# PS2 CS4501

ag9wse

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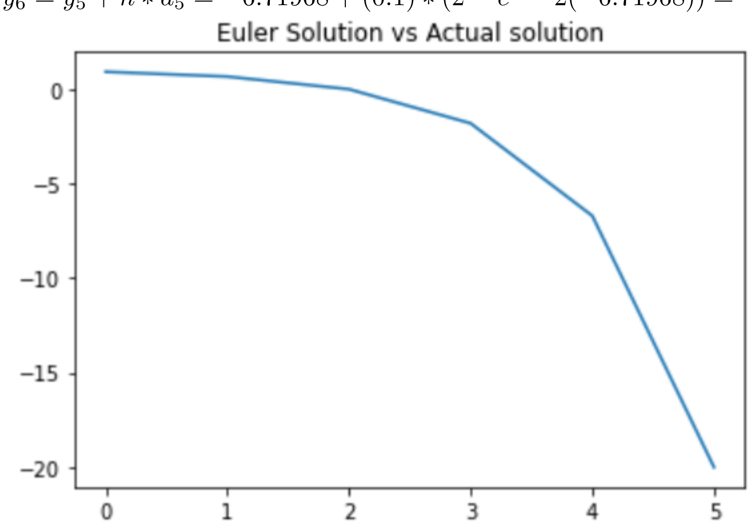
## 1 Problem 1

**1A: Closed Form Solution** $let d_n be (2 - e^{-4t} - 2y_n)$
$y_{n+1} = y_n + h * (2 - e^{-4t} - 2y_n)$
$= y_{n+1} = y_n + h * d_n$

**1B:** In this part, I assume h to be 0.1 and conduct the steps of Euler integration to get the next value. On the graph we have the plot of Euler values, the code for plotting the ODE is in the notebook but does not work on my machine.
$h = 0.1:$
$y_0 = 1, t = 0$
$d_0 = 2 - e^{-4*0} - 2(1) = -1$
$y_1 = y_0 + h * d_0 = 1 + (0.1 * -1) = 0.9$
$y_2 = y_1 + h * d_1 = 0.9 + (0.1)(2 - e^1 - 2(0.9)) = 0.64817$
$y_3 = y_2 + h * d_2 = 0.64817 + (0.1)(2 - e^2 - 2(0.64817)) = -0.02036$
$y_4 = y_3 + h * d_3 = -0.02036 + (0.1)(2 - e^3 - 2(-0.02036)) = -1.82484$
$y_5 = y_4 + h * d_4 = -1.82484 + (0.1) * (2 - e^4 - 2(-1.82484)) = -6.71968$
$y_6 = y_5 + h * d_5 = -6.71968 + (0.1) * (2 - e^5 - 2(-6.71968)) = -20.01705$



**1C:** In this problem, we are experimenting with different h values, as the h values decrease, we see that the curves become less steep and tend to 0, for a higher t. $h = 0.1$ :
$y_0 = 1, t = 0$
$d_0 = 2 - e^{-4*0} - 2(1) = -1$
$y_1 = y_0 + h * d_0 = 1 + (0.1 * -1) = 0.9$
$y_2 = y_1 + h * d_1 = 0.9 + (0.1)(2 - e^1 - 2(0.9)) = 0.64817$
$y_3 = y_2 + h * d_2 = 0.64817 + (0.1)(2 - e^2 - 2(0.64817)) = -0.02036$
$y_4 = y_3 + h * d_3 = -0.02036 + (0.1)(2 - e^3 - 2(-0.02036)) = -1.82484$
$y_5 = y_4 + h * d_4 = -1.82484 + (0.1) * (2 - e^4 - 2(-1.82484)) = -6.71968$
$y_6 = y_5 + h * d_5 = -6.71968 + (0.1) * (2 - e^5 - 2(-6.71968)) = -20.01705$

