$\int du = \quad -\sqrt{1-4x^2}$

$\int v = \int dx = x$

$x \arccos 2x + 2\int x \cdot \frac{1}{\sqrt{1-4x^2}} dx = x \arccos 2x + 2(\frac{1}{4} \cdot 2 \quad \sqrt{\lambda})$

$\frac{1}{2}$

Домашняя работа

№ 7.9

$f(z) = (z + \cos(2))^{-1} \qquad z = 0$

$f(z) = f(a) + \frac{f'a}{1!}(z-a) + \frac{f''(a)}{2!}(z-a)^2$

$f(0) = (0 + \cos 2\cdot 0)^{-1} = \underline{1}$

$f'(0) = -\frac{1}{(z+\cos(2z))^2} \cdot (1 + - \sin(2z)\cdot 2) = \frac{2\sin(2z) - 1}{(z + \cos(2z))^2} =$

$\frac{-1}{1} = -\underline{1}$

$f''(0) = \frac{2\cdot 2\cdot \cos(2z)(z+\cos(2z))^2 - (2\sin(2z)-1)(-4\sin(2z)+2)}{(z+\cos(2z))^4} =$

6.

$= 1 + (-1)x - 3x^2 + (O x^3)$

№ 7.4.

$f(z) = \frac{e^z}{1-z} = 1 + 2x + \frac{5}{2}x^2 + \frac{16}{6}x^3 + \ldots$

$f(z) = e^z = \sum_{n=0}^{\infty} \frac{1}{n!} z^n = 1 + z + \frac{z^2}{2} + \ldots$

$f(z) = \frac{1}{1-z} = \sum_{n=0}^{\infty} z^n = 1 + z + z^2 + \ldots$

$$f(z) = \frac{e^z}{1-z} = \sum_{n=0}^{\infty} \frac{1}{n!} \cdot z^n \cdot z^n = \sum_{n=0}^{\infty} \frac{z^{2n}}{n!}$$

№ 3586.

Разложить по формуле Маклорена

$$f(x,y) = \sqrt{1-x^2-y^2} = 1 + (0+0) + \frac{1}{2}\left(-1(x^2) + 0 + -1(y^2)\right) + \ldots = 1 - \frac{1}{2}(x^2+y^2) + \frac{1}{8}(x^2+y^2)^2$$

$f(a,b) = \sqrt{1} = 1$

$f'_x = \frac{1}{2\sqrt{1-x^2-y^2}} \cdot (-2x) =$

$f'_y = \frac{1}{2\sqrt{1-x^2-y^2}} \cdot (-2y)$

$f''_{xx} = \left(-\frac{x}{\sqrt{1-x^2-y^2}}\right)' = \frac{-\sqrt{1-x^2-y^2} + \frac{2 \cdot x^2}{2\sqrt{1-x^2-y^2}}}{1-x^2-y^2}$

$f''_{yy} = \frac{-\sqrt{1-x^2-y^2} + \frac{y^2}{\sqrt{1-x^2-y^2}}}{1-x^2-y^2}$

$f''_{xy} = + \frac{x \cdot 2y}{2\sqrt{1-x^2-y^2}} = \frac{xy}{\sqrt{1-x^2-y^2}}$

$f'''_{xxx} = \frac{(1-x^2-y^2) \cdot * - *}{(1-x^2-y^2)^2}$

$f'''_{yyy} = *$

$f'''_{xyx} = \frac{y\sqrt{1-x^2-y^2} - \frac{1}{2\sqrt{1-x^2-y^2}} \cdot 2y}{1-x^2-y^2} = *$

$f'''_{yxy} = *$

№ 3594

$$f(x,y) = \ln|1+x+y| = 0 + (x+y) + \frac{1}{2}(-x^2 - 2xy - y^2) \ldots$$

$f(a,b) = 0$

$f'_x = \frac{1}{1+x+y} = 1$

$f'_y = \frac{1}{1+x+y} = 1$

$f''_{yy} = f''_{xx} = -\frac{1}{(1+x+y)^2} = -1$

$f''_{xy} =$ "

$$\sum_{m,n=0}^{\infty} (-1)^{m+n-1} \frac{(m+n-1)!}{m!\,n!} \cdot x^m y^n$$

$*$ – обращается [illegible]

