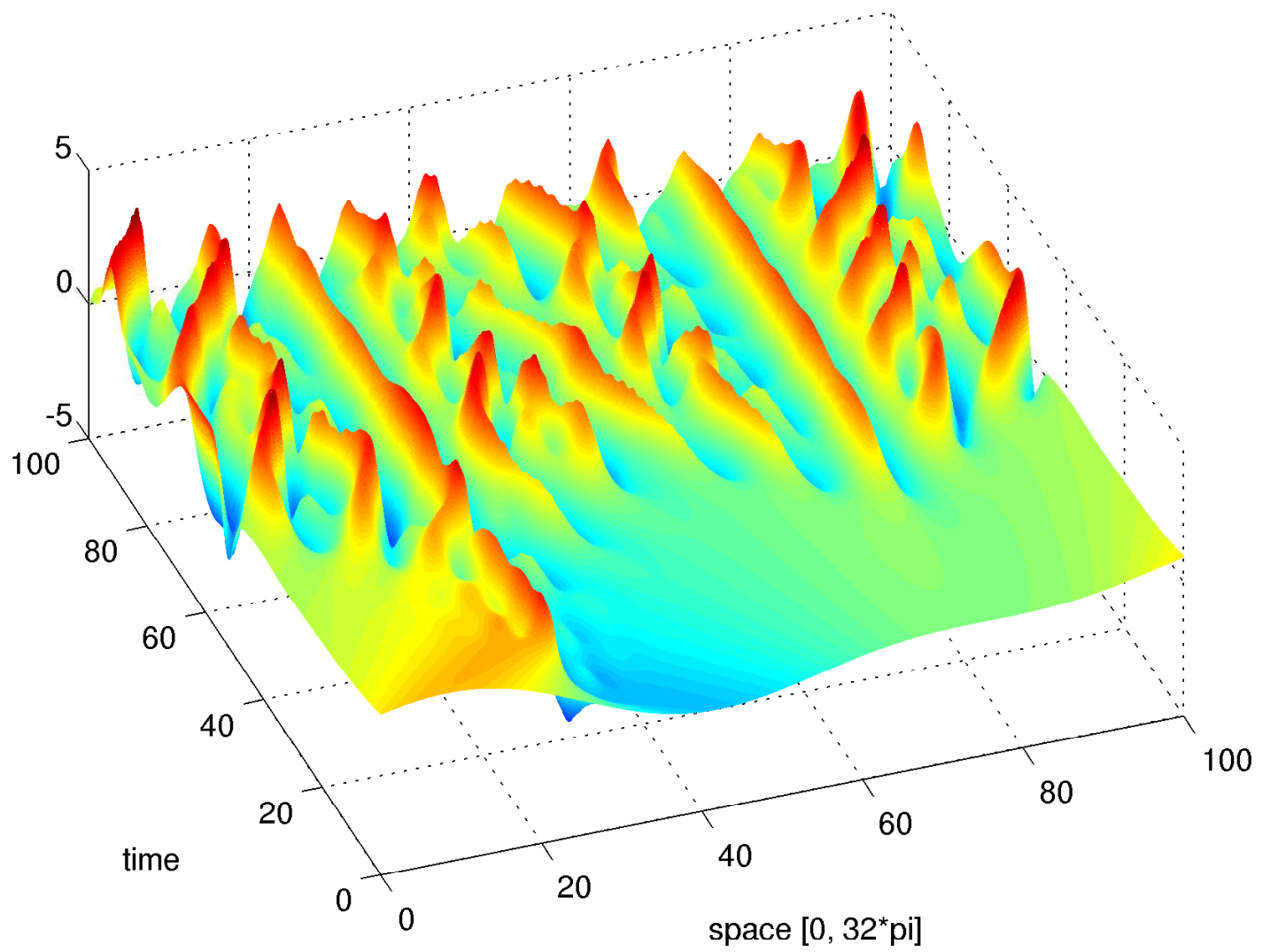
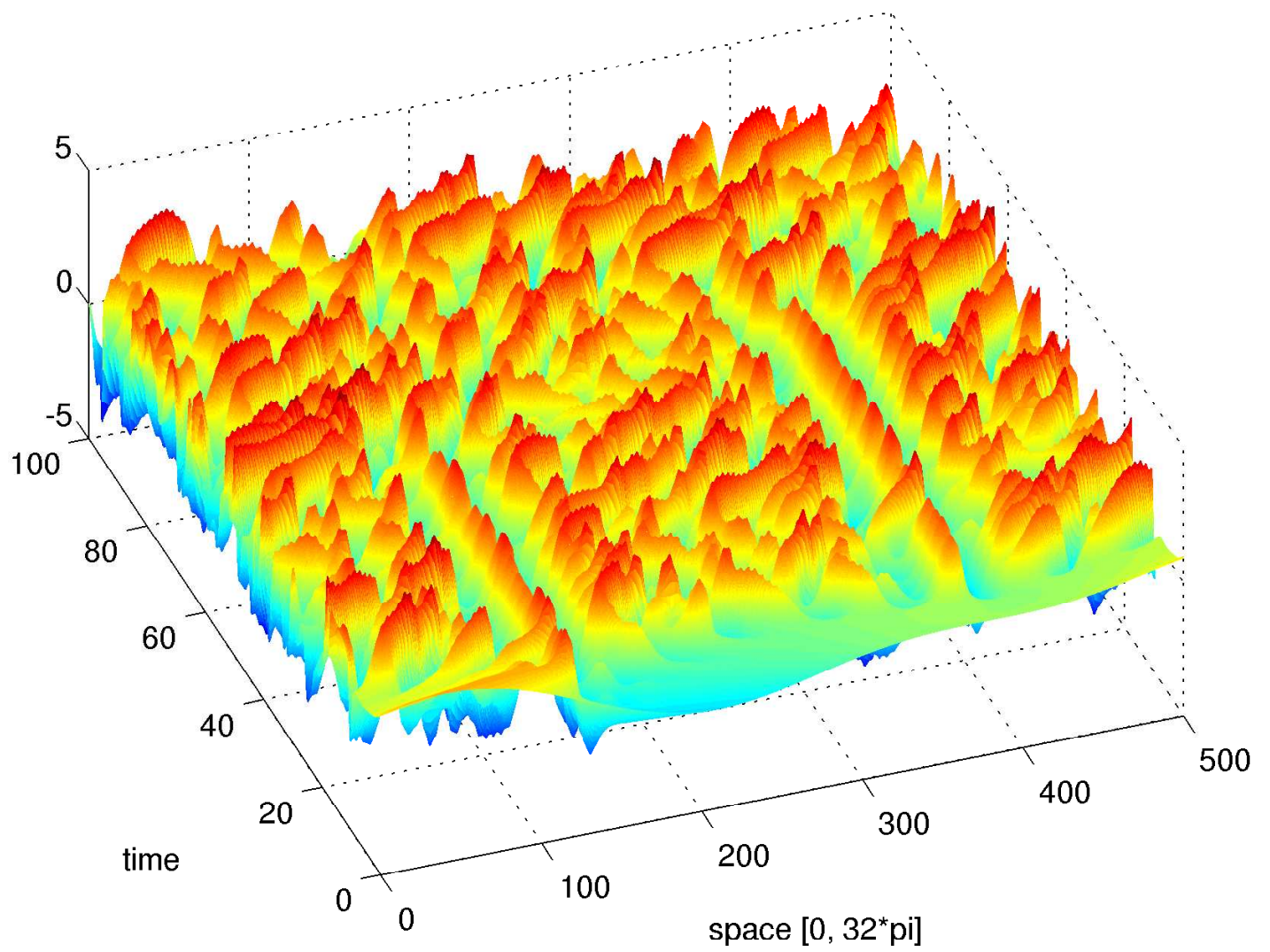


