

Pitch Class Sets

Paul Nelson - Revised: 1/20/2007

Pitch Class Sets are a method for describing harmonies in 20th century music. These notations and methods can describe and manipulate any type of chord that can be created within a 12-tone (equally tempered) scale. It is an extremely useful technique for composers to help understand and control the harmonies which make up their music.

Pitch class sets are the chemistry of harmonic color. Modern composers will use pitch class sets like chemistry, to mix and create interesting and vibrantly colorful harmonic sounds, which they then use to create works of music.

1 Basic Definitions

1.1 Pitches

- A "pitch" is any note that we hear.
- The standard piano can play 88 pitches: A0 to C8, where C4 = middle-C.
- For example, the notes above middle-C are as follows:
C4 (B \sharp 3), C \sharp 4 (D \flat 4), D4, D \sharp 4 (E \flat 4), E4 (F \flat 4), F4 (E \sharp 4),
F \sharp 4 (G \flat 4), G4, G \sharp 4 (A \flat 4), A4, A \sharp 4 (B \flat 4), B4 (C \flat 5)

1.2 Pitch Classes

- Pitch classes are used to discuss pitches *independent of octave displacement and enharmonic spelling*.
- Any two pitches which sound the same on an equal tempered scale (for example, C \sharp and D \flat) or are only different due to octave displacement are said to belong to the same "pitch class".
- For example, the following *pitches* all belong to *Pitch Class C*:
B \sharp 4, D \flat 4 (enharmonic equivalents), C0, C1, C2, C3, C4, C5, C6 (octave displacements)
- There are only 12 pitch classes in a system where each octave has 12 chromatic notes.
- Pitch classes can also be numbered: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.
These numbers are sometimes called "Pitch Class Representatives".
- For this tutorial, 0 = Pitch Class C (i.e. "fixed Do").
- All other pitch classes will be numbered by counting the half steps from pitch-class C.
- Therefore, C = 0, C \sharp = 1, D = 2, D \sharp = 3, E = 4, F = 5, F \sharp = 6, G = 7, G \sharp = 8, A = 9, A \sharp = 10, B = 11
- Sometimes the letter 'T' (for Ten) or 'A' is used instead of the number 10, and 'E' (for Eleven) or 'B' instead of 11.

1.3 Pitch Class Sets

- A "Pitch Class Set" is a list of pitch class numbers: [0, 4, 7, 10] (note the square brackets)
- These are also called "PC Sets".
- The PC Set for a C minor triad: [0, 3, 7]
- The PC Set for a G major triad: [7, 11, 2]
- In Pitch Class sets, octave doublings and displacements are ignored:
 - [0, 3, 7, 12] => [0, 3, 7] (see the section below on modulo math)
 - [14, 7, 11] => [2, 7, 11]
 - For example, all of the following can be described with Pitch Class Set [0, 1, 4]. The only difference in these chords are octave displacements or enharmonic spellings in the pitches.

Piano

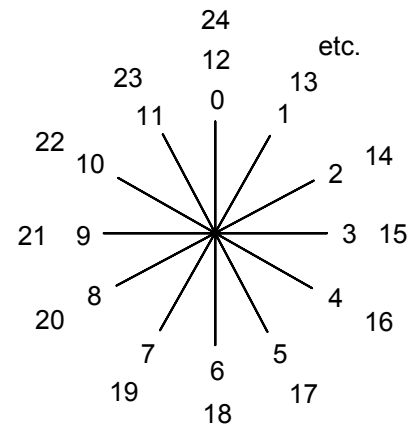
[0,1,4] [0,1,4] [0,1,4] [0,1,4] [0,1,4] [0,1,4] [0,1,4]

- Oftentimes PC Set notation is shown without the commas: [037]
(here is where A=T=10 and B=E=11 comes in handy, for example:
[0,4,7,10] = [047T] = [047A] (note: this is a C dominant 7th chord)

2 Simple Operations on Pitch Classes and Pitch Class Sets

2.1 "Clock" Math or Modulo Math

- When manipulating pitch classes, you will use a special operator, called the "modulo" operator.
- The "modulo" operator takes the remainder of an integer divided by some other integer.
- For example: $19 \text{ modulo } 12 = 7$
(i.e. 12 goes into 19 once, with 7 left over)
- Pitch class sets use "modulo 12". Any number above 12 should be reduced, using "mod 12", to a number from 0 to 11.
- The modulo operator can be visualized using a clock face:
- Some interesting characteristics of the clock face:
 - A tritone is made up of two notes which are opposite of each other (for example: $C = 0$ and $F^\sharp = 6$)
 - The notes of a cross make up a doubly-diminished 7th chord (for example: $C = 0$, $D^\sharp = 3$, $F^\sharp = 6$, $A = 9$)
 - The augmented triad ($C = 0$, $E = 4$, $G^\sharp = 8$) is also pleasingly symmetric.




2.2 Transposing Pitch Class Sets

- To transpose a pitch class set, add (or subtract) the same number to all elements of the list:
 $[0, 1, 4] \Rightarrow (\text{transpose up a major third}) [0+4, 1+4, 4+4] \Rightarrow [4, 5, 8]$
 In this example, the chord "C D \flat E" is transposed up to "E F G \sharp ".
- Remember to use "Module 12" when numbers are greater than or equal to 12:
 $[0, 1, 4] \Rightarrow (\text{transpose up a major 7}^{\text{th}}) [0+11, 1+11, 4+11] \Rightarrow [11, 12, 15] \Rightarrow [11, 0, 3]$

2.3 Inverting Pitch Class Sets

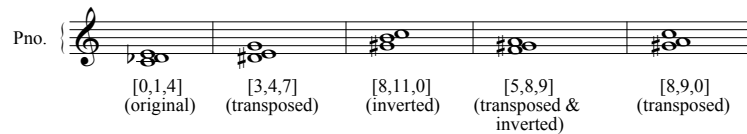
- To invert a PC Set, subtract each element of the list from 12:
 $[0, 1, 4] \Rightarrow [12 - 0, 12 - 1, 12 - 4] \Rightarrow [12, 11, 8] \Rightarrow [0, 11, 8]$
 (don't forget to use Mod 12 if any of the numbers are greater than 11)
 For example: The chord "C D \flat E" becomes "C B A \flat ".
- By convention, simple inversion is always around Pitch Class C (0). Therefore, any note of the chord which is N half-steps above C, will be flipped to be come a note N half-steps below C. In the above example, the note "E" (4 half-steps above C) was flipped to become "A \flat " (4 half-steps below C).
- Very often you will want to invert and transpose at the same time:
 $[0, 1, 4] \Rightarrow [(12-0) + 4, (12-1) + 4, (12-4) + 4] \Rightarrow [16, 15, 12] \Rightarrow [4, 3, 0]$
 This has a special notation: T4I (invert and then transpose up 4 half steps)
- Examples of the PC Sets shown above:

Pno. 

3 The Prime Form

3.1 Similar Pitch Class Sets: Set Classes & Prime Forms

- Some pitch class sets are very similar, for example: [0,1,4] is very similar to [3,4,7] (transposition), [8,11,0] (inversion), [5,8,9] (transposition and inversion), and [8,9,0] (transposition).
- For example, try playing the following chords. Can you hear that they all have something in common?



- A group of similar PC Sets like these is called a "Pitch Class Set Class", or more simply, a "Set Class".
- If two PC Sets differ only by transposition or inversion, then they belong to the same Set Class.
- There are only 208 different Set Classes!
- Each Set Class is represented by a "Prime Form" PC Set. For example: [0,1,4]; [3,4,7]; [0,3,4]; [5,8,9]; and [8,9,0] all belong to the Prime Form: (0,1,4)
- Note that parenthesis are used to denote Prime Forms in this tutorial. However, not everybody agrees on this syntax.

