Домашняя работа 21.02.2021

1879 ✓

$$\int \frac{x\,dx}{(x-1)^2(x^2+2x+2)} = \int \frac{x\,dx}{(x-1)^2(x^2+2x+2)} = \frac{2\ln|x-1| - \ln(x^2+2x+2) - 14\,\mathrm{arctg}(x+1)}{50} - \frac{1}{5(x-1)} + C$$

$$\frac{x}{(x-1)^2(x^2+2x+2)} = \frac{A_1}{(x-1)^2} + \frac{A_2}{(x-1)} + \frac{Bx+C}{(x^2+2x+2)}$$

$$x = A_1(x^2+2x+2) + A_2(x-1)(x^2+2x+2) + B\,x$$

$$x = A_1(x^2+2x+2) + A_2(x^3+2x^2+\cancel{2x}-x^2-\cancel{2x}-2) + (Bx+C)(x^2-2x+1)$$

$$x = A_1(x^2+2x+2) + A_2(x^3+x^2-2) + B(x^3-2x^2+x) + C(x^2-2x+1)$$

x^3 $\quad 0 = A_2 + B$

x^2 $\quad 0 = A_1 + A_2 - 2B + C$

x^1 $\quad 1 = 2A_1 + B - 2C$

x^0 $\quad 0 = 2A_1 - 2A_2 + C$

$C = -\frac{8}{25}$

$A_1 = \frac{1}{5}$

$A_2 = \frac{1}{25}$

$B = -\frac{1}{25}$

$$\int\left(\frac{1}{5(x-1)^2} + \frac{1}{25(x-1)} + \frac{\left(-\frac{1}{25}x - \frac{8}{25}\right)}{(x^2+2x+2)}\right)dx$$

1) $\frac{1}{5}\int\frac{1}{(x-1)^2}dx = \frac{1}{5}\left|\begin{array}{l} u = x-1 \\ du = dx\end{array}\right. = \frac{1}{5}\int u^{-2}du = -\frac{1}{5}\cdot\frac{1}{u} = -\frac{1}{5(x-1)}$

2) $\frac{1}{25}\int\frac{1}{x-1}dx = \frac{\ln|x-1|}{25}$

3) $\int\left(\frac{\frac{-x-8}{25}}{x^2+2x+2}\right)dx = \int\left(\frac{-(x+8)}{25(x^2+2x+2)}\right)dx = -\frac{1}{25}\int\frac{x+8}{x^2+2x+2}$

$\left|\begin{array}{l} u = x^2+2x+2 \\ du = (2x+2)dx \\ dx = \frac{du}{2x+2}\end{array}\right. = -\frac{1}{25}\int\left(\frac{\frac{1}{2}(2x+2)}{x^2+2x+2} + \frac{7}{x^2+2x+2}\right)dx =$

$-\frac{1}{25}\left(\int\left(\frac{x+1}{x^2+2x+2}\right)dx + \int\left(\frac{7}{x^2+2x+2}\right)dx\right) =$

• $\frac{1}{2}\int\frac{1}{u}du = \frac{1}{2}\cdot\ln|x^2+2x+2|$

• $7\int\frac{1}{(x+1)^2+1}dx = \left|\begin{array}{l} u = x+1 \\ du = dx\end{array}\right. = 7\cdot\int\frac{1}{u^2+1}du = 7\,\mathrm{arctg}(x+1)$

$-\frac{1\cdot\ln|x^2+2x+2|}{25\cdot 2} - \frac{1\cdot 7\,\mathrm{arctg}(x+1)}{25}$

2019 ✓

$$\int \frac{x\,dx}{x^4-3x^2+2} = \int \frac{x\,dx}{(x-1)(x+1)(x^2-2)}$$

$$\frac{x}{x^4-3x^2+2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-\sqrt{2}} + \frac{D}{x+\sqrt{2}}$$