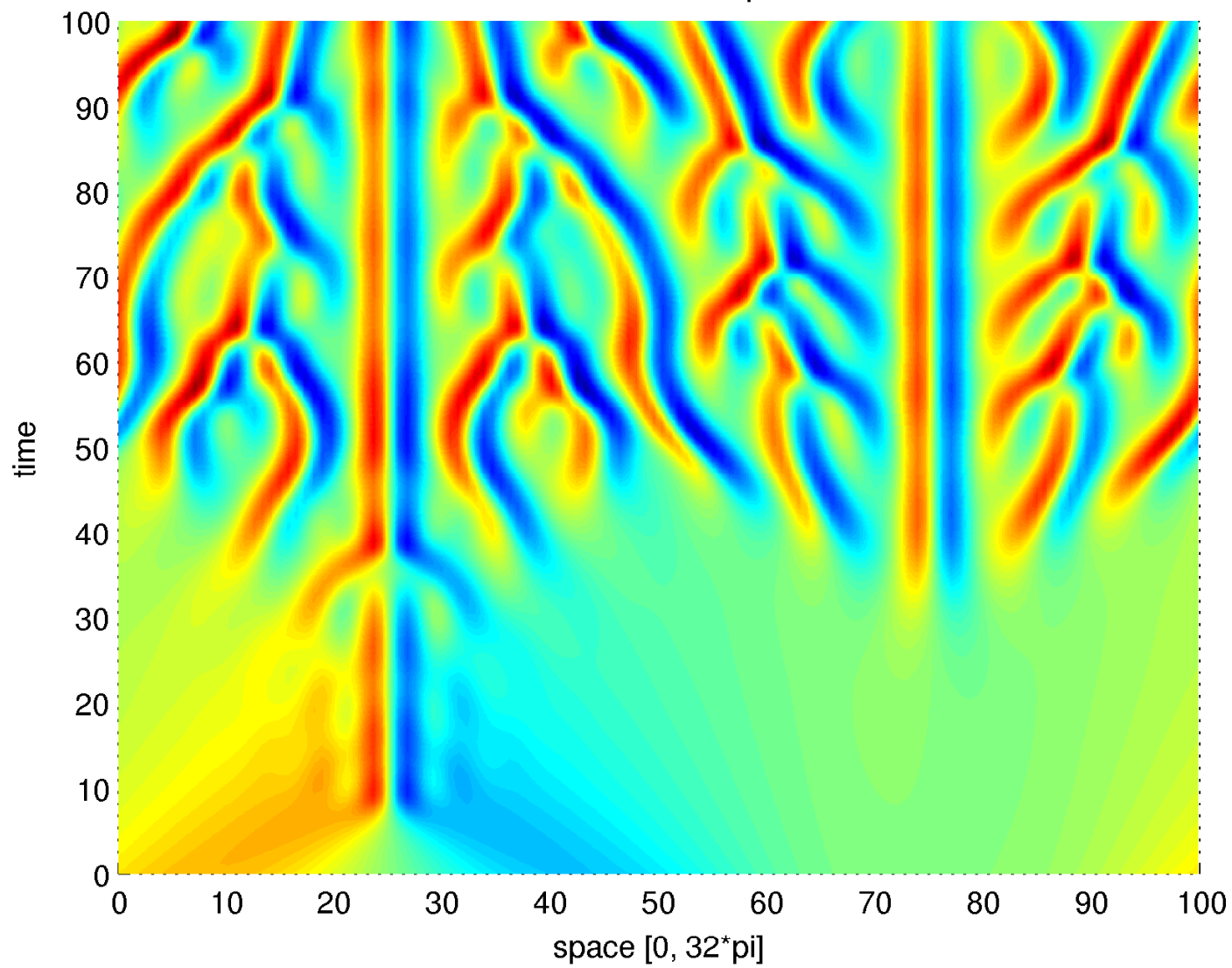
Numerical solution of the KS-eq: $M = 128$, $N = 2^{16}$



