Circuit analysis: DC Circuits (3 cr)

Fall 2009 / Class AS09

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Metropolia

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Slideset version: 1.1

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About the Course

- Lecturer: M.Sc. Vesa Linja-aho
- Lectures on Mon 11:00-14:00 and Thu 14:00-16:30, room P113
- To pass the course: Home assignments and final exam. The exam is on Monday 12th October 2009 at 11:00-14:00.
- All changes to the schedule are announced in the Tuubi-portal.

The Home Assignments

- There are 12 home assignments.
- Each assignment is graded with 0, 0,5 or 1 points.
- To pass the course, the student must have at least 4 points from the assignments.
- Each point exceeding the minimum of 4 points will give you 0,5 extra points in the exam.
- In the exam, there are 5 assignments, with maximum of 6 points each.
- To pass the exam, you need to get 15 points from the exam.
- All other grade limits (for grades 2-5) are flexible.

Example

The student has 8 points from the home assignments. He gets 13 points from the exam. He will pass the exam, because he gets extra points from the home assignments and his total score is $(8 - 4) \cdot 0{,}5 + 13 = 15$ points.

However, is one gets 8 of 12 points from the home assignments, he usually gets more than 13 points from the exam :-).

The Course Objectives

From the curriculum:

Learning outcomes of the course unit

Basic concepts and basic laws of electrical engineering. Analysis of direct current (DC) circuits.

Course contents

Basic concepts and basic laws of electrical engineering, analysis methods, controlled sources. Examples and exercises.

The Course Schedule

1. The basic quantities and units. Voltage source and resistance. Kirchhoff's laws and Ohm's law.
2. Conductance. Electric power. Series and parallel circuits. Node. Ground.
3. Current source. Applying the Kirchhoff's laws to solve the circuit. Node-voltage analysis.
4. Exercises on node-voltage analysis.
5. Source transformation.
6. Thévenin equivalent and Norton equivalent.
7. Superposition principle.
8. Voltage divider and current divider.
9. Inductance and capacitance in DC circuits.
10. Controlled sources.
11. Recap.
12. Recap.

The Course is Solid Ground for Further Studies in Electronics

The basic knowledge on DC circuits is needed on the courses *Circuit Analysis: Basic AC-Theory*, *Measuring Technology*, *Automotive Electronics 1*, *Automotive Electrical Engineering Labs*, ...

Important!

By studying this course well, **studying the upcoming courses will be easier**!

The basics of DC circuits are vital for automotive electronics engineer, just like the basics of accounting are vital for an auditor, and basics of strength of materials are vital for a bridge-building engineer etc.

What is Not Covered on This Course

The basic physical characteristics of electricity is not covered on this course. Questions like "What is electricity?" are covered on the course *Rotational motion and electromagnetism*.

Studying in Our School

- You have an opportunity to learn on the lectures. I can not force you to learn.
- You have more responsibility on your learning than you had in vocational school or senior high school.
- 1 cr $\approx$ 26,7 hours of work. 3 cr $=$ 80 hours of work. You will spend 39 hours on the lectures.
- Which means that you should use about 40 hours of your own time for studying!
- If I proceed too fast or too slow, please interject me (or tell me by email).
- Do not hesitate to ask. Ask also the "stupid questions".

What Is Easy and What Is Hard?

Different things are hard for different people. But my own experience shows that

- DC analysis is easy, because the math involved is very basic.
- DC analysis is hard, because the circuits are not as intuitive as, for example, mechanical systems are.

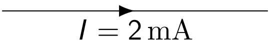
Studying your math courses well is important for the upcoming courses on circuit analysis. For example, in AC circuits analysis you have to use *complex arithmetics*.

Now, Let's Get into Business

Any questions on the practical arrangements of the course?

Electric Current

- Electric current is a flow of electric charge.
- The unit for electric current is the ampere (A).
- The abbreviation for the quantity is I.
- One may compare the electric current with water flowing in a pipe (so called *hydraulic analogy*).
- The current always circulates in a loop: current does not compress nor vanish.
- The current in a wire is denoted like this:

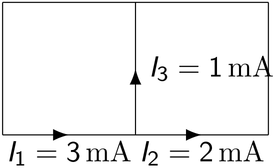


Kirchhoff's Current Law

- As mentioned on the previous slide, the current can not vanish anywhere.

Kirchhoff's Current Law (or: Kirchhoff's First Law)

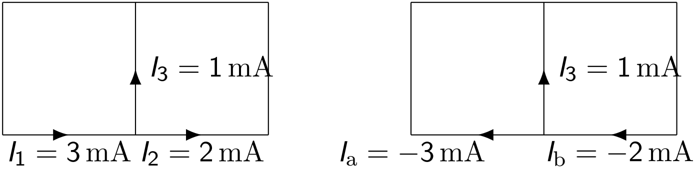
At any area in an electrical circuit, the sum of currents flowing into that area is equal to the sum of currents flowing out of that area.



If you draw a circle in any place in the circuit, you can observe that there is as the same amount of current flowing into the circle and out from the circle!

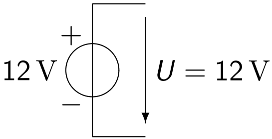
Be Careful with Signs

- One can say: "The balance of my account -50 euros" or equally "I owe 50 euros to my bank".
- One can say: "The profit of the company was -500000 euros" or equally "The loss of the company was 500000 euros".
- If you measure a current with an ammeter and it reads $-15\,\mathrm{mA}$, by reversing the wires of the ammeter it will show $15\,\mathrm{mA}$.
- The sign of the current shows the direction of the current. The two circuits below are exactly identical.



Voltage

- The potential difference between two points is called voltage.
- The abbreviation for the quantity is U.
- In circuit theory, it is insignificant how the potential difference is generated (chemically, by induction etc.).
- The unit of voltage is the volt (V).
- One may compare the voltage with a pressure difference in hydraulic system, or to a difference in altitude.
- Voltage is denoted with an arrow between two points.

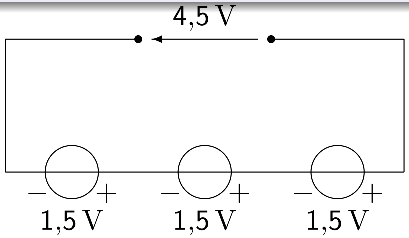


Kirchhoff's voltage law

- The voltage between two points is the same, regardless of the path chosen.
- This is easy to understand by using the analogy of differences in altitude. If you leave your home, go somewhere and return to your home, you have traveled uphill as much as you have traveled downhill.

Kirchhoff's Voltage Law (or: Kirchhoff's Second Law)

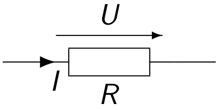
The directed **sum of the voltages around any closed circuit is zero**.



Ohm's law

- *Resistance* is a measure of the degree to which an object opposes an electric current through it.
- The larger the current, the larger the voltage – and vice versa.
- The abbreviation of the quantity is R and the unit is (Ω) (ohm).
- The definition of resistance is the ratio of the voltage over the element divided with the current through the element. $R = U/I$

$$U = RI$$



Definitions

Electric circuit A system consisting of compontents, in where electric current flows.

Direct current (DC) The electrical quantities (voltage and current) are constant (or nearly constant) over time.

Direct current circuit An electric circuit, where voltages and currents are constant over time.

Example

In a flashlight, there is a direct current circuit consisting of a battery/batteries, a switch and a bulb. In a bicycle there is an alternating current circuit (dynamo and bulb).

An alternate definition for direct current

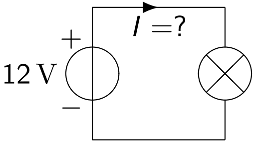
One may define also that direct current means a current, which does not change its direction (sign), but the magnitude of the current can vary over time. For example, a simple lead acid battery charger outputs a pulsating voltage, which varies between 0 V ... $\approx$ 18 V. This can be also called DC voltage.

Agreement

On this course, we define DC to mean constant voltage and current. The magnitude and sign are constant over time.

A simple DC circuit

- A light bulb is wired to a battery. The resistance of the filament is $10\,\Omega$.



A simple DC circuit

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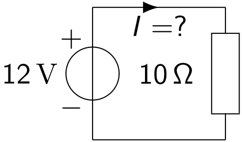
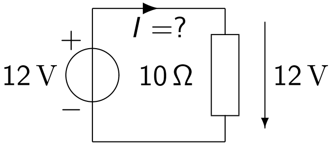


A simple DC circuit

- A light bulb is wired to a battery. The resistance of the filament is $10\,\Omega$.



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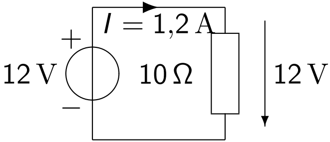


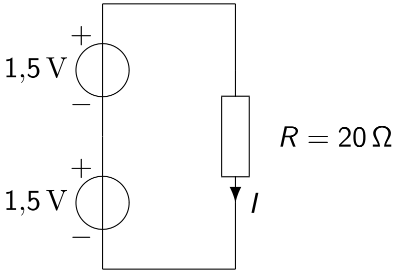
$U = RI$
$I = \frac{U}{R} = \frac{12\,\mathrm{V}}{10\,\Omega} = 1{,}2\,\mathrm{A}$

Homework 1 (released 31st Aug, to be returned 3rd Sep)

- The homework are to be returned at the beginning of the next lecture.
- Remember to include your name and student number.

Homework 1

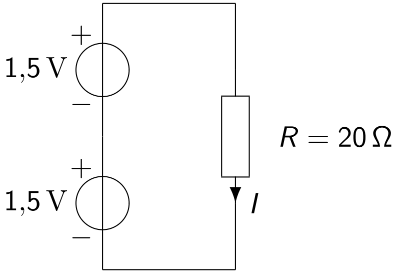
Find the current I.



Homework 1 - Model solution

Homework 1

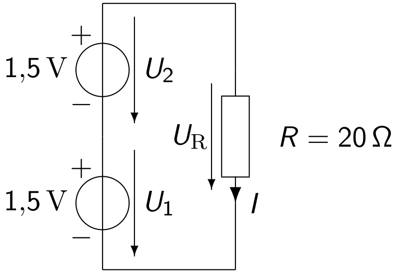
Find the current I.



