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DISCRETE MATH - ASSIGNMENT 1 (CHAPTER 1-4)

Exercise 1. Express the following sentences in propositional expressions. Set your own propositional variables or predicates if needed.

- 1. If I have money I will buy the new Iphone 6s.**

Let "I have money" is \mathbf{p}

Let "I will buy the new Iphone 6s" is \mathbf{q}

$$\mathbf{p} \longrightarrow \mathbf{q}$$

- 2. Passenger can pass the check-in door if the metal detector doesn't make sound.**

Let "Passenger can pass the check-in door" is \mathbf{p}

Let "The metal detector makes sound" is \mathbf{q}

$$\neg \mathbf{q} \longrightarrow \mathbf{p}$$

- 3. Susan has a friend who makes friend with everyone in school.**

Let "x has a friend" is $\mathbf{A(x,y)}$

Let "y makes friend with z in school" is $\mathbf{B(y,z)}$

$$\mathbf{A(Susan,y)} \wedge \forall \mathbf{zB(y,z)}$$

- 4. Patience can check out if and only if the doctor signs a permission document.**

Let "Patience can check out" is \mathbf{p}

Let "The doctor signs a permission document" is \mathbf{q}

$$\mathbf{p} \longleftrightarrow \mathbf{q}$$

- 5. Every one who doesn't like Donald Trump will vote for the other candidate.**

Let "Every one likes Donald Trump" is \mathbf{p}

Let "People will vote for the other candidate" is \mathbf{q}

$$\neg \mathbf{p} \longrightarrow \mathbf{q}$$

Exercise 2. Construct a truth table for each of these compound propositions.

a. $(p \vee q) \rightarrow \neg r$

p	q	r	$p \vee q$	$\neg r$	$(p \vee q) \rightarrow \neg r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	F	T	T	T
T	F	T	T	F	F
F	T	T	T	F	F
F	T	F	T	T	T
F	F	F	F	T	T
F	F	T	F	F	T

b. $(p \rightarrow q) \leftrightarrow (\neg p \vee q)$

p	q	$\neg p$	$p \rightarrow q$	$\neg p \vee q$	$(p \rightarrow q) \leftrightarrow (\neg p \vee q)$
T	T	F	T	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Exercise 3. Are $p \rightarrow (\neg q \vee r)$ and $q \rightarrow (\neg p \vee r)$ logically equivalent?

Prove your statement by using laws and verify by using truth table.

- $p \rightarrow (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ (Involving Conditional Statements)
 $\equiv \neg q \vee (\neg p \vee r)$ (Associative laws)
 $\equiv q \rightarrow (\neg p \vee r)$ (Involving Conditional Statements)

- $p \rightarrow (\neg q \vee r)$ (1)

p	q	$\neg q$	r	$\neg q \vee r$	$p \rightarrow (\neg q \vee r)$
T	T	F	T	T	T
T	T	F	F	F	F
T	F	T	F	T	T
T	F	T	T	T	T
F	T	F	T	T	T
F	T	F	F	F	F
F	F	T	F	T	T
F	F	T	T	T	T

• $q \longrightarrow (\neg p \vee r)$ (2)

p	q	$\neg p$	r	$\neg p \vee r$	$q \longrightarrow (\neg p \vee r)$
T	T	F	T	T	T
T	T	F	F	F	T
T	F	F	F	F	T
T	F	F	F	F	T
F	T	T	T	T	F
F	T	T	F	T	T
F	F	T	F	T	T
F	F	T	F	T	T

(1), (2) $\Rightarrow p \longrightarrow (\neg q \vee r) \equiv q \longrightarrow (\neg p \vee r)$

Exercise 4. Translate these sentences into English where $B(x)$ is "x is a teen boy", $G(x)$ is "x is a teen girl", $V(x)$ is "x likes video games", $L(x, y)$ is "x likes y", and the domain consists of all teenagers.

- a. $\forall x(G(x) \longrightarrow \neg V(x))$
Every teen girl doesn't like video games.
- b. $\exists x(G(x) \longrightarrow V(x))$
There is a teen girl who likes video games.
- c. $\forall x(B(x) \longrightarrow V(x))$
Every teen boy likes teen girl.
- d. $(G(x) \wedge V(x)) \longrightarrow (\forall y B(y) \longrightarrow L(y, x))$
Every teen boy like teen girls who play video games.

Exercise 5. For these sets of premises, what rules of inference used to obtain the conclusion from the premises.

- "There is a person who has read a book but doesn't remember it"
- "Everyone have read a book"
- Conclusion: "There is a person who doesn't remember a book"

Solution:

Let $A(x)$ is "x has read a book"

Let $B(x)$ is "x remember a book"

Premises:

$\exists x(A(x) \wedge \neg B(x))$

$\forall x(A(x))$

Conclusion:

$\exists x(\neg B(x))$

<i>Step</i>	<i>Reason</i>
1. $\exists x(A(x) \wedge \neg B(x))$	Permise
2. $A(e) \wedge \neg B(e)$	Existential instantiation from (1)
3. $\neg B(e) \wedge A(e)$	Commutative laws from (2)
4. $\neg B(e)$	Simplification from (3)
5. $\exists x(\neg B(x))$	Existential generalization from (4)

Exercise 6. Write you own at least 4 premises. Use rules of inferences to get a conclusion from these hypothesis.

- "I feel not good today and the test is really hard"
- "I wil get high score only if I feed good"
- "I get high score or I fail the exam"
- "If I fail the exam, then I will be punished"
- **Conclusion:** "I will be punished"

Solution:

Let p is "I feel good"
 Let q is "The test is hard"
 Let j is "I will get high score"
 Let k is "I fall the exam"
 Let s is "I will be punished"

Permises:

$\neg p \wedge q$
 $j \longrightarrow p$
 $j \vee k$
 $k \longrightarrow s$

Conclusion:

s

<i>Step</i>	<i>Reason</i>
1. $\neg p \wedge q$	Permise
2. $\neg p$	Simplification using (1)
3. $j \longrightarrow p$	Permise
4. $\neg j$	Modus tollens using (2) and (3)
5. $j \vee k$	Permise
5. k	Disjunctive syllogism using (4) and (5)
5. $k \longrightarrow s$	Permise
5. s	Modus ponens using (6) and (7)