

Acceleration model with 4 parameters

$$a(k-1) = -\bar{c}_1 a(k) + \bar{c}_2 (v(k+1) - v(k)) \quad (1)$$

$$a(k-1) = -\bar{c}_3 a(k) + \bar{c}_4 (s(k+1) - s(k) - dt \ v(k)) \quad (2)$$

Model in matrix notation

$$\begin{bmatrix} a(k-1) \\ a(k-1) \end{bmatrix} = \begin{bmatrix} -a(k) & v(k+1) - v(k) & 0 & 0 \\ 0 & 0 & -a(k) & s(k+1) - s(k) - dt \ v(k) \end{bmatrix} \begin{bmatrix} \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_3 \\ \bar{c}_4 \end{bmatrix} \quad (3)$$

Add x and y components

$$\begin{bmatrix} a_x(k-1) \\ a_x(k-1) \\ a_y(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -a_x(k) & v_x(k+1) - v_x(k) & 0 & 0 \\ 0 & 0 & -a_x(k) & s_x(k+1) - s_x(k) - dt \ v_x(k) \\ -a_y(k) & v_y(k+1) - v_y(k) & 0 & 0 \\ 0 & 0 & -a_y(k) & s_y(k+1) - s_y(k) - dt \ v_y(k) \end{bmatrix} \begin{bmatrix} \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_3 \\ \bar{c}_4 \end{bmatrix} \quad (4)$$

For easier implementation split into two models

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -a_x(k) & v_x(k+1) - v_x(k) \\ -a_y(k) & v_y(k+1) - v_y(k) \end{bmatrix} \begin{bmatrix} \bar{c}_1 \\ \bar{c}_3 \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -a_x(k) & s_x(k+1) - s_x(k) - dt \ v_x(k) \\ -a_y(k) & s_y(k+1) - s_y(k) - dt \ v_y(k) \end{bmatrix} \begin{bmatrix} \bar{c}_2 \\ \bar{c}_4 \end{bmatrix} \quad (6)$$

Mind dump

$$\Rightarrow \quad (7)$$

$$a_x(k-1) = -\bar{c}_1 a_x(k) + \bar{c}_2 (v_x(k+1) - v_x(k)) \quad (8)$$

$$a_x(k-1) = -\bar{c}_3 a_x(k) + \bar{c}_4 (s_x(k+1) - s_x(k) - dt \ v_x(k)) \quad (9)$$

$$a_y(k-1) = -\bar{c}_1 a_y(k) + \bar{c}_2 (v_y(k+1) - v_y(k)) \quad (10)$$

$$a_y(k-1) = -\bar{c}_3 a_y(k) + \bar{c}_4 (s_y(k+1) - s_y(k) - dt \ v_y(k)) \quad (11)$$

Extended forumal with added features (7 parameters)

$$v(k+1) = v(k) + c_0 v(k-1) + c_1 a(k) + c_2 a(k-1) \quad (12)$$

$$s(k+1) = s(k) + c_3 s(k-1) + dt \ v(k) + c_4 v(k-1) + c_5 a(k) + c_6 a(k-1) \quad (13)$$

$$(14)$$

$$c_2 a(k-1) = v(k+1) - v(k) - c_0 v(k-1) - c_1 a(k) \quad (15)$$

$$c_6 a(k-1) = s(k+1) - s(k) - c_3 s(k-1) - dt \ v(k) - c_4 v(k-1) - c_5 a(k) \quad (16)$$

$$(17)$$

$$a(k-1) = \frac{v(k+1) - v(k)}{c_2} - \frac{c_0}{c_2} v(k-1) - \frac{c_1}{c_2} a(k) \quad (18)$$

$$a(k-1) = \frac{s(k+1) - s(k)}{c_6} - \frac{c_3 s(k-1)}{c_6} - \frac{dt}{c_6} v(k) - \frac{c_4}{c_6} v(k-1) - \frac{c_5}{c_6} a(k) \quad (19)$$

$$(20)$$

Imagine all coefficient have dashes

$$a(k-1) = c_2 (v(k+1) - v(k)) - c_0 v(k-1) - c_1 a(k) \quad (21)$$

$$a(k-1) = c_6 (s(k+1) - s(k) - dt \ v(k)) - c_3 s(k-1) - c_4 v(k-1) - c_5 a(k) \quad (22)$$

$$(23)$$

Equation for the model

$$a(k-1) = -c_0 v(k-1) - c_1 a(k) + c_2 (v(k+1) - v(k)) \quad (24)$$

$$a(k-1) = -c_3 s(k-1) - c_4 v(k-1) - c_5 a(k) + c_6 (s(k+1) - s(k) - dt \ v(k)) \quad (25)$$

$$(26)$$

Here we solve the following system:

$$\begin{bmatrix} a(k-1) \\ a(k-1) \end{bmatrix} = \begin{bmatrix} v(k-1) & -a(k) & v(k+1) - v(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s(k-1) & -v(k-1) & -a(k) & +(s(k+1) - s(k) - dt v(k)) \end{bmatrix} \begin{bmatrix} \bar{c}_0 \\ \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_3 \\ \bar{c}_4 \\ \bar{c}_5 \\ \bar{c}_6 \end{bmatrix} \quad (27)$$

We add the x and y components (imagine the matrix also contains the indices)

$$\begin{bmatrix} a_x(k-1) \\ a_x(k-1) \\ a_y(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} v(k-1) & -a(k) & v(k+1) - v(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s(k-1) & -v(k-1) & -a(k) & +(s(k+1) - s(k) - dt v(k)) \\ v(k-1) & -a(k) & v(k+1) - v(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s(k-1) & -v(k-1) & -a(k) & +(s(k+1) - s(k) - dt v(k)) \end{bmatrix} \begin{bmatrix} \bar{c}_0 \\ \bar{c}_1 \\ \bar{c}_2 \\ \bar{c}_3 \\ \bar{c}_4 \\ \bar{c}_5 \\ \bar{c}_6 \end{bmatrix} \quad (28)$$

We split the model into two models for easier implementation

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} v_x(k-1) & -a_x(k) & v_x(k+1) - v_x(k) \\ v_y(k-1) & -a_y(k) & v_y(k+1) - v_y(k) \end{bmatrix} \begin{bmatrix} \bar{c}_0 \\ \bar{c}_1 \\ \bar{c}_2 \end{bmatrix} \quad (29)$$

$$\begin{bmatrix} a_x(k-1) \\ a_y(k-1) \end{bmatrix} = \begin{bmatrix} -s_x(k-1) & -v_x(k-1) & -a_x(k) & s_x(k+1) - s_x(k) - dt v_x(k) \\ -s_y(k-1) & -v_y(k-1) & -a_y(k) & s_y(k+1) - s_y(k) - dt v_y(k) \end{bmatrix} \begin{bmatrix} \bar{c}_3 \\ \bar{c}_4 \\ \bar{c}_5 \\ \bar{c}_6 \end{bmatrix} \quad (30)$$