Homework 1 - Model solution

Homework 1

Find the current I.

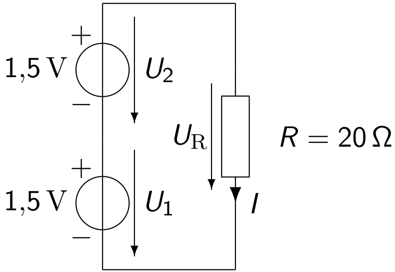


$$U_1 + U_2 - U_\mathrm{R} = 0 \Leftrightarrow U_\mathrm{R} = U_1 + U_2$$

Homework 1 - Model solution

Homework 1

Find the current I.



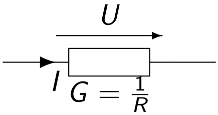
$$U_1 + U_2 - U_R = 0 \Leftrightarrow U_R = U_1 + U_2$$

$$U = RI \Rightarrow U_R = RI \Rightarrow I = \frac{U_R}{R} = \frac{U_1 + U_2}{R} = \frac{1{,}5\,\mathrm{V} + 1{,}5\,\mathrm{V}}{20\,\Omega} = 150\,\mathrm{mA}$$

Conductance

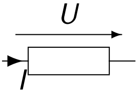
- Resistance is a measure of the degree to which an object opposes an electric current through it.
- The inverse of resistance is **conductance**. The symbol for conductance is G and the unit is Siemens (S).
- Conductance measures how easily electricity flows along certain element.
- For example, if resistance $R = 10\,\Omega$ then conductance $G = 0{,}1\,\mathrm{S}$.

$$G = \frac{1}{R} \qquad U = RI \Leftrightarrow GU = I$$



Electric Power

- In physics, power is the rate at which work is performed.
- The symbol for power is P and the unit is the Watt (W).
- The DC power consumed by an electric element is $P = UI$

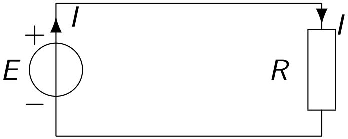


- If the formula outputs a positive power, the element is consuming power from the circuit. If the formula outputs a negative power, the element is delivering power to the circuit.

Electric Power

Energy can not be created nor destroyed

The power consumed by the elements in the circuit = the power delivered by the elements in the circuit.



$$I = \frac{U}{R}$$

$$P_R = UI = U\frac{U}{R} = \frac{U^2}{R}$$

$$P_E = U \cdot (-I) = U\frac{-U}{R} = -\frac{U^2}{R}$$

The power delivered by the voltage source is consumed by the resistor.

Series and Parallel Circuits

Definition: series circuit

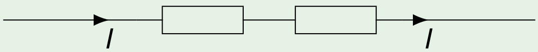
The elements are in series, if they are connected so that the same current flows through the elements.

Definition: parallel circuit

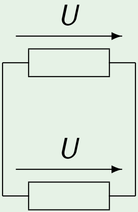
The elements are in parallel, if they are connected so that there is the same voltage across them.

Series and Parallel Circuits

Series circuit

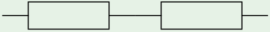


Parallel Circuit



Resistors in series and in parallel

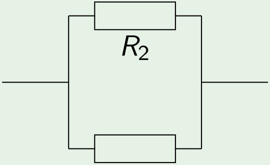
In series



$\Longleftrightarrow$

$R = R_1 + R_2$

In parallel



$\Longleftrightarrow$

$R = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$

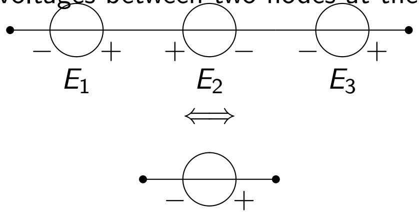
Or, by using conductances: $G = G_1 + G_2$.

Resistors in series and in parallel

- The formulae on the previous slide can be applied to an arbitrary number of resistors. For instance, the total resistance of five resistors in series is $R = R_1 + R_2 + R_3 + R_4 + R_5$.

Voltage Sources in Series

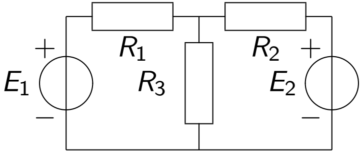
- The voltages can be summed like resistances, but be careful with correct signs.
- Voltage sources in parallel are inadmissible in circuit theory. There can not be two different voltages between two nodes at the same time.



$$E = E_1 - E_2 + E_3$$

What Series and Parallel Circuits are NOT

- Just the fact that two components seem to be one after the other, does not mean that they are in series.
- Just the fact that two components seem to be side by side, does not mean that they are in parallel.
- In the figure below, which of the resistors are in parallel and which are in series with each other?



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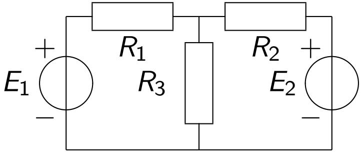
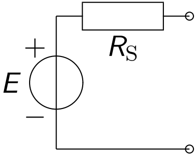


Solution

None! E_1 ja R_1 are in series and E_2 ja R_2 are in series. Both of these serial circuits are in parallel with R_3. But no two resistors are in parallel nor in series.

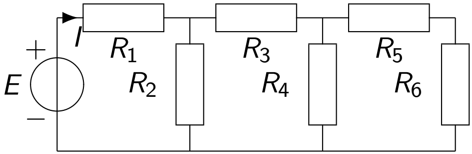
Terminal and Gate

- A point which provides a point of connection to external circuits is called a terminal (or pole).
- Two terminals form a gate.
- An easy example: a car battery with internal resistance.



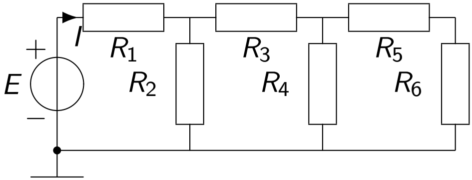
Node

- **A node** means an area in the circuit where there are no potential differences, or alternatively a place where two or more circuit elements meet.
- A "for dummies" –way to find nodes in the circuit: put your pen on a wire in the circuit. Start coloring the wire, and backtrack when your pen meets a circuit element. The area you colored is one node.
- How many nodes are there in the circuit below?



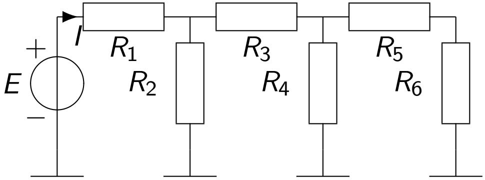
Ground

- One of the nodes in the circuit can be appointed the ground node.
- By selecting one of the nodes to be the ground node, the circuit diagram usually appear cleaner.
- The car battery is connected to the chassis of the car. Therefore it is convenient to handle the chassis as the ground node.
- When we say "the voltage of this node is 12 volts" it means that the voltage between that node and the ground node is 12 volts.



Ground

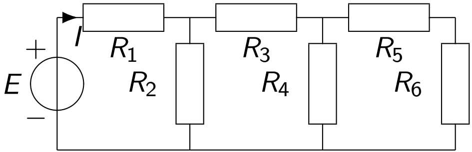
- The ground node can be connected to the chassis of the device or it can be leave not connected to the chassis.
- Therefore, the existence of the ground node does not mean that the device is "grounded".
- The circuit on the previous slide can be presented also like this:



Homework 2 (released 3rd Sep, to be returned 7th Sep)

Homework 2

Find the current I.

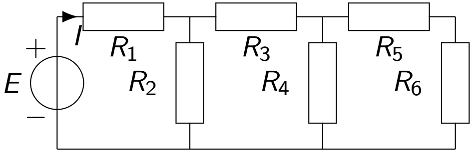


$$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 1\,\Omega \qquad E = 9\,\mathrm{V}$$

Homework 2 - Model solution

Homework 2

Find the current I.



$R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 1\,\Omega \qquad E = 9\,\mathrm{V}$

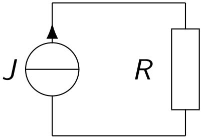
- R_5 ja R_6 are in series. The total resistance of the serial connection is $R_5 + R_6 = 2\,\Omega$.
- Furthermore, the serial connection is in parallel with R_4. The resistance of this parallel circuit is $\frac{1}{\frac{1}{1}+\frac{1}{2}}\,\Omega = \frac{2}{3}\,\Omega$.

Solution continues

- R_3 is in series with the parallel circuit calculated on the previous slide. The resistance for this circuit is $R_3 + \frac{2}{3}\,\Omega = \frac{5}{3}\,\Omega$.
- And the serial connection is in parallel with R_2. The resistance for the parallel circuit is $\frac{1}{(\frac{5}{3})^{-1}+\frac{1}{1}} = \frac{5}{8}\,\Omega$.
- Lastly, R_1 is in series with the resistance computed in the previous step. Therefore, the total resistance seen by voltage source E is $\frac{5}{8}\,\Omega + R_1 = \frac{13}{8}\,\Omega$.
- The current I is computed from Ohm's law $I = \frac{E}{\frac{13}{8}\,\Omega} = \frac{72}{13}\,\mathrm{A} \approx 5{,}5\,\mathrm{A}$.

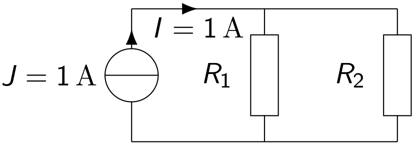
The Current Source

- The current source is a circuit element which delivers a certain current throught it, just like the voltage source keeps a certain voltage between its nodes.
- The current can be constant or it can vary by some rule.



The Current Source

- If there is a current source in a wire, you know the current of that wire.