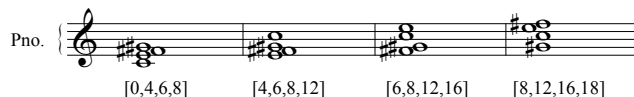
3.2 Uses for The Prime Form

- The prime form is considered to be the "simplest" version of the pitch class set.
- Generally, the "simplest" version of a PC set means that the pitches in the set are packed as tightly together possible, and as far to the left as possible.
- Once you know the prime form of a PC set, you can look it up in a table of prime forms to get more information about the PC Set, such as its interval vector and fellow related PC Sets (see appendix).
- You can also use the prime form to search for other, related PC Sets using other software tools. See <http://www.ComposerTools.com>.
- If you are a composer, you can use this information to help you better control, understand, and manipulate the harmonies in your music.

3.3 Determining the Prime Form: The Rigorous Method

- Goal: To identify the prime form for any PC set.
- Example: What is the prime form of [8,0,4,6] ?
- Step 1: Put the Pitch Classes in numerical order => [0,4,6,8]
- Step 2: List all of the rotations of the pitch class set. To rotate a PC Set, simply move the first number to the end and add 12 to it (i.e. shift it up an octave). For example, the Rotations of [0,4,6,8] are:

[0, 4, 6, 8]
[4, 6, 8, 12]
[6, 8, 12, 16]
[8, 12, 16, 18]



(note: numbers over twelve shown to demonstrate rotation, these should be reduced to 0-11 with "Modulo 12")

- Step 3: Determine which rotation of the PC Set has the minimum distance between the first and last numbers in the Set:
 - [0, 4, 6, 8] => (8 - 0) = 8
 - [4, 6, 8, 12] => (12 - 4) = 8
 - [6, 8, 12, 16] => (16 - 6) = 10
 - [8, 12, 16, 18] => (18 - 8) = 10

There is a tie! Versions [0,4,6,8] and [4,6,8,12] both have a minimum distance between first and last of 8

- Step 4: If there is a tie, choose the rotation which has a minimum distance between the first and second numbers:

Distances between the first and second numbers:

- $[0, 4, 6, 8] \Rightarrow (4 - 0) = 4$
- $[4, 6, 8, 12] \Rightarrow (6 - 4) = 2$

So, in our example, $[4,6,8,12]$ is preferred.

- Step 5: If there is still a tie, then check the first and third numbers, and so on until the tie is resolved.

The PC Set at this point is in "Normal" form.

- Step 6: Transpose the pitch class set so that the first number is zero:

$[4 - 4, 6 - 4, 8 - 4, 12 - 4] \Rightarrow [0, 2, 4, 8]$

- Step 7: Invert the pitch class set and reduce it using steps 1-5 above.

- Invert $[0,2,4,8] \Rightarrow [12-0, 12-2, 12-4, 12-8] \Rightarrow [12, 10, 8, 4] \Rightarrow [0, 10, 8, 4]$
- Put in numerical order: $[0, 4, 8, 10]$
- Find the best rotation:

PC Set	(last-first)	(second-first)
$[0, 4, 8, 10]$	10	4
$[4, 8, 10, 12]$	8	4
$[8, 10, 12, 16]$	8	2 << Preferred
$[10, 12, 16, 20]$	10	2

- Transpose down: $[8 - 8, 10 - 8, 12 - 8, 16 - 8] \Rightarrow [0, 2, 4, 8]$

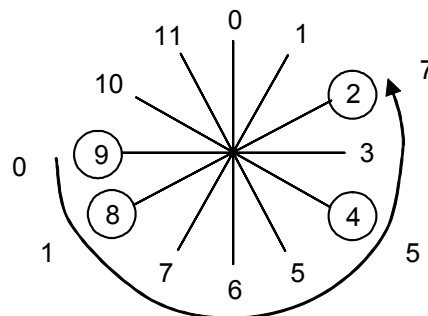
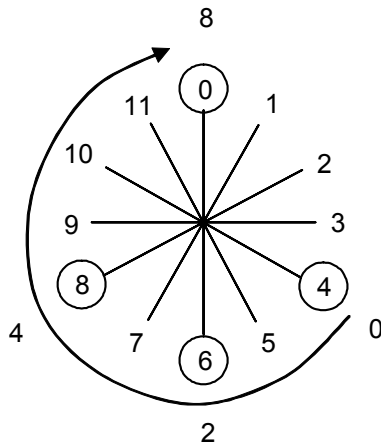
- Step 8: Which form, the original or the inverted, is most packed to the left (has the smallest numbers)? That will be the Prime Form.

In our example, both forms produced the same Prime Form (this is because the original PC Set was "inversionally symmetric"), and so the Prime Form is $(0, 2, 4, 8)$

3.4 Determining the Prime Form: Easier Methods

- **Option 1:** Use an online tool at <http://www.composertools.com>.
- **Option 2:** Figure it out on the piano
 - Step 1: Keep rotating your chord until it is as small as possible.
 - Step 2: If there are ties, then use the rotation that has the notes most packed towards the bottom.
 - Step 3: Check to see if the inversion is better packed.
- **Option 3:** Use the "Simplified Set List" at the back of *Post Tonal Theory* by Joseph N. Straus.
- **Option 4:** Use a MAX/MSP patch which displays the Prime form of a chord you play on your MIDI keyboard. See the URL: <http://www.euph0r1a.net/projects/?handler=etrof>.
- **Option 5:** Use the table of all prime forms. For example, 1) Find the interval vector first, then look it up in the table of all PC Sets (see below), or 2) skip steps 6 & 7 above and look up the inversion in the table.
- **Option 6:** Visualize the Pitch Class Set on a clock face and locate the prime form visually
 - Step 1: The shortest distance traveled around the clock.
 - Step 2: Numbers packed as close to the starting point as possible.

For example, the prime form of $[0,8,6,8]$ is $(0,2,4,8)$; and the prime form of $[2,4,8,9]$ is $(01,15,7)$:



4 Interval Vectors

An "Interval Vector" is a list of six numbers which summarizes the interval content in a PC Set. With a little experience, you will be able to get a sense for how a PC Set sounds when you see its interval vector. Further, once you know the interval content of a PC Set, you will also be able to manipulate the sound of the PC Set by inversion and octave displacement of pitches to emphasize certain intervals over others.

4.1 Pitch Intervals

- The distance between any two pitches is called a "pitch interval". This is the standard definition for an interval in music. For example:
- Ordered Intervals:

Ab3 to Db5 = Perfect 11th ascending = +17 half steps
 Db5 to Ab3 = Perfect 11th descending = -17 half steps

- Un-ordered Intervals:

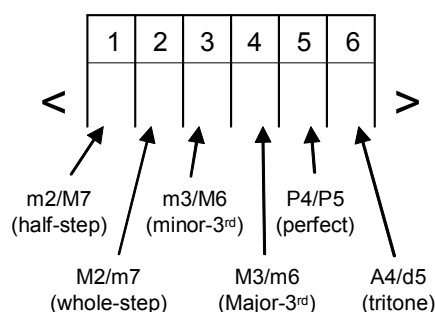
Ab3 to Db5 = Perfect 11th = 17 half steps
 Db5 to Ab3 = Perfect 11th = 17 half steps

4.2 Interval Classes

- In the same way that many *pitches* "sound alike" and are therefore put into the same *Pitch Class*, there are also many *intervals* which sound alike and so are put into the same *Interval Class*.
- There are six different interval classes which are numbered from 1 to 6.
 - m2 / M7 => 1 (half-steps)
 - M2 / m7 => 2 (whole-steps)
 - m3 / M6 => 3 (minor thirds)
 - M3 / m6 => 4 (major thirds)
 - P4 / P5 => 5 (perfect intervals)
 - A4 / d5 => 6 (tritones)
- The interval class number (1 to 6) is the count of half steps between two pitch classes. In other words, it is the minimum distance between two pitches ignoring the octave displacement of either pitch.
- For example, in the case of Ab3 to Db5, if you moved Ab3 up an octave to Ab4, then the distance between the two is a perfect 4th. And so the interval class is a '5', for a perfect interval.

