The Kuramoto-Sivashinsky Equation

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Abstract

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Introduction

The Kuramoto-Sivashinsky equation,

$$u_t + u_{xx} + u_{xxxx} + uu_x = 0 \tag{1}$$

is one of the simplest partial differential equations that exhibits complicated dynamics in both time and space, which is why the equation has been the attention for a lot of research. The equation was developed by two scientists at the same time in 1977 [1]. Gregory Sivashinsky determined an equation for a laminar flame front, while Yoshiki Kuramoto modeled a diffusion-induced chaos using the same equation. Because of this, the equation is named Kuramoto-Sivashinsky. The KS-equation also models the motion of a fluid going down a vertical wall, e.g. solitary pulses in a falling thin film. [2]

The reason for the complex behaviour comes from the second- and fourth-order derivatives in (1). While the second-order term acts as an energy source and has a destabilizing effect, the fourth-order term has a stabilizing effect. In addition to this, the nonlinear term transfers energy from low to high wave numbers. [3] The KS-equation is a stiff equation, i.e. an equation where numerical methods for solving it are numerically unstable, unless the step size is extremely small. u_{xxxx} is the main reason for this as it leads to rapid variation in the solution.

Numerical results

Initial conditions

In the solution of the KS-equation we had periodic boundary conditions, i.e. $u(0,t) = u(L,t)$. We also used L-periodic initial conditions. We experienced that a common initial condition used in several other reports was

$$u(x,0) = \cos(\frac{x}{16})(1 + \sin(\frac{x}{16}). \tag{2}$$

We also tried the initial condition

$$u(x,0) = \frac{1}{\sqrt{2}}\sin(x) - \frac{1}{8}\sin(2x), \tag{3}$$

which worked well. The L-periodic initial conditions is customarily taken [4] to satisfy

$$\int_0^L f(x)\,\mathrm{d}x = 0, \tag{4}$$

which both of our initial conditions satisfy. The same article also states that for L-periodic initial data, a unique solution for (1) exits, and is bounded as $t \to \infty$. The bound has been proven to be smaller than $O(L^{8/5})$. In our numerical tests, with $t = 5000$, the initial condition (2) did indeed not exceed the bound, nor did (3).

References

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