Week 3 Video 2

14 October 2021 18:16

# Characterisations of $\mathcal{M}$ (Lebesgue Measureable sets)

<u>Lemma</u> <u>Null sets are Lebaesgue measurable</u>

if $A \subseteq \mathbb{R}$ $\lambda^*(A) = 0$ then $A \in \mathcal{M}$

Pf/ This is a question on the assigment.

Hint: Remember to do the inequality both ways.

---

We also showed last week $\mathcal{B}(\mathbb{R}) \subseteq \mathcal{M}$

---

We can characterise all of $\mathcal{M}$ using these two groups of sets.

Prop$^n$ (1.2 in notes)

If $S \subseteq \mathbb{R}$ then $S \in \mathcal{M}$ iff $\exists$ $B \in \mathcal{B}(\mathbb{R})$ and $N$ a null set s.t. $S = B \Delta N$

everything in $B$ and not $N$ + everything in $N$ and not $B$.

"Lebesgue measurable sets differ from Borel sets by a null set"

---

In the excercise sheet this week show

If $A \in \mathcal{M}$ the $\exists$ $F$ and $F_\sigma$ set and $N$ a null set st. $A = F \cup N$. Recall: $F_\sigma$ set is a countable union of closed sets.

---

Key to proving this is the regularity property of Lebesgue measure.

---

In Lecture 1: we'll show there $\exists$ non-Lebesgue measurable sets.