Домашняя работа

№6 (3)

$\cos(xyz) + \operatorname{arctg}\left(\frac{y}{x}+1\right)(y^2+z^2)^{-1}$

$A(1;-1;0)$

$B(2;1;2)$

$\bar{l} = \overline{AB}\ (1;2;2)$

$$\frac{\partial f}{\partial l} = \frac{\partial f}{\partial x}\cdot\cos\alpha + \frac{\partial f}{\partial y}\cdot\cos\beta + \frac{\partial f}{\partial z}\cdot\cos\gamma =$$

$$\left(\frac{\partial f}{\partial x}\ \frac{\partial f}{\partial y}\ \frac{\partial f}{\partial z}\right)\begin{pmatrix}\cos\alpha\\ \cos\beta\\ \cos\gamma\end{pmatrix}$$

$$\left.\frac{\partial f}{\partial x}\right|_A = \underbrace{-\sin(xyz)\cdot yz}_{0} + \frac{1}{1+\underbrace{\left(\frac{\left(\frac{y}{x}+1\right)}{y^2+z^2}\right)^2}_{0}}\cdot -\frac{y}{(y^2+z^2)\,x^2} =$$

1

$$\left.\frac{\partial f}{\partial y}\right|_A = \underbrace{-\sin(xyz)\cdot xz}_{0} + \frac{1}{1+\underbrace{\left(\frac{\left(\frac{y}{x}+1\right)}{y^2+z^2}\right)^2}_{0}}\cdot\frac{\frac{1}{x}(y^2+z^2)-\left(\frac{y}{x}+1\right)\cdot 2y}{(y^2+z^2)^2} =$$

1

$$\left.\frac{\partial f}{\partial z}\right|_A = \underbrace{-\sin(xyz)\,xy}_{0} + \frac{1}{1+\underbrace{\left(\frac{\left(\frac{y}{x}+1\right)}{y^2+z^2}\right)^2}_{0}}\cdot -\frac{\left(\frac{y}{x}+1\right)\cdot 2z}{(y^2+z^2)^2} =$$

1

$$\frac{\partial f}{\partial l} = (1\ 1\ 1)\begin{pmatrix}1/3\\ -1/3\\ 0\end{pmatrix}$$

$|\overline{AB}| = \sqrt{1+4+4} = 3$

$\cos\alpha = \frac{1}{3}\quad \cos\beta = -\frac{1}{3}\quad \cos\gamma = 0$

$\frac{\partial f}{\partial l} = 0$

$|\operatorname{grad} f(A)| = \sqrt{1+1+1} = \sqrt{3}$

№ 3344

$z = 1 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)$

$M\left(\frac{a}{\sqrt{2}}; \frac{b}{\sqrt{2}}\right)$

$z = 1 - \left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)$

$\frac{\partial z}{\partial x}\Big|_M = (z_x' \cdot \cos\alpha + z_y' \cos\beta) =$

$\frac{2x}{a^2} \cdot \cos\alpha + \frac{2y}{b^2} \cdot \cos\beta$

$\text{grad } dz = \left(\frac{2x}{a^2}, \frac{2y}{b^2}\right)$

$l = \text{grad}\, dz(M) = \left(\frac{\sqrt{2}}{a}; \frac{\sqrt{2}}{b}\right)$

$|l| = |\text{grad } dz(M)| = \sqrt{\frac{2}{a^2} + \frac{2}{b^2}} = \frac{\sqrt{2}\sqrt{a^2+b^2}}{ab}$

$l_0 = \frac{l}{|l|} = \left(-\frac{b \text{ sign } a}{\sqrt{a^2+b^2}}; -\frac{a \text{ sign } b}{\sqrt{a^2+b^2}}\right)$

$= \left(\frac{\sqrt{2}\, b \text{ sign } a}{a} + \frac{\sqrt{2}\, a \text{ sign } b}{b}\right) \frac{1}{\sqrt{a^2+b^2}} = \frac{\sqrt{2(a^2+b^2)}}{ab}$

№ 3359

$z = z(x,y)$ $\quad \frac{d^2 z}{dy^2} = 2$ $\quad z(x,0) = 1$

$z_y'(x,0) = x$

$z' = \int 2\, dy = 2y + C$

1 $z = \int 2y\, dy = y^2 + C$ $\quad C = 1$ $\quad \Rightarrow z = 1 + xy + y^2$

2 $z_y' = 2y$ $\quad x = 0$

№ 3386

$z = \sqrt{x^2 - y^2}\; \text{tg}\, \frac{z}{\sqrt{x^2 - y^2}}$

$\sqrt{x^2 - y^2}\; \text{tg}\, \frac{z}{\sqrt{x^2 + y^2}} - z = 0$

1. z_x'

$z_x' = -\frac{F_x'}{F_z'}$ $\quad z_y' =$

$F - F_x' = \frac{\sqrt{x^2-y^2}}{\cos^2 \frac{z}{\sqrt{x^2-y^2}}} \cdot \frac{}{2\sqrt{x^2-y^2}} \cdot 2x + \frac{1}{\sqrt{x^2-y^2}} \cdot x \cdot \text{tg}\, \frac{z}{\sqrt{x^2-y^2}} - 0 =$

2) $F_z' = \dfrac{\cancel{\sqrt{x^2-y^2}}'}{\cos^2\frac{z}{\sqrt{x^2-y^2}}} \cdot \dfrac{1}{\cancel{\sqrt{x^2-y^2}}} - 1 = \dfrac{1}{\cos^2\frac{z}{\sqrt{x^2-y^2}}}$