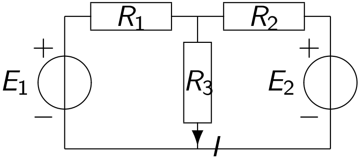
Applying Kirchhoff's Laws Systematically to the Circuit

When solving a circuit, it is highly recommended to use a systematic mehtod to find the voltages and/or currents. Otherwise it is easy to end up with writing a bunch of equations which can not be solved. One systematic method is called the **nodal analysis**:

1. Name each current in the circuit.
2. Select one node as the ground node. Assign a variable for each voltage between each node and ground node.
3. Write an equation based on Kirchhoff's current law for each node (except the ground node).
4. State the voltage of each resistor by using the node voltage variables in step 2. Draw the voltage arrows at the same direction you used for the current arrows (this makes it easier to avoid sign mistakes).
5. State every current by using the voltages and substitute them into the current equations in step 2.
6. Solve the set of equations to find the voltage(s) asked.
7. If desired, solve the currents by using the voltages you solved.

Example

Find the current I.



Example

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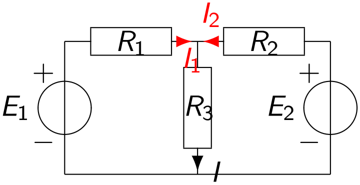

Example

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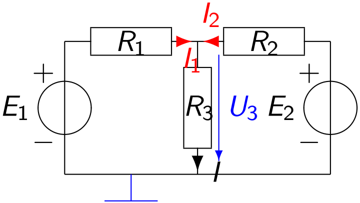
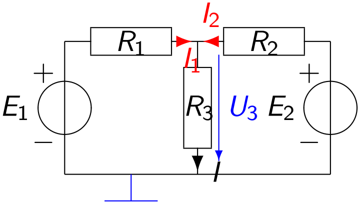


Example

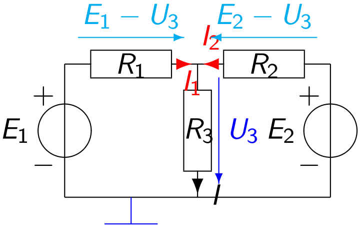
Find the current I.



$I = I_1 + I_2$

Example

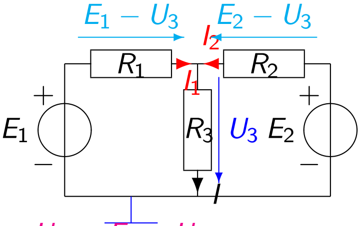
Find the current I.



$$I = I_1 + I_2$$

Example

Find the current I.

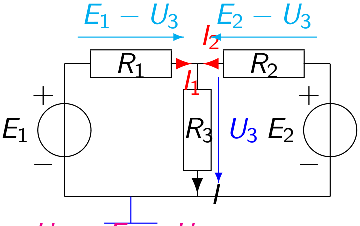


$$I = I_1 + I_2$$

$$\frac{U_3}{R_3} = \frac{E_1 - U_3}{R_1} + \frac{E_2 - U_3}{R_2}$$

Example

Find the current I.

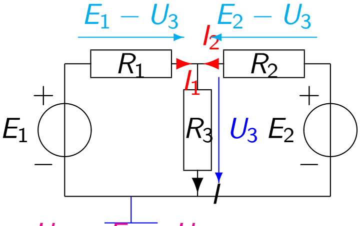


$$I = I_1 + I_2$$

$$\frac{U_3}{R_3} = \frac{E_1 - U_3}{R_1} + \frac{E_2 - U_3}{R_2} \implies U_3 = R_3 \frac{R_2 E_1 + R_1 E_2}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

Example

Find the current I.



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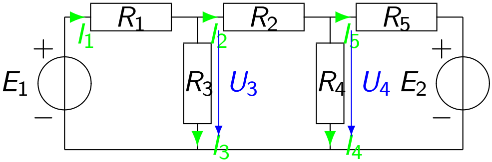
$$I = \frac{U_3}{R_3} = \frac{R_2 E_1 + R_1 E_2}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

Some Remarks

- There are many methods for writing the circuit equations, and there is no such thing as "right" method.
- The only requirement is that you follow Kirchhoff's laws and Ohm's law[1] and you have an equal number of equations and unknowns.
- If there is a current source in the circuit, it will (usually) make the circuit easier to solve, as you then have one unknown less to solve.
- By using conductances instead of resistances, the equations look a little cleaner.

[1] Ohm's law can only be utilized for resistors. If you have other elements, you must know their current-voltage equation.

Another Example



$I_1 = I_2 + I_3$
$I_2 = I_4 + I_5$

$$\frac{E_1 - U_3}{R_1} = \frac{U_3 - U_4}{R_2} + \frac{U_3}{R_3} \quad \text{ja} \quad \frac{U_3 - U_4}{R_2} = \frac{U_4}{R_4} + \frac{U_4 - E_2}{R_5}$$

$$G_1(E_1 - U_3) = G_2(U_3 - U_4) + G_3 U_3 \text{ ja } G_2(U_3 - U_4) = G_4 U_4 + G_5(U_4 - E_2)$$

Two equations, two unknowns $\rightarrow$ can be solved. Use conductances!

Some Remarks

- There are many other methods available too: mesh analysis, modified nodal analysis, branch current method ...
- If there are ideal voltage sources in the circuit (=voltage sources which are connected to a node without a series resistance), you need one more unknown (the current of the voltage source) and one more equation (the voltage source will determine the voltage between the nodes it is connected to).

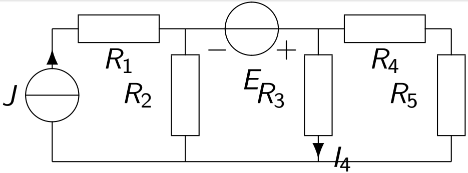
Homework 3 (released 7th Sep, to be returned 10th Sep)

Homework 3a)

Find the current I_4.

Homework 3b)

Verify your solution by writing down all the voltages and currents to the circuit diagram and checking that the solution does not contradict Ohm's and Kirchhoff's laws.



$$R_1 = R_2 = R_3 = R_4 = R_5 = 1\,\Omega \qquad E = 9\,\mathrm{V} \qquad J = 1\,\mathrm{A}$$

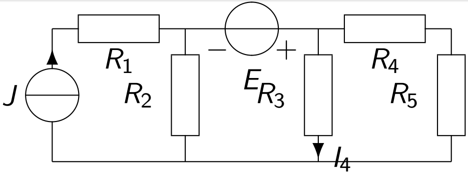
Homework 3 - Model Solution

Homework 3a)

Find current I_4.

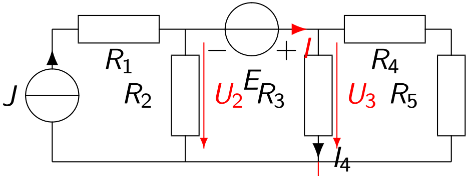
Homework 3b)

Verify your solution by writing down all the voltages and currents to the circuit diagram and checking that the solution does not contradict Ohm's and Kirchhoff's laws.



$R_1 = R_2 = R_3 = R_4 = R_5 = 1\,\Omega \qquad E = 9\,\mathrm{V} \qquad J = 1\,\mathrm{A}$

Solution



$R_1 = R_2 = R_3 = R_4 = R_5 = 1\,\Omega \qquad E = 9\,\mathrm{V} \qquad J = 1\,\mathrm{A}$

First we write two current equations and one voltage equation. The conductance of the series circuit formed by R_4 ja R_5 is denoted with G_{45}.

$$
\begin{aligned}
J &= U_2 G_2 + I \\
I &= U_3 G_3 + U_3 G_{45} \\
U_2 + E &= U_3
\end{aligned}
$$

By substituting I from the second equation to the first equation and then substituting U_2 from the third equation, we get

$$J = (U_3 - E)G_2 + U_3(G_3 + G_{45})$$

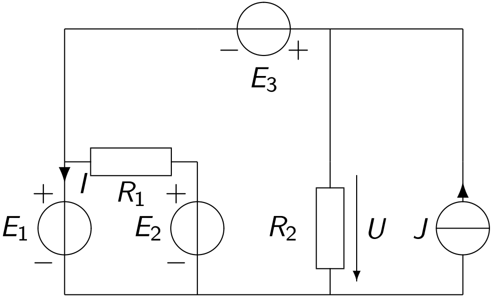
By substituting the component values, we get

$$U_3 = 4\,\mathrm{V}$$

- Therefore the current I is $4\,\mathrm{V} \cdot 1\,\mathrm{S} = 4\,\mathrm{A}$.
- From the voltage equation $U_2 + E = U_3$ we can solve $U_2 = -5\,\mathrm{V}$, therefore the current through R_2 is $5\,\mathrm{A}$ upwards.
- The current I is therefore $1\,\mathrm{A} + 5\,\mathrm{A} = 6\,\mathrm{A}$, of which $4\,\mathrm{A}$ goes through R_3:n and the remaining $2\,\mathrm{A}$ goes through R_4 ja R_5.
- There is no contradiction with Kirchhoff's laws and therefore we can be certain that our solution is correct.

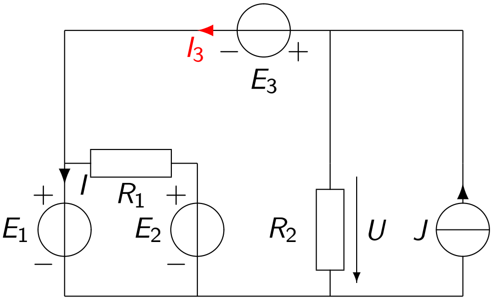
Example 1

Find I and U.



Example 1

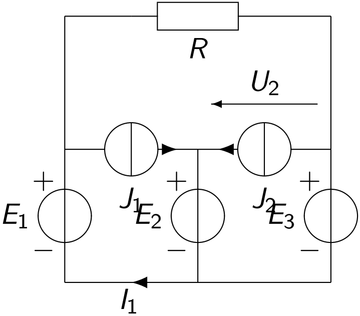
Find I and U.



$$
\begin{aligned}
J &= UG_2 + I_3 \\
I_3 &= I + (E_1 - E_2)G_1 \\
U &= E_1 + E_3
\end{aligned}
$$

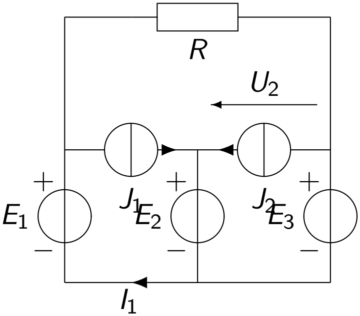
Example 2

Find U_2 and I_1.