4.3 Interval Vectors

- An *Interval Vector* is a summary of all of the intervals between all pairs of pitches in a pitch class set. It is, essentially, a histogram of all of the interval classes which can be found in a PC Set.
- For the purposes of this tutorial, an interval vector will be represented with angle brackets as follows:



- For example, a C major chord is represented by the PC Set: [047] and has the interval vector <001110>. This is because a C major chord contains one minor third (from E to G) one major third (from C to E) and one perfect interval from C to G). Since a major chord contains no half steps, whole steps, or tritones, these entries in the interval vector are all set to zero (0).
- Note that there is no agreed upon standard punctuation for representing an interval vector. The angle brackets appear to be the most common, but there are many other representations being used.

How to compute an interval vector:

- Step 1: Go through all pairs of pitches in your PC Set.

If your PC Set has:	It will contain:
2 pitches	1 interval
3 pitches	3 intervals
4 pitches	6 intervals
5 pitches	10 intervals
6 pitches	15 intervals

Overall, the formula for computing the number of intervals (Ni) from the number of pitches (Pi) is:

$$\text{Number of Intervals} = (N*(N-1)) / 2$$

- Step 2: For each pair, subtract the smaller number from the larger number.
- Step 3: Take the result of step 2 and increment the appropriate slot in the interval vector using the chart on the right:

11	10	9	8	7	
1	2	3	4	5	6

- Example: [0, 2, 7, 8]

- Step 1: 4 pitches in the pitch class set = 6 intervals = 6 pairs of pitches:
[0, 2] [0, 7] [0, 8] [2, 7] [2, 8] [7, 8]
- Step 2: For each pair, subtract the smaller number from the larger number:
[0, 2] = 2; [0, 7] = 7; [0, 8] = 8; [2, 7] = 5; [2, 8] = 6; [7, 8] = 1
- Step 3: For each difference in Step 2, increment the appropriate slot from the chart above:

11	10	9	8	7	
1	2	3	4	5	6
1	1	0	1	2	1

- Therefore, for our example, the interval vector is: <110121>
- This means that the Pitch Class Set [0, 2, 7, 8] contains the following interval classes:
1 half-step, 1 whole-step, 1 major third, 2 perfect intervals, and 1 tritone
- When I listen to this PC Set [0278], what I hear is a triad based on perfect fifths (0,2,7) = <010020> with an additional pitch (8) that adds some significant 'bite', via the half-step and tritone dissonance.

5 The Table of All Prime Forms of PC Sets

Please refer to the Appendix for a two-page table of all possible prime forms of Pitch Class Sets. This table is an indispensable aid for composers, since it is, essentially, a table of all possible types of chords. Not only does it contain all of the standard chords from tonal harmony such as triads (major, minor, diminished, and augmented) and seventh chords (dominant, major-minor sevenths, major-major sevenths, minor sevenths, etc.), but it also contains all chord types used by modern composers as well. Any chord which can be constructed using a 12-tone equal tempered scale is represented in the table.

5.1 The Columns of Data in the Table

For each prime form in the table, there are five columns of data:

- Column 1: The interval vector
- Column 2: The count of PC Sets which reduce to the prime form
- Column 3: The Forte code (see below)
- Column 4: The Prime Form PC Set
- Column 5: The inverted form (if different than the Prime Form)

5.2 The Layout of the PC Sets in the Table

- The PC Sets are grouped in the table by size, into 13 sections (from 0 pitches to 12 pitches per PC Set).
- Within each group the list is sorted by interval vector. Interval vectors with the most half-step intervals are listed first, then vectors with the most whole-step intervals, and so on.
- Z-related forms are listed together, one after the other (see section 5.4)
- Commonly known pitch class sets (e.g. well-known chord qualities, types of scales, etc.) are labeled with {curly braces}. For example, (0, 3, 7) is labeled as {min} because it is a minor triad.
- With the exception of the sets of 6 Pitch Classes, each set is listed opposite of its "complement". For example, set 4-16, (0,1,5,7) is listed to the left of set 8-16, (0,1,2,3,5,7,8,9). A set and its complement share many similar properties (see below for a discussion of Pitch Class Set complements).
- To conserve space, the table uses the letters A, B, and C for the numbers 10, 11, and 12.

5.3 Forte Names

- Allen Forte's book, *The Structure of Atonal Music*, published the first version of this table. In his table, he labeled each prime form of the PC Set with a unique designation, such as 5-20.
- The first number (5-) specifies the number of pitches in the pitch class set.
- The second number (20) is a unique number given to the prime form, which was sequentially assigned by Dr. Forte when he first created the table.
- When analyzing PC Sets, many music theorists will label them using the Forte designation, although simply using the prime form (e.g. (0,1,3,7,8) or (01378)) is becoming more common.

5.4 Z-Related Sets

- When two prime forms produce the same interval vector, and when one can not be reduced to the other (by inversion or transposition), they are said to be "Z-Related", or "Z Correspondents".
- The Forte Code for all PC Sets which are Z related contains a 'Z' in the PC Set ID. For example, 6-Z25.
- 'Z' doesn't stand for anything, it is just an identifier chosen by Dr. Forte when the table was first created.
- Z-related sets are "close cousins" to one another. They sound similar to each other, but not as similar as sets related by (say) transposition or inversion. For example, try playing the following PC Sets on the piano. Listen for the intervals they contain. Since the Z-related sets contain the same intervals, do they not sound at least somewhat similar?

Pno.

4-Z15	4-Z29	4-Z15	4-Z15	4-Z29	4-Z15	4-Z29	4-Z29
[0,1,4,6]	[0,1,3,7]	[0,1,4,6]	[1,3,6,7]	[6,7,9,1]	[0,1,4,6]	[9,10,0,4]	[0,4,6,7]

5.5 Other Comments on the Table

- When I first encountered the table, I was surprised that it contained so few interval vectors (200), prime forms (208) and chord qualities (351). For some reason, in my mind, I had always thought that the complete list of possible chord types was much much larger.
- Along the same lines, the number of chord types used by in common practice music is quite small, as few as a dozen different types chords, perhaps as many as 20 if you include Jazz chords.
 - This implies that there is a very number of chords yet to be thoroughly explored!
- The following music shows some very famous chords. With out PC Sets, how could the types of these chords be specified?

Pno.

7-32: (0,1,3,4,6,8,9)	7-Z36: (0,1,2,3,5,6,8)	4-19: (0,1,4,8)
Stravinsky: Rite of Spring	Stravinsky: Rite of Spring	Bernard Herrmann: "Psycho" Prelude

6 Subsets and Supersets

Any of the larger PC Sets can be divided into pieces. These pieces are, of course, also PC Sets in their own right. The smaller PC Sets are said to be "subsets" of the larger PC Set, which is the "superset".

- For Example, the superset (0,1,2,6,7,8) is quite dissonant and has the interval vector <420423>. It contains the following subsets:

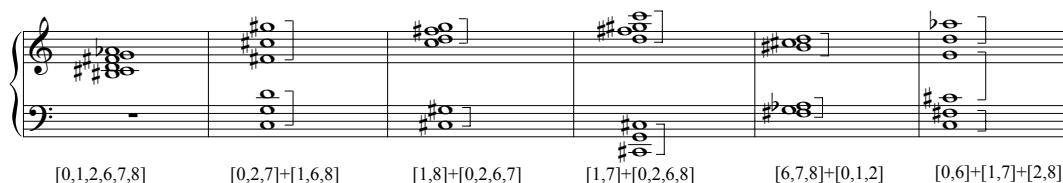
Subsets 1: => [0,2,7] + [1,6,8] = two quintal/quartal triads

Subsets 2: => [1,8] + [0,2,6,7] = a simple fifth + a complex chord (a dominant+tonic sound)

Subsets 3: => [1,7] + [0, 2, 6, 8] = a tritone + whole-tone-scale fragment

Subsets 4: => [6,7,8] + [0,1,2] = two chromatic clusters

Subsets 5: => [0,6] + [1,7] + [2,8] = three tritone intervals



PC subsets and supersets are a very useful compositional technique. Be sure to explore all of the subsets for PC Sets that you use (see ComposerTools.com). This will help you to use, space, and manipulate your harmonies.

