Week 3 Video 3

15 October 2021 10:11

Measure theoretic formulation of probability

Big example

Def^n $(\Omega, \mathcal{F}, \mathbb{P})$ is a probability space

set, σ-algebra, measure

must have $\mathbb{P}(\Omega) = 1$.

Call $\mathbb{P}$ a probability measure

Ω is called the event space or probability space

It is not often made very explict.

$A \in \mathcal{F}$ an event and $\mathbb{P}(A)$ is the probability of A.

So we've switched to looking at how likely thing
from looking at how big things are.

Def^n A random variable, X, is a measurable function

$(\Omega, \mathcal{F}, \mathbb{P}) \longrightarrow (E, \mathcal{A})$

prob space, measurable space

We can write $X = X(\omega)$ but normally its surpressed.

$B \in \mathcal{A}$

$$\mathbb{P}(X \in B) = \mathbb{P}(\{\omega \in \Omega : X(\omega) \in B\}) = \mathbb{P}(X^{-1}(B))$$

Def^n The law of a random variable X is a measure on $(E, \mathcal{A})$ given by $\mu_X(B) = \mathbb{P}(X \in B)$

μ_X is an object which allows us to understand both probability densities and discrete distributions in the same way.