Integration model:

$$v(k+1) = v(k) + c_1 a(k) + c_2 a(k-1) \tag{1}$$
$$s(k+1) = s(k) + dt\ v(k) + c_3 a(k) + c_4 a(k-1) \tag{2}$$

We have to split each entry into its x- and y-components

$$v_x(k+1) = v_x(k) + c_1 a_x(k) + c_2 a_x(k-1) \tag{3}$$
$$v_y(k+1) = v_y(k) + c_1 a_y(k) + c_2 a_y(k-1) \tag{4}$$
$$s_x(k+1) = s_x(k) + dt\ v_x(k) + c_3 a_x(k) + c_4 a_x(k-1) \tag{5}$$
$$s_y(k+1) = s_y(k) + dt\ v_y(k) + c_3 a_y(k) + c_4 a_y(k-1) \tag{6}$$

The constants are defined as such:

$$\frac{1}{c_2} = \overline{c}_2 \tag{7}$$
$$\frac{c_1}{c_2} = \overline{c}_1 \tag{8}$$
$$\frac{1}{c_4} = \overline{c}_4 \tag{9}$$
$$\frac{c_3}{c_4} = \overline{c}_3 \tag{10}$$

Rearange the constants:

$$c_2 = \frac{1}{\overline{c}_2} \tag{11}$$
$$c_1 = \overline{c}_1 \overline{c}_2 \tag{12}$$
$$c_4 = \frac{1}{\overline{c}_4} \tag{13}$$
$$c_3 = \overline{c}_3 \overline{c}_4 \tag{14}$$

Acceleration model with 4 parameters

$$a(k-1) = -\overline{c}_1 a(k) + \overline{c}_2 \big(v(k+1) - v(k)\big) \tag{15}$$
$$a(k-1) = -\overline{c}_3 a(k) + \overline{c}_4 \big(s(k+1) - s(k) - dt\ v(k)\big) \tag{16}$$

Model in matrix notation

$$\begin{bmatrix} a(k-1) \\ a(k-1) \end{bmatrix} = \begin{bmatrix} -a(k) & v(k+1)-v(k) & 0 & 0 \\ 0 & 0 & -a(k) & s(k+1)-s(k)-dt\ v(k) \end{bmatrix} \begin{bmatrix} \overline{c}_1 \\ \overline{c}_2 \\ \overline{c}_3 \\ \overline{c}_4 \end{bmatrix} \tag{17}$$

Add x and y components

$$\begin{bmatrix} a_x(k-1) \\ a_x(k-1) \\ a_y(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -a_x(k) & v_x(k+1)-v_x(k) & 0 & 0 \\ 0 & 0 & -a_x(k) & s_x(k+1)-s_x(k)-dt\ v_x(k) \\ -a_y(k) & v_y(k+1)-v_y(k) & 0 & 0 \\ 0 & 0 & -a_y(k) & s_y(k+1)-s_y(k)-dt\ v_y(k) \end{bmatrix} \begin{bmatrix} \overline{c}_1 \\ \overline{c}_2 \\ \overline{c}_3 \\ \overline{c}_4 \end{bmatrix} \tag{18}$$

For easier implementation split into two models

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -a_x(k) & v_x(k+1)-v_x(k) \\ -a_y(k) & v_y(k+1)-v_y(k) \end{bmatrix} \begin{bmatrix} \overline{c}_1 \\ \overline{c}_3 \end{bmatrix} \tag{19}$$

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -a_x(k) & s_x(k+1)-s_x(k)-dt\ v_x(k) \\ -a_y(k) & s_y(k+1)-s_y(k)-dt\ v_y(k) \end{bmatrix} \begin{bmatrix} \overline{c}_2 \\ \overline{c}_4 \end{bmatrix} \tag{20}$$

Mind dump

$$\Rightarrow \tag{21}$$

$$a_x(k-1) = -\overline{c}_1 a_x(k) + \overline{c}_2 \Big(v_x(k+1) - v_x(k) \Big) \tag{22}$$

$$a_x(k-1) = -\overline{c}_3 a_x(k) + \overline{c}_4 \Big(s_x(k+1) - s_x(k) - dt\ v_x(k) \Big) \tag{23}$$

$$a_y(k-1) = -\overline{c}_1 a_y(k) + \overline{c}_2 \Big(v_y(k+1) - v_y(k) \Big) \tag{24}$$

$$a_y(k-1) = -\overline{c}_3 a_y(k) + \overline{c}_4 \Big(s_y(k+1) - s_y(k) - dt\ v_y(k) \Big) \tag{25}$$

