FEM JOINTS

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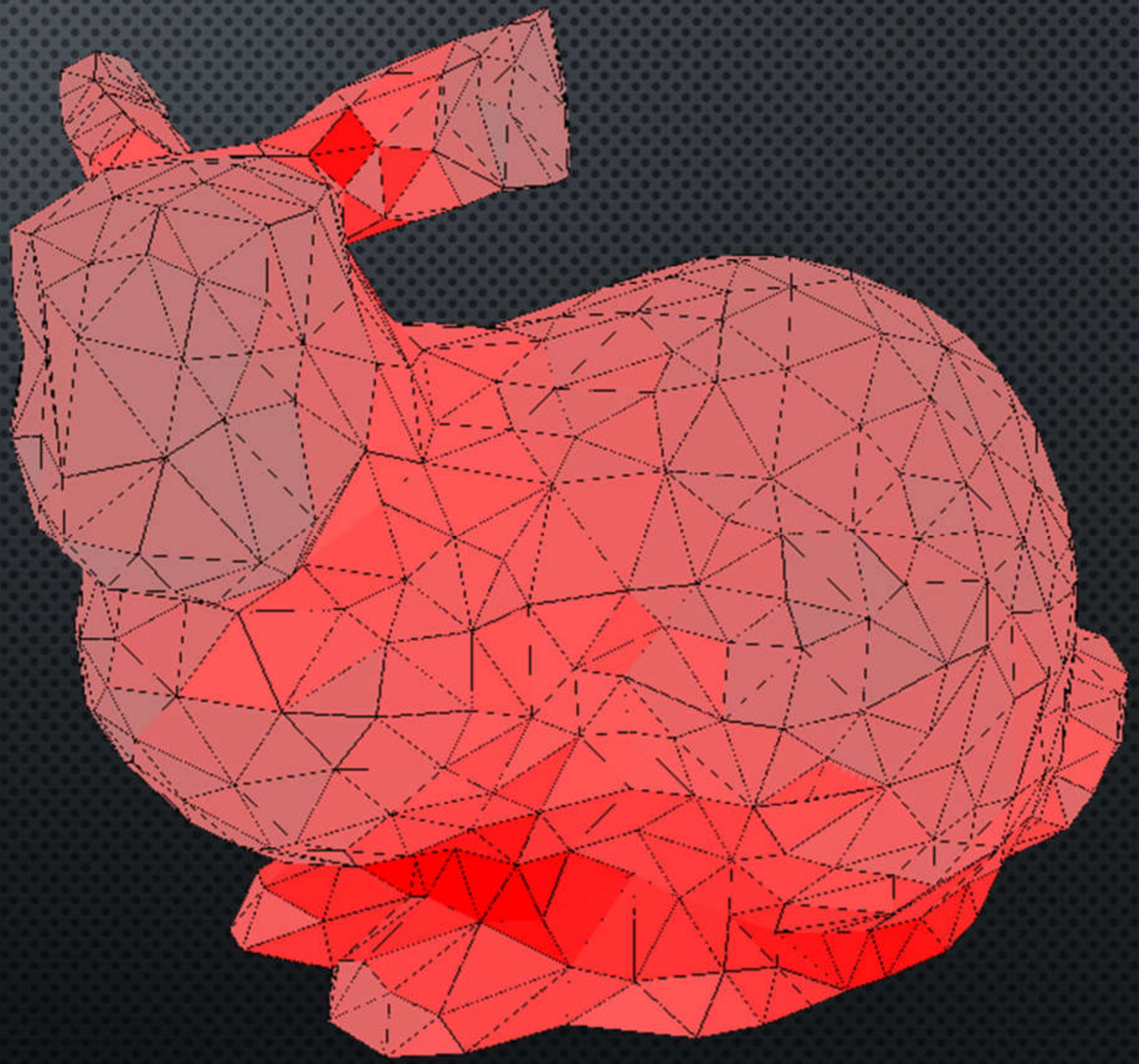
INTRODUCTION

- Real-time, GPU-accelerated FEM
- Coupled with constrained rigid body simulations



OUTLINE

- Mathematical formulation
  - FEM for Elastic Deformations
  - Coupling
  - Damping
  - Plasticity

- Alternative box-MLCP solver
  - Base algorithm
  - Box constraints
  - Reducing complexity
  - Improving convergence

WHY FEM ?

