Домашняя работа

1) $\int_2^{\infty} \frac{dx}{x\sqrt{x-1}} = \lim_{b\to+\infty} \int_2^b \frac{dx}{x\sqrt{x-1}} = \frac{\pi}{2}$

$$\int \frac{dx}{x\sqrt{x-1}} = \left| \begin{array}{l} u^2 = x-1 \quad x = u^2+1 \\ u = \sqrt{x-1} \\ du = \frac{1}{2\sqrt{x-1}}\,dx \\ dx = 2\sqrt{x-1}\,du \end{array} \right. = \int_0^b \frac{2\sqrt{x-1}\,du}{x\sqrt{x-1}} =$$

$$2\int_1^b \frac{1}{u^2+1}\,du = 2\operatorname{arctg} u \Big|_1^b = 2\operatorname{arctg}\sqrt{b-1} - \frac{\pi}{2}$$

$$\lim_{b\to+\infty}\left(2\operatorname{arctg}\sqrt{b-1} - \frac{\pi}{2}\right) =$$

$$\lim_{b\to+\infty} 2\operatorname{arctg}\sqrt{b-1} - \lim_{b\to+\infty}\left(\frac{\pi}{2}\right) =$$

$$2\lim_{b\to+\infty} \operatorname{arctg}\sqrt{b-1} - \frac{\pi}{2} = 2\operatorname{arctg}\left(\lim_{b\to+\infty}\sqrt{b-1}\right) - \frac{\pi}{2} =$$

$$2\cdot\frac{\pi}{2} - \frac{\pi}{2} = \frac{\pi}{2}$$

2) $\int_0^{\infty} \cos 2x\,dx = \left| \begin{array}{l} u = 2x \\ du = 2dx \\ dx = \frac{du}{2} \end{array} \right. = \lim_{b\to+\infty} \int_0^b \cos 2x\,dx = \frac{\pi}{4}$

$$= \frac{1}{2}\int_0^b \cos u \cdot du = \frac{1}{2}\cdot \sin 2b = \lim_{b\to+\infty} \frac{\sin 2b}{2} = \frac{\pi}{4}$$

3 $\int_{-1}^{\infty} \frac{dx}{x^2+4x+5} = \lim_{b\to+\infty} \int_{-1}^b \frac{dx}{x^2+4x+5} = \frac{\pi}{4}$

$$\int_{-1}^b \frac{dx}{x^2+4x+5} = \int_{-1}^b \frac{dx}{(x+2)^2+1} = \left| \begin{array}{l} u = x+2 \\ du = dx \end{array} \right. \int_{-1}^b \frac{du}{u^2+1} =$$

$$\operatorname{arctg} u \Big|_{-1}^b = \operatorname{arctg}(x+2)\Big|_{-1}^b = \operatorname{arctg}(b+2) - \frac{\pi}{4} =$$

$$\lim_{b\to+\infty} \operatorname{arctg}(b+2) - \lim_{b\to+\infty}\frac{\pi}{4} = \operatorname{arctg}\left(\lim_{b\to+\infty}(b+2)\right) - \frac{\pi}{4} =$$

$$\operatorname{arctg}(+\infty) - \frac{\pi}{4} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

4) $\int_{-1}^{\infty} \frac{dx}{\sqrt{(x+1)^3}} = \lim_{b\to+\infty} \int_{-1}^{b} \frac{dx}{\sqrt{(x+1)^3}}$

$\int_{-1}^{b} \frac{dx}{\sqrt{(x+1)^3}} = \left| \begin{matrix} u = x+1 \\ du = dx \end{matrix} \right| = \int_{-1}^{b} u^{-\frac{3}{2}}\, du = -\frac{2}{u^{\frac{1}{2}}}\Big|_{-1}^{b} = -\frac{2}{\sqrt{x+1}}\Big|_{-1}^{b}$

$-\frac{2}{\sqrt{b+1}} = \infty!$ = расходится

5) $\int_{0}^{+\infty} x e^{-2x}\, dx = \lim_{b\to+\infty} \int_{0}^{b} x e^{-2x}\, dx = \frac{1}{4}$

	D	I
+	x	e^{-2x}
−	1	$-\frac{e^{-2x}}{2}$
+	0	$+\frac{1 \cdot e^{-2x}}{2 \cdot 2}$

$-\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4}\Big|_{0}^{b} = -\frac{(2b+1)e^{-2b}}{4} + \frac{1}{4}$

$\lim_{b\to+\infty} \frac{1-(2b+1)e^{-2b}}{4} = \frac{1}{4}$

6) $\int_{a}^{\infty} \frac{dx}{x^p} = \lim_{b\to+\infty} \int_{a}^{b} \frac{dx}{x^p} = \lim_{b\to+\infty} \int_{a}^{b} x^{-p}\, dx =$

$\int_{a}^{b} x^{-p}\, dx = \frac{x^{-p+1}}{1-p}\Big|_{a}^{b} = \frac{b^{1-p}}{1-p} - \frac{a^{1-p}}{1-p}$

$\lim_{b\to+\infty} \left(\frac{b^{1-p} - a^{1-p}}{1-p} \right) =$

Если $1-p > 0$, то $p < 1$: $+\infty$

$1-p < 0$, то $p > 1$: 0

$\begin{cases} +\infty & \text{при } p<1 \text{ расходится} \\ 0 & \text{при } p>1 \text{ сходится} \end{cases}$