Example 2

Find U_2 and I_1.



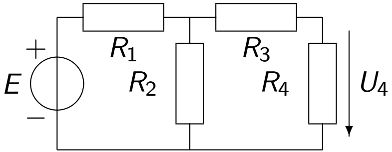
$$\begin{aligned} I_1 &= (E_1 - E_3)G + J_1 \\ E_2 + U_2 &= E_3 \end{aligned}$$

How To Get Extra Exercise?

- There are plenty of problems with solutions available at `http://users.tkk.fi/~ksilvone/Lisamateriaali/lisamateriaali.htm`
- For example, you can find 175 DC circuit problems at `http://users.tkk.fi/~ksilvone/Lisamateriaali/teht100.pdf`
- At the end of the pdf file you can find the model solutions, so you can check your solution.
- If you are enthusiastic, you can install and learn to use a circuit simulator:
  `http://www.linear.com/designtools/software/ltspice.jsp`

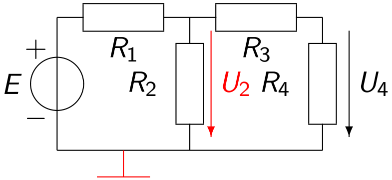
Example 3

Find U_4.



Example 3

Find U_4.

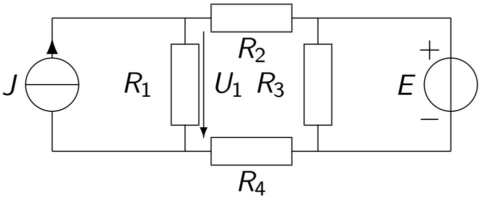


$$
\begin{aligned}
(E - U_2)G_1 &= U_2 G_2 + (U_2 - U_4)G_3 \\
(U_2 - U_4)G_3 &= G_4 U_4
\end{aligned}
$$

Homework 4 (released 10th Sep, to be returned 14th Sep)

Homework 4

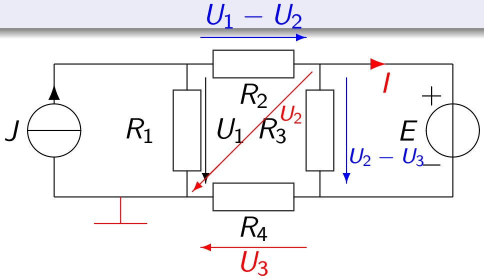
Find the voltage U_1. All resistors have the value $10\,\Omega$, $E = 10\,\mathrm{V}$ ja $J = 1\,\mathrm{A}$.



Homework 4 - Model Solution

Homework 4

Find the voltage U_1. All resistors have the value $10\,\Omega$, $E = 10\,\mathrm{V}$ ja $J = 1\,\mathrm{A}$.



$$
\begin{aligned}
J &= U_1 G_1 + (U_1 - U_2) G_2 \\
(U_1 - U_2) G_2 &= (U_2 - U_3) G_3 + I \\
G_3 (U_2 - U_3) + I &= U_3 G_4 \\
U_2 - U_3 &= E
\end{aligned}
$$

Solution

$$\begin{aligned} J &= U_1 G_1 + (U_1 - U_2) G_2 \\ (U_1 - U_2) G_2 &= E G_3 + I \\ G_3 E + I &= U_3 G_4 \\ U_2 - U_3 &= E \end{aligned}$$

I is solved from the third equation and substituted into the second equation, then U_3 is solved from the equation and substituted.

$$\begin{aligned} J &= U_1 G_1 + (U_1 - U_2) G_2 \\ (U_1 - U_2) G_2 &= E G_3 + (U_2 - E) G_4 - G_3 E \end{aligned}$$

$$\begin{aligned} 1 &= 0{,}2 U_1 - 0{,}1 U_2 \\ 0{,}1 U_1 - 0{,}1 U_2 &= 0{,}1 U_2 - 1 \end{aligned}$$

Solution

$$
\begin{aligned}
1 &= 0{,}2U_1 - 0{,}1U_2 \\
0{,}1U_1 - 0{,}1U_2 &= 0{,}1U_2 - 1
\end{aligned}
$$

Which is solved

$$
\begin{aligned}
U_1 &= 10 \\
U_2 &= 10
\end{aligned}
$$

Therefore the voltage U_1 is 10 Volts. This is easy to verify with a circuit simulator.

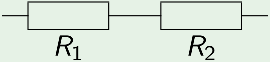
Circuit Transformation

1. An operation which transforms a part of the circuit into an internally different, but externally equally acting circuit, is called a *circuit transformation*.
2. For example, combining series resistors or parallel resistors into one resistor, is a circuit transformation. Combining series voltage sources into one voltage source is a circuit transformation too.
3. On this lecture, we learn dealing with parallel current sources and the *source transformation*, with which we can transform a voltage source with series resistance into a current source with parallel resistance.

An Example of a Circuit Transformation

Two (or more) resistors are combined to a single resistor, which acts just like the original circuit of resistors.

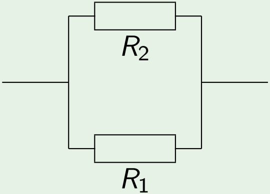
Resistors in series



$\Longleftrightarrow$

$R = R_1 + R_2$

Resistors in parallel



$\Longleftrightarrow$

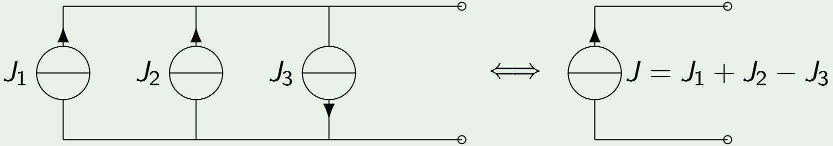
$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Or, by using conductance: $G = G_1 + G_2$.

Current Sources in Parallel

One or more current sources are transformed into single current source, which acts just like the original parallel circuit of current sources.

Current sources in parallel

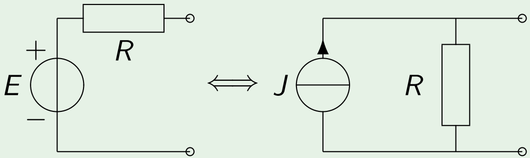


Just like connecting two or more voltage sources in parallel, connecting current sources in series is an undefined (read: forbidden) operation in circuit theory, just like divide by zero is undefined in mathematics. There can not be two currents in one wire!

The Source Transformation

A voltage source with series resistance acts just like **current source with parallel resistance**.

The source transformation



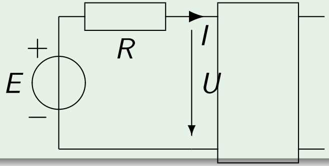
$E = RJ$

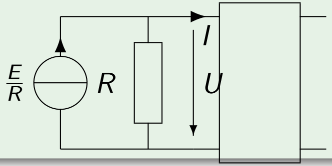
Important

- Note that an ideal voltage or current source can not be transformed like in previous slide. The voltage source to be transformed must have series resistance and the current source must have parallel resistance.
- The resistance remains the same, and the value for the source is found from formula $E = RJ$, which is based on Ohm's law.
- The source transform is not just a curiosity. It can save from many lines of manual calculations, for example when analyzing a transistor amplifier.

Rationale for the Source Transformation

The source transformation





In the figure left:

$$I = \frac{E - U}{R} \qquad U = E - RI$$

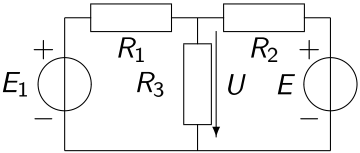
In the figure right:

$$I = \frac{E}{R} - \frac{U}{R} = \frac{E - U}{R} \qquad U = (\frac{E}{R} - I)R = E - RI$$

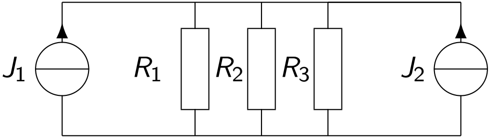
Both the circuits function equally.

Example

Solve U.



The circuit is transformed



And we get the result:

$$U = \frac{J_1 + J_2}{G_1 + G_2 + G_3}$$

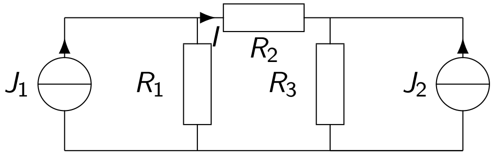
A Very Important Notice!

- The value of the resistance remains the same, but the resistor is not the same resistor! For instance, in the previous example the current through the original resistor is not same as current through the transformed resistor!

Homework 5 (released 14th Sep, to be returned 17th Sep)

Homework 5

Find current I by using source transformation. $J_1 = 10\,\mathrm{A}$, $J_2 = 1\,\mathrm{A}$, $R_1 = 100\,\Omega$, $R_2 = 200\,\Omega$ ja $R_3 = 300\,\Omega$.

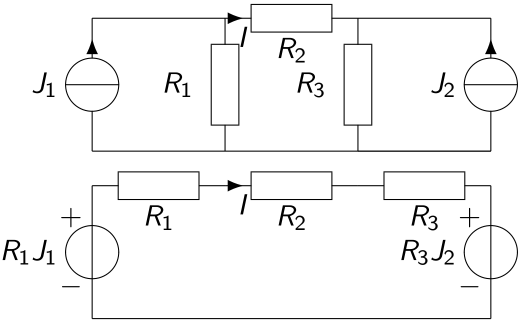


This is easy and fast assignment. If you find yourself writing many lines of equations, you have done something wrong.