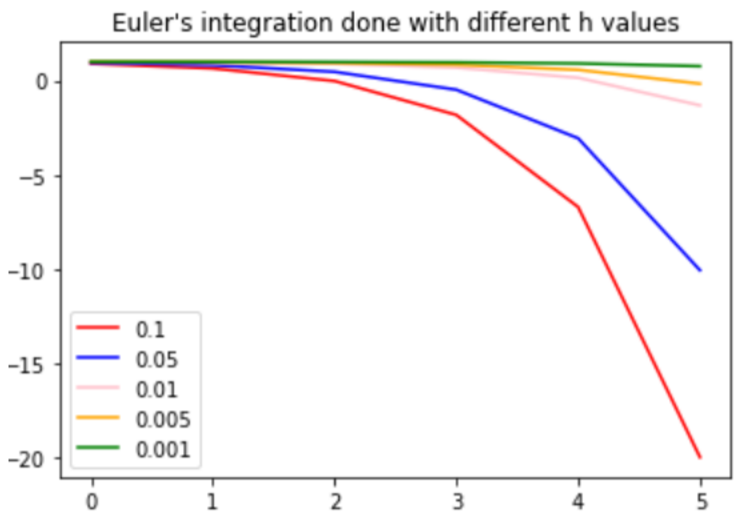
$h = 0.05$ :
$y_0 = 1, t = 0$
$d_0 = 2 - e^{-4*0} - 2(1) = -1$
$y_1 = y_0 + h * d_0 = 1 + (0.05 * -1) = 0.95$
$y_2 = y_1 + h * d_1 = 0.95 + (0.05)(2 - e^1 - 2(0.95)) = 0.81908$
$y_3 = y_2 + h * d_2 = 0.81908 + (0.05)(2 - e^2 - 2(0.81908)) = 0.46771$
$y_4 = y_3 + h * d_3 = 0.46771 + (0.05)(2 - e^3 - 2(0.46771)) = -0.48333$
$y_5 = y_4 + h * d_4 = -0.48333 + (0.05) * (2 - e^4 - 2(-0.48333)) = -3.06490$
$y_6 = y_5 + h * d_5 = -3.06490 + (0.05) * (2 - e^5 - 2(-3.06490)) = -10.079067$

$h = 0.01$ :
$y_0 = 1, t = 0$
$d_0 = 2 - e^{-4*0} - 2(1) = -1$
$y_1 = y_0 + h * d_0 = 1 + (0.01 * -1) = 0.99$
$y_2 = y_1 + h * d_1 = 0.99 + (0.01)(2 - e^1 - 2(0.99)) = 0.96301$
$y_3 = y_2 + h * d_2 = 0.96301 + (0.01)(2 - e^2 - 2(0.96301)) = 0.88985$
$y_4 = y_3 + h * d_3 = 0.88985 + (0.01)(2 - e^3 - 2(0.88985)) = 0.69119$
$y_5 = y_4 + h * d_4 = 0.69119 + (0.01) * (2 - e^4 - 2(0.69119) = 0.15138$
$y_6 = y_5 + h * d_5 = 0.15138 + (0.01) * (2 - e^5 - 2(0.15138)) = -1.31577$

$h = 0.005$ :
$y_0 = 1, t = 0$
$d_0 = 2 - e^{-4*0} - 2(1) = -1$
$y_1 = y_0 + h * d_0 = 1 + (0.005 * -1) = 0.995$
$y_2 = y_1 + h * d_1 = 0.995 + (0.005)(2 - e^1 - 2(0.995)) = 0.981458$
$y_3 = y_2 + h * d_2 = 0.981458 + (0.005)(2 - e^2 - 2(0.981458)) = 0.94469$
$y_4 = y_3 + h * d_3 = 0.94469 + (0.005)(2 - e^3 - 2(0.94469)) = 0.84481$
$y_5 = y_4 + h * d_4 = 0.84481 + (0.005) * (2 - e^4 - 2(0.84481) = 0.57337$
$y_6 = y_5 + h * d_5 = 0.57337 + (0.005) * (2 - e^5 - 2(0.57337)) = -0.16442$

