



Content

01. Longest common subsequence
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Q Given 2 strings, find the length of longest common subsequence.

$S_1 = abc \rightarrow \{a, b, c, ab, ac, abc, bc\}$ Ans = 2

$S_2 = ace \rightarrow \{a, c, e, ac, ae, ace, ce\}$

$S_1 = abbc dgh$
 $S_2 = bach egf$ { ach, acg, bfg } Ans = 3

$S_1 = abbc dgh$
 $S_2 = ach egf$ { acgh } Ans = 4

$S_1 = abbc dgh$
0 1 2 3 4 5 6

$S_2 = ach egf$
0 1 2 3 4 5

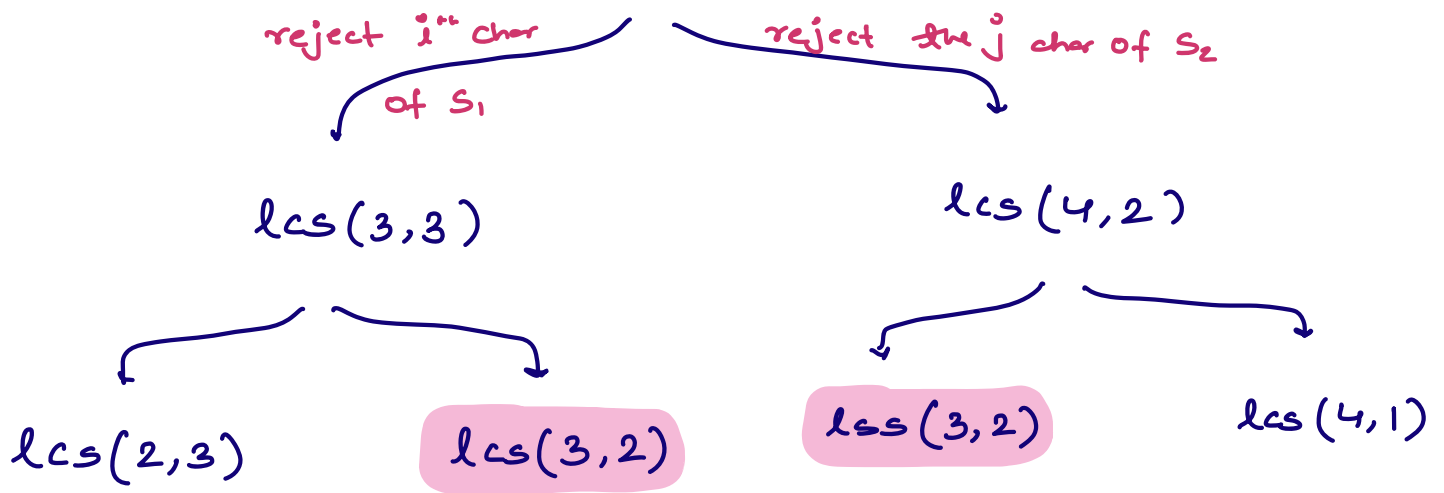
$lcs(6, 5)$

\downarrow if $(S_1[6] == S_2[5])$

$lcs(5, 4) + 1$

\downarrow $S_1[5] == S_2[4]$

$lcs(4, 3) + 1$



$$\text{lcs}(i, j) = \begin{cases} \text{lcs}(i-1, j-1) + 1; & \text{if } s_1[i] == s_2[j] \\ \text{Max} \begin{cases} \text{lcs}(i-1, j) \\ \text{lcs}(i, j-1) \end{cases} & \text{if } s_1[i] \neq s_2[j] \end{cases}$$

int [][] dp = new int[n][m] → (-1)

int lcs (String s₁, String s₂, i-1, j-1)

if (i < 0 || j < 0) return 0;

if (dp[i][j] != -1) return dp[i][j];
ans = 0

if (s₁[i] == s₂[j]) {

ans = lcs (s₁, s₂, i-1, j-1) + 1

}

else {

int a = lcs (s₁, s₂, i-1, j);

TC: O(n*m)