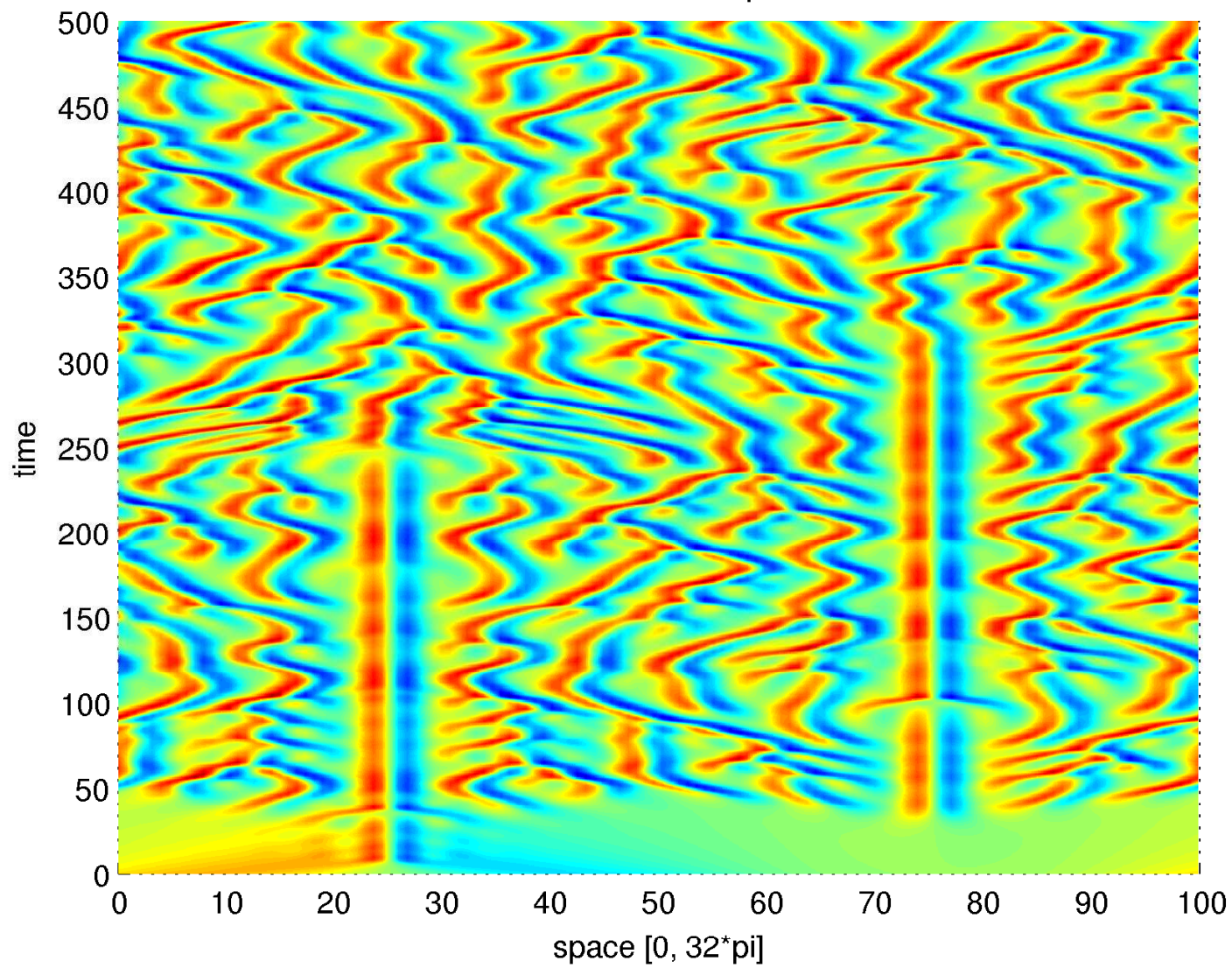
Numerical solution of the KS-eq: $M = 128$, $N = 2^{16}$



