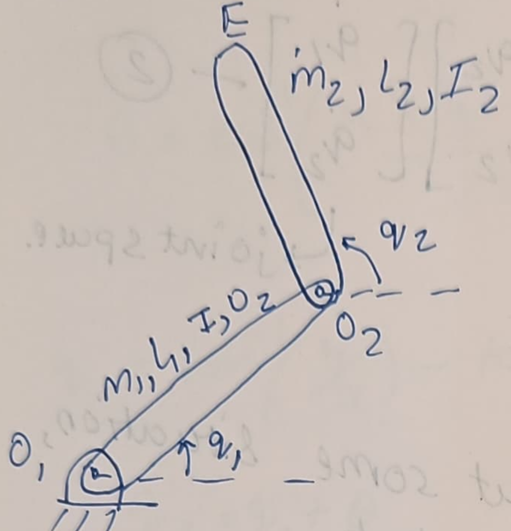
24/08/22

MINI-PROJECT

AVNISH P. CHOKSHI
22210009
Mtech Mech

Task-0



$m_1, m_2 \Rightarrow$ mass of link 1 & link 2 resp.

$l_1, l_2 \Rightarrow$ lengths of link 1 & link 2 resp.

$I_1, I_2 \Rightarrow$ Moment of Inertia (mass) link 1 & link 2

$q_1, q_2 \rightarrow$ Angles made by horizontal with link 1 & link 2 resp.

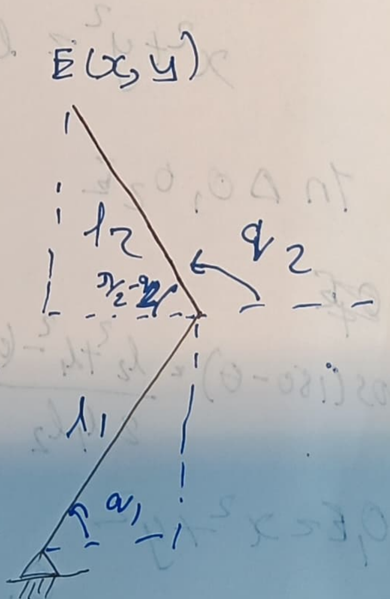
$E \rightarrow$ End effector

Motors connected at O_1, O_2 providing torque τ_1 & τ_2 an controlling the angles q_1 & q_2 are desired.

Position of $E(x, y)$

$$x = l_1 \cos q_1 + l_2 \cos q_2$$

$$y = l_1 \sin q_1 + l_2 \sin q_2$$



Simplify,

$$\left.\begin{aligned} x &= l_1 c q_1 + l_2 c q_2 \\ y &= l_1 s q_1 + l_2 s q_2 \end{aligned}\right\} \text{— (1)}$$

Differentiate Eq^n 1

$\dot{x} = -l_1 s q_1 \dot{q}_1 - l_2 s q_2 \dot{q}_2$

$\dot{y} = l_1 \dot{q}_1 c q_1 + l_2 \dot{q}_2 \cdot c q_2$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & -l_2 s q_2 \\ l_1 c q_1 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad \text{— } 2$$

cartesian / Task space / space

joint space.

If we want to figure out some situation, that requires particular output.
we require position & torque.