SC: O(n*m)

```
int b = lcs(s1, s2, i, j-1);
```

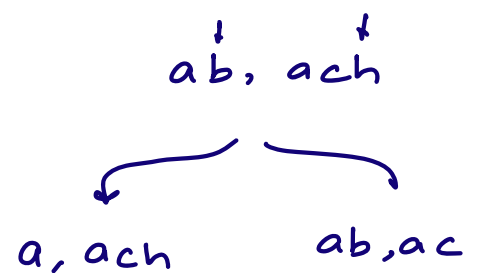
```
ans = Max(a, b)
```

```
dp[i][j] = ans;
```

```
return ans;
```

$dp[i][j] \Rightarrow$ max length of common subsequence
in $(0-i)$ of s_1 & $(0-j)$ of s_2

		a	c	h	e	g	f
		0	1	2	3	4	5
a	0	1	1	1	1	1	1
b	1	1	1	1	1	1	1
b	2	1	1	1	1	1	1
c	3	1	2	2	2	2	2
d	4	1	2	2	2	2	2
g	5	1	2	2	2	3	3
f	6	1	2	2	2	3	4



```
for (i=0; i<n; i++)
```

```
    for (j=0; j<m; j++) {
```

```
        if (s1[0] == s2[0]) {
```

```
            dp[0][0] = 1;
```

```
        else if (j==0) {
```

```
            if (s1[i] == s2[j]) dp[i][j] = 1
```

```
            else dp[i][j] = dp[i-1][j];
```

```
        else if (i==0) {
```

```
            if (s1[i] == s2[j]) dp[i][j] = 1
```

```
            else dp[i][j] = dp[i][j-1];
```

```
        else {
```

```
            if (s1[i] == s2[j])
```

```
                dp[i][j] = dp[i-1][j-1] + 1;
```

```
            else {
```

```
                dp[i][j] = max(dp[i-1][j], dp[i][j-1]);
```

```
    return dp[n-1][m-1];
```

Q Given a string, find longest palindromic subsequence

A = "scalar" Ans = 3

A = "abcdefb" Ans = 3

A = "abdcgba" Ans = 5

palindrome of A = palindrome of reverse of A

$$\left. \begin{array}{l} S_1 = \text{scalar} \\ S_2 = \text{ralacsa} \end{array} \right\} \text{find LCS} \rightarrow 3$$

$$\left. \begin{array}{l} A = \text{abcdefb} \\ B = \text{bfedcba} \end{array} \right\} \underline{\underline{\text{Ans} = 3}}$$

$$\left. \begin{array}{l} A = \text{abdcgba} \\ B = \text{abgcdba} \end{array} \right\} \underline{\underline{\text{Ans} = 5}}$$

Q Given 2 strings S_1 & S_2 , find min operations to be performed in S_1 so that it becomes equal to S_2 .

Operations allowed

01. Insert \rightarrow we can insert any character
02. Replace \rightarrow we can replace any character
03. Delete \rightarrow we can delete any character

Eg:- $S_1 = \cancel{d} f \cancel{a} \cancel{c} l$ \rightarrow delete d
 $S_2 = f g l$ \rightarrow delete a
 \rightarrow replace c } Ans = 3

Eg:- $S_1 = \cancel{d} f \cancel{a} \cancel{x} \cancel{x} z$ } Ans = 4
 $S_2 = f x z$

0 1 2 3 4
 $S_1 = d f a c l$

$S_2 = f g l$
0 1 2

$ed(4, 2)$

$S_1[4] == S_2[2]$

$ed(3, 1)$

$\rightarrow ans = \min(a, b, c) + 1;$

insert

$ed(3, 0)$

$S_1 = dfacg$

$S_2 = fg$

a

delete

$ed(2, 1)$

$S_1 = dfac$

$S_2 = fg$

b

replace

$ed(2, 0)$

$S_1 = dfac$

$S_2 = fg$

c

$if(S_1[i] == S_2[j])$

$(i-1, j-1)$

* (i, j)

$\min \left\{ \begin{array}{l} \text{insert}(i, j-1) \\ \text{delete}(i-1, j) \\ \text{replace}(i-1, j-1) \end{array} \right\} + 1;$