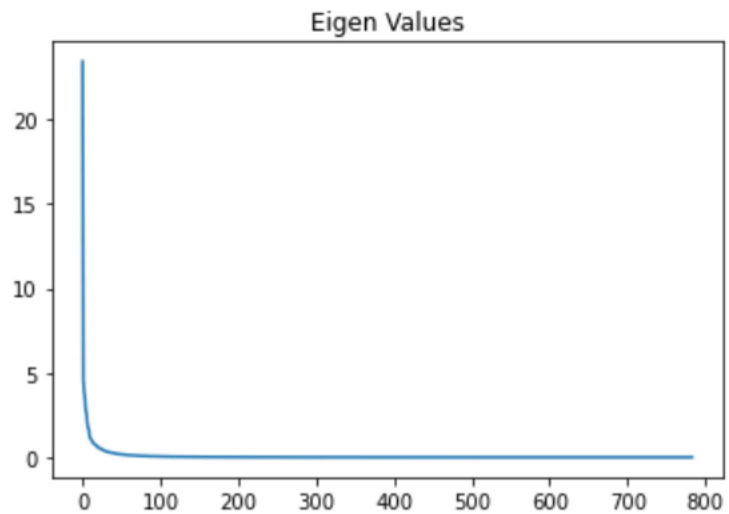
$h = 0.001$ :
$y_0 = 1, t = 0$
$d_0 = 2 - e^{-4*0} - 2(1) = -1$
$y_1 = y_0 + h * d_0 = 1 + (0.001 * -1) = 0.999$

$y_2 = y_1 + h * d_1 = 0.999 + (0.001)(2 - e^1 - 2(0.999)) = 0.99628$
$y_3 = y_2 + h * d_2 = 0.99628 + (0.001)(2 - e^2 - 2(0.99628)) = 0.98889$
$y_4 = y_3 + h * d_3 = 0.98889 + (0.001)(2 - e^3 - 2(0.98889)) = 0.96882$
$y_5 = y_4 + h * d_4 = 0.96882 + (0.001) * (2 - e^4 - 2(0.96882) = 0.91428$
$y_6 = y_5 + h * d_5 = 0.91428 + (0.001) * (2 - e^5 - 2(0.91428)) = 0.76603$

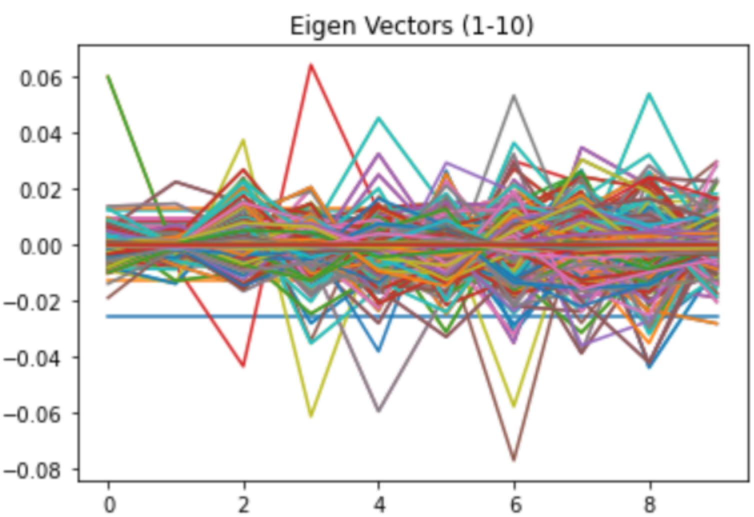



## 1.1 Problem 2:

I used np.linalg.eig to retrieve the eigenvalues and eigenvectors. On plot 1, we have the plot of eigenvalues for the decomposition of the image. Next, we have a plot of the first 10 eigenvectors and finally we have a plot of the 53 principal components chosen to gather about 90% of the data.

