~~Домашняя работа~~

$$\int_1^{\infty} \frac{1+x^2}{x^3}dx = \lim_{b\to+\infty} \int_1^b \frac{1+x^2}{x^3}dx =$$

$$\int_1^b \left(\frac{1}{x^3}+\frac{1}{x}\right)dx = \int_1^b \frac{1}{x^3}dx + \int_1^b \frac{1}{x}dx = \ln(x) - \frac{1}{2x^2}\Big|_1^b =$$

$$\ln(b) - \frac{1}{2b^2} + \frac{1}{2}$$

$$\ominus \lim_{b\to+\infty}\left(\ln(b) - \frac{1}{2b^2} + \frac{1}{2}\right) = \infty$$

$$\int_0^{\infty} e^{-px}dx = \lim_{b\to\infty+} \int_0^b e^{-px}dx = \lim_{b\to\infty}\left(-\frac{1}{p}e^{-px}\right)\Big|_0^b$$

$$\lim_{b\to+\infty}\left(-\frac{1}{p}\cdot e^{-bp} + \frac{1}{p}\right) = \frac{1}{p}$$

$$\int_1^{+\infty} \frac{\operatorname{arctg} 3x}{1+9x^2}dx = \lim_{b\to\infty}\left(\int_1^b \frac{\operatorname{arctg} 3x}{1+9x^2}dx\right)$$

$$\int_1^b \frac{\operatorname{arctg} 3x}{1+9x^2}dx = \left| \begin{array}{l} u = \operatorname{arctg}(3x) \\ du = \frac{3}{1+9x^2}dx \\ dx = \frac{9x^2+1}{3}du \end{array}\right.$$

$$\frac{1}{3}\int_1^b u\,du = \frac{1}{3}\cdot\frac{\operatorname{arctg}^2(3x)}{2}\Big|_1^b = \frac{\operatorname{arctg}^2 3b}{6} - \frac{\operatorname{arctg}^2(3)}{6}$$

$$\ominus \lim_{b\to+\infty}\left(\frac{\operatorname{arctg}^2(3b)}{6} - \frac{\operatorname{arctg}^2(3)}{6}\right) = \frac{\left(\frac{\pi}{2}\right)^2}{6} - \frac{\operatorname{arctg}^2(3)}{6} =$$

$$\frac{\pi}{24} - \frac{\operatorname{arctg}^2(3)}{6}$$

$$\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)(x^2+4)} = \int_{-\infty}^{0} \frac{dx}{(x^2+1)(x^2+4)} + \int_0^{+\infty} \frac{dx}{(x^2+1)(x^2+4)} = \frac{\pi}{6}$$

1) $\lim_{a\to-\infty} \int_a^0 \frac{dx}{(x^2+1)(x^2+4)}$; $\lim_{a\to\infty} \int_0^a \frac{dx}{(x^2+1)(x^2+4)}$

$$\int_a^0 \frac{dx}{(x^2+1)(x^2+4)} = \int_a^0 \left(\frac{1}{3(x^2+1)} - \frac{1}{3(x^2+4)}\right)dx = \frac{\operatorname{arctg}(x)}{3} - \frac{\operatorname{arctg}\left(\frac{x}{2}\right)}{6}\Big|_a^0$$

$-\frac{\operatorname{arctg}(a)}{3} + \frac{\operatorname{arctg}\left(\frac{a}{2}\right)}{6}$, для $\int_0^a \frac{dx}{(x^2+1)(x^2+4)} = \frac{\operatorname{arctg}(a)}{3} - \frac{\operatorname{arctg}\left(\frac{a}{2}\right)}{6}$

$$\lim_{a\to-\infty}\left(-\frac{\operatorname{arctg}(a)}{3} + \frac{\operatorname{arctg}\left(\frac{a}{2}\right)}{6}\right) + \lim_{a\to\infty}\left(\frac{\operatorname{arctg}(a)}{3} - \frac{\operatorname{arctg}\left(\frac{a}{2}\right)}{6}\right) =$$

$$\frac{\pi}{12} + \frac{\pi}{12} = \frac{\pi}{6}$$

Демидович № 3906

$$\int_0^1 dx \int_0^1 (x+y)\,dy = 1$$

1) $\int_0^1 (x+y)\,dy = xy + \frac{y^2}{2}\Big|_0^1 = x + \frac{1}{2}$

2) $\int_0^1 \left(x+\frac{1}{2}\right)dx = \frac{x^2}{2} + \frac{x}{2}\Big|_0^1 = \underline{1}$

№ 3907

$$\int_0^1 dx \int_{x^2}^{x} xy^2\,dy = \frac{1}{40}$$

1) $\int_{x^2}^{x} xy^2\,dy = x\frac{y^3}{3}\Big|_{x^2}^{x} = \frac{x^4}{3} - \frac{x^7}{3}$

2) $\int_0^1 \left(\frac{x^4}{3} - \frac{x^7}{3}\right)dx = \frac{x^5}{15} - \frac{x^8}{24}\Big|_0^1 = \frac{1}{15} - \frac{1}{24} = \frac{1}{40}$

3908

$$\int_0^{2\pi} d\varphi \int_0^a r^2 \sin^2\varphi\,dr =$$

1) $\int_0^a r^2 \sin^2\varphi\,dr = \frac{\sin^2\varphi \cdot r^3}{3}\Big|_0^a = \frac{\sin^2\varphi \cdot a^3}{3}$

2) $\int_0^{2\pi} \frac{a^3}{3}\cdot \sin^2\varphi\,d\varphi = \frac{a^3}{3}\left(\frac{1}{2}x - \frac{\sin 2x}{4}\right)\Big|_0^{2\pi} =$

$\frac{\pi a^3}{3}$