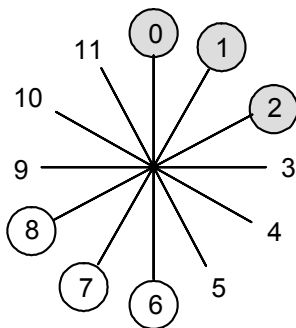
- Other things to experiment with:
 - Use subsets for growth; i.e. restrict sections of your music to use only portions of a larger PC set and then grow the PC set over time, making your harmonies denser and more complex.
 - Put the sub-sets in different registers to emphasize their unique sounds (see examples below).
 - Construct melodies from sub-sets which can be combined together to create

6.1 Definition: Transpositional Combination of Two Common Subsets

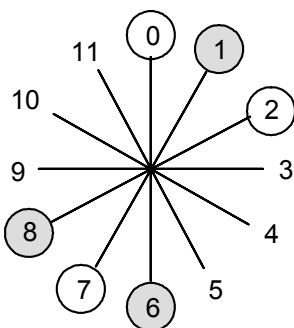
- Transpositional combination: When a superset is created from two equal subsets, where one is transposed.
- Example 1: [0,1,2] + [0,1,2]{transposed by 6 halfsteps} => [0,1,2] + [6,7,8] => [0,1,2,6,7,8]
- Example 2: [0,2,7] + [0,2,7]{transposed by 6 halfsteps} => [0,2,7] + [6,8,1] => [0,1,2,6,7,8]

6.2 Definition: Inversional Combination of Two Common Subsets

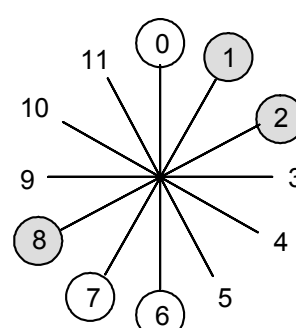
- Inversional combination: When a superset is created from two equal subsets, where one is inverted (and possibly transposed)
- Example: [0,1,6] + [0,1,6]{invert and transpose by 8 half steps} => [0,1,6] + [12-0+8, 12-1+8, 12-6+8] => [0,1,6] + [8,19,14] => [0,1,6] + [8,7,2] => [0,1,2,6,7,8]
- Note: The result of an inversive combination will always be "inversionally symmetric" (see below for a discussion of inversive symmetry)



Transpositional Combination



Transpositional Combination



Inversional Combination

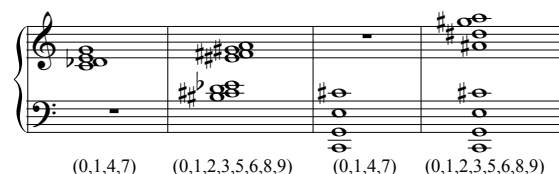
7 PC Set Complements

7.1 Definitions

- *Literal Complement*: When one PC Set contains all of the Pitch Classes not in some other PC Set.
Example: [0,1,4,7] and [2,3,5,6,8,9,10,11] are literal complements of each other
- *Abstract Complement*: When two PC Sets would be complements of each other, except that one is transposed or inverted from the other. When someone says that a PC Set is the *complement* of some other PC Set, it usually means that they are Abstract Complements of each other.
Example: (0,1,4,7) and (0,1,2,3,5,6,8,9) are abstract complements of each other
- The prime forms of abstract complements are listed side-by-side in the PC Set table found in the Appendix (except for the sets of 6 pitch classes).
- Note that the Forte designation for a PC Set and its complement will always have the same PC Set ID number (after the dash). For example, 4-18 and 8-18 are abstract complements of each other.

7.2 PC Set Complements and Their Interval Vectors

- A Pitch Class Set and its complement will have very similar interval vectors.
- In fact, there is a simple formula for computing the interval vector of a complement:
 - How many **more** pitch classes does the complement have? Call this 'D'.
 - Note: If the original PC set has X pitch classes, its complement will have (12-X) pitch classes, and the difference between the two will be: $D = (12-X) - X = (12-X*2)$
 - For example, if the original PC set has 5 pitches, the complement will have (12-5) pitches (i.e. 7 pitch classes) and the difference (D) between 5 and 7 is $(12-5*2) = 2$.
 - If the interval vector for the original PC Set is $\langle I_1, I_2, I_3, I_4, I_5, I_6 \rangle$
 - Then the interval vector for the complement will be:
 $\langle I_1+D, I_2+D, I_3+D, I_4+D, I_5+D, I_6+(D/2) \rangle$
 - Note that the tritone is special because it divides the 12-tone chromatic scale exactly in half. For this reason, its interval vector grows by D/2.
 - Also note that D is always an even number {0, 2, 4, 6, 8, 10}, and so D/2 will always be an integer number (never a fraction).
- Example:
 - 4-18:(0,1,4,7) has 4 pitch classes and an interval vector of $\langle 102111 \rangle$
 - Its complement is 8-18:(0,1,2,3,5,6,8,9)
 - $8 - 4 = D = 4$
 - The complement's interval vector is: $\langle 1+4, 0+4, 2+4, 1+4, 1+4, 1+(4/2) \rangle = \langle 546553 \rangle$

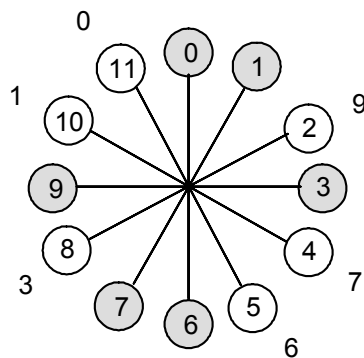


- Some famous complements:
 - Pentatonic Scale (5 Pitches) : $\langle 032140 \rangle \Leftrightarrow$ Diatonic Scale (7 Pitches) : $\langle 254361 \rangle$
 - Octatonic Scale (8 Pitches) : $\langle 448444 \rangle \Leftrightarrow$ doubly-diminished 7th chord (4 Pitches) : $\langle 004002 \rangle$

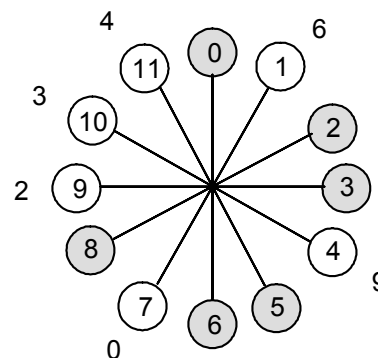
7.3 6-note complements

- The complement of a set with 6 Pitch Classes will itself have 6 Pitch Classes
- Therefore, the difference in number of Pitch Classes is always 0 (zero).
- Therefore, a 6-note complement will always have the same interval vector as its complement!
- True!

- Since all PC Sets with 6 pitches have a complement with the same interval vector, there are only two ways that one of these PC Sets can be related to its complement:
 - The set is "self complementary", that is, the set and its complement have the same prime form.
 - The set and its complement are Z-related: Two sets with the same interval vector but which can not be reduced to the same Prime Form by transposition or inversion.