№ 3598

$$f(x,y) = \cos x \operatorname{ch} y$$

$$f(x) = \cos(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{(2m)!} x^{2m}$$

$$f(y) = \operatorname{ch}(y) = \sum_{n=0}^{\infty} \frac{1}{(2n)!} y^{2n}$$

$$f(x,y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} (-1)^m \frac{x^{2m} y^{2n}}{2m!\,2n!}$$

№ 3599

$$f(x,y) = \sin(x^2+y^2)$$

$$f(x) = \sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$f(x,y) = \sin(x^2+y^2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot (x^2+y^2)^{2n+1}$$

№ 3587 (а)

$$f(x,y) = \frac{\cos x}{\cos y} = 1 + \frac{1}{2}\left(-1(x^2) + y^2\right) = 1 - \frac{1}{2}(x^2 - y^2)$$

$$f(a,b) = 1$$

$$f'_x = -\frac{1}{\cos y} \cdot \sin x = 0$$

$$f'_y = -\frac{1}{(\cos y)^2} \cdot \sin y \cdot \cos x = 0$$

$$f''_{xx} = -\frac{1}{\cos y} \cdot \cos x = -1$$

$$f''_{yy} = \cancel{*} \cos x \cdot \frac{\cos y - \cos^2 y + 2\sin^2 y}{\cos^4 y} = +1$$

$$f''_{xy} = +\sin x \cdot \frac{1}{\cos^3 y} \cdot (-\sin y) = 0$$

(б)

$$f(x,y) = \operatorname{arctg} \frac{1+x+y}{1-x+y} = \frac{\pi}{4} + x - xy$$

$$f(a,b) = \operatorname{arctg} 1 = \frac{\pi}{4}$$

$$f'_x = \frac{1}{1+\left(\frac{1+x+y}{1-x+y}\right)^2} \cdot \left(\frac{(1-x+y)+(1+x+y)}{(1-x+y)^2}\right) = 1$$

$$f'_y = \frac{1}{1+\left(\frac{1+x+y}{1-x+y}\right)^2} \cdot \frac{(1-x+y)-(1+x+y)}{(1-x+y)^2} = 0$$

$$f''_{xx\,(a,b)} = f''_{yy\,(a,b)} = 0$$

$$f''_{xy} = -\frac{1}{2}$$

№ 3388

$f(x,y,z) = \cos(x+y+z) - \cos x \cos y \cos z = \quad -(xy + xz + yz)$

$f'_z = f'_y = f'_x = -\sin(x+y+z) + \cos y \cos z \sin x = 0$

$f''_{yy} = f''_{xx} = -\cos(x+y+z) + \cos y \cos z \cos x = 0$

f''_{zz}

$f''_{xy} = -\cos(x+y+z) + -\sin y \sin x \cos z = -1$

$f''_{xz} = f''_{yz}$

№ 3389

$f(x,y,z) = 0 \qquad x=1;\ y=-2;\ z=-1$

$x^2 + 2y^2 + 3z^2 + xy - z - 8 = 0$

$z'_x,\ z'_y\ -?$

$z''_{xx},\ z''_{yy},\ z''_{xy}$

$\frac{\partial z}{\partial x} = z'_x$

$\boxed{z'_x = -\frac{F'_x}{F'_z} =} - \frac{2x+y}{6z-1}$

$z'_y = -\frac{4y+x}{6z-1}$

$d(x^2) + d(2y^2) + d(3z^2) + d(xy)$

$z''_{xy} = -\frac{1}{6z-1} = -\frac{1}{5}$

$z''_{xx} = -\frac{1}{6z-1} \cdot 2 = -\frac{2}{6-1} = -\frac{2}{5}$

$z''_{yy} = -\frac{1}{6z-1} \cdot 4 = -\frac{4}{5}$

Ответ: $-\frac{1}{5};\ -\frac{2}{5};\ -\frac{4}{5}$

№ 3392

$\frac{x}{z} = \ln\frac{z}{y} + 1$

$\frac{x}{z} - \ln\frac{z}{y} - 1 = 0$

$\boxed{dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy}$

$\frac{\partial z}{\partial y} = + \frac{\frac{y}{z} \cdot \frac{1}{y^2} \cdot z}{\left(\frac{x+z}{z^2}\right)} = \frac{z^2}{y(x+z)}$