$int[][] dp = \text{new int}[n][m] \rightarrow -1$


```
int ed (String s1, s2, i, j)
```

```
if (i < 0 & j < 0) return 0;
```

```
if (i < 0) return j+1 // j+1 no. of char to insert
```

```
if (j < 0) return i+1 // i+1 char to be deleted
```

```
if (dp[i][j] != -1) return dp[i][j];
```

```
int ans = 0
```

```
if (s1[i] == s2[j])
```

```
    | ans = ed(s1, s2, i-1, j-1);  
    |  
    | 3
```

```
else {
```

```
    | int a = ed(s1, s2, i, j-1); // insert
```

```
    | int b = ed(s1, s2, i-1, j); // delete
```

```
    | int c = ed(s1, s2, i-1, j-1); // replace
```

```
    | ans = min(a, b, c) + 1;
```

```
    | 3
```

```
dp[i][j] = ans;
```

```
return ans;
```

TC: $O(n \times m)$

SC: $O(n \times m)$

* Regular Expression Matching

Given an input string S & pattern P , implement pattern matching with support for $?$ & $*$

$?$ \rightarrow matches any single character

$*$ \rightarrow matches any sequence of character

$S = "aa"$

$P = "a"$

Ans = false

$S = "aa"$

$P = "*"$

Ans = true

$S = "cb"$

$P = "?a"$

Ans = false

$S = "ab"$

$P = "a*b"$

} Ans = true

$S = a$

$P = a*b$

} Ans = false

$$\left. \begin{array}{l} S = axyzpqb \\ P = a * b \end{array} \right\} \text{Ans} = \text{true}$$

$$\left. \begin{array}{l} S = abc \\ P = a ? * \end{array} \right\} \text{Ans} = \text{true}$$

Approach

Case 1 $\begin{array}{c} 0 \ 1 \ 2 \ 3 \ 4 \ 5 \\ S_1 = a b c d e f \end{array} \rightarrow$

