The Kuramoto-Sivashinsky Equation

Anders, Elisabeth og Espen

March 31, 2014

Abstract

This is the abstract. Write smart things here.

Introduction

The Kuramoto-Sivashinsky equation,

$$u_t + u_{xx} + u_{xxxx} + uu_t = 0 \tag{1}$$

written in integral form as

$$h_t + h_{xx} + h_{xxxx} + \frac{1}{2}h_x^2 = 0, u = h_x$$

is one of the simplest partial differential equations that exhibits complicated dynamics in both time and space, which is why the equation has been the attention for a lot of research. The equation was developed by two scientists at the same time in 1977 [1]. Gregory Sivashinsky determined an equation for a laminar flame front, while Yoshiki Kuramoto modeled a diffusion-induced chaos using the same equation. Because of this, the equation is named Kuramoto-Sivashinsky. The KS-equation also models the motion of a fluid going down a vertical wall, e.g. solitary pulses in a falling thin film. [2]

The reason for the complex behaviour comes from the second- and fourth-order derivatives in (1). While the second-order term acts as an energy source and has a destabilizing effect, the fourth-order term has a stabilizing effect. In addition to this, the nonlinear term transfers energy from low to high wave numbers. [3] The KS-equation is a stiff equation, i.e. an equation where numerical methods for solving it are numerically unstable, unless the step size is extremely small. u_{xxxx} is the main reason for this.

References

[1] Scott Arthur Gasner, Fall 2004, *Integrating the Kuramoto-Sivashinsky equation: A simulation of the hopping state* http://terminus.sdsu.edu/thesis_repository/ScottGasner_2004_Fall_MS_Comp_Sci.pdf, 03/31-2014

[2] Mehrdad Lakestani and Mehdi Dehghan, February 2012, *Numerical solutions of the generalized*

Kuramoto-Sivashinsky equation using B-spline functions, http://www.sciencedirect.com/science/article/pii/S0307904X11004082, 03/31-2014

[3] Marjan Uddin and Sardar Ali, January 2013, *RBF-PS method and Fourier Pseudospectral method for solving stiff nonlinear partial differential equations*, http://www.naturalspublishing.com/files/published/ed38fj6n3xt187.pdf, 03/31-2014