$z_x' = \dfrac{1)}{2)}$

$z_y' = \dfrac{1)}{2)}$

$\dfrac{\partial z}{\partial x} = \dfrac{xz}{x^2-y^2}, \quad \dfrac{\partial z}{\partial y} = -\dfrac{yz}{x^2-y^2}$

$\dfrac{\partial z}{\partial x^2} = -\dfrac{y^2 z}{(x^2-y^2)^2}$

$\dfrac{\partial^2 z}{\partial x \partial y} = \dfrac{xyz}{(x^2-y^2)^2}$

№ 344

$z = x^2+y^2$, где $y(x) = x^2 - xy + y^2 = 1$

$y = \dfrac{x \pm \sqrt{4-3x^2}}{2}$

$\dfrac{dz}{dx} \quad \dfrac{d^2z}{dx^2}$ - ?

$z = f(u, v)$, где $u = \varphi(x)$; $v = \psi(x)$

$\dfrac{dz}{dx} = \dfrac{\partial z}{\partial u} \cdot \dfrac{du}{dx} + \dfrac{\partial z}{\partial v} \cdot \dfrac{dv}{dx} \qquad u = x;\ v = y$

$\dfrac{dz}{dx} = \dfrac{\partial z}{\partial x} \cdot \dfrac{dx}{dx} + \dfrac{\partial z}{\partial y} \cdot \dfrac{dy}{dx} = \dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} \cdot \dfrac{dy}{dx}$

$\dfrac{\partial z}{\partial x} = 2x \qquad \dfrac{\partial z}{\partial y} = 2y$

$\dfrac{dz}{dx} = 2x - 2y \cdot \dfrac{2x-y}{2y-x} = \dfrac{2x(2y-x) - \cancel{4x}y + 2y^2}{2y-x} = \dfrac{2(-x^2+y^2)}{2y-x}$

$x^2 - xy + y^2 = 1$

$x^2 - xy + y^2 - 1 = 0$

$f(x,y) = x^2 - xy + y^2 - 1$

$\dfrac{dy}{dx} = -\dfrac{F_x}{F_y}$

$F_x = 2x - y$

$F_y = 2y - x$

$\dfrac{dy}{dx} = -\dfrac{2x-y}{2y-x}$

$\dfrac{d^2z}{dx^2} = \dfrac{4x+2y}{x-2y} + \dfrac{6x}{(x-2y)^3}$

№ 345

$x = e^u + u\sin v \quad , \quad y = e^u - u\cos v.$ $\qquad \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$

$$\frac{\partial u}{\partial x} = \frac{\sin v}{e^u(\sin v - \cos v) + 1};$$

$$\frac{\partial u}{\partial y} = \frac{-\cos v}{e^u(\sin v - \cos v) + 1}$$

$$\frac{\partial v}{\partial x} = \frac{-(e^u - \cos v)}{u(e^u(\sin v - \cos v) + 1)}$$

$$\frac{\partial v}{\partial y} = \frac{e^u + \sin v}{u(e^u(\sin v - \cos v) + 1)}$$

№ 3454

$$x^2 y'' + x y' + y = 0$$

$$F(x_{tt}'', x_t', t)$$

$x = e^t$

$t = \ln(x)$

$t_x' = \frac{1}{x}$

$$t_{xx}'' = -\frac{1}{x^2}$$

$$x_t' = \frac{1}{t_x'}$$

$$x_t' = x$$

$$-\frac{x_{tt}''}{(x_t')^3} = -\frac{1}{x^2}$$

$$+\frac{x_{tt}''}{x^3} = \frac{1}{x^2}$$

$$x_{tt}'' = x$$

$$y_{tt}'' + y = 0$$

№ 3459

$$y\frac{\partial z}{\partial x} - x\frac{\partial z}{\partial y} = 0$$

$\xi = x \qquad \eta = x^2 + y^2$

$$z = \varphi(x^2 + y^2)$$

№ 3528

$x = a\sin^2 t \quad , \quad y = b\sin t\cos t \quad , \quad z = c\cos^2 t \qquad t = \frac{\pi}{4}$

$$\frac{x - x(t)}{x_t'|_{t=\frac{\pi}{4}}} = \frac{y - y(t)}{y_t'|_{t=\frac{\pi}{4}}} = \frac{z - z(t)}{z_t'|_{t=\frac{\pi}{4}}}$$

$x'_t = 2a\sin(t)\cos(t)$

$y'_t = b(\cos^2(t) - \sin^2(t))$

$z'_t = -2c\cos(t)\sin(t)$

$\sin(t) = \frac{\sqrt{2}}{2}$

$\cos(t) = \frac{\sqrt{2}}{2}$

$\sin^2(t) = \frac{2}{4} = \frac{1}{2}$

$x'_t = \cancel{2} \cdot a \cdot \frac{\sqrt{2}}{2} \cdot \frac{\cancel{\sqrt{2}}}{\cancel{2}}$

$y'_t = b\left(\frac{1}{2} - \frac{1}{2}\right) = 0$

$z'_t = -\cancel{2}c \cdot \frac{1}{\cancel{2}}$

$\frac{x}{a} = -\frac{z}{c}$

$\frac{x}{a} + \frac{z}{c} = 1, \quad y = \frac{b}{2}; \quad ax - cz = \frac{a^2 - c^2}{2}$