$\varphi'_y = \frac{1}{b}$

$L''_{xx} = 2$

$L''_{yy} = 2$

$L''_{xy} = 0$

$\acute{z}_{min} =$ в. м, при $a, b \neq 0$

Домашняя работа

№ 7.6

$f(x) = \frac{3-z^2}{(1+z)(2+z)}$ → в $z=0$ $= -1 + \frac{3z+5}{(1+z)(2+z)} = -1 + 2 - 2z + 2z^2 + \frac{1}{2} - \frac{z}{4} + \frac{z^2}{8}$

$\frac{3}{2} - 2\frac{1}{4}z + 2\frac{1}{8}z^2$

$-z^2 + 3 \,|\, z^2+3z+2$

$-z^2 - 3z - 2 \,|\, -1$

$\frac{3z+5}{(1+z)(2+z)} = \frac{A}{(1+z)} + \frac{B}{(2+z)} = \frac{2}{1+z} + \frac{1}{2+z}$

$3z+5 = A(2+z) + B(1+z)$

$\begin{cases} 3 = A + B \\ 5 = 2A + B \end{cases}$ $\quad A=2 \quad B=1$

- $2\sum_{n=0}^{\infty}(-z)^n = 2(1 + (-z) + z^2)$
- $\frac{1}{2}\sum_{n=0}^{\infty}\left(-\frac{z}{2}\right)^n = \frac{1}{2}\left(1 + \left(-\frac{z}{2}\right) + \frac{z^2}{4}\right)$ $\quad |z| < 1$

№ 7.8

$f(x) = e^z(1+z)^{10}$ → в $z=0$ $= 1 + 11x + \frac{111}{2}x^2 + \frac{1021}{6}x^3$ $\quad z=0$

$f(x) = e^z = \sum_{n=0}^{\infty} \frac{1}{n!} z^n = 1 + z + \frac{z^2}{2} + \frac{z^3}{6}$

$f(x) = (1+z)^{10} = \sum_{n=0}^{\infty} \frac{\mu(\mu-1)\ldots(\mu-n+1)}{n!} z^n = 1 + 10z + 45z^2 + 120x^3$

№ 7.7

$f(x) = z^{-2}$ $\quad z_0 = -1$

$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2$

$f(x) = x^{-2} = \frac{1}{x^2}$

$f(a) = 1$

$f'(x) = -2\frac{1}{x^3}$

$f'(a) = 2$

$f''(x) = +2 \cdot 3 \frac{1}{x^4}$

$f''(a) = 6$

$f(x) = 1 + 2(x+1) + 3(x+1)^2$ $\qquad -1 < z < 0$

№ 1383

$\sqrt[3]{\sin x^3}$ до члена x^{13}

$\sin(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{9!}x^9$

$\sqrt[3]{\sin x^3} = x - \frac{x^7}{18} - \frac{x^{13}}{3240} \ldots$

№ 1388

$f(x) = \sqrt{x}$ при $x = 1$ $\qquad$ т. 1

$f(x) = \sqrt{x}$

$f(a) = 1$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(a) = \frac{1}{2}$

$f''(x) = \cancel{-\frac{1}{4}\frac{1}{x^{\frac{3}{2}}}} = \left(\frac{1}{2} \cdot x^{-\frac{1}{2}}\right)' = \frac{1}{2} \cdot -\frac{1}{2} \cdot$

$f''(a) = -\frac{1}{8}$

$f(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2$

№ 1396 (g, e)

g) $\sin 18^\circ \approx \sin 30^\circ + (\sin' x) \cdot -12^\circ$
$\qquad\qquad\qquad\qquad\qquad\qquad 30$

$-12^\circ = \quad = \frac{12\pi}{180} = \frac{\pi}{15}$

$\pi = 180 \qquad \frac{\pi}{\pi} = \frac{180}{\pi}$

$\frac{1}{2} + (-\cos 30^\circ) \cdot \frac{\Pi}{15} = \frac{1}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\Pi}{15} \approx 0{,}318.$

$\sin(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5$

$\sin(18^\circ) = \frac{\Pi}{10} - \frac{1}{6}\cdot\left(\frac{\Pi}{10}\right)^3 + \frac{1}{120}\cdot\left(\frac{\Pi}{10}\right)^5 \approx 0{,}297$

$18^\circ = \frac{\Pi}{10}$

e) $\ln 1{,}2 =$

$1+0{,}2$

$\ln(1+y) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

$\ln(1{,}2) = 0{,}2 - \frac{0{,}2^2}{2} + \frac{0{,}2^3}{3} - \frac{0{,}2^4}{4} + \frac{0{,}2^5}{5} \ldots$

