$$a(k-1) = -\overline{c}_1 a(k) + \overline{c}_2 (v(k+1) - v(k)) \tag{1}$$
$$a(k-1) = -\overline{c}_3 a(k) + \overline{c}_4 (s(k+1) - s(k) - dt\ v(k)) \tag{2}$$

$$\begin{bmatrix} a(k-1) \\ a(k-1) \end{bmatrix} = \begin{bmatrix} -a(k) & v(k-1) - v(k) & 0 & 0 \\ 0 & 0 & -a(k) & s(k+1) - s(k) - dt\ v(k) \end{bmatrix} \begin{bmatrix} \overline{c}_1 \\ \overline{c}_2 \\ \overline{c}_3 \\ \overline{c}_4 \end{bmatrix} \tag{3}$$

$$\Rightarrow \tag{4}$$

$$a_x(k-1) = -\overline{c}_1 a_x(k) + \overline{c}_2 \Big(v_x(k+1) - v_x(k) \Big) \tag{5}$$
$$a_x(k-1) = -\overline{c}_3 a_x(k) + \overline{c}_4 \Big(s_x(k+1) - s_x(k) - dt\ v_x(k) \Big) \tag{6}$$
$$a_y(k-1) = -\overline{c}_1 a_y(k) + \overline{c}_2 \Big(v_y(k+1) - v_y(k) \Big) \tag{7}$$
$$a_y(k-1) = -\overline{c}_3 a_y(k) + \overline{c}_4 \Big(s_y(k+1) - s_y(k) - dt\ v_y(k) \Big) \tag{8}$$

$$\begin{bmatrix} a_x(k-1) \\ a_x(k-1) \\ a_y(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -a_x(k) & v_x(k-1) - v_x(k) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_x(k) & s_x(k+1) - s_x(k) - dt\ v_x(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -a_y(k) & v_y(k-1) - v_y(k) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -a_y(k) & s_y(k+1) - s_y(k) - dt\ v_y(k) \end{bmatrix} \begin{bmatrix} \overline{c}_{x1} \\ \overline{c}_{x2} \\ \overline{c}_{x3} \\ \overline{c}_{x4} \\ \overline{c}_{y1} \\ \overline{c}_{y2} \\ \overline{c}_{y3} \\ \overline{c}_{y4} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} a_x(k-1) \\ a_x(k-1) \\ a_y(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -a_x(k) & v_x(k-1) - v_x(k) & 0 & 0 \\ 0 & 0 & -a_x(k) & s_x(k+1) - s_x(k) - dt\ v_x(k) \\ -a_y(k) & v_y(k-1) - v_y(k) & 0 & 0 \\ 0 & 0 & -a_y(k) & s_y(k+1) - s_y(k) - dt\ v_y(k) \end{bmatrix} \begin{bmatrix} \overline{c}_1 \\ \overline{c}_2 \\ \overline{c}_3 \\ \overline{c}_4 \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -a_x(k) & v_x(k-1) - v_x(k) \\ -a_y(k) & v_y(k-1) - v_y(k) \end{bmatrix} \begin{bmatrix} \bar{c}_1 \\ \bar{c}_2 \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -a_x(k) & s_x(k+1) - s_x(k) - dt\ v_x(k) \\ -a_y(k) & s_y(k+1) - s_y(k) - dt\ v_y(k) \end{bmatrix} \begin{bmatrix} \bar{c}_3 \\ \bar{c}_4 \end{bmatrix} \tag{12}$$