$\frac{\partial z}{\partial x} = -\frac{\frac{1}{z}}{-\frac{x}{z^2} - \frac{y}{z} \cdot \frac{1}{y}} = +\frac{1}{z\left(\frac{x+z}{z^2}\right)} = \frac{z}{x+z}$

$$dz = \left(\frac{z}{x+z}\right)dx + \left(\frac{z^2}{y(x+z)}\right)dy$$

$$\frac{yz\,dx + z\,dy}{y(x+z)} = \frac{z(y\,dx + z\,dy)}{y(x+z)}$$

$$d^2 z = -\frac{z^2 (y\,dx - x\,dy)^2}{y^2 (z+x)^3}$$

№ 3394

du $\quad u^3 - 3(x+y)u^2 + z^3 = 0$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$\frac{\partial u}{\partial x} = u'_x = + \frac{+3u^2}{3u^2 - 6(x+y)u} = \frac{3u^2}{3u(u-2(x+y))}$$

$$\frac{\partial u}{\partial y} = u'_y = \frac{u}{u-2(x+y)}$$

$$\frac{\partial u}{\partial z} = + \frac{3z^2}{3(u^2 - 2u(x+y))} = \frac{z^2}{3u(u-2(x+y))}$$

$$du = \frac{u^2(dx+dy) - z^2\,dz}{u(u-2(x+y))}$$

* № 3399.1

$$z^3 - xz + y = 0$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$\frac{\partial z}{\partial x} = + \frac{+z}{3z^2 - x}$$

$$\frac{\partial z}{\partial y} = - \frac{1}{3z^2 - x}$$

$$dz = \frac{z}{3z^2 - x}\cdot dx + -\frac{1}{3z^2 - x}\cdot dy$$

$x = 3 \quad y = -2 \quad z = 2$

$$dz = \frac{2}{9}dx - \frac{1}{9}dy = \frac{1}{9}(2dx - dy)$$

№ 3401

$\frac{dx}{dz}$ $\quad x + y + z = 0$

$\frac{dy}{dz}$

$$\begin{cases} \frac{dx}{dz} + \frac{dy}{dz} + 1 = 0 \\ 2x\frac{dx}{dz} + 2y\frac{dy}{dz} + 2z = 0 \end{cases}$$

$$\frac{dx}{dz} = \frac{y-z}{x-y}, \quad \frac{dy}{dz} = \frac{z-x}{x-y}$$

* Определенный интеграл

а) $$\int_{-1}^{0} \frac{dx}{4x^2-9} = \int_{-1}^{0} \frac{1}{(2x-3)(2x+3)}\,dx =$$

$$\int \frac{1}{(2x-3)(2x+3)}\,dx = \frac{A}{2x-3} + \frac{B}{2x+3}$$

$$1 = A(2x+3) + B(2x-3)$$

$$\begin{cases} x: 0 = 2A + 2B \\ 1: 1 = 3A + (-3B) \end{cases} \qquad \begin{cases} A = \frac{1}{6} \\ B = -\frac{1}{6} \end{cases}$$

$$\int_{1}^{0} \left(\frac{1}{6(2x-3)} - \frac{1}{6(2x+3)}\right) dx = \left| \begin{array}{l} u = 2x-3 \\ du = 2dx \\ dx = \frac{1}{2}du \end{array} \right.$$

$$\frac{1}{2}\int_{1}^{0} \frac{1}{6u} - \frac{1}{6u}\,du$$

$$\frac{\ln|2x-3|}{12} - \frac{\ln|2x+3|}{12} \Bigg|_{-1}^{0} = \frac{\ln\left|\frac{2x-3}{2x+3}\right|}{12} \Bigg|_{-1}^{0} = ?$$

$$\frac{\ln(-5)}{12} = -\frac{\ln 5}{12}$$

б) $$\int_{1}^{\sqrt{e}} \frac{dx}{x\sqrt{1-\ln^2 x}} = \arcsin \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \\ dx = x\,du \end{array} \right.$$

$$\int_{1}^{\sqrt{e}} \frac{x\,du}{x\sqrt{1-u^2}} = \arcsin(\ln(x)) = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

в) $$\int_{0}^{1} 3(x^2 + x \cdot e^{x^2})\,dx$$

$$\int x^2 dx = \frac{x^3}{3} + C \qquad \int x \cdot e^{x^2} dx = \left| \begin{array}{l} u = x^2 \\ du = 2x\,dx \\ dx = \frac{du}{2x} \end{array} \right. = \int \frac{x \cdot e^u\,du}{2x} = \frac{1}{2} \cdot e^u$$

$$3\left(\frac{x^3}{3} + \frac{e^{x^2}}{2}\right)\Bigg|_0^1 = 3 \cdot \frac{3e-1}{6} = \frac{3e-1}{2}$$

a) $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$ $\left| \begin{array}{l} u = \frac{1}{x} \\ du = -\frac{1}{x^2} dx \end{array} \right.$