Homework 5 - Model Solution

Homework 5

Find current I by using source transformation. $J_1 = 10\,\mathrm{A}$, $J_2 = 1\,\mathrm{A}$, $R_1 = 100\,\Omega$, $R_2 = 200\,\Omega$ ja $R_3 = 300\,\Omega$.



$$I = \frac{R_1 J_1 - R_3 J_2}{R_1 + R_2 + R_3} = \frac{1000\,\mathrm{V} - 300\,\mathrm{V}}{600\,\Omega} = \frac{7}{6}\,\mathrm{A} \approx 1{,}17\,\mathrm{A}.$$

Thévenin's Theorem and Norton's Theorem

- So far we have learned the following circuit transformations: voltage sources in series, current sources in parallel, resistances in parallel and in series and the source transformation.
- Thévenin's theorem and Norton's theorem relate to circuit transformations too.
- By Thévenin's and Norton's theorems an arbitrary circuit constisting of voltage sources, current sources and resistances can be transformed into a single voltage source with series resistance (Thévenin's equivalent) or a single current source with parallel resistance (Norton's equivalent).

Thévenin's Theorem and Norton's Theorem

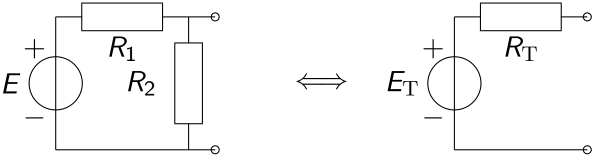
Thévenin's Theorem

An arbitrary linear circuit with two terminals is electrically equivalent to a single voltage source and a single series resistor, called Thévenin's equivalent.

Norton's Theorem

An arbitrary linear circuit with two terminals is electrically equivalent to a single current source and a single parallel resistor, called Norton's equivalent.

Calculating the Thévenin Equivalent

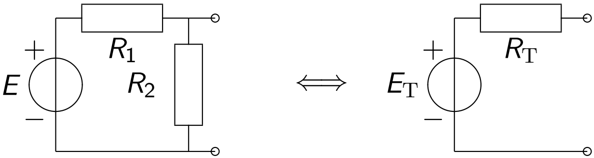


The voltage E_{T} in the Thévenin equivalent is solved simply by calculating the voltage between the terminals. For solving R_{T}, there are two ways:

- By turning off all independent (= non-controlled) sources in the circuit, and calculating the resistance between the terminals.
- By calculating the short circuit current of the port and applying Ohm's law.

Independent source is a source, whose value does not depend on any other voltage or current in the circuit. All sources we have dealt with for now, have been independent. Controlled sources covered later in this course.

Calculating the Thévenin Equivalent

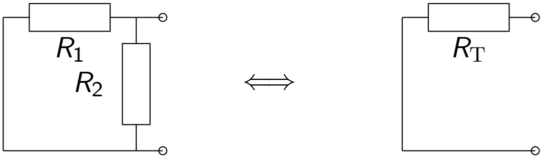


The voltage at the port is found by calculating the current through the resistors and multiplying it with R_2. The voltage at the port, called also the *idle voltage* of the port, is equal to E_{T}.

$$E_{\mathrm{T}} = \frac{E}{R_1 + R_2} R_2$$

Calculating the Thévenin Equivalent

There are two ways to solve R_{T}. Way 1: turn off all the (independent) sources, and calculate the voltage at the port. A turned-off voltage source is a voltage source, whose voltage is zero, which is same as just a wire:



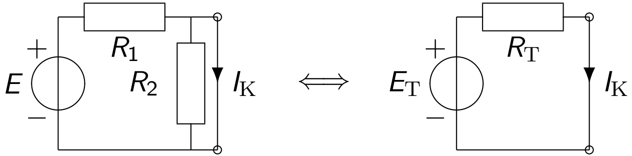
Now it is easy to solve the resistance between the nodes of the port: R_1 ja R_2 are in parallel, and therefore the resistance is

$$R_{\mathrm{T}} = \frac{1}{G_1 + G_2} = \frac{R_1 R_2}{R_1 + R_2}.$$

This method is usually simpler than the other way with short circuit current!

Solving R_{T} by Using the Short-circuit Current

There are two ways to solve R_{T}. Way 2: short-circuit the port, and calculate the current through the short-circuit wire. This current is called the *short-circuit current*:



The value for the short-circuit current is

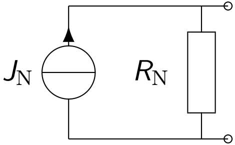
$$I_{\mathrm{K}} = \frac{E}{R_1}$$

and the resistance R_{T} is (by applying Ohm's law to the figure on the right):

$$R_{\mathrm{T}} = \frac{E_{\mathrm{T}}}{I_{\mathrm{K}}} = \frac{E_{\mathrm{T}}}{\frac{E}{R_1}} = \frac{\frac{E}{R_1+R_2}R_2}{\frac{E}{R_1}} = \frac{R_1 R_2}{R_1 + R_2}$$

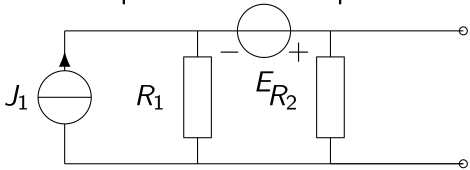
Norton Equivalent

The Norton equivalent is simply a Thévenin equivalent, which has been source transformed into a current source and parallel resistance (or vice versa). The resistance has the same value in both equivalents. The value of the current source is the same as the short-circuit current of the port.



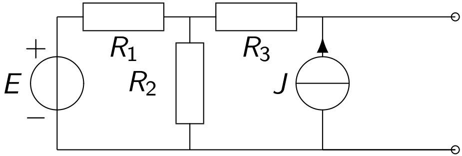
Example 1

Calculate the Thévenin equivalent. All component values = 1.



Example 2

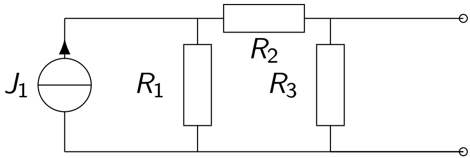
Calculate the Thévenin equivalent. All component values = 1.



Homework 6 (released 17th Sep, to be returned 21th Sep)

Homework 6

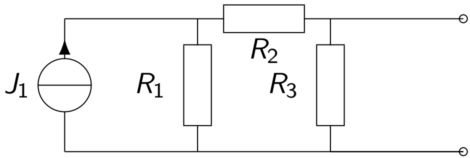
Calculate the Thévenin equivalent. All the component values are 1. (Every resistance is $1\,\Omega$ and the current source is $J_1 = 1\,\mathrm{A}$.)



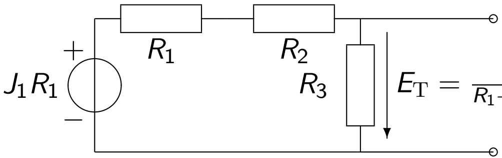
Homework 6 - Model Solution

Homework 6

Calculate the Thévenin equivalent. All the component values are 1. (Every resistance is $1\,\Omega$ and the current source is $J_1 = 1\,\mathrm{A}$.)



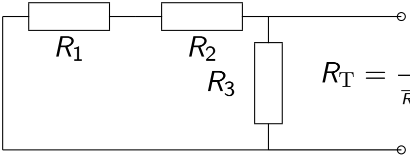
First we solve the voltage E_T. This can be done by using applying source transformation:



$$E_\mathrm{T} = \frac{J_1 R_1}{R_1 + R_2 + R_3} R_3 = \frac{1}{3}\,\mathrm{V}$$

Solution

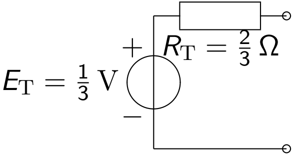
Next we solve the resistance R_{T} of the Thévenin equivalent. The easiest way to do it is to turn off all the sources and calculate the resistance between the output port. (The other way is to find out the short-circuit current.) The resistance can be solver either from the original or the transformed circuit. Let's use the transformed circuit and turn off the voltage source:



$$R_{\mathrm{T}} = \frac{1}{\frac{1}{R_1+R_2}+\frac{1}{R_3}} = \frac{2}{3}\,\Omega$$

Now the resistors R_1 ja R_2 are in series, and the series circuit is in parallel with R_3. Now we know both E_{T} ja R_{T} and we can draw the Thévenin equivalent (on the next slide).

The Final Circuit



Superposition Principle

- A circuit consisting of resistances and constant-valued current and voltage sources is *linear*.
- If a circuit is linear, all the voltages and currents can be solved by calculating the effect of each source one at the time.
- This principle is called the **method of superposition**.

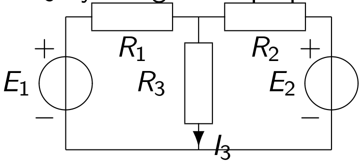
Superposition Principle

The method of superposition is applied as follows

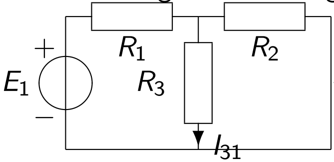
- The current(s) and/or voltage(s) caused by each source is calculated one at a time so that all other sources are turned off.
- A turned-off voltage source = short circuit (a wire), a turned-off current source = open circuit (no wire).
- Finally, all results are summed together.

Superposition Principle: an Example

Find current I_3 by using the superposition principle.



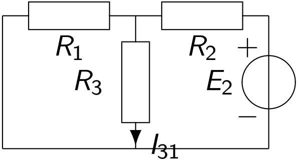
First, we turn off the rightmost voltage source:



$$I_{31} = \frac{E_1}{R_1 + \frac{1}{G_2+G_3}} \frac{1}{G_2+G_3} G_3$$

Next, we turn off the leftmost voltage source:

$$I_{32} = \frac{E_2}{R_2 + \frac{1}{G_1+G_3}} \frac{1}{G_1+G_3} G_3$$



Superposition Principle: an Example

The current I_3 is obtained by summing the partial currents I_{31} and I_{32}.

$$I_3 = I_{31} + I_{32} = \frac{E_1}{R_1 + \frac{1}{G_2+G_3}} \frac{1}{G_2 + G_3} G_3 + \frac{E_2}{R_2 + \frac{1}{G_1+G_3}} \frac{1}{G_1 + G_3} G_3$$

When Is It Handy to Use the Superposition Principle?

