

$$s_i(k+1) - s_i(k) = \begin{bmatrix} \bar{\theta}_{s_1} & \bar{\theta}_{s_2} & \bar{\theta}_{s_3} - \bar{\theta}_{v_1} & \bar{\theta}_{s_4} - \bar{\theta}_{v_2} & \bar{\theta}_{v_4} & \bar{\theta}_{v_3} \end{bmatrix} \begin{bmatrix} s_i(k-1) \\ s_i(k-2) \\ v_i(k) \\ v_i(k-1) \\ v_i(k+1) \\ -v_i(k-2) \end{bmatrix} \quad (1)$$

*Given Variables :*

$$\begin{aligned} &\bar{\theta}_{s_1} \\ &\bar{\theta}_{s_2} \\ &\bar{\theta}_{s_3} \\ &\bar{\theta}_{s_4} \\ &\bar{\theta}_{v_4} \\ &\bar{\theta}_{v_3} \end{aligned} \quad (2)$$

*To be solved :*

$$\begin{aligned} &\theta_{s_1} \\ &\theta_{s_2} \\ &\theta_{s_3} \\ &\theta_{s_4} \\ &\theta_{s_5} \\ &\theta_{v_1} \\ &\theta_{v_2} \\ &\theta_{v_3} \\ &\theta_{v_4} \end{aligned} \quad (3)$$

$$s_i(k+1) - s_i(k) = \begin{bmatrix} \bar{\theta}_{s_1} & \bar{\theta}_{s_2} & \bar{\theta}_{s_3} - \bar{\theta}_{v_1} & \bar{\theta}_{s_4} - \bar{\theta}_{v_2} & \bar{\theta}_{v_4} & \bar{\theta}_{v_3} \end{bmatrix} \begin{bmatrix} s_i(k-1) \\ s_i(k-2) \\ v_i(k) \\ v_i(k-1) \\ v_i(k+1) \\ -v_i(k-2) \end{bmatrix} \quad (4)$$

*Definitions :*

$$\begin{aligned}
\bar{\theta}_{s_1} &= \theta_{s_1} \\
\bar{\theta}_{s_2} &= \theta_{s_2} \\
\bar{\theta}_{s_3} &= \theta_{s_3} \\
\bar{\theta}_{s_4} &= \theta_{s_4} \\
\bar{\theta}_{v_1} &= \frac{\theta_{v_1} \theta_{s_5}}{\theta_{v_4}} \\
\bar{\theta}_{v_2} &= \frac{\theta_{v_2} \theta_{s_5}}{\theta_{v_4}} \\
\bar{\theta}_{v_3} &= \frac{\theta_{v_3} \theta_{s_5}}{\theta_{v_4}} \\
\bar{\theta}_{v_4} &= \frac{\theta_{s_5}}{\theta_{s_4}} \\
\overline{\bar{\theta}_{s_3}} &= \bar{\theta}_{s_3} - \bar{\theta}_{v_1} \\
\overline{\bar{\theta}_{s_4}} &= \bar{\theta}_{s_4} - \bar{\theta}_{v_2}
\end{aligned} \tag{5}$$

$$s_i(k+1) - s_i(k) = \begin{bmatrix} \bar{\theta}_{s_1} & \bar{\theta}_{s_2} & \bar{\theta}_{s_3} & \bar{\theta}_{s_4} & \bar{\theta}_{v_4} & \bar{\theta}_{v_3} \end{bmatrix} \begin{bmatrix} s_i(k-1) \\ s_i(k-2) \\ v_i(k) \\ v_i(k-1) \\ v_i(k+1) \\ -v_i(k-2) \end{bmatrix} \tag{6}$$

*Definitions :*

$$\begin{aligned}
\bar{\theta}_{s_1} &= \theta_{s_1} \\
\bar{\theta}_{s_2} &= \theta_{s_2} \\
\bar{\theta}_{v_3} &= \frac{\theta_{v_3} \theta_{s_5}}{\theta_{v_4}} \\
\bar{\theta}_{v_4} &= \frac{\theta_{s_5}}{\theta_{s_4}} \\
\bar{\bar{\theta}}_{s_4} &= \theta_{s_4} - \frac{\theta_{v_2} \theta_{s_5}}{\theta_{v_4}} \\
\bar{\bar{\theta}}_{s_3} &= \theta_{s_3} - \frac{\theta_{v_1} \theta_{s_5}}{\theta_{v_4}}
\end{aligned} \tag{7}$$

Normalisation step

$$s_i(k+1) - s_i(k) = \begin{bmatrix} \theta_{s_1} & \theta_{s_2} & \theta_{s_3} - \frac{\theta_{v_1} \theta_{s_5}}{\theta_{v_4}} & \theta_{s_4} - \frac{\theta_{v_2} \theta_{s_5}}{\theta_{v_4}} & \frac{\theta_{s_5}}{\theta_{s_4}} & \frac{\theta_{v_3} \theta_{s_5}}{\theta_{v_4}} \end{bmatrix} \begin{bmatrix} s_i(k-1) \\ s_i(k-2) \\ v_i(k) \\ v_i(k-1) \\ v_i(k+1) \\ -v_i(k-2) \end{bmatrix} \tag{8}$$

$$||t - Ax||^2 \tag{9}$$

$$||t - Ax||^2 + lambda ||x||^2 \tag{10}$$

$$\| \begin{bmatrix} t & 0 \end{bmatrix} - diag(A, I * \sqrt{\lambda}) x \|^2 \tag{11}$$

$$s_i(k+1) - s_i(k) = \begin{bmatrix} s_i(k-1) & s_i(k-2) & v_i(k) & v_i(k-1) & v_i(k+1) & v_i(k) & v_i(k-1) & -v_i(k-2) \end{bmatrix} \tag{12}$$

tsdkfbsdjf

sldk

sdf

sdf

Distance:

$$\begin{aligned} s_i(k+1) &= s_i(k) + \theta_{s_1} s_i(k-1) + \theta_{s_2} s_i(k-2) + \theta_{s_3} v_i(k) + \theta_{s_4} v_i(k-1) + \theta_{s_5} a_i(k) \\ a_i(k) &= \frac{s_i(k+1) - s_i(k) - \theta_{s_1} s_i(k-1) - \theta_{s_2} s_i(k-2) - \theta_{s_3} v_i(k) - \theta_{s_4} v_i(k-1)}{\theta_{s_5}} \end{aligned} \quad (13)$$

