

## Difference Scheme

The Kuramoto-Sivashinsky equation

$$u_t + u_{xx} + u_{xxxx} + uu_x = 0$$

$$u_t + u_{xx} + u_{xxxx} + \frac{1}{2}(u^2)_x = 0$$

Difference scheme:

$$\begin{aligned} u_t &\approx \frac{\Delta u}{k} = \frac{u^{n+1} - u^n}{k} \\ u_{xx} &\approx \frac{\delta^2 u}{h^2} = \frac{u_{m+1} - 2u_m + u_{m-1}}{h^2} = \frac{1}{h^2} Au \\ u_{xxxx} &\approx \frac{\delta^4 u}{h^4} = \frac{u_{m+2} - 4u_{m+1} + 6u_m - 4u_{m-1} + u_{m-2}}{h^4} = \frac{1}{h^4} AAu \\ (u^2)_x &\approx \frac{\mu \delta u^2}{h} = \frac{(u_{m+1})^2 - (u_{m-1})^2}{2h} = \frac{1}{2h} D \end{aligned}$$

$$U^{n+1} = U^n - \frac{k}{h^2} AU^n - \frac{k}{h^4} AAU^n - \frac{k}{4h} D(U^n \odot U^n)$$

$\odot$  = Element-wise multiplication

## Consistency

Consistency

$$u_t = \frac{\Delta u}{k} + O(k)$$

$$u_{xx} = \frac{\delta^2 u}{h^2} + O(h^2)$$

$$u_{xxxx} = \frac{\delta^4 u}{h^4} + O(h^2)$$

$$\begin{aligned}\mu\delta[u(x)^2] &= \mu[u(x + \frac{h^2}{2}) - u(x - \frac{h}{2})^2] \\ &= 2huu_x + O(h^3)\end{aligned}$$

$$uu_x = \frac{\mu\delta u^2}{4h} + O(h^2)$$

Local Truncation Error

$$\tau^n = O(h^2) + O(k) \xrightarrow{k,h \rightarrow 0} 0 \tag{1}$$

$\Rightarrow$  Consistent