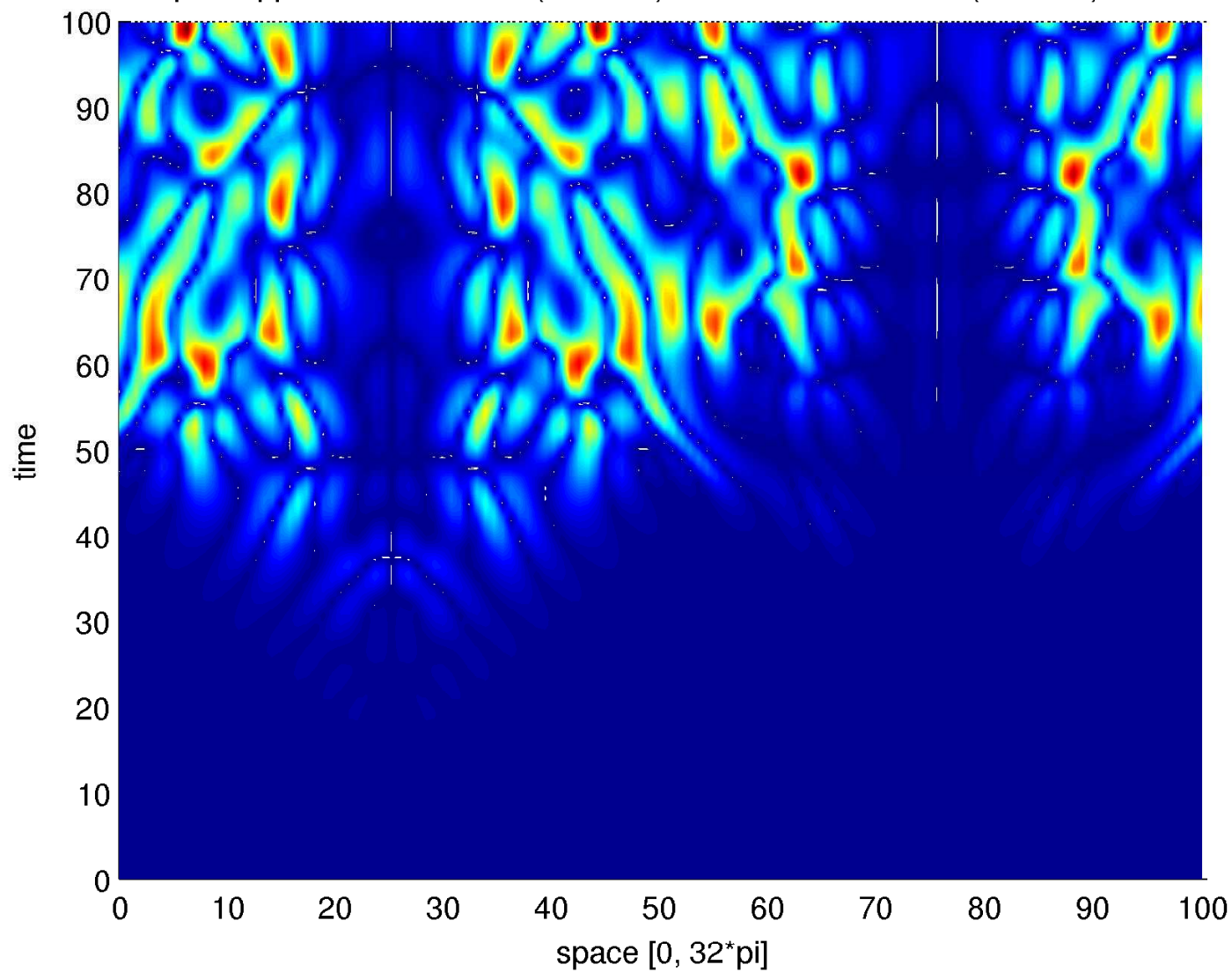
Numerical solution of the KS-eq: $M = 128$, $N = 2^{16}$



