1 Measure Theory: Assignment Two - Constructing new measurable functions from old measurable functions

This sheet is all about building new measurable functions from existing measurable functions. Throughout this sheet we consider functions taking values in $\mathbb{R}$. We are interested in their measurability with respect to the Borel σ-algebra.

Question 1.1. Suppose that f is a measurable function. Show that $-f$ and λf are both measurable functions, where λ is a strictly positive contant. *2 marks*

Question 1.2. Suppose that f_1 and f_2 are measurable. Show that $f_1 \wedge f_2 = \min\{f_1, f_2\}$ is also measurable. *3 marks*

Question 1.3. Suppose that $f_1, f_2, f_3, \ldots$ is a sequence of measurable functions. Show that $\inf_n f_n$ is also a measurable function. Use this to show that $\sup_n f_n$ is a measurable function as well. *5 marks*

Question 1.4. Again let $f_1, f_2, f_3, \ldots$ be a sequence of measurable functions. Show that $\limsup_n f_n$ and $\liminf_n f_n$ are both measurable functions. Why does this mean that $\lim_n f_n$ will be a measurable function if it exists. *5 marks*

Question 1.5. Suppose that f is a measurable function, show that f^2 is measurable. *2 marks*

Question 1.6. Using the previous question, or otherwise, show that if f_1 an f_2 are measurable the $f_1 f_2$ is also measurable. *3 marks*

Question 1.7. Let f be a measurable function with $f > 0$ everywhere. Show that $1/f$ is also measurable. *5 marks*