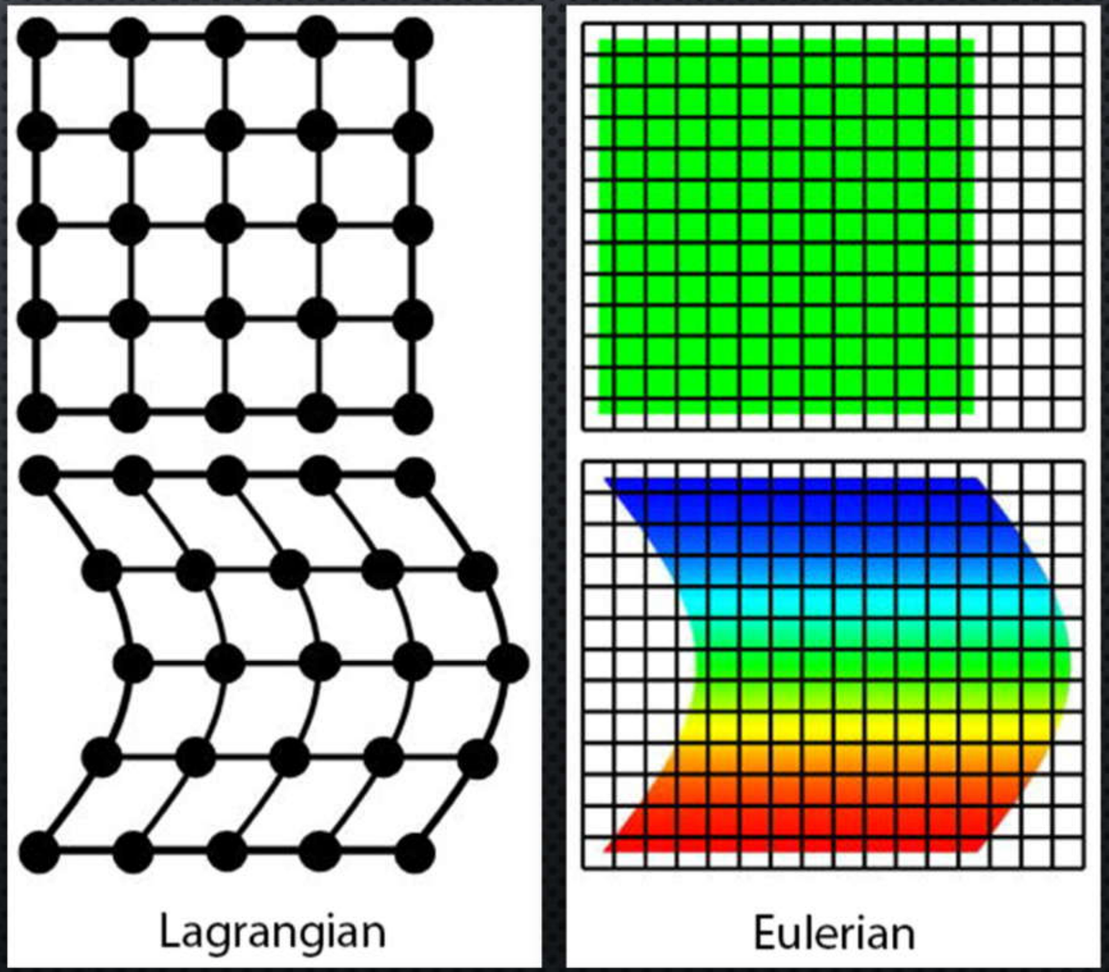
- Better accuracy
- Inherently volumetric
- "Real" material properties
- Basis for advanced materials