A $\quad x=1$

$$\frac{1}{2(1-\sqrt{2})(1+\sqrt{2})} = \frac{1}{2(1-2)} = -\frac{1}{2}$$

B $= x=-1$

$$+\frac{1}{2(-1-\sqrt{2})(-1+\sqrt{2})} = -\frac{1}{2}$$

C $\quad x=\sqrt{2}$

$$\frac{\sqrt{2}}{(\sqrt{2}-1)(1+\sqrt{2})\,2\sqrt{2}} = \frac{1}{2(2-1)} = \frac{1}{2}$$

D $= -\sqrt{2}$

$$\frac{-\sqrt{2}}{-2\sqrt{2}\,(2-1)} = \frac{1}{2}$$

$$\int \left(-\frac{1}{2(x-1)} - \frac{1}{2(x+1)} + \frac{1}{2(x-\sqrt{2})} + \frac{1}{2(x+\sqrt{2})}\right) dx$$

$$-\frac{1}{2}\cdot \ln|x-1| - \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-\sqrt{2}| + \frac{1}{2}\cdot\ln|x+\sqrt{2}| + C$$

$$-\frac{1}{2}\ln|x^2-1| + \frac{1}{2}\ln|x^2-2| + C$$

2021 ✓

$$\int \left(\frac{x^6-2x^4+3x^3-9x^2+4}{x^5-5x^3+4x}\right)dx = \int \left(x + \frac{3x^4+3x^3-13x^2+4}{x^5-5x^3+4x}\right)dx$$

$$\frac{3x^4+3x^3-13x^2+4}{x(x^4-5x^2+4)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x-1} + \frac{D}{x+1} + \frac{E}{x+2}$$

$$3x^4+3x^3-13x^2+4 = A(x^4-3x^2+4)+(Bx+C)(x^3+\sqrt{7}x^2+2x)+(Dx+E)(x^3-\sqrt{7}x^2+2x)$$

$$+D(x^4-\sqrt{7}x^3+2x^2)$$

$$A(x^4-3x^2+4)+B(x^4+\sqrt{7}x^3+2x^2)+C(x^3+\sqrt{7}x^2+2x)$$

$$+E(x^3-\sqrt{7}x^2+2x)$$

$A=1$
$B=1$
$C=2$
$D=\frac{3}{2}$
$E=1$

X

$$\int\left(\frac{1}{x}+\frac{1}{x-2}+\frac{2}{x-1}+\frac{3}{2(x+1)}+\frac{1}{x+2}\right)dx = \frac{x^2}{2}+\ln x+\ln(x-2)+\frac{3\ln(x+1)}{2}+\ln(x+2)+C$$

N 2029 ✓

$$\int\left(\frac{x^3-6x^2+9x+7}{(x-2)^3(x-5)}\right)dx$$

$$\frac{x^3-6x^2+9x+7}{(x-2)^3(x-5)} = \frac{A_1}{(x-2)^3}+\frac{A_2}{(x-2)^2}+\frac{A_3}{(x-2)}+\frac{B}{x-5}$$

$$x^3-6x^2+9x+7 = A_1(x-5)+A_2(x^2-7x+10)+A_3(x^2-4x+4)(x-5)+B(x-2)^3$$

$$x^3-6x^2+9x+7 = A_1(x-5)+A_2(x^2-7x+10)+A_3(x^3-9x^2+24x-20)+B(x^3-6x^2+12x-8)$$

x^3 $\quad 1 = A_3+B$

x^2 $\quad -6 = A_2-9A_3-6B$

x^1 $\quad 9 = A_1-7A_2+24A_3+12B$

x^0 $\quad 7 = -5A_1+10A_2-20A_3-8B$

$B=1$
$A_1=-3$
$A_2=0$
$A_3=0$

$$\int\left(\frac{-3}{(x-2)^3}+\frac{1}{x-5}\right)dx = +\frac{3}{2}\cdot\frac{1}{(x-2)^2}+\ln|x-5|+C$$

