Project 2: Analysis of a Roll Forge

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1. Executive Summary

Static Failure Factor of Safety	η = 3.34
Dynamic Failure Factor of Safety	η = 1.94
Deflection	-0.868 mm
Gear Train Value	0.1989
Minimum Rated Power	1.039 kW

Table 1.1. Executive Summary.

2. Introduction

This paper holds the analysis of a roll forge. In order to reduce flash when creating steel connecting rods, roll forging is used to redistribute the weight or “preform” the rod. This is important because excess flash can lead to an unfavorable microstructure within the rod, produce marginally more waste, and use a lot more forces than if the rod was preformed. Roll forging takes a hot billet of material and runs it through two counter rotating rollers with tool steel dies attached. In this paper, we will be discussing longitudinal forging where the smaller end of the billet is run through the rollers first and the die is semi cylindrical.

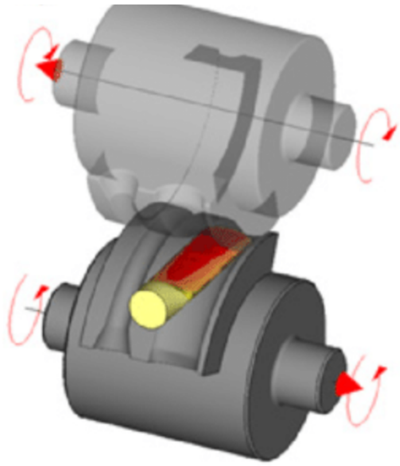

Figure 2.0.1: Figure of roll forging from Powerpoint in ME41.

The preforms from this process are two connecting rods made of AISI 1055 at hot working conditions of 1100℃. It takes four passes to create one preform. This cycle takes about 10 seconds creating 360 preforms per hour. For simplicity, the passes are assumed to exert the same torque and radial load. The peak force is 1.41MN and the peak torque is 25.7 kN*m. The roll forge is run 8 hours a day, 260 days a year, for an assumed 5 years. This totals around 3 million passes for its lifetime equalling 1.5 million connecting rods.

There are two types of roll forges. The one analyzed in this paper is the most common. There is an A/C motor driving a flywheel with V-belts at the top left of the machine. A clutch engages the flywheel with the gear train on the left side of the machine. In turn, the gear train is connected to the rollers. On either side of the rollers, there are bearings to hold the die in place. The gears are assumed to be on the outside of the bearings. See figure below for details.