THEORY

- STRAIN AND STRESS
- GENERALIZED HOOKE'S LAW

$$\vec{F} = [K]\partial\vec{X}$$

- FINITE ELEMENTS
- LAGRANGIAN VS EULERIAN



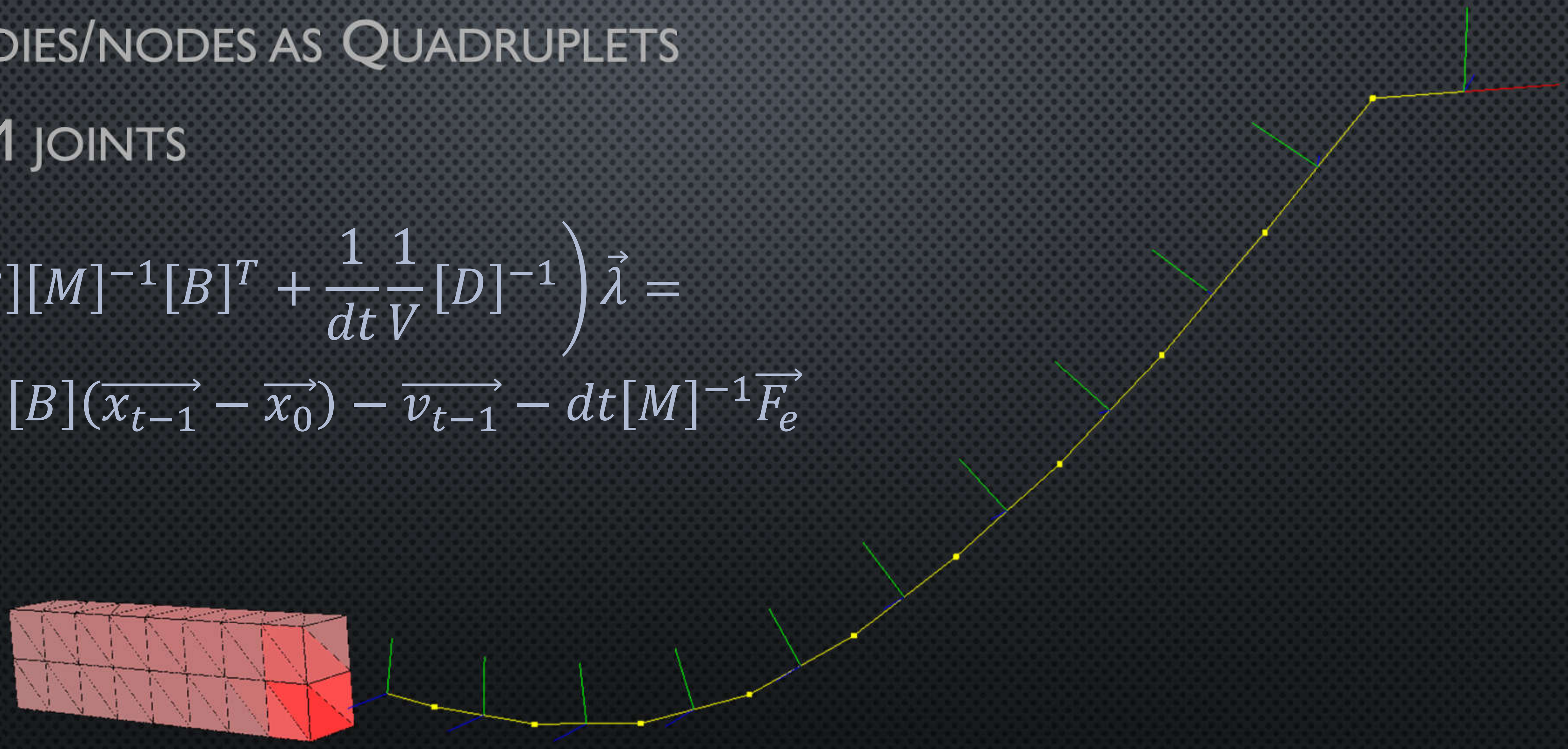
ORIGINAL PICTURE BY S. TAKAGI ET AL.

Coupling with Rigid Body Dynamics

- Bodies/nodes as Quadruplets
- FEM joints

$$dt\left([B][M]^{-1}[B]^{T}+\frac{1}{dt}\frac{1}{V}[D]^{-1}\right)\vec{\lambda}=$$

$$=-\frac{1}{dt}[B](\overrightarrow{x_{t-1}}-\overrightarrow{x_0})-\overrightarrow{v_{t-1}}-dt[M]^{-1}\overrightarrow{F_e}$$



STRAIN TENSOR

- Green Strain Tensor

$$[u]_{ik} = \frac{1}{2}\left(\frac{\partial \overrightarrow{u_i}}{\partial \overrightarrow{x_k}} + \frac{\partial \overrightarrow{u_k}}{\partial \overrightarrow{x_i}} + \sum_{l=1}^{3} \frac{\partial \overrightarrow{u_i}}{\partial \overrightarrow{x_k}} \frac{\partial \overrightarrow{u_k}}{\partial \overrightarrow{x_i}}\right)$$

- Cauchy Infinitesimal Strain Tensor

$$[u]_{ik} = \frac{1}{2}\left(\frac{\partial \overrightarrow{u_i}}{\partial \overrightarrow{x_k}} + \frac{\partial \overrightarrow{u_k}}{\partial \overrightarrow{x_i}}\right)$$

Rotation Artifacts

- Fixed by Stiffness Warping

$$\vec{F} = [R][B]^T[D][B]V[R]^T(\vec{x} - \overrightarrow{x_0})$$

- Rotation Extraction
  - Polar Decomposition

  or

  - Extracting from deterministically chosen basis

References | "Interactive virtual materials", M. Müller et al.