2045 ✓

$$\int \left(\frac{x^5 + 2x^3 + 4x + 4}{x^4 + 2x^3 + 2x^2} \right) dx \cdot \int \left((x-2) + \frac{4x^3 + 4x^2 + 4x + 4}{x^4 + 2x^3 + 2x^2} \right) dx$$

$$\frac{4x^3 + 4x^2 + 4x + 4}{x^4 + 2x^3 + 2x^2} = \frac{A_1}{x^2} + \frac{A_2}{x} + \frac{Bx + C}{x^2 + 2x + 2}$$

$$4x^3 + 4x^2 + 4x + 4 = A_1(x^2 + 2x + 2) + A_2(x^3 + 2x^2 + 2x) + Bx^3 + Cx^2$$

x^3 $4 = A_2 + B$ $C = 2$

x^2 $4 = A_1 + 2A_2 + C$ $A_1 = 2$

x^1 $4 = 2A_1 + 2A_2$ $A_2 = 0$

x^0 $4 = 2A_1$ $B = 4$

$$\int \left(\frac{2}{x^2} + \frac{4x + 2}{x^2 + 2x + 2} \right) dx = 2 \int \frac{1}{x^2} dx + \int \left(\frac{4x + 2}{x^2 + 2x + 2} \right) dx = -2 \cdot \frac{1}{x} + 2 \int \frac{2x + 1}{x^2 + 2x + 2} dx$$

$$\cdot \int \left(\frac{2x + 2}{x^2 + 2x + 2} - \frac{1}{(x+1)^2 + 1} \right) dx \ ; \ 2 \cdot \frac{1}{2} \int \frac{1}{u} du = \ln(x^2 + 2x + 2) \ ; \ \int \frac{1}{(x+1)^2 + 1} dx =$$

$\operatorname{arctg}(x+1)$

$$\frac{x^2}{2} - 2x - \frac{2}{x} + 2(\ln(x^2 + 2x + 2) - \operatorname{arctg}(x+1)) + C = \frac{4(\ln(x^2 + 2x + 2) - \operatorname{arctg}(x+1)) + x(x-4)}{2} - \frac{2}{x} + C$$

2051 ✓

$$\int \frac{(x+1)^4}{(x^2 + 2x + 2)^3} dx = \int \frac{(x^2 + 2x + 1)(x^2 + 2x + 1)}{(x^2 + 2x + 2)^3} dx$$

$$\frac{x^4 + 2x^3 + x^2 + 2x^3 + 4x^2 + 2x + x^2 + 2x + 1}{(x^2 + 2x + 2)^3} = \frac{x^4 + 4x^3 + 6x^2 + 4x + 1}{(x^2 + 2x + 2)^3}$$

$$A_1x + A_2 + (B_1x + B_2)(x^2 + 2x + 2) + (C_1x + C_2)(x^2 + 2x + 2)(x^2 + 2x + 2) = x^4 + 4x^3 + 6x^2 + 4x + 1$$

$$A_1x + A_2 + B_1(x^3 + 2x^2 + 2x) + B_2(x^2 + 2x + 2) + C_1(x^5 + 4x^4 + 8x^3 + 8x^2 + 4x) + C_2(x^4 + 4x^3 + 8x^2 + 8x + 4) =$$

$$= x^4 + 4x^3 + 6x^2 + 4x + 1$$

x^5 $0 = C_1$ $A_1 = 0$

x^4 $1 = 4C_1 + C_2$ $A_2 = 1$

x^3 $4 = B_1 + 8C_1 + 4C_2$ $B_1 = 0$

x^2 $6 = 2B_1 + B_2 + 8C_1 + 8C_2$ $B_2 = -2$

x^1 $4 = A_1 + 2B_1 + 2B_2 + 4C_1 + 8C_2$ $C_1 = 0$

x^0 $1 = A_2 + 2B_2 + 4C_2$ $C_2 = 1$

$$\int \left(\frac{1}{(x^2 + 2x + 2)^3} - \frac{2}{(x^2 + 2x + 2)^2} + \frac{1}{(x^2 + 2x + 2)} \right) dx$$

- $\int \frac{1}{x^2+2x+2}\,dx = \left|\begin{array}{l} u = x+1 \\ du = dx \\ dx = du \end{array}\right. = \int \frac{1}{u^2+1}\,du = \operatorname{arctg}(x+1)$

- $-2\int \frac{1}{(x^2+2x+2)^2}\,dx = -2\int \frac{1}{(u^2+1)^2}\,du = -2\left(\frac{u}{2(u^2+1)} + \frac{1}{2}\int \frac{1}{u^2+1}\,du\right)$