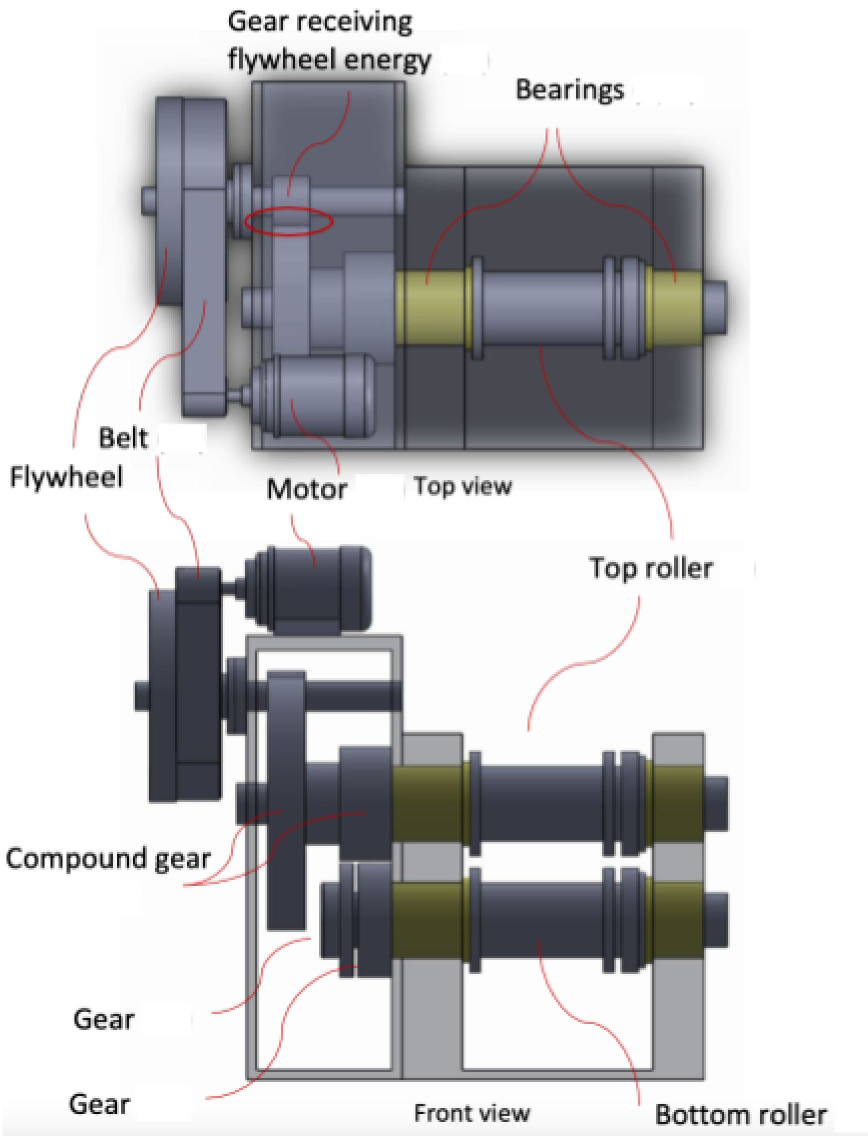

Figure 2.0.2: General overview of roll forge components, as provided.

The goal of this paper is to analyze the roll forge in different states and to find possible points of failure. There will be an analysis of static and dynamic failure as well as a deeper dive into the bearings and gears that make up some of the main components of the forge. This is all in order to produce continuous, controlled parts coming from the forge ensuring reliability in the machine.

It is assumed there is a dedicated roll forge for creating the preforms. For this paper, the rollers are treated as shafts with a semicircular cross section. Because of this, grooves and keyways on the die can be ignored. The load from the radial force is assumed to be at the center of the roller with two identical bearings on either side. Both the top and bottom roller are assumed to be loaded identically. The dies are connected to the rollers and the material is assumed to be AISI 1095 steel[1].

2.1 Support at Bearings

One can begin by modeling the forces on the top roller.

$$\sum F_z = 0 = F - R_{Az} - R_{Bz}$$

Due to symmetry, and a lack of rotation, $R_{AZ} = R_{BZ} = F/2$. As the maximum force on the roller is 1.41 MN, the maximum reaction force at each support is 0.705 MN, or 705 kN.
The roller undergoes a fluctuating torsional force over the course of a cycle, the maximum of which is at 25.7 kN m. For this project, it was assumed the bearings are a point force, not a distributed load. Based on this assumption, the length of the part was then assumed to be 0.6628 m with the forces from the bearings acting on the end.

2.2 Free Body Diagram

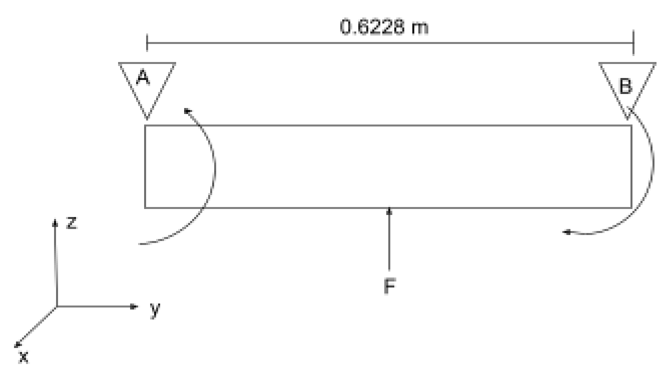

Figure 2.2.1: Free Body Diagram, Top Roller

[1] *AISI 1095 Hot Rolled Steel*. MatWeb. (n.d.). Retrieved December 6, 2021, from http://www.matweb.com/search/DataSheet.aspx?MatGUID=21bc72229925455db41e3cea6bb7625a&ckck=1.

