Fixed vs. Collider Physics

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1 Center of Mass Comparison

1.1 Fixed

First we use the definition of the s-channel

$$s = (P_p c^2 + P_e c^2)^2$$

Since P_p is fixed $\vec{P}_p = 0$

$$s = M_p^2 c^4 + m_e^2 c^4 + 2E_p E_e$$

$E_e \gg M_p c^2$ and $m_e c^2$, therefore

$$s \approx 2E_p E_e$$

Using the Energy-Momentum Relation and knowing $\vec{P}_p = 0$ we can use $E_p = M_p c^2$ to get

$$s \approx 2E_e M_p c^2$$

The center of mass is defined as $\sqrt{s}$ so the center of mass energy is

$$\sqrt{s} \approx c\sqrt{2E_e M_p}$$

1.2 Collider

Again using the definition of the s-channel

$$s = (P_p c^2 + P_e c^2)^2 = M_p^2 c^4 + m_e^2 c^4 + 2(E_p E_e - \vec{P}_p \cdot \vec{P}_e c^2)$$

Using the Energy-Momentum Relation and the fact that the two beams are antiparallel

$$s = E_p^2 - |\vec{P}_p|^2 + E_e^2 - |\vec{P}_e|^2 + 2(E_p E_e + |\vec{P}_p||\vec{P}_e| c^2)$$

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Knowing $E_p \gg M_p c^2$ to get $E \approx |\vec{P}_p|c$, therefore

$$s \approx 4E_pE_e$$

The center of mass is defined as $\sqrt{s}$ so the center of mass energy is

$$\sqrt{s} \approx 2\sqrt{E_pE_e}$$

1.3 Ratio

$$\frac{\sqrt{s_{fixed}}}{\sqrt{s_{collider}}} = c\sqrt{\frac{M_p}{2E_p}}$$

2 Max Q^2

$$Q^2_{max} = \frac{s}{c^2}$$

2.1 Fixed

$$Q^2_{max,fixed} = 2E_eM_p$$

2.2 Collider

$$Q^2_{max,collider} = 4\frac{E_pE_e}{c^2}$$

3 Spacial Resolution

$$\lambda = \Delta x \approx \frac{h}{\sqrt{Q^2}}$$

3.1 Fixed

$$\lambda_{fixed} \approx \frac{h}{\sqrt{2E_eM_p}}$$

3.2 Collider

$$\lambda_{collider} \approx \frac{hc^2}{2E_pE_e}$$

4 t-channel

The t-channel is defined as

$$t = (P-N)^2 = M_p^2c^4 + M_N^2c^4 - 2(E_pE_N - \vec{P}\cdot\vec{N}c^2) = M_p^2c^4 + M_N^2c^4 - 2(E_pE_N - |\vec{P}||\vec{N}|c^2 cos\theta)$$

4.1 Fixed

Since P is fixed $\vec{P} = 0$

$$t = M_p^2c^4 + M_N^2c^4 - 2E_pE_N$$

4.2 Collider

Using the Energy-Momentum Relation and knowing $E_N, E_p \gg M_N c^2$, M_pc^2 to get $E_N \approx |\vec{N}|c$, $E_p \approx |\vec{P}|c$, therefore

$$t \approx -2E_pE_N(1 - cos\theta)$$

Lets check this answer. For the pion theory says $t \lesssim 0.6$ GeV^2 and we know that $E_p \sim E_N$

$$-t \sim 2E_p^2(1 - cos\theta) \lesssim 0.6 \text{ GeV}^2$$

If we test this result against 4on41, 10on100, and 18on275 GeV^2. Theta, θ, must be

- 5on41, $-1.08\,\text{deg} < \theta < 1.08\,\text{deg}$
- 10on100, $-0.44\,\text{deg} < \theta < 0.44\,\text{deg}$

- 18on275, $-0.16\,\text{deg} < \theta < 0.16\,\text{deg}$

Using the most up to date schematics for the EIC detector designs, the Zero Degree Calorimeter (ZDC) will be 38 meters from IP6. Using the angle and this distance we can obtain the required ZDC size for each energy

- 5on41, $\sim$ 70 cm
- 10on100, $\sim$ 30 cm
- 18on275, $\sim$ 10 cm

The trends are just as expected based off the simulation results. Now the exact geometry is not this trivial but as an exercise to check the results there is consistency.

5 Elastic vs Inelastic

At invariant masses $W \gtrsim 2.5GeV/c^2$ individual resounances can not be distinguished from excitations and thus hadrons are produced. In elastic collisions, there is only one free parameter for any fixed beam energy following from the relationships

$$2M\upsilon - Q^2 = 0,\, W = M,\, x = 1$$

where $\upsilon = \frac{Pq}{M} = E - E'$ is the energy transfer and $x = \frac{Q^2}{2Pq} = \frac{Q^2}{2M\upsilon}$ is the Bjorken scaling variable. Inelastic collisions, instead, follows

$$2M\upsilon - Q^2 > 0,\, W > M,\, 0 < x < 1$$

because of the extra degree of freedom added by the excited proton. This yields two distinct quantities; the elastic form factor (1D function) and the inelastic structure function (2D function).