$\approx 0{,}182$

№ 3628

$z = xy + \frac{50}{x} + \frac{20}{y}$

$\begin{cases} z'_x = y - \frac{50}{x^2} = 0 \\ z'_y = x - \frac{20}{y^2} = 0 \end{cases}$ $\begin{cases} x = 5 \\ y = 2 \end{cases}$ $M(5;2)$

$A = z''_{xx} = +50 \cdot 2\frac{1}{x^3} = 100/x^3$ $(x^{-2})' = -2x^{-3}$

$B = z''_{xy} = 1$

$C = z''_{yy} = +20 \cdot 2\frac{1}{y^3} = \frac{40}{y^3}$

$D = \frac{100 \cdot 40}{x^3 y^3} - 1 = 4 - 1 = 3 > 0$ экстремум существует

$A = \frac{100}{125} > 0$ => экстрем. (min)

Ответ: $z_{min}(M) = 30$ в $M(5;2)$

№ 3631

$z = 1 - \sqrt{x^2+y^2}$

$\begin{cases} z'_x = -\frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = 0 \\ z'_y = -\frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = 0 \end{cases}$ $\begin{cases} x = 0 \\ y = 0 \end{cases}$ $M(0;0)$

$z = 1$

$A = z''_{xx} = \left(-\frac{x}{\sqrt{x^2+y^2}}\right)' = -\frac{\sqrt{x^2+y^2} - x \cdot \frac{1 \cdot 2x}{2\sqrt{x^2+y^2}}}{x^2+y^2}.$

$B = z''_{xy} = +x \cdot \frac{1}{2(x^2+y^2)^{+\frac{3}{2}}} \cdot 2y$

~~3657.1~~

$$C = -\frac{y}{\sqrt{x^2+y^2}} = \frac{-\sqrt{x^2+y^2} - \frac{x^2}{\sqrt{x^2+y^2}}}{x^2+y^2}$$

~~$(D = 0 - 0 = 0)$~~

Ответ: $z(max) = 1$ в т. $M(0;0)$

№ 3657.1

$z = Ax^2 + 2Bxy + Cy^2$ $\qquad x^2+y^2=1$

Уравнение Лагранжа $\qquad x^2+y^2-1=0$

$L = f(x,y) + \lambda\varphi(x,y)$

$L = Ax^2 + 2Bxy + Cy^2 + \lambda(x^2+y^2-1)$

$L'_x = 2Ax + 2By + 2\lambda x$

$L'_y = 2Bx + 2Cy + 2\lambda y$

$$\begin{cases} Ax + By + \lambda x = 0 \\ Bx + Cy + \lambda y = 0 \\ x^2+y^2=1 \end{cases}$$

$z = x^2 + 12xy + 2y^2 \qquad 4x^2+y^2=25$

Уравнение Лагранжа

$L = f(x,y) + \lambda\varphi(x,y)$

$L = x^2 + 12xy + 2y^2 + \lambda(4x^2+y^2-25)$

$L'_x = 2x + 12y + 8\lambda x = 0$

$L'_y = 12x + 4y + 2\lambda y = 0$

$$\begin{cases} x + 6y + 4\lambda x = 0 \\ 6x + 2y + \lambda y = 0 \\ 4x^2+y^2=25 \end{cases}$$

$x = \pm 2; y = \pm 3$ при $z_{min} = -50$

$x = \pm\frac{3}{2}$

$y = \pm 4$ при $z_{max} = 106$

$\varphi'_x = 8x \qquad L''_{xx} = 2 + 8\cdot -\frac{5}{2} = -18$

$\varphi'_y = 2y \qquad L''_{yy} = 4 + 2\cdot -\frac{5}{2} = -1$

$L''_{xy} = 12$

$$A = \begin{vmatrix} 0 & 16 & 6 \\ 16 & -18 & 12 \\ 6 & 12 & -1 \end{vmatrix} < 0 \Rightarrow \cancel{A}\ z_{min} = -50\ (\pm 2; \pm 3)$$

$$z = x^2 + 6x + y^2 + 2y + 9 \qquad -4 \le x \le -2$$
$$-2 \le y \le 0$$

$$\cancel{h x} = x^2 + 6x + y^2 + 2y + 9 + \lambda (\quad)$$

$$\begin{cases} z'_x = 2x + 6 \\ z'_y = 2y + 2 \end{cases} \qquad \begin{cases} x = -3 \\ y = -1 \end{cases}$$

$A = z''_{xx} = 2$

$B = z''_{xy} = 0$

$C = z''_{yy} = 2$

$D = 4 - 0 > 0$

Ответ: $z_{min} = -1$ при $-4 \le x \le -2$ $\quad -2 \le y \le 0$

$z_{max} = 1$