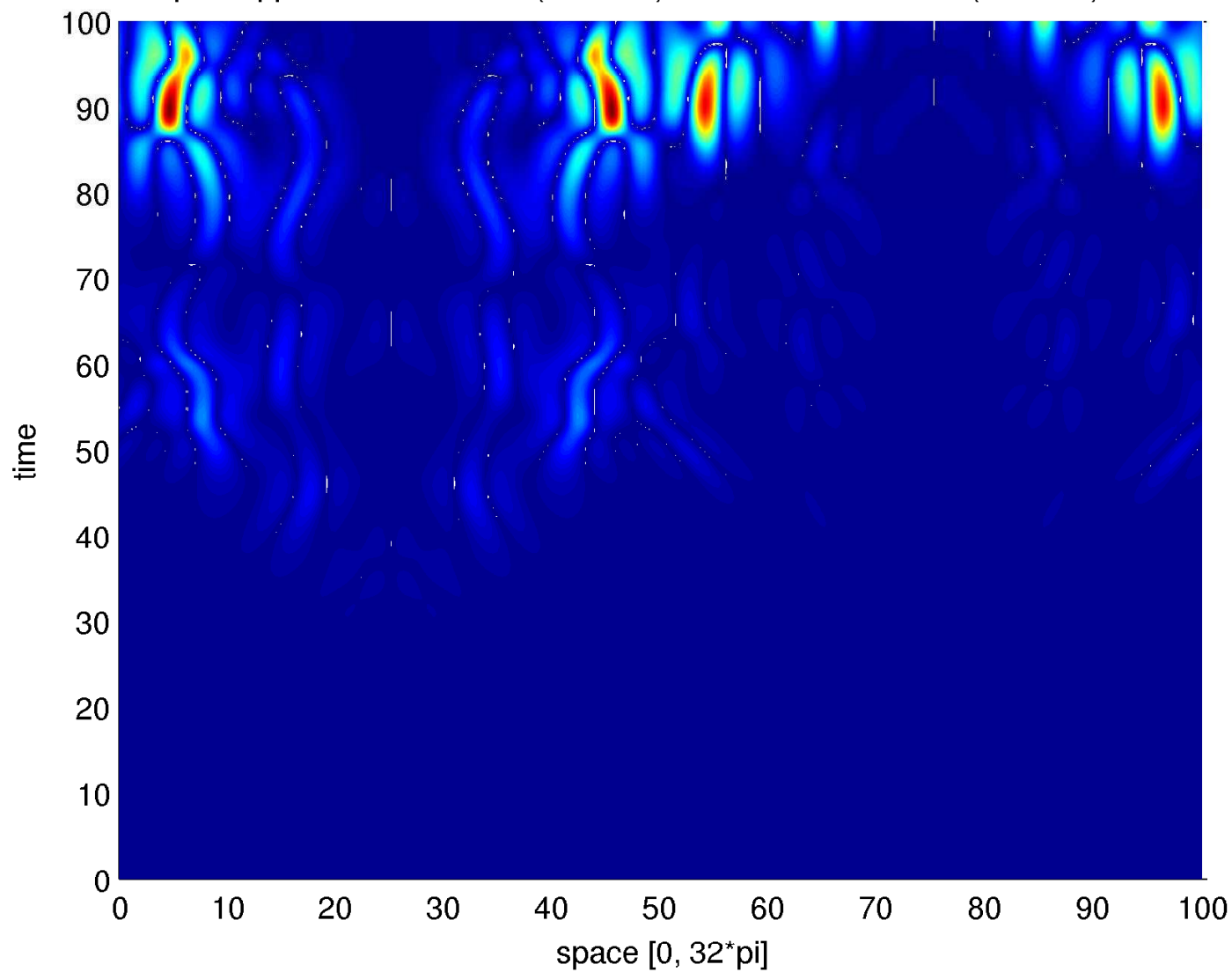
Numerical solution of the KS-eq: $M = 128$, $N = 2^{16}$