$\int -\frac{e^u \cdot x^2}{x^2} du = \int -e^u du = -e^u =$

$-e^{\frac{1}{x}} \Big|_1^2 = e - \sqrt{e}$

б) $\int_0^1 \operatorname{arctg} \sqrt{x}\, dx = \frac{\pi - 2}{2}$

$\left| \begin{array}{l} u = \operatorname{arctg} \sqrt{x} \\ d\sigma = \int 1\, dx \\ du = \frac{1}{2\sqrt{x}\,(x+1)} \\ \sigma = x \end{array} \right.$ $= \operatorname{arctg}(\sqrt{x})\, x - \int \frac{\sqrt{x}}{2(x+1)} dx$

$\frac{1}{2} \int \frac{\sqrt{x}}{x+1} = \sqrt{x} - \operatorname{arctg}(\sqrt{x})$

$\operatorname{arctg} 1 - (1 - \operatorname{arctg} 1) - 0 + \operatorname{arctg} 0 =$

б) $\int_2^3 \frac{x+2}{x^2(x-1)} dx = \int_2^3 \left(-\frac{3}{x} - \frac{2}{x^2} + \frac{3}{x-1}\right) dx$

$x + 2 = A(x-1) + B_1 x^2 + B_2 x$

$x^2 \quad 0 = B_1$

$x^1 \quad 1 = A + B_2$

$x^0 \quad 2 = -A$

$-3\ln(x) + \frac{2}{x} + 3\ln(x-1) \Big|_2^3 = \underline{-9\ln(3) + \frac{18}{3}\ln(2) - 1}$

a) $\int_1^{\sqrt{2}} \frac{x\, dx}{\sqrt{4 - x^2}} = \left| \begin{array}{l} u = 4 - x^2 \\ du = -2x\, dx \end{array} \right. = -\sqrt{4 - x^2} \Big|_1^{\sqrt{2}} =$

$-\sqrt{2} + \sqrt{3} = \sqrt{3} - \sqrt{2}$

б) $\int_1^e \frac{\ln^2 x}{x^2} dx = \frac{-\ln^2(x) + 2\ln(x) + 2}{x} \Big|_1^e = 2 - \frac{5}{e}$

a) $\int_1^{e^3} \frac{dx}{x\sqrt{1 + \ln x}} = \left| \begin{array}{l} u = \ln x \\ du = \frac{1}{x} dx \end{array} \right.$

$$2\sqrt{\ln(x)+1}\Big|_1^{e^3} = 2$$

б) $$\int_2^3 \frac{2x^4-5x^2+3}{x^2-1}\,dx = \int_2^3 (2x^2-3)\,dx =$$

$$2\frac{x^3}{3} - 3x\Big|_2^3 = \frac{29}{3}$$

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} x\frac{\sin x}{\cos^2 x}\,dx = \ldots\ldots \quad x\sec(x) - \ln|tg(x) - \sec(x)|\Big|_{-\frac{\pi}{3}}^{\frac{\pi}{3}} =$$

$$-\ln(\sqrt{3}+2) + \ln(2-\sqrt{3}) + \frac{4\pi}{3}$$

$$\int_1^e (x+1)\ln x\,dx = \left| \begin{array}{ll} u = \ln(x) & dv = x+1\,dx \\ du = \frac{1}{x}dx & v = \int(x+1)dx = \frac{x^2}{2}+x \end{array} \right. =$$

$$\frac{x(2(x+2)\ln x - x - 4)}{4}\Big|_1^e = \frac{e^2}{4} + \frac{5}{4} = \frac{e^2+5}{4}$$

№ 3.1

$$\int_2^3 x(3-x)^7\,dx = \left| \begin{array}{l} u = x-3 \\ du = dx \end{array} \right.$$

$$-\int_2^3 (u+3)u^7\,du = -\int_2^3 u^8\,du - \int_2^3 3u^7\,du =$$

$$-\frac{(x-3)^9}{9} - \frac{3(x-3)^8}{8}\Big|_2^3 = +\frac{19}{72}$$

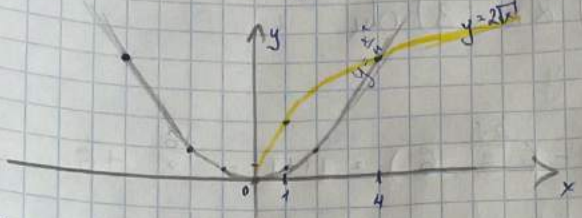
№ 3.2.