Extended forumal with added features (7 parameters)

$$
\begin{align}
v(k+1) &= v(k) + c_0 v(k-1) + c_1 a(k) + c_2 a(k-1) \tag{26}\\
s(k+1) &= s(k) + c_3 s(k-1) + dt\ v(k) + c_4 v(k-1) + c_5 a(k) + c_6 a(k-1) \tag{27}\\
& \tag{28}
\end{align}
$$

$$
\begin{align}
c_2 a(k-1) &= v(k+1) - v(k) - c_0 v(k-1) - c_1 a(k) \tag{29}\\
c_6 a(k-1) &= s(k+1) - s(k) - c_3 s(k-1) - dt\ v(k) - c_4 v(k-1) - c_5 a(k) \tag{30}\\
& \tag{31}
\end{align}
$$

$$
\begin{align}
a(k-1) &= \frac{v(k+1) - v(k)}{c_2} - \frac{c_0}{c_2} v(k-1) - \frac{c_1}{c_2} a(k) \tag{32}\\
a(k-1) &= \frac{s(k+1) - s(k)}{c_6} - \frac{c_3 s(k-1)}{c_6} - \frac{dt}{c_6} v(k) - \frac{c_4}{c_6} v(k-1) - \frac{c_5}{c_6} a(k) \tag{33}\\
& \tag{34}
\end{align}
$$

Imagine all coefficient have dashes

$$
\begin{align}
a(k-1) &= c_2\big(v(k+1) - v(k)\big) - c_0 v(k-1) - c_1 a(k) \tag{35}\\
a(k-1) &= c_6\big(s(k+1) - s(k) - dt\ v(k)\big) - c_3 s(k-1) - c_4 v(k-1) - c_5 a(k) \tag{36}\\
& \tag{37}
\end{align}
$$

Equation for the model

$$
\begin{align}
a(k-1) &= -c_0 v(k-1) - c_1 a(k) + c_2\big(v(k+1) - v(k)\big) \tag{38}\\
a(k-1) &= -c_3 s(k-1) - c_4 v(k-1) - c_5 a(k) + c_6\big(s(k+1) - s(k) - dt\ v(k)\big) \tag{39}\\
& \tag{40}
\end{align}
$$

Here we solve the following system:

$$\begin{bmatrix} a(k-1) \\ a(k-1) \end{bmatrix} = \begin{bmatrix} v(k-1) & -a(k) & v(k+1)-v(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s(k-1) & -v(k-1) & -a(k) & +\big(s(k+1)-s(k)-dt\ v(k)\big) \end{bmatrix} \begin{bmatrix} \overline{c}_0 \\ \overline{c}_1 \\ \overline{c}_2 \\ \overline{c}_3 \\ \overline{c}_4 \\ \overline{c}_5 \\ \overline{c}_6 \end{bmatrix} \tag{41}$$

We add the x and y components (imagine the matrix also contains the indices)

$$\begin{bmatrix} a_x(k-1) \\ a_x(k-1) \\ a_y(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} v(k-1) & -a(k) & v(k+1)-v(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s(k-1) & -v(k-1) & -a(k) & +\big(s(k+1)-s(k)-dt\ v(k)\big) \\ v(k-1) & -a(k) & v(k+1)-v(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s(k-1) & -v(k-1) & -a(k) & +\big(s(k+1)-s(k)-dt\ v(k)\big) \end{bmatrix} \begin{bmatrix} \overline{c}_0 \\ \overline{c}_1 \\ \overline{c}_2 \\ \overline{c}_3 \\ \overline{c}_4 \\ \overline{c}_5 \\ \overline{c}_6 \end{bmatrix} \tag{42}$$

We split the model into two models for easier implementation

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} v_x(k-1) & -a_x(k) & v_x(k+1)-v_x(k) \\ v_y(k-1) & -a_y(k) & v_y(k+1)-v_y(k) \end{bmatrix} \begin{bmatrix} \overline{c}_0 \\ \overline{c}_1 \\ \overline{c}_2 \end{bmatrix} \tag{43}$$

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -s_x(k-1) & -v_x(k-1) & -a_x(k) & s_x(k+1)-s_x(k)-dt\ v_x(k) \\ -s_y(k-1) & -v_y(k-1) & -a_y(k) & s_y(k+1)-s_y(k)-dt\ v_y(k) \end{bmatrix} \begin{bmatrix} \overline{c}_3 \\ \overline{c}_4 \\ \overline{c}_5 \\ \overline{c}_6 \end{bmatrix} \tag{44}$$