New forumal with added features

$$v(k+1) = v(k) + c_1 a(k) + c_2 a(k-1) \tag{13}$$
$$s(k+1) = s(k) + dt v(k) + c_3 a(k) + c_4 a(k-1) \tag{14}$$
$$\tag{15}$$

$$v(k+1) = v(k) + c_0 v(k-1) + c_1 a(k) + c_2 a(k-1) \tag{16}$$
$$s(k+1) = s(k) + c_3 s(k-1) + dt\ v(k) + c_4 v(k-1) + c_5 a(k) + c_6 a(k-1) \tag{17}$$
$$\tag{18}$$

$$c_2 a(k-1) = v(k+1) - v(k) - c_0 v(k-1) - c_1 a(k) \tag{19}$$
$$c_6 a(k-1) = s(k+1) - s(k) - c_3 s(k-1) - dt\ v(k) - c_4 v(k-1) - c_5 a(k) \tag{20}$$
$$\tag{21}$$

$$a(k-1) = \frac{v(k+1) - v(k)}{c_2} - \frac{c_0}{c_2} v(k-1) - \frac{c_1}{c_2} a(k) \tag{22}$$
$$a(k-1) = \frac{s(k+1) - s(k)}{c_6} - \frac{c_3 s(k-1)}{c_6} - \frac{dt}{c_6} v(k) - \frac{c_4}{c_6} v(k-1) - \frac{c_5}{c_6} a(k) \tag{23}$$
$$\tag{24}$$

Imagine all coefficient have dashes

$$a(k-1) = c_2 \big(v(k+1) - v(k) \big) - c_0 v(k-1) - c_1 a(k) \tag{25}$$
$$a(k-1) = c_6 \big(s(k+1) - s(k) - dt\ v(k) \big) - c_3 s(k-1) - c_4 v(k-1) - c_5 a(k) \tag{26}$$
$$\tag{27}$$

Equation for the model

$$a(k-1) = -c_0 v(k-1) - c_1 a(k) + c_2\big(v(k+1) - v(k)\big) \tag{28}$$

$$a(k-1) = -c_3 s(k-1) - c_4 v(k-1) - c_5 a(k) + c_6\big(s(k+1) - s(k) - dt\ v(k)\big) \tag{29}$$

$$\tag{30}$$

Here we solve the following system:

$$\begin{bmatrix} a(k-1) \\ a(k-1) \end{bmatrix} = \begin{bmatrix} v(k-1) & -a(k) & v(k+1) - v(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s(k-1) & -v(k-1) & -a(k) & +\big(s(k+1) - s(k) - dt\ v(k)\big) \end{bmatrix} \begin{bmatrix} \bar{c}_0 \\ \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_3 \\ \bar{c}_4 \\ \bar{c}_5 \\ \bar{c}_6 \end{bmatrix} \tag{31}$$

We add the x and y components (imagine the matrix also contains the indices)

$$\begin{bmatrix} a_x(k-1) \\ a_x(k-1) \\ a_y(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} v(k-1) & -a(k) & v(k+1) - v(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s(k-1) & -v(k-1) & -a(k) & +\big(s(k+1) - s(k) - dt\ v(k)\big) \\ v(k-1) & -a(k) & v(k+1) - v(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s(k-1) & -v(k-1) & -a(k) & +\big(s(k+1) - s(k) - dt\ v(k)\big) \end{bmatrix} \begin{bmatrix} \bar{c}_0 \\ \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_3 \\ \bar{c}_4 \\ \bar{c}_5 \\ \bar{c}_6 \end{bmatrix} \tag{32}$$

We split the model into two models for easier implementation

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} v_x(k-1) & -a_x(k) & v_x(k+1) - v_x(k) \\ v_y(k-1) & -a_y(k) & v_y(k+1) - v_y(k) \end{bmatrix} \begin{bmatrix} \bar{c}_0 \\ \bar{c}_1 \\ \bar{c}_2 \end{bmatrix} \tag{33}$$

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -s_x(k-1) & -v_x(k-1) & -a_x(k) & s_x(k+1) - s_x(k) - dt\ v_x(k) \\ -s_y(k-1) & -v_y(k-1) & -a_y(k) & s_y(k+1) - s_y(k) - dt\ v_y(k) \end{bmatrix} \begin{bmatrix} \bar{c}_3 \\ \bar{c}_4 \\ \bar{c}_5 \\ \bar{c}_6 \end{bmatrix} \tag{34}$$