$$\int_0^{\frac{\pi}{6}} \sin^3 x\cos x\,dx = \left| \begin{array}{l} u = \sin x \\ du = \cos x\,dx \end{array} \right. = \int_0^{\frac{\pi}{6}} u^3 \cdot \frac{\cos x \cdot du}{\cos x} =$$

$$\frac{u^4}{4}\Big|_6^{\pi} = \frac{\sin^4(x)}{4}\Big|^{\frac{\pi}{6}} = \frac{\left(\frac{1}{2}\right)^4}{4} = \frac{1}{2^4 \cdot 2^2} = \frac{1}{2^6} = \frac{1}{64}$$

6.1 a)

$y = \frac{x^2}{4}$; $y = 2\sqrt{x}$



$[0; 4]$

$2\sqrt{x} \geq \frac{x^2}{4}$

$0 \leq x \leq 4$

$S = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx = 2\int_0^4 x^{\frac{1}{2}} dx - \frac{1}{4}\int_0^4 x^2 dx =$

$2 \cdot \frac{2\sqrt{x^3}}{3}\Big|_0^4 - \frac{1}{4} \cdot \frac{x^3}{3}\Big|_0^4 = \frac{2 \cdot 2 \cdot 4 \cdot 2}{3} - \frac{4^3}{12} = \frac{16}{3}$ ед^2

6.5

$r = \sqrt{3} \sin\varphi$ $\qquad r = 1 + \cos\varphi$

$\begin{cases} \rho = \sqrt{3}\sin\varphi \\ r = 1 + \cos\varphi \end{cases}$

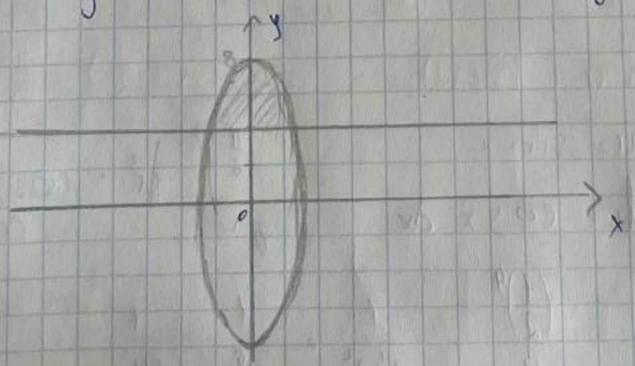
$\varphi_1 = \frac{\pi}{3}$ $\quad \varphi_2 = \pi$

$S = \frac{1}{2} \int_0^{\frac{\pi}{3}} 3\sin^2\varphi \, d\varphi + \frac{1}{2} \int_{\frac{\pi}{3}}^{\pi} (1+\cos\varphi)^2 \, d\varphi = \frac{3}{4}\left(\varphi - \frac{\sin 2\varphi}{2}\right)\Big|_0^{\frac{\pi}{3}} +$

$\frac{1}{2}\left(\varphi + 2\sin\varphi + \frac{\varphi}{2} + \frac{\sin 2\varphi}{4}\right)\Big|_{\frac{\pi}{3}}^{\pi} = \frac{3}{4}(\pi - \sqrt{3})$ ед^2

6.3 a

$x = 3\cos t$ $\quad y = 8\sin t$ $\quad$ эллипс $\quad y = 4$ $\quad y \geq 4$



$\frac{x^2}{9} + \frac{y^2}{64} = 1$

$$\sqrt{64-\frac{64}{9}x^2} = 4$$

$$x = \pm\frac{3\sqrt{3}}{2}$$

$$\int = \int_{-\frac{3\sqrt{3}}{2}}^{\frac{3\sqrt{3}}{2}} \left(\sqrt{64-\frac{64}{9}x^2} - 4\right)dx = \frac{x\sqrt{64-\frac{64x^2}{9}}}{2} - 4x + 12\arcsin\left(\frac{x}{3}\right)\Bigg|_{-\frac{3\sqrt{3}}{2}}^{\frac{3\sqrt{3}}{2}} = 8\pi - 6\sqrt{3}$$