Damping and Stability

- Purely elastic systems → infinite oscillations
- Dissipating Symplectic Euler
- Damping

$$\vec{F} = -k\vec{x} - c\vec{v}$$

- Stability issues due to lack of convergence
- Damping as form of regularization

PLASTICITY

- Residual Deformation
- Decomposing Total Strain

$$[u_{total}] = [u_{elastic}] + [u_{plastic}]$$

- Coefficients: yield, creep, max plastic strain

REFERENCES

"Interactive virtual materials", M. Müller et al.

"Graphical Modeling and Animation of Ductile Fracture", J. O'Brien et al.

Constrained Convex Optimization

- Linear Complementarity Problem
- Common approach: Projected Gauss-Seidel
- Not good for FEM simulation

Alternative: CG-Based Solver

- Modified Proportioning with Reduced Gradient Projections
- Issues with MPRGP
  - Quadratic time complexity
  - Lower bounds only

Gist of the MPRGP

1. Trial CG
2. CG
3. Expansion
4. Proportioning

- Either steps 1-3 or step 4
- Requires Specrtal Radius estimation

References

"Optimal Quadratic Programming Algorithms With Applications to Variational Inequalities", Z. Dostál

INTRODUCING BOX CONSRTAINTS

- Terminology
  - Feasible set
  - Active set
  - Free set

- Modifications
  - Chopped (active set) gradient
  - Reduced free gradient
  - Trial CG step
  - Proportioning step

Pseudocode in the paper

Reducing Complexity

- Common PGS trick: matrix decomposition and sparsity

$$[A] = [J][M]^{-1}[J]^T + [C]$$

- Successive matrix-vector multiplications made linear
- Trick applied to FEM simulation

Improving Convergence

- CG methods depend on condition number
- Simplest preconditioner – Jacobi

$$[A][P]^{-1}[P]\vec{x} = \vec{b}$$

- Violates PSD
- Symmetric Jacobi preconditioner

$$[P_1]^{-1}[A][P_2]^{-1}[P_2]\vec{x} = [P_1]^{-1}\vec{b}$$

$$[P_1] = [P_2] = [diag([A])]^{\frac{1}{2}}$$

- Near-singular matrices and regularization

GPU Acceleration

- PGS methods are sequential by nature
- Parallel PGS often jitter
- Mass-Splitting solves jittering
- MS convergence is insufficient for FEM simulation
- But CG is parallel by nature

References

"Mass Splitting for Jitter-Free Parallel Rigid Body Simulation", R. Tonge et al.

