Meson Splitting Functions

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This is just a quick breakdown of Patrick's email within the context of the paper he sent (10.1103/PhysRevD.94.094035). This will then get a bit into Tim Hobb's splitting function code.

1 A comparison of π and K structure functions

Almost nothing is known about K PDFs, except for one plot about the ratio of up quarks in the π and K. PDFs are not directly observable though, so it is a dubious claim since the methods used to "extract" them as data is questionable.

Current thinking would start with the assumption that the charged π and K structure functions are the same, which follows from the assumption that their PDFs are the same (of course, more data is needed to confirm this). This allows a way to relate the π (K) structure function to the tagged structure function through a comparision of splitting functions.

The tagged structure function is defined

$$F_2^T = \kappa f F_2$$

where κ is a kinematic factor, f is the splitting function, and F_2 is the inclusive structure function of the meson. The kinematic factor, κ, is the same for both π and K and the splitting function, f, is defined below (see eqns. 58-60 of 10.1103/PhysRevD.94.094035).

$$f_{KY}^{(rbw)}(y) = \frac{C_{KY}^2 \bar{M}^2}{(4\pi f_\phi)^2}[f_Y^{(on)}(y) + f_K^{(\delta)}(y)]$$

The reaction we care about is $N \to K\Lambda$ so only the on-shell splitting needs to be kept, since the δ-function term is independent of the hyperon.

$$f_Y^{(on)}(y) = y \int dk_\perp^2 \frac{k_\perp^2 + (M_Y + \Delta)^2}{(1-y)^2 D_{KY}^2} F^{(on)}$$

where $\Delta = M_Y - M$ is the difference of the hyperon and nucleon mass, repsectively.

The propagator, D_{KY}, in the denominator is defined as

$$D_{KY} = -[\frac{k_\perp^2 + yM_Y^2 + (1-y)m_K^2 - y(1-y)M^2}{1-y}]$$

and $F^{(on)}$ is the regulating function which regulates the ultraviolet divergences of $k_\perp^2$ integration. This is required because the splitting function for pointlike particles are UV divergent. The regulator is where further investigation will need done, which leads nicely into Tim Hobb's code. Before getting into Tim's code, it is good to mention the theoretical constaints. For SU(3) coupling, $D = 0.85$ and $F = 0.41$ so that to get a triplet axial charge of $g_A = D + F = 2\bar{\alpha}^{(1)} - \bar{\beta}^{(1)}/3 = 1.26$ and a octet axial charge of $g_8 = 3F - D = \bar{\alpha}^{(1)} + \bar{\beta}^{(1)} = 0.38$.

2 Tim Hobb's Splitting Function Code

Originally written in Fortran, it has been rewritten in C++ so it can easily be called into the EMMC (EIC meson MC) framework. There are splitting functions for a variety of reactions, but, for our purposes, there is a function for the $N \to \pi N'$ and $N \to K\Lambda$ processes. This section will be a work in progress as some areas will need clarification and opinions from Patrick et al.

Since the kaon splitting function was laid out above, only the kaon will be outlined here but the two are analogous with minor differences. The general equation for gthe K splitting function is

$$f(y) = \frac{g_{K\Lambda}}{16\pi^2} \int dk_\perp \frac{k_\perp^2 + (M_\Lambda - yM_N)^2}{y} * ((1-y)(s_{K\Lambda} - M_N))^2 * (2k_\perp) F^2$$

where $s_{K\Lambda} = \frac{k_\perp^2 + M_K^2}{1-y} + \frac{k_\perp^2 + M_\Lambda^2}{y}$, $g_{K\Lambda} = 4\pi * 15.56$ is the coupling (inferred from Mueller-Groeling et al.), and F are form factors with that are defined by different regulation perscriptions. The limits of integration are from $0 \to k\perp, max = 10$ and the y limits are $0 \geq y \geq 0.999$. There are five perscriptions defined

1. Monopole

$$F = \frac{\Lambda^2 + M_N^2}{\Lambda^2 + s_{K\Lambda}}$$

1. Dipole

$$F = [\frac{\Lambda^2 + M_N^2}{\Lambda^2 + k_\perp}]^2$$

1. Exponential

$$F = exp\frac{M_N^2 - k_\perp}{\Lambda^2}$$

1. Covariant Dipole

$$F = \frac{\Lambda^2 - M_K^2}{\Lambda^2 - t}$$

$$t = \frac{-k_\perp^2 - {M_N}^2 y^2}{1 - y}$$

1. Dipole s-channel lambda exchange

$$F = \frac{\Lambda^4 + M_\Lambda^4}{\Lambda^4 + s_M^2}$$

$$s_M = \frac{k_\perp^2 + (1+y)M_K^2}{y} + \frac{k_\perp^2 + yM_\Lambda^2}{1-y} + M_N^2$$

There is a good explainer for the pion perscriptions in 10.1103/PhysRevD.93.054011. This is where I think our discussion with Patrick et al. should begin as it is difficult to parse through this for a good starting point.