_2 g\frac{l_2}{2}q_2$$
$$- m_2\frac{l_1 l_2}{2}\dot{q}_1(\dot{q}_2-\dot{q}_1)\sin(q_2-q_1)$$

$\Rightarrow$ Inverse kinematics

$$\left[m_1 l_{c_1}^2 + m_2(l_1^2 + l_{c_2}^2 + 2l_1 l_{c_2}^2 + 2l_1 l_{c_2}\cos q_2)\right]\ddot{q}_1 +$$
$$[m_2(l_{c_2}^2 + l_1 l_{c_2}\cos q_2)]\ddot{q}_2 - m_2 l_1 l_{c_2}\sin q_2\,\dot{q}_1\dot{q}_2$$
$$- m_2 l_1 l_{c_2}\sin q_2\,\dot{q}_1\dot{q}_2$$
$$- m_2 l_1 l_{c_2}\sin q_2\,\dot{q}_2^2$$
$$+ (m_1 l_{c_1} + m_2 l_1)g\cos q_1 + m_2 l_{c_2} g\cos(q_1+q_2) = \tau_1$$

$$m_2(l_{c_2}^2 + l_1 l_{c_2}\cos q_2)\ddot{q}_1 + m_2 l_{c_2}^2\ddot{q}_2 - m_2 l_1 l_{c_2}\sin q_2\,\dot{q}_1^2$$
$$+ m_2 l_{c_2}\cos(q_1+q_2) = \tau_2$$

In Miniproject, we have not taken in account the gravity & mass of ~~len~~ centre of mass for the length.

In Dynamic Equation, velocity ~~is~~ analysis is done with the help of Jacobian and coordinate frame are taken in ~~s~~ such a way that at the joint ~~and~~. velocity and angular velocity are assumed to be acted at centre of mass & joint respectively.

9. Review done.

10. For given $D(q)$ and $V(q)$

$\boxed{\text{Step 1:}}$ Write down each and every element of $D(q)$ matrix seperate

like $d_{11} =$ ___ $d_{12} =$ ___

$d_{21} =$ ___ $d_{22} =$ ___

$\boxed{\text{Step 2:}}$ Find $c_{ijk} = \frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ik}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}$

For $i = 1,2,3$ ; $j = 1,2,3$ ; $k = 1,2,3$

from which majority of term will be zero.

~~then~~

$\boxed{\text{Step 3:}}$ find $\frac{\partial V(q)}{\partial q}$.

$\boxed{\text{Step 5:}}$ Put all the values in $D(q)\ddot{q} + C(q,\dot{q})\dot{q} + \phi_k(q) = \tau$