<u>Squaring</u> & <u>adding</u> Eq^n 1

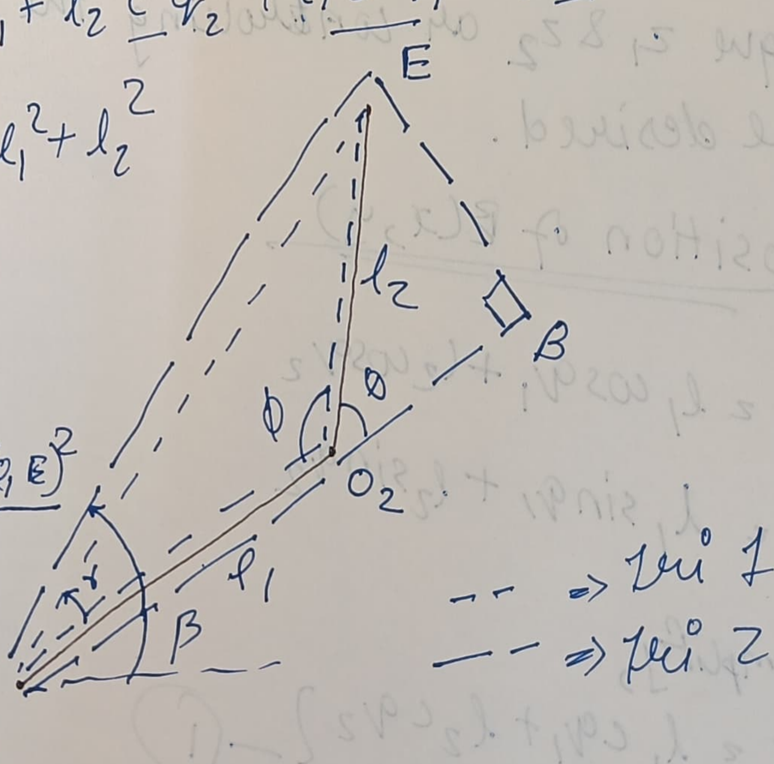
$x^2 + y^2 = l_1^2 c^2 q_1 + l_2^2 c^2 q_2 + l_1^2 s^2 q_1 + l_2^2 s^2 q_2$

$x^2 + y^2 = l_1^2 + l_2^2$

In $\Delta O_1 O_2 E$

$\cos(180 - \theta) = \dfrac{l_2^2 + l_1^2 - (O_1 E)^2}{2 l_1 l_2}$

$O_1 E = x^2 + y^2$



$q_2 = q_1 + \theta$

$l_1^2 + l_2^2 = h^2 \Rightarrow$ Right angle Triangle (Tri-2)

$l_1^2 + l_2^2 - 2 l_1 l_2 \cos\theta = h^2$ (Tri-1)

$$\theta = \cos^{-1}\left(\frac{x^2 + y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

$$q_1 = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2 \sin\theta}{l_1 + l_2 \cos\theta}\right) \quad (\because q_1 = \beta - \gamma)$$

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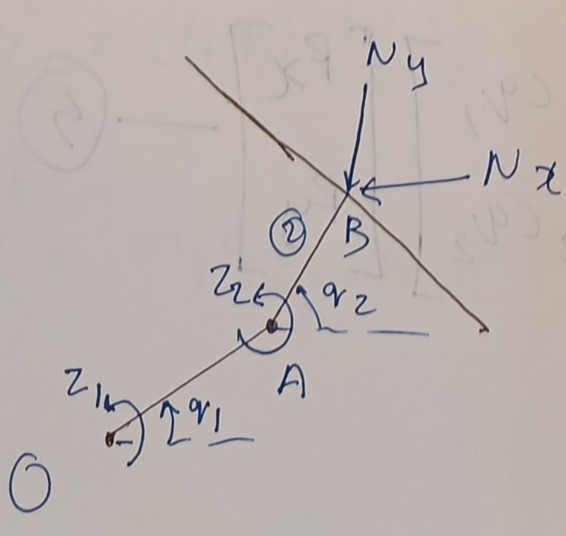
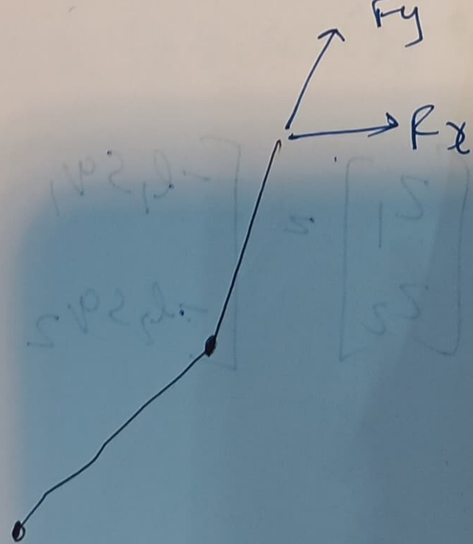
$q_2 = q_1 + \theta$