Self-Complementary Set {6-30}
with 6 Pitch Classes



Z-Related Sets with 6 Pitch Classes

6-Z23

6-Z45

7.4 PC Set Complements Used in Twelve Tone Composition

- PC Set complements are critically important when composing music with 12-tone rows, because:
 - If you take any 12-tone row and divide it up into two pieces at any point, then
 - the two pieces will have similar (or exactly the same) interval content.
- This is one of the reasons why a 12-tone composition has a "built-in" harmonic cohesiveness.
- For example, consider the following 12-tone row:

Schoenberg, Op. 25

3-2:(0,1,3) <111000> 9-2:(0,1,2,3,4,5,6,7,9) <876663>

4-12:(0,2,3,6) <112101> 8-12:(0,1,3,4,5,6,7,9) <556543>

5-28:(0,2,3,6,8) <122212> 7-28:(0,1,3,5,6,7,9) <344433>

6-Z43:(0,1,2,5,6,8) <322332> 6-Z17:(0,1,2,4,7,8) <322332>

Note: This is just a sample, many more combinations are possible

- By definition, the last 6 notes of a 12-tone row are the PC Set complement of the first 6 notes
- For more harmonic *cohesiveness*, make the first and last 6 notes of the row the same PC Set, i.e. self complementary and possibly inversionally related.
- For more harmonic *variety*, make the first and last 6 notes of the row Z-related PC-sets.
- This is the first step towards hexachordal combinatoriality: where a 12-tone row is made up of two similar halves, for example, where the 2nd half is a transposed inversion of the first half (further discussion is beyond the scope of this tutorial). This is a favored technique of late Schoenberg compositions.

8 More Properties of Pitch Class Sets and Interval Vectors

8.1 Common Tones when Transposed

- If two PC Sets contain the same pitches, these are called the "common tones".
 - For example, the common tones between [0, 3, 4, 7] and [1, 3, 4, 8] are 3 and 4.
- The interval vector can tell you how many common tones you will have after transposing a pitch class set. Simply look up the transposing interval in the interval vector, and the number you find will be the number of common tones after transposing the pitch class set.
- For example, if the interval vector of the PC Set is: <324222> (6-Z13), and if you transpose the PC Set by a minor third, you will have 4 common tones between the original PC Set and the transposed PC Set.
- How does the interval vector help? Take any Pitch Class Set
- Algorithm:
 - Step 1: Find the Prime Form of your PC Set. Suppose it is (0,3,4,7)
 - Step 2: Lookup the PC Set, (0,3,4,7), in the Prime Forms table and find its interval vector, <102210>.
 - Step 3: The elements of the PC Set will tell you how many common tones to expect as you transpose the PC Set.
 - For example, if [0,3,4,7] is transposed by a half-step it becomes [1,4,5,8]. The original PC Set and the transposed PC Set have one common tone (4 = E).
 - As a second example, if [0,3,4,7] is transposed by a major third it becomes [4,7,8,11], which has two common tones (4 and 7).
 - Based on the interval vector <102210>, here is a complete list of how many common tones to expect when the PC Set is transposed:
 - a half-step => 1 pitch class remains the same
 - a whole-step => All new pitch classes
 - a minor-third => 2 pitch classes remains the same
 - a major third => 2 pitch classes remains the same
 - a perfect 4th => 1 pitch class remains the same
 - a tritone => All new pitch classes



- **Except:** For tritones (it would figure). When transposing by a tritone, you get double the number of common pitches as specified in the interval vector. For example, if you transpose [0,1,6] by a tritone, you would get two common pitch class sets, rather than one (see above for an example).
- You can use this fact for composition to either make transitions smoother or more abrupt. If two adjacent harmonies in your music have many common tones, they will transition smoothly from one to the other. If they have few common tones, then the transition will be less smooth. For example:
 - Use for common tone transposition / modulation: Transpose a PC Set around a common tone for smoother transitions.



- Alternatively, transpose with all new notes to emphasize the difference.
- Also, the interval vector can be used to help identify when a PC Set can be combined with itself to make larger PC Sets with all unique pitches.
 - For example, the PC Set 6-8 has 6 pitches and the interval vector <343230>. This PC Set can be combined with itself by transposing it a tritone to make up a complete twelve tone row.

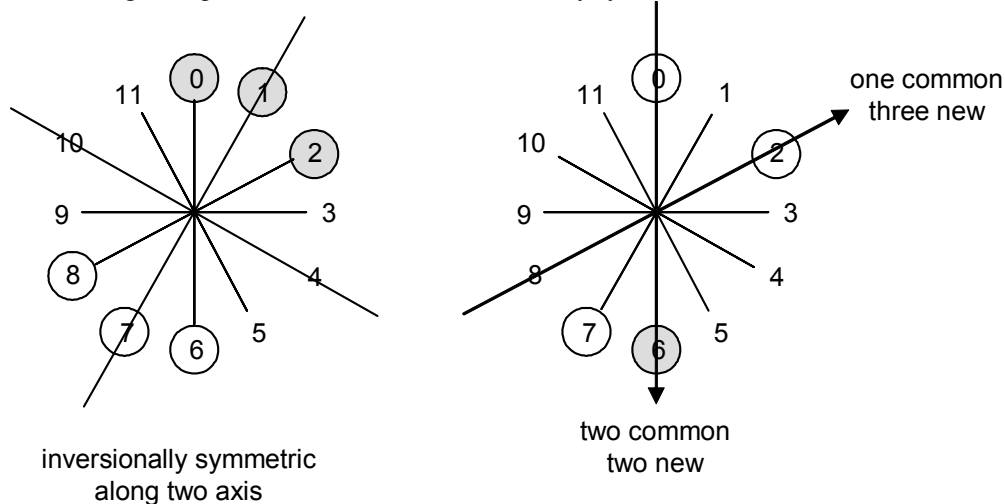
8.1.1 More Examples and Transpositional Symmetry

Let us consider two interesting PC Sets: The diatonic scale and the whole tone scale. Both of these scales have some rather interesting properties when they are transposed.

- Diatonic Scale, 7-35:(0,1,3,5,6,8,10) which has the interval vector <254361>
 - Transpose the scale by a fifth or fourth (i.e. modulate to the dominant or the sub-dominant) and there will be 6 common pitch classes, and 1 new pitch class.
 - Transpose the scale by a half step, and there will be only 2 common pitches and 5 new ones. For example: C Major to C[♯] Major, or C Major to B major.
 - This gives rise, in tonal music, to the notion of "near" and "distant" keys.
- The whole tone scale: 6-35:(0,2,4,6,8,10) has interval vector <060603>
 - Transpose this scale by any interval and either 1) all the pitch classes will be new or 2) all the pitch classes will be different.
 - Remember to double the value of the tritones entry (from 3 to 6).
- If any entry of the interval vector is equal to the number of pitch classes in the set, then the PC Set can be transposed to itself with all pitch classes in common. This is called "Transpositional Symmetry."
- In the Pitch Class Set Table, any PC Set with a "Count" column smaller than 12 has some Transpositional Symmetry.

8.2 Inversal Symmetry

- The best way to see if (and how) a PC Set contains common pitch classes when inverted is to visualize the PC set on a clock face, and then look for one or more axis of symmetry.
- OR: when looking at the table of all prime forms in the Appendix, if a PC Set has no entry in the "inversion" column, then it is inversionally symmetric on at least one axis.
- Otherwise, there is no special math involved to determine inversional symmetry.
- A PC Set which can invert to itself (on some axis of inversion) is said to be "Inversionally Symmetric".
- Looking for inversions and inversional symmetry is another way to manipulate PC Sets to get new sounds.
- In the following examples, the first PC Set is inversionally symmetric, and the second is not.



9 Other PC Set Similarity Relations

This section covers other ways in which two PC Sets can be related. Again, this can be a useful compositional technique. For example, you could choose a PC Set and compose a work which is made up of just the original PC Set plus other, closely related sets. Such a composition should have a fairly consistent harmonic color throughout. Similarly, if you are looking for dramatic color contrasts, you will likely want to avoid similarly related PC Sets.

Note that you can explore many of these similarity relations at <http://www.ComposerTools.com>.