Error plot: approximate solution ($M = 128$) vs reference solution ($M = 512$). $N = 2^{18}$.



Error plot: approximate solution ($M = 256$) vs reference solution ($M = 512$). $N = 2^{18}$.



Difference Scheme

The Kuramoto-Sivashinsky equation

$$u_t + u_{xx} + u_{xxxx} + uu_x = 0$$

$$u_t + u_{xx} + u_{xxxx} + \frac{1}{2}(u^2)_x = 0$$

Difference scheme:

$$\begin{aligned} u_t &\approx \frac{\Delta u}{k} = \frac{u^{n+1} - u^n}{k} \\ u_{xx} &\approx \frac{\delta^2 u}{h^2} = \frac{u_{m+1} - 2u_m + u_{m-1}}{h^2} = \frac{1}{h^2} Au \\ u_{xxxx} &\approx \frac{\delta^4 u}{h^4} = \frac{u_{m+2} - 4u_{m+1} + 6u_m - 4u_{m-1} + u_{m-2}}{h^4} = \frac{1}{h^4} AAu \\ (u^2)_x &\approx \frac{\mu \delta u^2}{h} = \frac{(u_{m+1})^2 - (u_{m-1})^2}{2h} = \frac{1}{2h} D \end{aligned}$$

$$U^{n+1} = U^n - \frac{k}{h^2} AU^n - \frac{k}{h^4} AAU^n - \frac{k}{4h} D(U^n \odot U^n)$$

\odot = Element-wise multiplication

Consistency

Consistency

$$\begin{aligned}u_t &= \frac{\Delta u}{k} + O(k) \\u_{xx} &= \frac{\delta^2 u}{h^2} + O(h^2) \\u_{xxxx} &= \frac{\delta^4 u}{h^4} + O(h^2) \\\mu \delta[u(x)^2] &= \mu[u(x + \frac{h^2}{2}) - u(x - \frac{h}{2})^2] \\&= 2huu_x + O(h^3) \\uu_x &= \frac{\mu \delta u^2}{4h} + O(h^2)\end{aligned}$$

Local Truncation Error

$$\tau^n = O(h^2) + O(k) \xrightarrow{k, h \rightarrow 0} 0 \tag{1}$$

\Rightarrow Consistent