- If one doesn't like solving equations but likes fiddling with the circuit.
- If there are many sources and few resistors, the method of superposition is usually fast.
- If there are sources with different frequencies (as we learn on the AC Circuits course), the analysis of such a circuit is based on the superposition principle.

Linearity and the Justification for the Superposition Principle

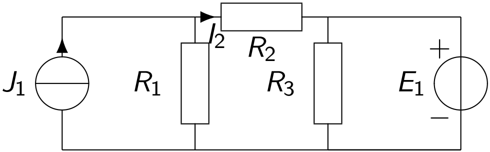
- The method of superposition is based on the linearity of the circuit, which means that every source affects every voltage and current with a constant factor.
- This means that if there are sources E_1, E_2, E_3, J_1, J_2 in the circuit, then every voltage and current is of form $k_1E_1 + k_2E_2 + k_3E_3 + k_4J_1 + k_5J_2$, where constants k_n are real numbers.
- If all the sources are turned off ($= 0$), then all currents and voltages in the circuit are zero. Therefore, by nullifying all sources except one, we can find out the multiplier for the source in question.

Homework 7 (released 21st Sep, to be returned 24th Sep)

Homework 7

Find current I_2 by using the superposition principle.

$$J_1 = 1\,\mathrm{A} \quad R_1 = 10\,\Omega \quad R_2 = 20\,\Omega \quad R_3 = 30\,\Omega \quad E_1 = 5\,\mathrm{V}$$

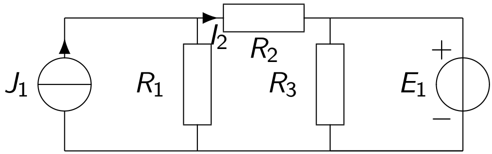


Homework 7 - Model Solution

Homework 7

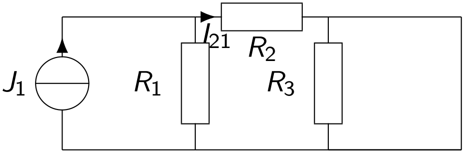
Find current I_2 by using the superposition principle.

$$J_1 = 1\,\mathrm{A} \quad R_1 = 10\,\Omega \quad R_2 = 20\,\Omega \quad R_3 = 30\,\Omega \quad E_1 = 5\,\mathrm{V}$$



Ratkaisu

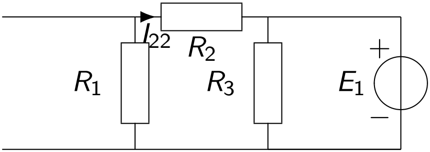
First, we find the effect of the current source:



The voltage over R_1 and R_2 is the same (they are in parallel) and R_2 is twice as large as R_1 and therefore the current through R_2 is half of the current of R_1. Because the total current through the resistors is $J_1 = 1\,\mathrm{A}$, the current through R_1:n is $2/3\,\mathrm{A}$ and the current through R_2 is$I_{21} = 1/3\,\mathrm{A}$.

Ratkaisu

Next, we find out the effect of the voltage source:



The resistors R_1 and R_2 are now in series and the total voltage over them is $E = 5\,\mathrm{V}$, and therefore

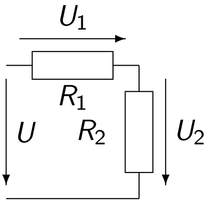
$$I_{22} = -\frac{E}{R_1 + R_2} = -\frac{5\,\mathrm{V}}{10\,\Omega + 20\,\Omega} = -\frac{1}{6}\,\mathrm{V}.$$

The minus sign comes from the fact that the direction of the current I_{22} is upwards and the direction of the voltage E is downwards.

Finally, we sum the partial results:

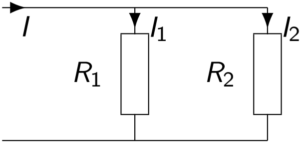
$$I_2 = I_{21} + I_{22} = \frac{1}{3}\,\mathrm{A} - \frac{1}{6}\,\mathrm{A} = \frac{1}{6}\,\mathrm{A}.$$

Voltage Divider



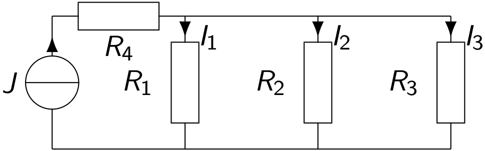
- $U_1 = U\frac{R_1}{R_1+R_2}$ ja $U_2 = U\frac{R_2}{R_1+R_2}$
- It is quite common in electronic circuit design, that we need a reference voltage formed from another voltage in the circuit.
- The formula is valid also for multiple resistors in series. The denominator is formed by summing all the resistances and the resistor whose voltage is to be solved is in the numerator.

Current Divider

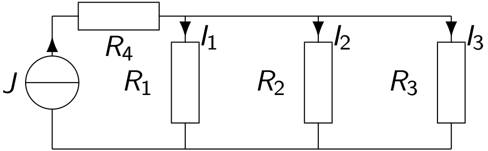


- $I_1 = I\frac{G_1}{G_1+G_2}$ ja $I_2 = I\frac{G_2}{G_1+G_2}$
- The formula is valid also for multiple resistors in parallel.
- The formula for current divider is not used as frequently as the voltage divider, but it is natural to discuss it in this concept.

Example 1

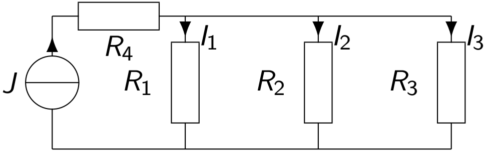


Example 1



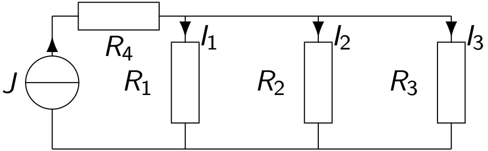
$$I_1 = J\frac{G_1}{G_1+G_2+G_3}$$

Example 1



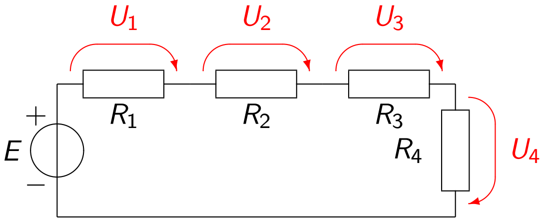
$I_1 = J\frac{G_1}{G_1+G_2+G_3}$ $\quad$ $I_2 = J\frac{G_2}{G_1+G_2+G_3}$

Example 1

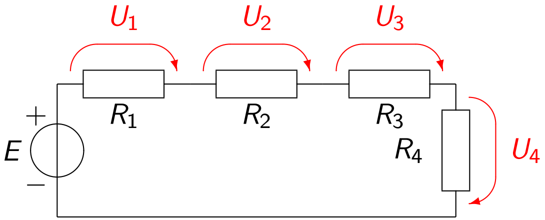


$$I_1 = J\frac{G_1}{G_1+G_2+G_3} \qquad I_2 = J\frac{G_2}{G_1+G_2+G_3} \qquad I_3 = J\frac{G_3}{G_1+G_2+G_3}$$

Example 2

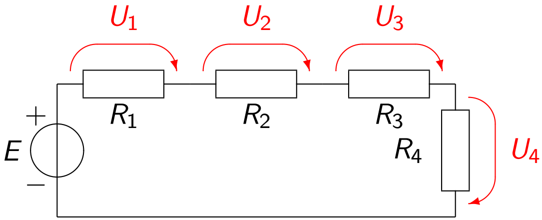


Example 2



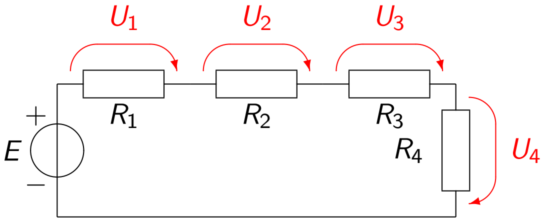
$$U_1 = E\frac{R_1}{R_1+R_2+R_3+R_4}$$

Example 2



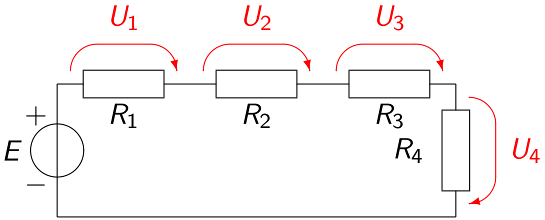
$$U_1 = E\frac{R_1}{R_1+R_2+R_3+R_4} \qquad U_2 = E\frac{R_2}{R_1+R_2+R_3+R_4}$$

Example 2



$$U_1 = E\frac{R_1}{R_1+R_2+R_3+R_4} \qquad U_2 = E\frac{R_2}{R_1+R_2+R_3+R_4} \qquad U_3 = E\frac{R_3}{R_1+R_2+R_3+R_4}$$

Example 2



$U_1 = E\frac{R_1}{R_1+R_2+R_3+R_4}$ $U_2 = E\frac{R_2}{R_1+R_2+R_3+R_4}$ $U_3 = E\frac{R_3}{R_1+R_2+R_3+R_4}$
$U_4 = E\frac{R_4}{R_1+R_2+R_3+R_4}$

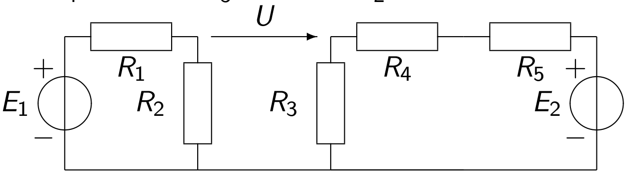
Homework 8 (released 24th Sep, to be returned 28th Sep)

Homework 8

Find the voltage U by applying the voltage divider formula.

$$E_1 = 10\,\mathrm{V} \quad R_1 = 10\,\Omega \quad R_2 = 20\,\Omega \quad R_3 = 30\,\Omega$$

$$R_4 = 40\,\Omega \quad R_5 = 50\,\Omega \quad E_2 = 15\,\mathrm{V}$$



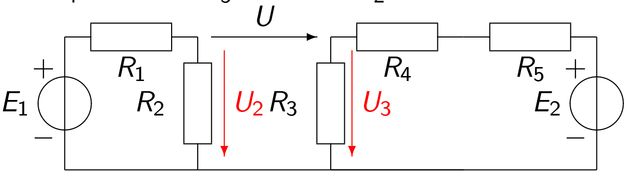
Homework 8 - Model Solution

Homework 8

Find the voltage U by applying the voltage divider formula.