9.1 Special Purpose Relations: R_p , R_0 , R_1 , R_2

- $R_p \Rightarrow$ When two PC Sets are the same except for one different pitch class, i.e. one note different
 - Very useful for composers, this is one way to "morph" PC sets. For example, you can go from PC Set 1 to PC Set 2 by changing a single note, as long as the two sets are related by R_p .
 - But not too useful for analysis, since this relates many PC sets to many many other PC sets
- $R_0 \Rightarrow$ When two PC Sets have the same number of pitch classes, but no interval vector entries in common, for example:
 - 4-2: (0,1,2,4) has interval vector $\langle 221100 \rangle$
 - 4-13: (0,1,3,6) has interval vector $\langle 112011 \rangle$
 - There is no interval which has the same count in both interval vectors.
 - Not a very useful measure, since it has to do with the relative strengths of the intervals, rather than the presence or total absence of intervals.
- $R_1 \Rightarrow$ When two PC Sets have the same number of pitch classes, and their interval vectors are as similar as they can be without being equal
 - This will be the case when the 4 of the 6 entries in the interval vector are the same, **and** the remaining two entries are simply exchanged, for example:
 - 4-2: (0,1,2,4) has interval vector $\langle 221100 \rangle$
 - 4-3: (0,1,3,4) has interval vector $\langle 212100 \rangle$
 - Note the highlighted entries in the interval vector are the only ones which are different, and the two entries are merely exchanged from one to the other.
- $R_2 \Rightarrow$ Just like R_1 , except that the two different entries are not merely an exchange of numbers. For example:
 - 5-10: (0,1,3,4,6) has interval vector $\langle 223111 \rangle$
 - 5-Z12: (0,1,3,5,6) has interval vector $\langle 222121 \rangle$
- Note that R_1 and R_2 are also R_p .

9.2 Other techniques for generating related PC Sets

- Rotational arrays: Used by Oliver Knussen and Igor Stravinsky
- Intervallic projection to relate subsets and supersets:
 - Add notes to a PC Set by projecting up from the top note by a certain interval
 - For example: Quartal / Quintal harmony is created by projecting by adding a note to a PC set which is a perfect 4th or 5th above the last note added
 - Or this can be done with alternating intervals (i.e. first add a 5th, then a tritone, etc)

Appendix - Table of All PC Set Prime Forms

interval vector	Forte count	prime code	prime form	inverted form
Sets of 0 pitch classes, 0 intervals (1 vector, 1 quality, 1 total)				
<000000>	(1)		{silence}	
Sets of 1 pitch classes, 0 intervals (1 vector, 1 quality, 12 total)				
<000000>	(12)		{single-note}	
Sets of 2 pitch classes, 1 intervals (6 vectors, 6 qualities, 66 total)				
<100000>	(12)		(0,1){half-step}	
<010000>	(12)		(0,2){whole-step}	
<001000>	(12)		(0,3){minor-third}	
<000100>	(12)		(0,4){major-third}	
<000010>	(12)		(0,5){perfect}	
<000001>	(6)		(0,6){tritone}	
Sets of 3 pitch classes, 3 intvls (12 vectors, 19 qualities, 220 total)				
<210000>	(12)	3-1:	(0,1,2)	
<111000>	(24)	3-2:	(0,1,3)	[0,2,3]
<101100>	(24)	3-3:	(0,1,4)	[0,3,4]
<100110>	(24)	3-4:	(0,1,5)	[0,4,5]
<100011>	(24)	3-5:	(0,1,6)	[0,5,6]
<020100>	(12)	3-6:	(0,2,4)	
<011010>	(24)	3-7:	(0,2,5)	[0,3,5]
<010101>	(24)	3-8:	(0,2,6){It.}	[0,4,6]
<010020>	(12)	3-9:	(0,2,7){quar-3}	
<002001>	(12)	3-10:	(0,3,6){dim}	
<001110>	(24)	3-11:	(0,3,7){min}	[0,4,7]{maj}
<000300>	(4)	3-12:	(0,4,8){aug}	
Sets of 4 pitch classes, 6 intvls (28 vectors, 43 qualities, 495 total)				
<321000>	(12)	4-1:	(0,1,2,3)	
<221100>	(24)	4-2:	(0,1,2,4)	[0,2,3,4]
<212100>	(12)	4-3:	(0,1,3,4)	
<211110>	(24)	4-4:	(0,1,2,5)	[0,3,4,5]
<210111>	(24)	4-5:	(0,1,2,6)	[0,4,5,6]
<210021>	(12)	4-6:	(0,1,2,7)	
<201210>	(12)	4-7:	(0,1,4,5)	
<200121>	(12)	4-8:	(0,1,5,6)	
<200022>	(6)	4-9:	(0,1,6,7)	
<122010>	(12)	4-10:	(0,2,3,5)	
<121110>	(24)	4-11:	(0,1,3,5)	[0,2,4,5]
<112101>	(24)	4-12:	(0,2,3,6)	[0,3,4,6]
<112011>	(24)	4-13:	(0,1,3,6)	[0,3,5,6]
<111120>	(24)	4-14:	(0,2,3,7)	[0,4,5,7]
<111111>	(48)	4-Z15:	(0,1,4,6)	[0,2,5,6]
		4-Z29:	(0,1,3,7)	[0,4,6,7]
<110121>	(24)	4-16:	(0,1,5,7)	[0,2,6,7]
<102210>	(12)	4-17:	(0,3,4,7)	
<102111>	(24)	4-18:	(0,1,4,7)	[0,3,6,7]
<101310>	(24)	4-19:	(0,1,4,8){mM7}	[0,3,4,8]
<101220>	(12)	4-20:	(0,1,5,8){maj7}	
<030201>	(12)	4-21:	(0,2,4,6)	
<021120>	(24)	4-22:	(0,2,4,7)	[0,3,5,7]
<021030>	(12)	4-23:	(0,2,5,7){quar-4}	
<020301>	(12)	4-24:	(0,2,4,8){7+5}	
<020202>	(6)	4-25:	(0,2,6,8){fr.,7-5}	
<012120>	(12)	4-26:	(0,3,5,8){min7,maj6}	
<012111>	(24)	4-27:	(0,2,5,8){hd7}	[0,3,6,8]{dom7}
<004002>	(3)	4-28:	(0,3,6,9){dd7}	