Velocity:

$$\begin{aligned} v_i(k+1) &= \theta_{v_1} v_i(k) + \theta_{v_2} v_i(k-1) + \theta_{v_3} v_i(k-2) + \theta_{v_4} a_i(k) \\ a_i(k) &= \frac{v_i(k+1) - \theta_{v_1} v_i(k) - \theta_{v_2} v_i(k-1) - \theta_{v_3} v_i(k-2)}{\theta_{v_4}} \end{aligned} \quad (14)$$

Equate by acceleration  $a_i(k)$ :

$$\theta_{v_4} \left( s_i(k+1) - s_i(k) - \theta_{s_1} s_i(k-1) - \theta_{s_2} s_i(k-2) - \theta_{s_3} v_i(k) - \theta_{s_4} v_i(k-1) \right) = \theta_{s_5} \left( v_i(k+1) - \theta_{v_1} v_i(k) - \theta_{v_2} v_i(k-1) - \theta_{v_3} v_i(k-2) \right) \quad (15)$$

Solve for  $s_i(k+1) - s_i(k)$ :

$$\theta_{v_4} s_i(k+1) - \theta_{v_4} s_i(k) = \theta_{v_4} \theta_{s_1} s_i(k-1) + \theta_{v_4} \theta_{s_2} s_i(k-2) + \theta_{v_4} \theta_{s_3} v_i(k) + \theta_{v_4} \theta_{s_4} v_i(k-1) \quad (16)$$

$$+ \theta_{s_5} v_i(k+1) - \theta_{s_5} \theta_{v_1} v_i(k) - \theta_{s_5} \theta_{v_2} v_i(k-1) - \theta_{s_5} \theta_{v_3} v_i(k-2) \quad (17)$$

$$\begin{aligned} s_i(k+1) - s_i(k) &= \theta_{s_1} s_i(k-1) + \theta_{s_2} s_i(k-2) + \theta_{s_3} v_i(k) + \theta_{s_4} v_i(k-1) \\ &+ \frac{\theta_{s_5}}{\theta_{v_4}} v_i(k+1) - \frac{\theta_{s_5} \theta_{v_1}}{\theta_{v_4}} v_i(k) - \frac{\theta_{s_5} \theta_{v_2}}{\theta_{v_4}} v_i(k-1) - \frac{\theta_{s_5} \theta_{v_3}}{\theta_{v_4}} v_i(k-2) \end{aligned}$$

Solve system:

$$s_i(k+1) - s_i(k) = \begin{bmatrix} s_i(k-1) & s_i(k-2) & v_i(k) & v_i(k-1) & v_i(k+1) & v_i(k) & v_i(k-1) & v_i(k-2) \end{bmatrix} \begin{bmatrix} \theta_{s_1} \\ \theta_{s_2} \\ \theta_{s_3} \\ \theta_{s_4} \\ \frac{\theta_{s_5}}{\theta_{v_4}} \\ \frac{\theta_{s_5} \theta_{v_1}}{\theta_{v_4}} \\ \frac{\theta_{s_5} \theta_{v_2}}{\theta_{v_4}} \\ \frac{\theta_{s_5} \theta_{v_3}}{\theta_{v_4}} \end{bmatrix} \quad (18)$$

To solve this we use:

$$||t - Ax||^2 \quad (19)$$

Where:

$$t = s_i(k+1) - s_i(k) \quad (20)$$

$$A = [s_i(k-1) \quad s_i(k-2) \quad v_i(k) \quad v_i(k-1) \quad v_i(k+1) \quad v_i(k) \quad v_i(k-1) \quad v_i(k-2)] \quad (21)$$

$$x = \begin{bmatrix} \theta_{s_1} \\ \theta_{s_2} \\ \theta_{s_3} \\ \theta_{s_4} \\ \theta_{s_5} \\ \theta_{v_4} \\ \frac{\theta_{s_5} \theta_{v_1}}{\theta_{v_4}} \\ \frac{\theta_{v_4}}{\theta_{s_5} \theta_{v_2}} \\ \frac{\theta_{v_4}}{\theta_{s_5} \theta_{v_3}} \\ \theta_{v_4} \end{bmatrix} \quad (22)$$

To solve for 9 parameters in  $x$ , we normalize the system:

$$\left\| \begin{bmatrix} t \\ 0 \end{bmatrix} - \begin{bmatrix} A & 0 \\ 0 & \mathbb{I} * \sqrt{\lambda} \end{bmatrix} \begin{bmatrix} x \\ x'' \end{bmatrix} \right\|^2 \quad (23)$$

New equation:

We want to solve this system

$$a(k-1) = -c_1 a(k) + c_2 (v(k-1) - v(k)) \quad (24)$$

$$a(k-1) = -c_3 a(k) + c_4 (s(k+1) - s(k) - v(k)) \quad (25)$$