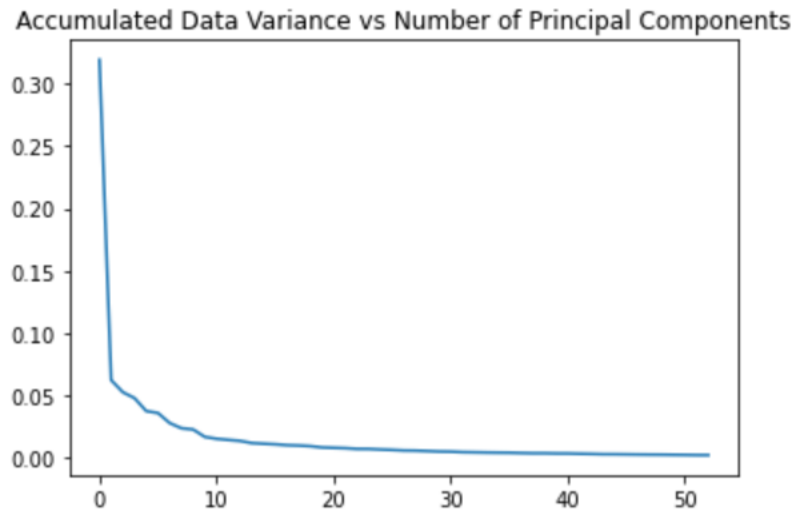



**Eigenvalues**




**Eigenvectors**

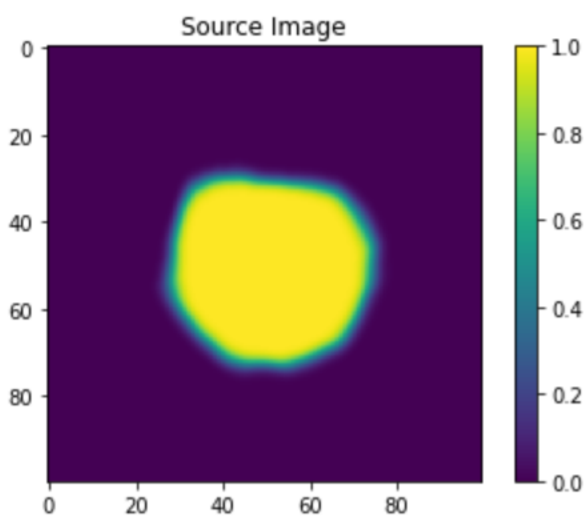
```
Ratio of data selected: (0.9009071274431155+0j)
Number of components selected: 53
```

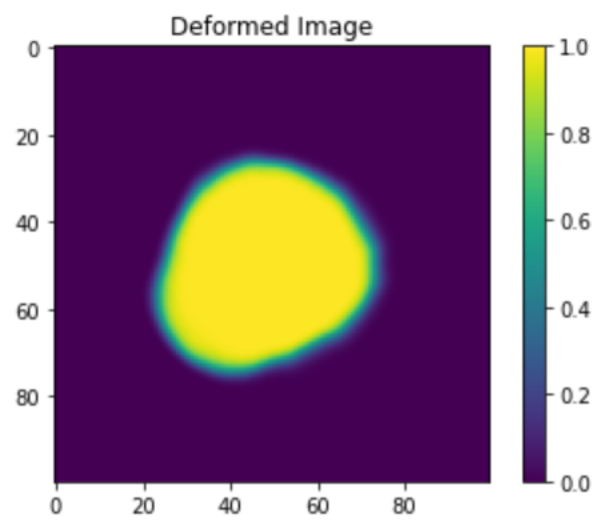


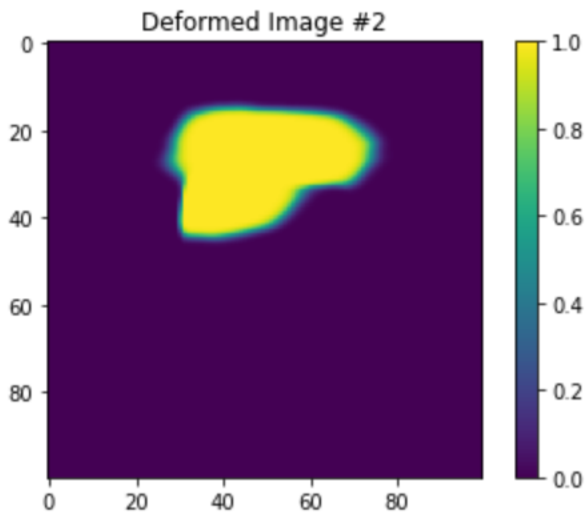

**2C:**

## 1.2 Problem 3:

Here we have the source image, the first deformation and another deformation from subtracting $\phi_0$ from $\phi_1$




Deformed Image #2
0
20
40
60
80
0
20
40
60
80
1.0
0.8
0.6
0.4
0.2
0.0