$= \frac{x+1}{(x+1)^2+1} + \operatorname{arctg}(x+1)$

- $\int \frac{1}{(x^2+2x+2)^3}\,dx = \int \frac{1}{(u^2+1)^3}\,du =$

$$\boxed{\int \frac{1}{(ax^2+b)^n}\,dx = \frac{2n-3}{2b(n-1)}\int \frac{1}{(ax^2+b)^{n-1}}\,dx + \frac{x}{2b(n-1)(ax^2+b)^{n-1}}}$$

$= \frac{u}{4(u^2+1)^2} + \frac{3}{4}\int \frac{1}{(u^2+1)^2}\,du$

- $\int \frac{1}{(u^2+1)^2}\,du = \frac{1}{2}\int \frac{1}{u^2+1}\,du + \frac{u}{2(u^2+1)} = \frac{\operatorname{arctg}(x+1)}{2} + \frac{x+1}{2((x+1)^2+1)}$

$= \frac{x+1}{4((x+1)^2+1)^2} + \frac{3}{4}\left(\frac{\operatorname{arctg}(x+1)}{2} + \frac{x+1}{2((x+1)^2+1)}\right) + C$

N 1896 ✓

$\int \left(\frac{x^2+3x-2}{(x-1)(x^2+x+1)^2}\right)dx = \frac{Ax+B}{x^2+x+1} + \int \frac{Cx^2+Dx+E}{x^3-1}$

$Q_1 = x^2+x+1$

$Q_2 = x^3-1$

$P = Ax+B$

$L = Cx^2+Dx+E$

$\left(\frac{Ax+B}{x^2+x+1}\right)' = \frac{A(x^2+x+1)-(Ax+B)(2x+1)}{(x^2+x+1)^2}$

$\frac{x^2+3x-2}{(x-1)(x^2+x+1)^2} = \frac{A(x^2+x+1)-A(2x^2+x)-B(2x+1)}{(x^2+x+1)^2} + \frac{Cx^2+Dx+E}{x^3-1}$

$x^2+3x-2 = -Ax^3+Ax^2+Ax-A-2Bx^2+3Bx-B+C(x^4+x^3+x^2)+D(x^3+x^2+x)+$

$E(x^2+x+1)$

$C=0$

$-A+C+D=0$

$A-2B+C+D+E=1$

$A+3B+D+E=3$

$-2=-A-B+E$

$A=D=\frac{5}{3}$

$C=0$

$B=\frac{2}{3}$

$E=-1$

$$= \frac{5x+2}{3(x^2+x+1)} + \frac{1}{3}\int\left(\frac{5x-3}{x^3-1}\right)dx$$

$$\int \frac{5x-3}{(x-1)(x^2+x+1)}\,dx = \int\left(\frac{2}{3(x-1)} - \frac{2x-11}{3(x^2+x+1)}\right)dx$$

$$= \frac{2}{3}\ln|x-1| - \frac{1}{3}\left(\ln(x^2+x+1) - 8\sqrt{3}\,\text{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)\right) =$$

$$\frac{5x+2}{3(x^2+x+1)} + \frac{2\ln(x-1)}{9} - \frac{\ln(x^2+x+1)}{9} + \left(\frac{8\sqrt{3}\,\text{arctg}\,\frac{2x+1}{\sqrt{3}}}{3}\right) + C$$

№ 1890 ✓

$$\int \frac{ax^2+bx+c}{x^3(x-1)^2}\,dx = \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{x^3}$$