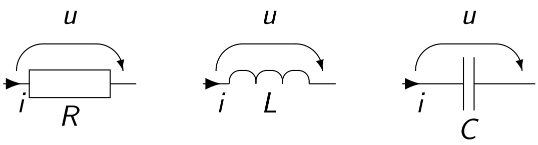
$$E_1 = 10\,\mathrm{V} \quad R_1 = 10\,\Omega \quad R_2 = 20\,\Omega \quad R_3 = 30\,\Omega$$

$$R_4 = 40\,\Omega \quad R_5 = 50\,\Omega \quad E_2 = 15\,\mathrm{V}$$



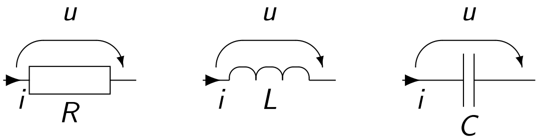
$$U_2 = E_1 \frac{R_2}{R_1+R_2} = 10\,\mathrm{V} \frac{20\,\Omega}{10\,\Omega+20\,\Omega} = 6\tfrac{2}{3}\,\mathrm{V}$$
$$U_3 = E_2 \frac{R_3}{R_3+R_4+R_5} = 15\,\mathrm{V} \frac{30\,\Omega}{30\,\Omega+40\,\Omega+50\,\Omega} = 3{,}75\,\mathrm{V}$$
$$U = U_2 - U_3 = 2\tfrac{11}{12}\,\mathrm{V} \approx 2{,}92\,\mathrm{V}.$$

Inductors and Capacitors



$u = Ri$ $\qquad$ $u = L\frac{\mathrm{d}i}{\mathrm{d}t}$ $\qquad$ $i = C\frac{\mathrm{d}u}{\mathrm{d}t}$

Inductors and Capacitors in DC Circuit

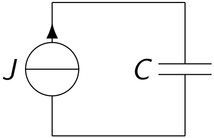


$u = Ri$ $\qquad$ $u = L\frac{\mathrm{d}i}{\mathrm{d}t}$ $\qquad$ $i = C\frac{\mathrm{d}u}{\mathrm{d}t}$

DC voltage and current remain constant as function of time or the time derivatives of the voltage and current is zero. Therefore the voltage of an inductor and the current of a capacitor is zero in a DC circuit.

Exception 1

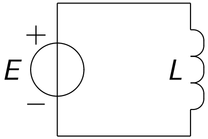
The capacitor is fed with DC current so that the current has no other route.



$i = C\frac{\mathrm{d}u}{\mathrm{d}t} \Rightarrow J = C\frac{\mathrm{d}u}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}t} = \frac{J}{C}$. The voltage of the capacitor rises at constant speed.

Exception 2

A constant voltage source is connected to the terminals of an inductor.



$u = L\frac{\mathrm{d}i}{\mathrm{d}t} \Rightarrow E = L\frac{\mathrm{d}i}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}i}{\mathrm{d}t} = \frac{E}{L}$. The current of the inductor rises at constant speed.

Dealing with all other cases involving inductors and capacitors in DC circuits

Inductors are replaced with short circuits and capacitors are replaced with open circuits.

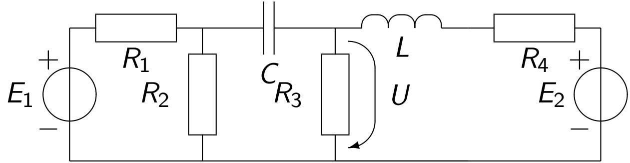
Homework 9 (released 28th Sep, to be returned 1st Oct)

Homework 9

Solve the voltage U from this DC circuit.

$$E_1 = 10\,\mathrm{V} \quad R_1 = 10\,\Omega \quad R_2 = 20\,\Omega \quad R_3 = 30\,\Omega$$

$$R_4 = 40\,\Omega \quad L = 500\,\mathrm{mH} \quad C = 2\,\mathrm{F} \quad E_2 = 15\,\mathrm{V}$$



Homework 9 - Model Solution

Homework 9

Solve the voltage U from this DC circuit.

$$E_1 = 10\,\mathrm{V} \quad R_1 = 10\,\Omega \quad R_2 = 20\,\Omega \quad R_3 = 30\,\Omega$$

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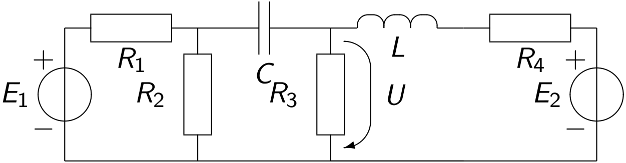
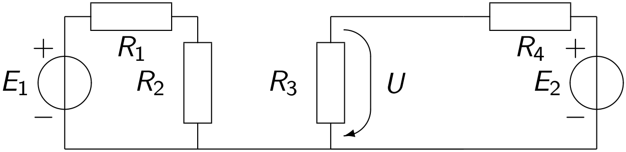


Homework 9 - Model Solution

Because there are no parallel connections of inductors and voltage sources and no serial connections of capacitors and current sources and the circuit is a DC circuit (= constant voltages and currents), we can replace the inductors with short circuits and the capacitors with open circuits.



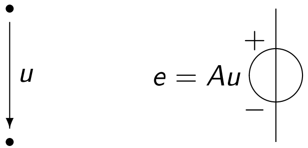
in which case we obtain U easily by applying the voltage divider formula:

$$U = E_2 \frac{R_3}{R_3 + R_4} = 6\frac{3}{7}\,\mathrm{V} \approx 6{,}4\,\mathrm{V}$$

Controlled Sources

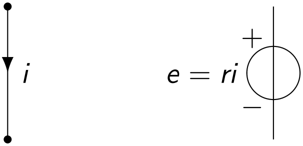
- So far, all of our sources have been constant valued.
- If the value of a source does not depend on any of the voltages or currents in the circuit, the source is an independent source. For example, constant valued sources and sources varying as function of time (only) are independent sources.
- If the value of a source is a function of a voltage and/or current in the circuit, the source is a *controlled source*.

Voltage Controlled Voltage Source (VCVS)



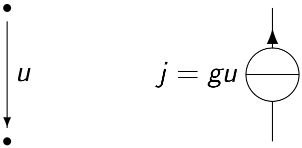
- The voltage e of VCVS is dependent of some voltage u.
- The multiplier A is called *voltage gain*.
- A real-world example: an audio amplifier.

Current Controlled Voltage Source (CCVS)



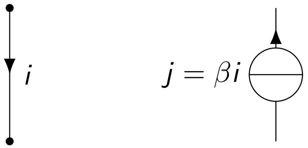
- The voltage e of CCVS is dependent of some current i.
- The multiplier r is called *transresistance*.
- There is no good everyday example of this source available (of course we can construct this kind of source by using an *operational amplifier*).

Voltage Controlled Current Source (VCCS)



- The current j of VCCS is dependent of some voltage u.
- The multiplier g is called *transconductance*.
- A real-world example: a field-effect transistor (JFET or MOSFET).

Current Controlled Current Source (CCCS)



- The current j of CCCS is dependent of some current i.
- The multiplier β is called *current gain*.
- A real-world example: a (bipolar junction) transistor.

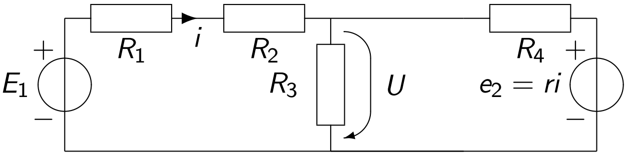
Homework 10 (released 1st Oct, to be returned 5th Oct)

Homework 10

Find the voltage U.

$$E_1 = 10\,\mathrm{V} \quad R_1 = 10\,\Omega \quad R_2 = 20\,\Omega \quad R_3 = 30\,\Omega$$

$$R_4 = 40\,\Omega \qquad r = 2\,\Omega$$



Note that the source on the right is a controlled source.

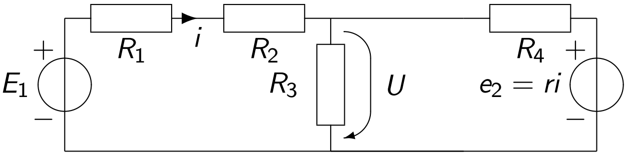
Homework 10 - Model Solution

Homework 10

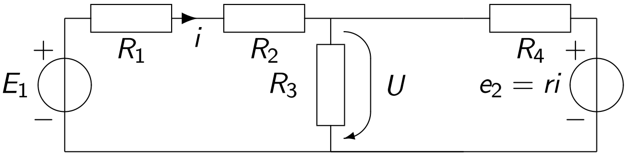
Find the voltage U.

$$E_1 = 10\,\mathrm{V} \quad R_1 = 10\,\Omega \quad R_2 = 20\,\Omega \quad R_3 = 30\,\Omega$$

$$R_4 = 40\,\Omega \qquad r = 2\,\Omega$$



Note that the source on the right is a controlled source.