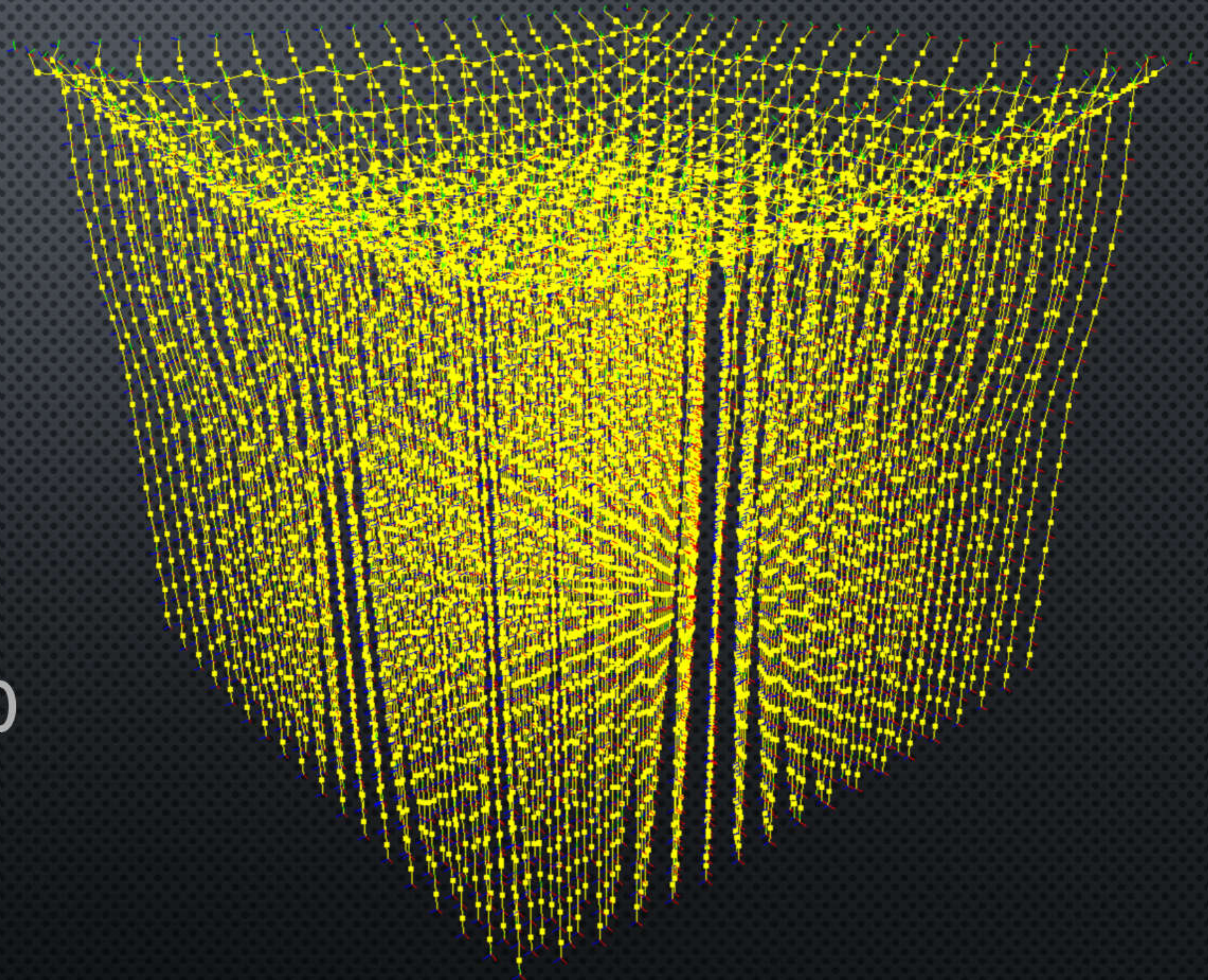
CG-BASED METHODS: DRAWBACKS

- Computational cost
  - More complex iteration
  - Estimating Spectral Radius
  - Addressed by GPU-accelerations

- MPRGP "noisy" on low iterations *vs* PGS

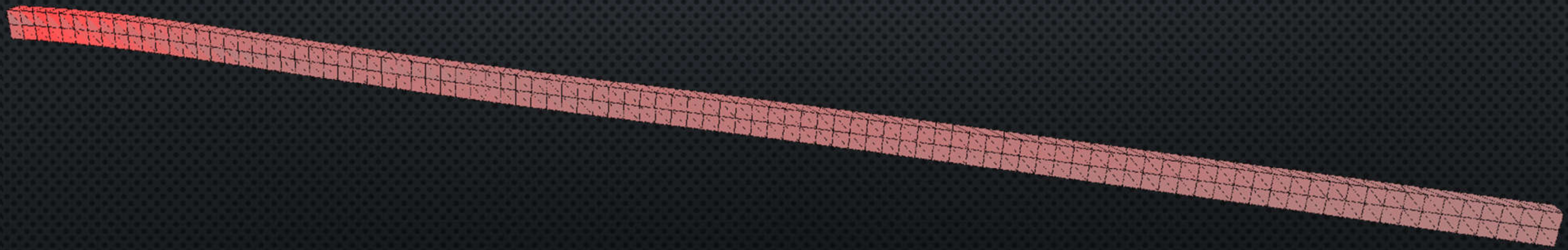
RESULTS: MASSIVE BALL-JOINT GRID

- > 20K ROWS
- MPRGP:
45MS, 140 ITERATIONS MAX
- PGS:
3110MS, 840 ITERATIONS MAX
- NVIDIA GEFORCE GTX 260
VS INTEL CORE I7-920



Results: FEM Stress Test

- > 10K rows
- MPRGP: 50ms, 220 iterations max
- PGS: > 6s, > 2000 iterations max
- NVIDIA GeForce GTX 260 *vs* Intel Core i7-920



Future Work

- Incorporating Fluid Simulation
- SPH-based continuum mechanics
- Advanced materials

References

"Constraint Fluids", K. Bodin et al.

"Point Based Animation of Elastic, Plastic and Melting Objects", M. Müller et al.

THANK YOU!