$A: x=1 \Rightarrow a+b+c$

$C: x=0 \Rightarrow c$

$$\int \frac{a+b+c}{(x-1)^2} = -\frac{a+b+c}{(x-1)}$$

$$\int \frac{c}{x^3} = -\frac{c}{2x^2}$$

Условие если $a+2b+3c=0$

№ 1902 ✓

При $a\gamma + c\alpha = 2b\beta$

№ 1903 ✓

$$\int \frac{x^3}{(x-1)^{100}}\,dx = \left| \begin{array}{l} u = x-1 \\ du = dx \\ x^3 = (u+1)^3 \end{array} \right. = \int \frac{(u+1)^3}{u^{100}}\,du$$

$$\int \frac{u^3 + 3u^2 + 3u + 1}{u^{100}}\,du = \int \left(\frac{1}{u^{97}} + \frac{3}{u^{98}} + \frac{3}{u^{99}} + \frac{1}{u^{100}} \right) du =$$

$$-\frac{1}{96(x-1)^{96}} - \frac{3}{97(x-1)^{97}} - \frac{3}{98(x-1)^{98}} - \frac{1}{99(x-1)^{99}} + C$$

№ 1904 ✓

$$\int \frac{x\,dx}{x^8-1} = \left| \begin{array}{l} u = \frac{x^2}{2} \\ du = x\,dx \\ dx = \frac{du}{x} \end{array} \right. = \int \frac{1}{16u^4-1}\,du$$

$$16u^4 = x^8$$

$$\int \frac{1}{(2u-1)(2u+1)(4u^2+1)}\,du \quad \left| \begin{array}{l} A = \frac{1}{4} \\ B = -\frac{1}{4} \\ C = -\frac{1}{2} \end{array} \right.$$

$$\int \left(-\frac{1}{2(4u^2+1)} - \frac{1}{4(2u+1)} + \frac{1}{4(2u-1)} \right) du =$$

$$-\frac{1}{4} \cdot \operatorname{arctg}(x^2) - \frac{1}{8} \cdot \ln(x^2+1) + \frac{1}{8} \ln(x^2-1) + C$$

№ 1905 ✓

$$\int \frac{x^3\,dx}{x^8+3} \quad \left| \begin{array}{l} u = \frac{x^4}{\sqrt{3}} \\ du = \frac{4x^3}{\sqrt{3}}\,dx \\ dx = \frac{\sqrt{3}}{4x^3}\,du \end{array} \right.$$

$$\int \frac{\sqrt{3}}{4(3u^2+3)} = \frac{1}{4\sqrt{3}} \int \frac{1}{u^2+1}\,du = \frac{1}{4\sqrt{3}} \operatorname{arctg}\left(\frac{x^4}{\sqrt{3}}\right) + C$$

№ 1906

$$\int \frac{x^2+x}{(x^2+1)(x^4-x^2+1)}\,dx = \int \left(\frac{x-1}{3(x^2+1)} - \frac{x^3-x^2-2x-1}{3(x^4-x^2+1)} \right) dx$$

1) $$\frac{1}{3} \int \frac{x-1}{x^2+1}\,dx = \frac{1}{3} \int \left(\frac{x}{x^2+1} - \frac{1}{x^2+1} \right) dx =$$

$$\frac{\ln(x^2+1)}{6} - \frac{\operatorname{arctg} x}{3}$$

2) $$\int \left(\frac{x^3-x^2-2x-1}{3(x^4-x^2+1)} \right) dx = \frac{\ln(x^2+\sqrt{3}x+1)}{12} + \frac{\ln(x^2-\sqrt{3}x+1)}{12} + \frac{\left(\frac{3}{2}-\sqrt{3}\right)\operatorname{arctg}(2x+\sqrt{3})}{3^{\frac{3}{2}}} + \frac{\left(-\sqrt{3}-\frac{3}{2}\right)\operatorname{arctg}(2x-\sqrt{3})}{3^{\frac{3}{2}}}$$

192)

$$J_3 = \int \frac{dx}{(x^2+x+1)^3} = \frac{2x+1}{2\cdot 3(x^2+x+1)^2} + \frac{2\cdot\cancel{3}\cdot\cancel{2}}{\cancel{2}(1\cdot\cancel{3})}\cdot J_2$$

$$J_2 = \frac{2x+1}{3(x^2+x+1)} + \frac{1}{1}\cdot\frac{2}{3}\cdot J_1$$

$$J_1 = \int \frac{dx}{(x^2+x+1)} = \frac{2\,\mathrm{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

$$= \frac{2x+1}{6(x^2+x+1)^2} + \frac{2x+1}{3(x^2+x+1)} + \frac{4\,\mathrm{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}} + C$$