Домашняя работа 27.2.2022

№ 1935

$$\int \frac{dx}{1+\sqrt{x}+\sqrt{1+x}} = \left| \begin{array}{l} t^2 = x+1 \\ x = t^2-1 \\ dx = \frac{1}{2\sqrt{t^2-1}}\cdot 2t\cdot dt \end{array} \right.$$

$$-\frac{\ln\left(\left|\frac{\sqrt{x+1}}{\sqrt{x}}-1\right|\right)}{2} + \frac{\ln\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)}{2} + \frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x}-1\right)} - x - 2\sqrt{t} + C$$

$$x = \left(\frac{u^2-1}{2u}\right)^2 \quad dx = \frac{4u^2-2u^2-2}{4u^2}\cdot\frac{2(u^2-1)}{2u} = \frac{2(u^2-1)}{4u^2}\cdot\frac{(u^2-1)}{u} = \frac{(u^2-1)^2}{2u^3}du$$

$$1+\frac{u^2-1}{2u}+\sqrt{1+\frac{u^4-2u^2+1}{4u^2}} = 1+\frac{u^2-1}{2u}+\frac{\sqrt{(u^2-1)^2+4u^2}}{2u} = \frac{u^2+2u-1+\sqrt{(u^2-1)^2+4u^2}}{2u}$$

$$\int\frac{(u^2-1)^2}{2u^3}\cdot\frac{2u}{(u^2+2u-1+\sqrt{(u^2-1)^2+4u^2})}du = \frac12\int\frac{(u-1)^2(u+1)^2}{u(u^2+u)}du =$$

$$\int\frac{(u-1)^2(u+1)}{u^2}du = \int\left(u-\frac1u+\frac{1}{u^2}-1\right)du =$$

$$\frac{u^2}{2}-\ln|u|+\left(-\frac1u\right)-u+C, \text{ где } u = \left(\sqrt{x}+\sqrt{x+1}\right)$$

№ 1940

$$\int\frac{\sqrt{x^2+2x+2}}{x}dx = \left| \begin{array}{l} \sqrt{x^2+2x+1+1} = \sqrt{(x+1)^2+1} \\ (x+1) = t \\ dt = dx \end{array} \right.$$

$$\int\frac{\sqrt{t^2+1}}{t-1}dt = \frac{\sqrt{tg^2 t+1}}{(tg\, t-1)\cos^2 t} = w = tg\left(\frac t2\right) \Rightarrow$$

$$2\int\frac{(w^2+1)^2}{(w-1)^2(w+1)^2(w^2+2w-1)}dw =$$

$$4\int\frac{1}{w^2+2w-1}dw + \int\frac{1}{w+1}dw - \int\frac{1}{(w+1)^2}dw\int\frac{1}{w-1}dw -$$

$$\int\frac{1}{(w-1)^2}dw = \sqrt2\ln(w-\sqrt2+1) - \sqrt2\ln(w+\sqrt2+1) + \ln(w+1) + \frac{1}{w+1} - \ln(w-1) +$$

$$+\frac{1}{w-1}+C$$

$$R = \sqrt{x^2+2x+2}$$

$$R+\ln(x+1+R)-\sqrt2\ln\left(\frac{x+2+\sqrt2 R}{x}\right)+C$$

№ 1939

$$\int\frac{dx}{(1-x)^2\sqrt{1-x^2}} = \left| \begin{array}{l} x = \cos u \\ dx = -\sin u\, du \end{array} \right.$$

$$\int \frac{-\sin u}{(1-\cos u)^2 \sqrt{1-\cos^2 u}}\, du = \int \frac{-\sin u}{(1-\cos u)^2 \sin u}\, du =$$

$$-\int \frac{1}{(1-\cos u)^2}\, du = \left| \begin{array}{l} \sigma = \operatorname{tg} \frac{u}{2} \\ 2d\sigma = (\sigma^2+1)\, du = \\ du = \frac{2\, d\sigma}{\sigma^2+1} \end{array} \right.$$

$$-\int \frac{1 \cdot 2d\sigma}{\left(1 - \frac{1-\sigma^2}{1+\sigma^2}\right)(\sigma^2+1)} = -\int \frac{2d\sigma}{\left(\frac{2\sigma^2(\sigma^2+1)}{(1+\sigma^2)}\right)} = -\int \frac{d\sigma}{\sigma^2} =$$

$$+\frac{1}{\operatorname{tg}\frac{u}{2}} + C = +\frac{\sin u}{1-\cos u} + C = \frac{\sin u}{1-x} + C$$