interval vector	Forte count	prime code	prime form	inverted form
Sets of 12 pitch classes, 66 intervals (1 vector, 1 quality, 1 total)				
<CCCCC6>	(1)		(0,1,2,3,4,5,6,7,8,9,A,B){chromatic}	
Sets of 11 pitch classes, 55 intervals (1 vector, 1 quality, 12 total)				
<AAAAA5>	(12)		(0,1,2,3,4,5,6,7,8,9,A)	
Sets of 10 pitch classes, 45 intervals (6 vectors, 6 qualities, 66 total)				
<988884>	(12)		(0,1,2,3,4,5,6,7,8,9)	
<898884>	(12)		(0,1,2,3,4,5,6,7,8,A)	
<889884>	(12)		(0,1,2,3,4,5,6,7,9,A)	
<888984>	(12)		(0,1,2,3,4,5,6,8,9,A)	
<888894>	(12)		(0,1,2,3,4,5,7,8,9,A)	
<888885>	(6)		(0,1,2,3,4,6,7,8,9,A)	
Sets of 9 pitch classes, 36 intvls (12 vectors, 19 qualities, 220 total)				
<876663>	(12)	9-1:	(0,1,2,3,4,5,6,7,8)	
<777663>	(24)	9-2:	(0,1,2,3,4,5,6,7,9)	[0,2,3,4,5,6,7,8,9]
<767763>	(24)	9-3:	(0,1,2,3,4,5,6,8,9)	[0,1,3,4,5,6,7,8,9]
<766773>	(24)	9-4:	(0,1,2,3,4,5,7,8,9)	[0,1,2,4,5,6,7,8,9]
<766674>	(24)	9-5:	(0,1,2,3,4,6,7,8,9)	[0,1,2,3,5,6,7,8,9]
<686763>	(12)	9-6:	(0,1,2,3,4,5,6,8,A)	
<677673>	(24)	9-7:	(0,1,2,3,4,5,7,8,A)	[0,1,3,4,5,6,7,8,A]
<676764>	(24)	9-8:	(0,1,2,3,4,6,7,8,A)	[0,1,2,4,5,6,7,8,A]
<676683>	(12)	9-9:	(0,1,2,3,5,6,7,8,A)	
<668664>	(12)	9-10:	(0,1,2,3,4,6,7,9,A)	
<667773>	(24)	9-11:	(0,1,2,3,5,6,7,9,A)	[0,1,2,4,5,6,7,9,A]
<666963>	(4)	9-12:	(0,1,2,4,5,6,8,9,A)	
Sets of 8 pitch classes, 28 intvls (28 vectors, 43 qualities, 495 total)				
<765442>	(12)	8-1:	(0,1,2,3,4,5,6,7)	
<665542>	(24)	8-2:	(0,1,2,3,4,5,6,8)	[0,2,3,4,5,6,7,8]
<656542>	(12)	8-3:	(0,1,2,3,4,5,6,9)	
<655552>	(24)	8-4:	(0,1,2,3,4,5,7,8)	[0,1,3,4,5,6,7,8]
<654553>	(24)	8-5:	(0,1,2,3,4,6,7,8)	[0,1,2,4,5,6,7,8]
<654463>	(12)	8-6:	(0,1,2,3,5,6,7,8)	
<645652>	(12)	8-7:	(0,1,2,3,4,5,8,9)	
<644563>	(12)	8-8:	(0,1,2,3,4,7,8,9)	
<644464>	(6)	8-9:	(0,1,2,3,6,7,8,9)	
<566452>	(12)	8-10:	(0,2,3,4,5,6,7,9)	
<565552>	(24)	8-11:	(0,1,2,3,4,5,7,9)	[0,2,4,5,6,7,8,9]
<556543>	(24)	8-12:	(0,1,3,4,5,6,7,9)	[0,2,3,4,5,6,8,9]
<556453>	(24)	8-13:	(0,1,2,3,4,6,7,9)	[0,2,3,5,6,7,8,9]
<555562>	(24)	8-14:	(0,1,2,4,5,6,7,9)	[0,2,3,4,5,7,8,9]
<555553>	(48)	8-Z15:	(0,1,2,3,4,6,8,9)	[0,1,3,5,6,7,8,9]
		8-Z29:	(0,1,2,3,5,6,7,9)	[0,2,3,4,6,7,8,9]
<554563>	(24)	8-16:	(0,1,2,3,5,7,8,9)	[0,1,2,4,6,7,8,9]
<546652>	(12)	8-17:	(0,1,3,4,5,6,8,9)	
<546553>	(24)	8-18:	(0,1,2,3,5,6,8,9)	[0,1,3,4,6,7,8,9]
<545752>	(24)	8-19:	(0,1,2,4,5,6,8,9)	[0,1,3,4,5,7,8,9]
<545662>	(12)	8-20:	(0,1,2,4,5,7,8,9)	
<474643>	(12)	8-21:	(0,1,2,3,4,6,8,A)	
<465562>	(24)	8-22:	(0,1,2,3,5,6,8,A)	[0,1,3,4,5,6,8,A]
<465472>	(12)	8-23:	(0,1,2,3,5,7,8,A)	
<464743>	(12)	8-24:	(0,1,2,4,5,6,8,A)	
<464644>	(6)	8-25:	(0,1,2,4,6,7,8,A)	
<456562>	(12)	8-26:	(0,1,2,4,5,7,9,A)	
<456553>	(24)	8-27:	(0,1,2,4,5,7,8,A)	[0,1,3,4,6,7,8,A]
<448444>	(3)	8-28:	(0,1,3,4,6,7,9,A){octatonic}	