Let's denote the total resistance of R_1:n ja R_2 with symbol R_{12} and write a nodal equation:

$$UG_3 = (E_1 - U)G_{12} + (ri - U)G_4$$

There are two unknowns in the circuit and therefore we need another equation with the same unknowns:

$$i = (E_1 - U)G_{12}$$

Then we substitute i to the first equation:

$$E_1G_{12} - UG_{12} + rG_4G_{12}E_1 - rG_4G_{12}U - UG_4 = UG_3$$

$$E_1 G_{12} - UG_{12} + rG_4 G_{12} E_1 - rG_4 G_{12} U - UG_4 = UG_3$$

from which we get

$$G_{12} E_1 (1 + rG_4) = U(G_3 + G_{12} + G_4 + rG_4 G_{12}).$$

Then we substitute the component values and solve U:

$$U = \frac{\frac{10}{30}(1 + \frac{2}{40})}{\frac{1}{30} + \frac{1}{30} + \frac{1}{40} + \frac{2}{40 \cdot 30}} = 3{,}75\,\mathrm{V}$$

Recapitulation

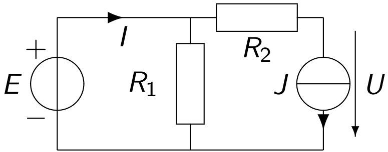
On this lesson, we solve some refresher assignments. If you have solved all the circuits, solve the home assignment.

Recap assignment 1

Recap assignment 1

Find U and I first by using the method of superposition and then by some other method of your choice.

$$R_1 = 1\,\Omega \quad R_2 = 2\,\Omega \quad J = 1\,\mathrm{A} \quad E = 3\,\mathrm{V}$$



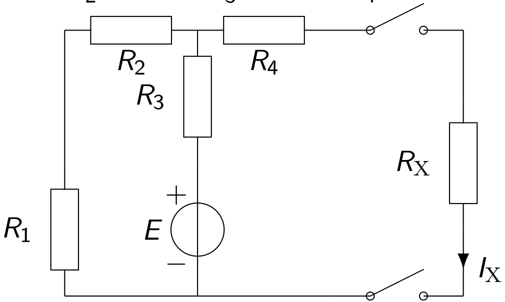
Vastaus: $I = 4\,\mathrm{A}$ ja $U = 1\,\mathrm{V}$.

Recap assignment 2

Recap assignment 2

Form a Thévenin equivalent of the circuit on the left. Then, compute the current I_{X}, when the switches are closed and R_{X} is a) $0\,\Omega$, b) $8\,\Omega$ ja c) $12\,\Omega$.

$R_1 = 5\,\Omega \quad R_2 = 3\,\Omega \quad R_3 = 8\,\Omega \quad R_4 = 4\,\Omega \quad E = 16\,\mathrm{V}$



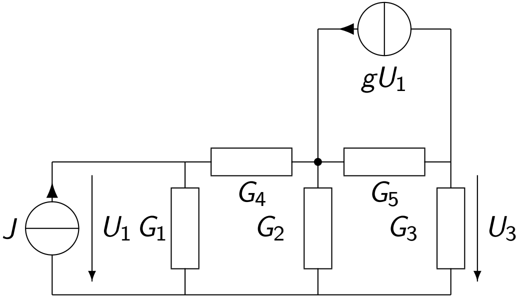
: Vastaus: $R_{\mathrm{T}} = 8\,\Omega$, $E_{\mathrm{T}} = 8\,\mathrm{V}$. a) $1\,\mathrm{A}$ b) $0{,}5\,\mathrm{A}$ c)$0{,}4\,\mathrm{A}$.

Recap assignment 3

Recap assignment 3

Find U_3.

$$G_1 = 1\,\mathrm{S} \quad G_2 = 2\,\mathrm{S} \quad G_3 = 3\,\mathrm{S} \quad G_4 = 4\,\mathrm{S} \quad G_5 = 5\,\mathrm{S} \quad g = 6\,\mathrm{S} \quad J = 3\,\mathrm{A}$$



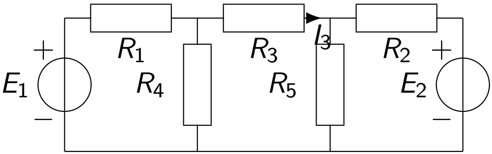
$U_3 = -\frac{48}{115}\,\mathrm{V} \approx -417\,\mathrm{mV}$

Homework 11 (released 5th Oct, to be returned 8th Oct)

Homework 11

We are given a fact that the current $I_3 = 0\,\mathrm{A}$. Find E_1.

$$R_1 = 5\,\Omega \quad R_2 = 4\,\Omega \quad R_3 = 2\,\Omega \quad R_4 = 5\,\Omega \quad R_5 = 6\,\Omega \quad E_2 = 30\,\mathrm{V}$$

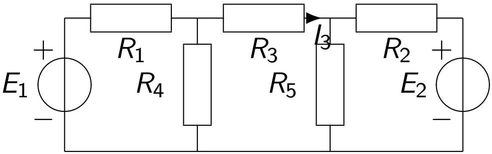


Homework 11 - Model Solution

Homework 11

We are given a fact that the current $I_3 = 0\,\mathrm{A}$. Find E_1.

$$R_1 = 5\,\Omega \quad R_2 = 4\,\Omega \quad R_3 = 2\,\Omega \quad R_4 = 5\,\Omega \quad R_5 = 6\,\Omega \quad E_2 = 30\,\mathrm{V}$$

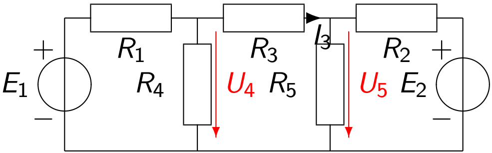


Homework 11 - Model Solution

Homework 11

We are given a fact that the current $I_3 = 0\,\mathrm{A}$. Find E_1.

$R_1 = 5\,\Omega \quad R_2 = 4\,\Omega \quad R_3 = 2\,\Omega \quad R_4 = 5\,\Omega \quad R_5 = 6\,\Omega \quad E_2 = 30\,\mathrm{V}$



Because $I_3 = 0\,\mathrm{A}$, the current through R_1 equals the current through R_4 and the current through R_2 equals the current through R_5. Therefore, the resistors are in series[2] and we may use the voltage divider formula to find voltages over R_4 and R_5. The voltage over R_5 is $U_5 = E_2 \frac{R_5}{R_2+R_5} = 18\,\mathrm{V}$. Therefore the voltage over R_4 is $18\,\mathrm{V}$ too. Now, by the voltage divider rule:

$$U_4 = E_1 \frac{R_4}{R_1 + R_4} \quad \Rightarrow \quad 18\,\mathrm{V} = E_1 \frac{5\,\Omega}{5\,\Omega + 5\,\Omega}$$

from which we can solve $E_1 = 36\,\mathrm{V}$. Note: it is completely correct to write nodal equations for the circuit and solve E_1 from them, too.

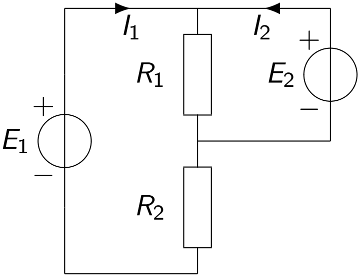
[2] Because and only because we know that I_3 is zero.

Recapitulation

On this lesson, we solve some refresher assignments. I can also demonstrate some examples on the blackboard or to your booklets, too.

Recap assignment 4

$$R_2 = 5\,\Omega \quad E_1 = 3\,\mathrm{V} \quad E_2 = 2\,\mathrm{V}$$

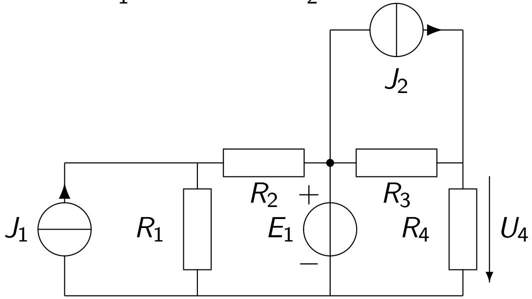


a) How should we choose R_1, if we want I_2 to be $0\,\mathrm{A}$?
b) How large is I_1 then?
a) $10\,\Omega$ ja b) $0{,}2\,\mathrm{A}$.

Recap assignment 5

$$R_1 = 100\,\Omega \quad R_2 = 500\,\Omega \quad R_3 = 1{,}5\,\mathrm{k}\Omega \quad R_4 = 1\,\mathrm{k}\Omega \quad E_1 = 5\,\mathrm{V}$$

$$J_1 = 100\,\mathrm{mA} \quad J_2 = 150\,\mathrm{mA}$$

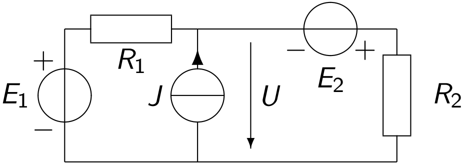


Find U_4.

$U_4 = 92\,\mathrm{V}$

Recap assignment 6

$$R_1 = 12\,\Omega \quad R_2 = 25\,\Omega \quad J = 1\,\mathrm{A} \quad E_1 = 1\,\mathrm{V} \quad E_2 = 27\,\mathrm{V}$$



Find voltage U.

Solution: $\frac{1}{37}\,\mathrm{V} \approx 27\,\mathrm{mV}$

Homework 12 (released 8th Oct, to be returned 12th Oct)

Write a **short** essay on following subjects:

- What did you learn on the course?
- Did the course suck or was it worthwhile?
- What could the lecturer do better?
- How should this course be improved?

The essay will not affect the grading of the exam — please give honest feedback[3].

How to return this homework: Write the essay as a plain text email (no attachments) and send it to me no later than the exam day at 18:00. **The subject of the email message must be 'DC Circuits course feedback 2009 Firstname Surname'.**

[3] I am really interested in how I could make the course better.

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- The slides are licensed with CC By 1.0 [4]. In short: you can use and modify the slides freely as long as you mention my name (= Vesa Linja-aho) somewhere.
- Single examples and circuits can be of course used without any name mentioning, because they are not an object of copyright (legal term: "Threshold of originality").
- The origin of these slides is the DC Circuits course in Metropolia polytechnic in Helsinki, Finland.
- If you find typos, misspellings or errors in facts, please give me feedback.

[4] http://creativecommons.org/licenses/by/1.0/