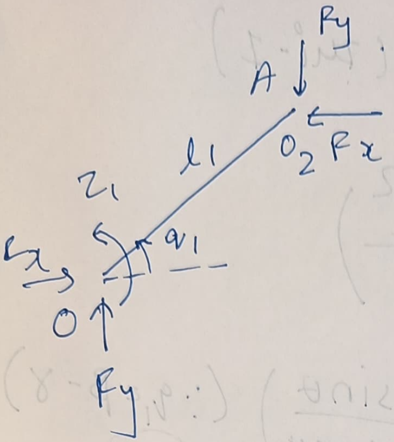
Force & Torque

In 2R elbow manipulator, Force acting on end effector is N_y & N_x by the wall and Force exerted by endeffector on the wall is F_y & F_x.





FBD of link 1

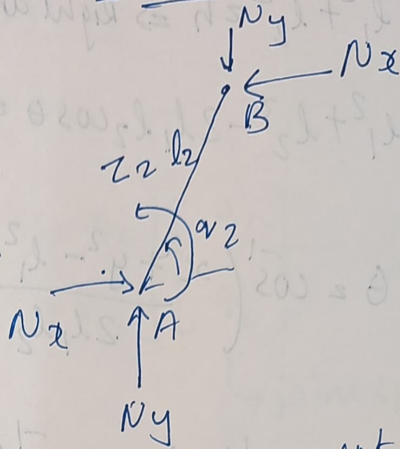


Taking moment about O

$$\Sigma M_O = 0$$

$$\boxed{-F_x l_1 S q_1 + F_y l_1 C q_1 = z_1}$$

FBD of link 2



Taking moment about A

$$\Sigma M_A = 0$$

$$\therefore z_2 = N_y l_2 C q_2 - N_x l_2 S q_2$$

$|N_y| = |F_y|$ &

$|N_x| = |F_x|$

$$\boxed{\therefore z_2 = F_y l_2 C q_2 - F_x l_2 S q_2}$$

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} -l_1 S q_1 & l_1 C q_1 \\ -l_2 S q_2 & l_2 C q_2 \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix} \quad \text{--- } (5)$$

Lagrange's Eq^n

$$L = K - V$$

K.E (for K), P.E (for V)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) - \frac{\partial L}{\partial q_1} = Q_i, \quad i = 1, 2, \ldots, n$$

Q_i → generalized forces.

$$K = \frac{1}{2} I_1 \omega_1^2 + \frac{1}{2} I_2 \omega_2^2 + \frac{1}{2} m_2 v_2^2$$

$\frac{1}{2} I_1 \omega_1^2$ → Pure rotation of link 1

$\frac{1}{2} I_2 \omega_2^2$ → Pure rotation of link 2

$\frac{1}{2} m_2 v_2^2$ → translation of center of mass of link 2.

about centre of mass

$$K = \frac{1}{2}\left(\frac{1}{2} m_1 l_1^2\right)\dot{q}_1^2 + \frac{1}{2}\left(\frac{1}{12} m_2 l_2^2\right)\dot{q}_2^2 + \frac{1}{2} m_2 v_{c_2}^2$$

$$v_{c_2}^2 = (l_1 \dot{q}_1)^2 + \left(\frac{l_2}{2}\dot{q}_2\right)^2 + 2 l_1 \dot{q}_1 \frac{l_2}{2}\dot{q}_2 \cos(q_2 - q_1)$$

P.E

$$V = m_1 g \frac{l_1}{2} s q_1 + m_2 g \left(l_1 s q_1 + \frac{l_2}{2} s q_2\right)$$

$$\tau_1 = \frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + m_2 \frac{l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1)$$

$$- m_2 \frac{l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1)$$

$$+ m_1 g \frac{l_1}{2} c q_2 + m_2 g l_1 c q_2$$

$$\tau_2 = \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + m_2 \frac{l_2^2}{4} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1)$$

$$- m_2 \frac{l_1 l_2}{2} q_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + m_2 g \frac{l_2}{2} q_2$$

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