№ 1969

$$\int \frac{x - \sqrt{x^2+3x+2}}{x + \sqrt{x^2+3x+2}}\, dx$$

$$\int \frac{x - x + z}{x + x + z}\, dx \quad \left| \begin{array}{l} x = \frac{z^2-2}{3-2z} \\ dx = \frac{2z(3-2z) + 2(z^2-2)}{(3-2z)^2}\, dz \end{array} \right.$$

$$\int \frac{z}{\frac{2z^2-4+z(3-2z)}{3-2z}} \cdot \frac{2(z+1)(z+2)}{(3-2z)^2}\, dz =$$

$$\int \frac{2z(z+1)(z+2)}{(3z-4)(3-2z)}\, dz = \frac{-9\ln|2+(\sqrt{(x+1)(x+2)}+3)| - 40\ln\left(\frac{1}{\sqrt{x+2}}\sqrt{x+2} - \sqrt{x+1}\right)}{108} + C$$

$$+\frac{32\left(\ln\left(\frac{1}{\sqrt{x+2}}\sqrt{x+2} - 2\sqrt{x+1}\right) + \ln(3x+2)\right) - 32\ln\left(\left(\sqrt{x+2} + 2\sqrt{x+1}\right)\frac{1}{\sqrt{x+2}}\right)}{108} +$$

$$+ \frac{40\ln\left(\left(\sqrt{x+2} + \sqrt{x+1}\right)\left|\frac{1}{\sqrt{x+2}}\right|\right)}{108} + 48\sqrt{x+1}\sqrt{x+2} - 6\left(3\sqrt{x+1}\cdot\sqrt{x+2} - 2x(3x+3\right.$$

$+ C$.

№ 1986

$$\int \frac{dx}{\sqrt[4]{1+x^4}} = \int x^0 \cdot (1+x^4)^{-\frac{1}{4}}$$

$$p = -\frac{1}{4} \quad m = 0 \quad n = 4$$

Случай №2 $\quad \frac{1}{4} - \varnothing$

Случай №3 $\quad \frac{1}{4} + -\frac{1}{4} = 0$

$$1 + x^4 = x^4 y^4 \qquad y = \frac{\sqrt[4]{1+x^4}}{x} \qquad x = (y^4-1)^{-\frac{1}{4}}$$

$$dx = -\frac{1}{4}(y^4-1)^{-\frac{5}{4}}\, 4y^3\, dy$$

$$dx = -y^3(y^4-1)^{-\frac{5}{4}}\, dy$$

$$1+x^4 = 1+\frac{1}{y^4-1} = \frac{y^4}{y^4-1}$$

$$-\int\left(\frac{y^4}{y^4-1}\right)^{-\frac{1}{4}} y^3(y^4-1)^{-\frac{5}{4}}\, dy = \int y^{-1}\cdot(y^4-1)^{\frac{1}{4}}(y^4-1)^{-\frac{5}{4}}\, dy =$$

$$\int\frac{1}{y^2(y^4-1)}\, dy = -\frac{\ln(y+1)}{4} + \frac{\operatorname{arctg}(y)}{2} + \frac{1}{y} + \frac{\ln(y-1)}{4} + C$$

где $y = \frac{\sqrt[4]{1+x^4}}{x}$

№ 1987 ✓

$$\int\frac{dx}{x\sqrt[6]{1+x^6}} = \int x^{-1}(1+x^6)^{-\frac{1}{6}}\, dx$$

$p = -\frac{1}{6}$ $m = -1$ $n = 6$

Случай № 2 $\frac{-1+1}{6} = 0$

$$1+x^6 = y^6$$

$$y = \sqrt[6]{1+x^6}$$

$$x = \sqrt[6]{y^6-1}$$

$$dx = \frac{1}{6}(y^6-1)^{-\frac{5}{6}}\, 6y^5\, dy$$

$$dx = y^5(y^6-1)^{-\frac{5}{6}}\, dy$$

$$1+x^6 = \cancel{1} + y^6 \cancel{-1}$$

$$\frac{1}{x} = \frac{1}{\sqrt[6]{y^6-1}} = (y^6-1)^{-\frac{1}{6}}$$

$$\int (y^6-1)^{-\frac{1}{6}}\, y^{-1}\, y^5\, (y^6-1)^{-\frac{5}{6}}\, dy =$$

$$\int y^{-4}(y^6-1)^{-1}\, dy = \frac{\ln(y+1)+\ln(y^2+y+1)-\ln(y^2-y+1)+\ln(y-1)}{6}$$