$\begin{array}{c} P = a * d ? f \\ 0 \ 1 \ 2 \ 3 \ 4 \end{array} \rightarrow$

$\text{isMatch}(5, 5) \rightarrow \text{ismatch}(4, 3)$

Case 2 $\begin{array}{c} 0 \ 1 \ 2 \ 3 \ 4 \\ S = a b c d e \end{array}$

$\begin{array}{c} P = a * d ? \\ 0 \ 1 \ 2 \ 3 \end{array}$

$\text{ismatch}(4, 3) \rightarrow \text{ismatch}(3, 2)$

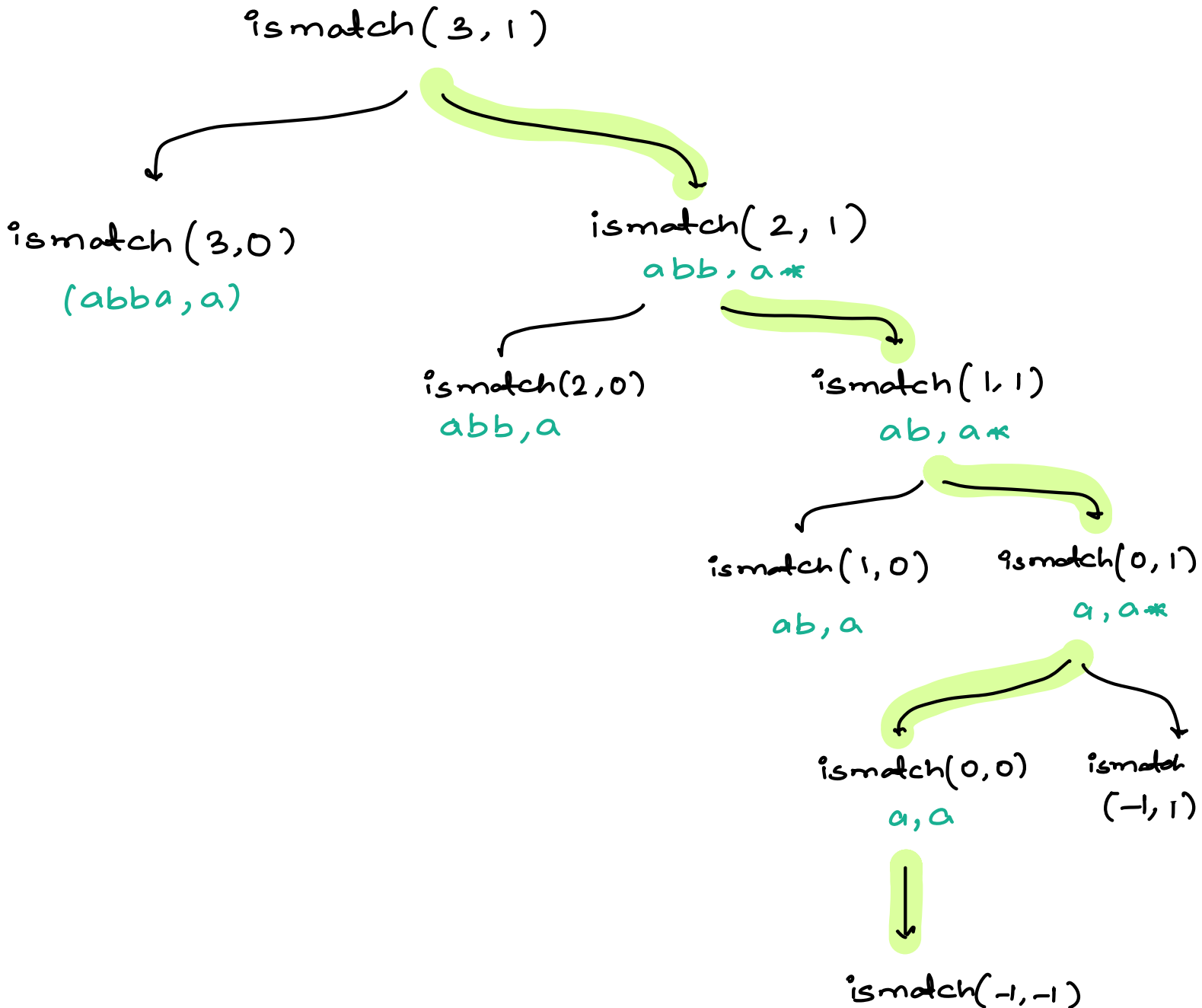
Case 3 $\begin{array}{c} 0 \ 1 \ 2 \ 3 \\ S = a b b a \end{array}$

$\begin{array}{c} P = a * \\ 0 \ 1 \end{array}$

* \rightarrow it can match with 0 characters

* \rightarrow can match with one or more character

$\text{ismatch}(\overset{0}{a}\overset{1}{b}\overset{2}{b}\overset{3}{a}, \overset{0}{a}\overset{1}{*}) \longrightarrow \underline{\text{True}}$



$S = \overset{0}{a}\overset{1}{c}\overset{2}{b}$

$P = a? * b$
 $\overset{0}{a}\overset{1}{?}\overset{2}{*}\overset{3}{b}$

True

ismatch(2, 3)

acb, a?*b

ismatch(1, 2)

ac, a?*

* matches with
0 character

ismatch(1, 1)

ac, a?

ismatch(0, 0)

a, a

ismatch(-1, -1)

ismatch(0, 2)

a, a?*

ismatch(0, 1)

a, a?

ismatch(-1, 0)

_, a

ismatch(-1, 2)

_, a?*

S = _

P = *

* Case 4

S = abc

P = a?*d

Ans = false

boolean ismatch (String S, String P, int i, int j)

```
if (i < 0 && j < 0) return true;
else if (j < 0) return false
```

```
if (i < 0) {
    for (k = 0; k ≤ j; k++) {
        if (P[k] != '*') return false;
    }
    return true;
}
```

```
if (S[i] == P[j])
    return ismatch(S, P, i-1, j-1);
}
```

```
else if (P[j] == '?')
    return ismatch(S, P, i-1, j-1);
}
```

```
else if (P[j] == '*') {
    return ismatch(S, P, i, j-1) ||
           ismatch(S, P, i-1, j);
}
```

```
else {
    return false;
}
```

Doubles

a b c d
0 1 2 3
k c d
 ↑

$lcs(l, m)$

Max

for ($i=l; i<n; i++$)

for ($j=m; j<m; j++$)

if ($match(i, j)$)

$m = \boxed{lcs(i+1, j+1)} + 1$

3

else

3

3

3