Quantum Algorithm for Linear Systems of Equations

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***Abstract*—This document is a model and instructions for LaTeX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.**

***Index Terms*—component, formatting, style, styling, insert**

I. Ease of Use

A. Maintaining the Integrity of the Specifications

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Keep your text and graphic files separate until after the text has been formatted and styled. Do not number text heads—LaTeX will do that for you.

A. Abbreviations and Acronyms

Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
- Avoid combining SI and CGS units, such as current in amperes and magnetic field in oersteds. This often leads to confusion because equations do not balance dimensionally. If you must use mixed units, clearly state the units for each quantity that you use in an equation.
- Do not mix complete spellings and abbreviations of units: "Wb/m^2" or "webers per square meter", not "webers/m^2". Spell out units when they appear in text: ". . . a few henries", not ". . . a few H".
- Use a zero before decimal points: "0.25", not ".25". Use "cm^3", not "cc".)

C. Equations

Number equations consecutively. To make your equations more compact, you may use the solidus (/), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \tag{1}$$

Be sure that the symbols in your equation have been defined before or immediately following

D. LaTeX-Specific Advice

Please use "soft" (e.g., `\eqref{Eq}`) cross references instead of "hard" references (e.g., `(1)`). That will make it possible to combine sections, add equations, or change the order of figures or citations without having to go through the file line by line.

Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

III. Template

Please note that the `{subequations}` environment in LaTeX will increment the main equation counter even when there are no equation numbers displayed. If you forget that, you might write an article in which the equation numbers skip from (17) to (20), causing the copy editors to wonder if you've discovered a new method of counting.

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IV. Discussion

In this section, we evaluate the performance of the HHL algorithm by comparing it to the classical conjugate gradient descent algorithm, focusing on time complexity and other factors that impact its efficiency.

A. Conjugate Gradient Descent

The classical conjugate gradient descent algorithm is very common method for solving linear systems of equations. It uses the conjugate direction method, which locates the minimum of a quadratic function efficiently. By iteratively updating the solution vector, the algorithm converges towards the exact solution of the linear system. It serves as a suitable benchmark for evaluating the performance of the HHL algorithm. Not only is it one of the fastest classical algorithms, it has very similar constraints sets and calculates similar results (eg. $\vec{x}^{\dagger} M \vec{x}$).

B. Time Complexity

TABLE I
TIME COMPLEXITY COMPARISON

Conjugate Gradien Descent	HHL Algorithm
$\mathcal{O}\left(\frac{\kappa^2 s^2}{\epsilon} \log N\right)$	$\mathcal{O}(\kappa s \log\left(\frac{1}{\epsilon}\right) N)$

Classical conjugate gradient descent, achieves a time complexity of $\mathcal{O}(\kappa s \log\left(\frac{1}{\epsilon}\right) N)$, where s is the sparsness, ϵ is the accuracy, and κ is the condition number. The condition number describes how sensitiv the output of a function is on the error of the input. That means a function is well conditioned if the output of a function does not change a lot with bigger errors in the input.

On the other hand, the HHL algorithm exhibits a time complexity of $\mathcal{O}\left(\frac{\kappa^2 s^2}{\epsilon} \log N\right)$, which is comparable to the classical conjugate gradient descent algorithm when considering the logarithmic dependence on N. Here, s denotes the sparsity of the matrix, and ϵ represents the desired accuracy.