$$+\frac{1}{3x^3} + C$$, где $y = \sqrt[6]{1+x^6}$

№ 1989

$$\int \sqrt[3]{3x - x^3}\,dx = \int (3x - x^3)^{\frac{1}{3}} dx = \int x^{\frac{1}{3}} (3 - x^2)^{\frac{1}{3}} dx$$

$$p = \frac{1}{3} \quad m = \frac{1}{3} \quad n = 2$$

$$Сл\ №2 \quad \frac{\frac{1}{3}+1}{2} = \frac{\frac{4}{3}}{2} = \frac{4}{6} \quad \times$$

$$Сл\ № \quad \frac{4}{6} + \frac{1}{3} = \frac{4}{6} + \frac{2}{6} = \underline{1}$$

$$3 - x^2 = x^2 y^3$$

$$y = \frac{\sqrt[3]{3x - x^3}}{x}$$

$$x = \sqrt{\frac{3 + 3y^3}{1 + y^3}} = \sqrt{\frac{3}{1 + y^3}}$$

$$dx = -\frac{3\sqrt{3}\,xy^2}{2(xy^3+1)^{\frac{3}{2}}}\,dy$$

$$3 - x^2 = 3 - \frac{3}{1+y^3} = \frac{y^3}{1+y^3}$$

$$-\frac{3\sqrt{3}}{2}\int \left(\frac{3}{1+y^3}\right)^{\frac{1}{6}} \cdot \left(\frac{y^3}{1+y^3}\right)^{\frac{1}{3}} \cdot \frac{y^2}{(y^3+1)^{\frac{3}{2}}}\,dy$$

$$\sqrt[6]{3} \int y^3 \cdot (y^3+1)^{-2}\,dy =$$

$$-\frac{3\sqrt{3}\sqrt[6]{3}}{2}\left(\frac{\ln(y+1)}{9} - \frac{\ln(y^2 - y + 1)}{18} + \frac{\operatorname{arctg}\left(\frac{2y-1}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{2\operatorname{arctg}\frac{2y-1}{\sqrt{3}}}{3\sqrt{3}} - \right.$$

$$\left. - \frac{y}{3(y^3+1)}\right) + C \quad , где \quad y = \frac{\sqrt[3]{3x - x^3}}{x}$$

№ 1996

$$\int \sin^5 x \cos^5 x\,dx \qquad \sin x = t$$

$$\int t^5 \cdot t(1-t^2)^2\,dt = \int t^6 (1 - t^2)^2\,dt =$$

$$\frac{t^{11}}{11} - \frac{2t^9}{9} + \frac{t^7}{7} + C \quad , где \quad t = \sin x$$

№1999

$$\int \frac{dx}{\sin^3 x} = \int \frac{\sin^2 x + \cos^2 x}{\sin^3 x} dx =$$

$$\int \frac{1}{\sin x} dx + \int \frac{\cos^2 x}{\sin^3 x} dx =$$

$$\left(\int \sin x \, dx + \int \cos^2 x \sin^{-1} x \, dx\right) + \int \cos^2 x \sin^{-3} x \, dx$$

$$= \frac{\ln(1-\cos(x))}{4} - \frac{\ln(\cos(x)+1)}{4} + \frac{\cos(x)}{2\cos^2(x)-2}$$

$$\int \frac{1-\sin^2 x}{\sin^3 x} dx = \int \frac{1}{\sin^3 x} dx - \int \frac{1}{\sin x} dx$$

№2000 Аналогично

$$0. \quad \frac{\ln(\sin x + 1)}{4} - \frac{\ln(1-\sin x)}{4} - \frac{\sin x}{2\sin^2 x - 2} + C$$

№2132

$$\int \operatorname{ch} x \, dx = \sinh x + C$$

№2133

$$\int \operatorname{sh} x \, dx = \cosh(x) + C$$

№2134

$$\int \frac{dx}{\operatorname{ch}^2 x} = \frac{4}{2e^{-2x}+2} + C$$