interval vector	Forte count	prime code	prime form	inverted form	interval vector	Forte count	prime code	prime form	inverted form
Sets of 5 pitch classes, 10 intvls (35 vectors, 66 qualities, 792 total)					Sets of 7 pitch classes, 21 intvls (35 vectors, 66 qualities, 792 total)				
<432100>	(12)	5-1:	(0,1,2,3,4)		<654321>	(12)	7-1:	(0,1,2,3,4,5,6)	
<332110>	(24)	5-2:	(0,1,2,3,5)	[0,2,3,4,5]	<554331>	(24)	7-2:	(0,1,2,3,4,5,7)	[0,2,3,4,5,6,7]
<322210>	(24)	5-3:	(0,1,2,4,5)	[0,1,3,4,5]	<544431>	(24)	7-3:	(0,1,2,3,4,5,8)	[0,3,4,5,6,7,8]
<322111>	(24)	5-4:	(0,1,2,3,6)	[0,3,4,5,6]	<544332>	(24)	7-4:	(0,1,2,3,4,6,7)	[0,1,3,4,5,6,7]
<321121>	(24)	5-5:	(0,1,2,3,7)	[0,4,5,6,7]	<543342>	(24)	7-5:	(0,1,2,3,5,6,7)	[0,1,2,4,5,6,7]
<311221>	(24)	5-6:	(0,1,2,5,6)	[0,1,4,5,6]	<533442>	(24)	7-6:	(0,1,2,3,4,7,8)	[0,1,4,5,6,7,8]
<310132>	(24)	5-7:	(0,1,2,6,7)	[0,1,5,6,7]	<532353>	(24)	7-7:	(0,1,2,3,6,7,8)	[0,1,2,5,6,7,8]
<232201>	(12)	5-8:	(0,2,3,4,6)		<454422>	(12)	7-8:	(0,2,3,4,5,6,8)	
<231211>	(24)	5-9:	(0,1,2,4,6)	[0,2,4,5,6]	<453432>	(24)	7-9:	(0,1,2,3,4,6,8)	[0,2,4,5,6,7,8]
<223111>	(24)	5-10:	(0,1,3,4,6)	[0,2,3,5,6]	<445332>	(24)	7-10:	(0,1,2,3,4,6,9)	[0,2,3,4,5,6,9]
<222220>	(24)	5-11:	(0,2,3,4,7)	[0,3,4,5,7]	<444441>	(24)	7-11:	(0,1,3,4,5,6,8)	[0,2,3,4,5,7,8]
<222121>	(36)	5-Z12:	(0,1,3,5,6)		<444342>	(36)	7-Z12:	(0,1,2,3,4,7,9)	
		5-Z36:	(0,1,2,4,7)	[0,3,5,6,7]			7-Z36:	(0,1,2,3,5,6,8)	[0,2,3,5,6,7,8]
<221311>	(24)	5-13:	(0,1,2,4,8)	[0,2,3,4,8]	<443532>	(24)	7-13:	(0,1,2,4,5,6,8)	[0,2,3,4,6,7,8]
<221131>	(24)	5-14:	(0,1,2,5,7)	[0,2,5,6,7]	<443352>	(24)	7-14:	(0,1,2,3,5,7,8)	[0,1,3,5,6,7,8]
<220222>	(12)	5-15:	(0,1,2,6,8)		<442443>	(12)	7-15:	(0,1,2,4,6,7,8)	
<213211>	(24)	5-16:	(0,1,3,4,7)	[0,3,4,6,7]	<435432>	(24)	7-16:	(0,1,2,3,5,6,9)	[0,1,3,4,5,6,9]
<212320>	(24)	5-Z17:	(0,1,3,4,8)		<434541>	(24)	7-Z17:	(0,1,2,4,5,6,9)	
		5-Z37:	(0,3,4,5,8)				7-Z37:	(0,1,3,4,5,7,8)	
<212221>	(48)	5-Z18:	(0,1,4,5,7)	[0,2,3,6,7]	<434442>	(48)	7-Z18:	(0,1,4,5,6,7,9)	[0,2,3,4,5,8,9]
		5-Z38:	(0,1,2,5,8)	[0,3,6,7,8]			7-Z38:	(0,1,2,4,5,7,8)	[0,1,3,4,6,7,8]
<212122>	(24)	5-19:	(0,1,3,6,7)	[0,1,4,6,7]	<434343>	(24)	7-19:	(0,1,2,3,6,7,9)	[0,1,2,3,6,8,9]
<211231>	(24)	5-20:	(0,1,3,7,8)	[0,1,5,7,8]	<433452>	(24)	7-20:	(0,1,2,4,7,8,9)	[0,1,2,5,7,8,9]
<202420>	(24)	5-21:	(0,1,4,5,8)	[0,3,4,7,8]	<424641>	(24)	7-21:	(0,1,2,4,5,8,9)	[0,1,3,4,5,8,9]
<202321>	(12)	5-22:	(0,1,4,7,8)		<424542>	(12)	7-22:	(0,1,2,5,6,8,9){hungar-min}	
<132130>	(24)	5-23:	(0,2,3,5,7)	[0,2,4,5,7]	<354351>	(24)	7-23:	(0,2,3,4,5,7,9)	[0,2,4,5,6,7,9]
<131221>	(24)	5-24:	(0,1,3,5,7)	[0,2,4,6,7]	<353442>	(24)	7-24:	(0,1,2,3,5,7,9)	[0,2,4,6,7,8,9]
<123121>	(24)	5-25:	(0,2,3,5,8)	[0,3,5,6,8]	<345342>	(24)	7-25:	(0,2,3,4,6,7,9)	[0,2,3,5,6,7,9]
<122311>	(24)	5-26:	(0,2,4,5,8)	[0,3,4,6,8]	<344532>	(24)	7-26:	(0,1,3,4,5,7,9)	[0,2,4,5,6,8,9]
<122230>	(24)	5-27:	(0,1,3,5,8)	[0,3,5,7,8]{min9}	<344451>	(24)	7-27:	(0,1,2,4,5,7,9)	[0,2,4,5,7,8,9]
<122212>	(24)	5-28:	(0,2,3,6,8)	[0,2,5,6,8]	<344433>	(24)	7-28:	(0,1,3,5,6,7,9)	[0,2,3,4,6,8,9]
<122131>	(24)	5-29:	(0,1,3,6,8)	[0,2,5,7,8]	<344352>	(24)	7-29:	(0,1,2,4,6,7,9)	[0,2,3,5,7,8,9]
<121321>	(24)	5-30:	(0,1,4,6,8)	[0,2,4,7,8]	<343542>	(24)	7-30:	(0,1,2,4,6,8,9)	[0,1,3,5,7,8,9]
<114112>	(24)	5-31:	(0,1,3,6,9)	[0,2,3,6,9]{7-9}	<336333>	(24)	7-31:	(0,1,3,4,6,7,9)	[0,2,3,5,6,8,9]
<113221>	(24)	5-32:	(0,1,4,6,9)	[0,2,5,6,9]{7+9}	<335442>	(24)	7-32:	(0,1,3,4,6,8,9){harm-min}	[0,1,3,5,6,8,9]
<040402>	(12)	5-33:	(0,2,4,6,8,9){9+5,9-5}		<262623>	(12)	7-33:	(0,1,2,4,6,8,A)	
<032221>	(12)	5-34:	(0,2,4,6,9){dom9}		<254442>	(12)	7-34:	(0,1,3,4,6,8,A)	
<032140>	(12)	5-35:	(0,2,4,7,9){pentatonic,Quar-5}		<254361>	(12)	7-35:	(0,1,3,5,6,8,A){diatonic}	
Sets of 6 pitch classes, 15 intervals (35 vectors, 80 qualities, 924 total)									
<543210>	(12)	6-1:	(0,1,2,3,4,5)		<322242>	(24)	6-18:	(0,1,2,5,7,8)	[0,1,3,6,7,8]
<443211>	(24)	6-2:	(0,1,2,3,4,6)	[0,2,3,4,5,6]	<313431>	(48)	6-Z19:	(0,1,3,4,7,8)	[0,1,4,5,7,8]
<433221>	(48)	6-Z3:	(0,1,2,3,5,6)	[0,1,3,4,5,6]			6-Z44:	(0,1,2,5,6,9)	[0,1,4,5,6,9]
		6-Z36:	(0,1,2,3,4,7)	[0,3,4,5,6,7]	<303630>	(4)	6-20:	(0,1,4,5,8,9)	
<432321>	(24)	6-Z4:	(0,1,2,4,5,6)		<242412>	(24)	6-21:	(0,2,3,4,6,8)	[0,2,4,5,6,8]
		6-Z37:	(0,1,2,3,4,8)		<241422>	(24)	6-22:	(0,1,2,4,6,8)	[0,2,4,6,7,8]
<422232>	(24)	6-5:	(0,1,2,3,6,7)	[0,1,4,5,6,7]	<234222>	(24)	6-Z23:	(0,2,3,5,6,8)	
<421242>	(24)	6-Z6:	(0,1,2,5,6,7)				6-Z45:	(0,2,3,4,6,9)	
		6-Z38:	(0,1,2,3,7,8)		<233331>	(48)	6-Z24:	(0,1,3,4,6,8)	[0,2,4,5,7,8]
<420243>	(6)	6-7:	(0,1,2,6,7,8)				6-Z46:	(0,1,2,4,6,9)	[0,2,4,5,6,9]
<343230>	(12)	6-8:	(0,2,3,4,5,7)		<233241>	(48)	6-Z25:	(0,1,3,5,6,8)	[0,2,3,5,7,8]
<342231>	(24)	6-9:	(0,1,2,3,5,7)	[0,2,4,5,6,7]			6-Z47:	(0,1,2,4,7,9)	[0,2,3,4,7,9]
<333321>	(48)	6-Z10:	(0,1,3,4,5,7)	[0,2,3,4,6,7]	<232341>	(24)	6-Z26:	(0,1,3,5,7,8)	
		6-Z39:	(0,2,3,4,5,8)	[0,3,4,5,6,8]			6-Z48:	(0,1,2,5,7,9)	
<333231>	(48)	6-Z11:	(0,1,2,4,5,7)	[0,2,3,5,6,7]	<225222>	(24)	6-27:	(0,1,3,4,6,9)	[0,2,3,5,6,9]
		6-Z40:	(0,1,2,3,5,8)	[0,3,5,6,7,8]	<224322>	(24)	6-Z28:	(0,1,3,5,6,9)	
<332232>	(48)	6-Z12:	(0,1,2,4,6,7)	[0,1,3,5,6,7]			6-Z49:	(0,1,3,4,7,9)	
		6-Z41:	(0,1,2,3,6,8)	[0,2,5,6,7,8]	<224232>	(24)	6-Z29:	(0,1,3,6,8,9)	
<324222>	(24)	6-Z13:	(0,1,3,4,6,7)				6-Z50:	(0,1,4,6,7,9)	
		6-Z42:	(0,1,2,3,6,9)		<224223>	(12)	6-30:	(0,1,3,6,7,9)	[0,2,3,6,8,9]
<323430>	(24)	6-14:	(0,1,3,4,5,8)	[0,3,4,5,7,8]	<223431>	(24)	6-31:	(0,1,3,5,8,9)	[0,1,4,6,8,9]
<323421>	(24)	6-15:	(0,1,2,4,5,8)	[0,3,4,6,7,8]	<143250>	(12)	6-32:	(0,2,4,5,7,9){min11}	
<322431>	(24)	6-16:	(0,1,4,5,6,8)	[0,2,3,4,7,8]	<143241>	(24)	6-33:	(0,2,3,5,7,9)	[0,2,4,6,7,9]{dom11}
<322332>	(48)	6-Z17:	(0,1,2,4,7,8)	[0,1,4,6,7,8]	<142422>	(24)	6-34:	(0,1,3,5,7,9)	[0,2,4,6,8,9]
		6-Z43:	(0,1,2,5,6,8)	[0,2,3,6,7,8]	<060603>	(2)	6-35:	(0,2,4,6,8,A){wholetone}	
Totals:	Total unique interval vectors:	200							
	Total prime forms:	208							
	Total unique chord qualities:	351		(according to the Forte designations, does not include 0, 1, 2, 10, 11, 12 element PC sets)					
	Total pitch collections:	4095		(the prime forms plus inversions of all PC sets shown above)					
				(includes all transpositions and inversions of all PC sets shown above)					