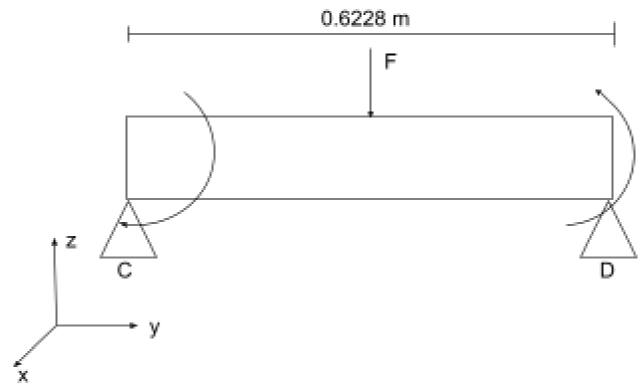

Figure 2.2.2: Free Body Diagram, Bottom Roller

2.3 Shear - Moment - Torque Diagrams

Based on the free body diagram, we found the shear diagram. We then used the shear diagram to find the moment diagram. The torsion diagram was constructed separately. Because the beam is simply supported on each side by the bearings, with the force from the part going between the rollers pushing in the opposite direction. This will result in the maximum given force being in the middle. Thus, the shear diagram for the bottom roller will look like this:

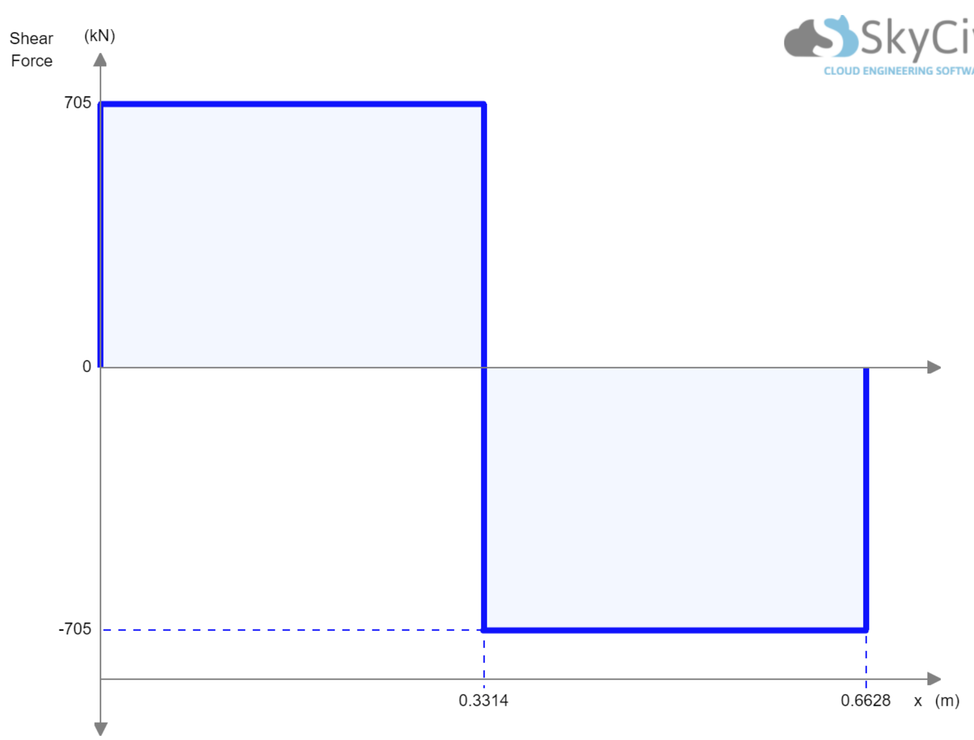

Figure 2.3.1: Shear Diagram

The moment diagram for the bottom roller is $\int_0^L V dx$ which is equivalent to:

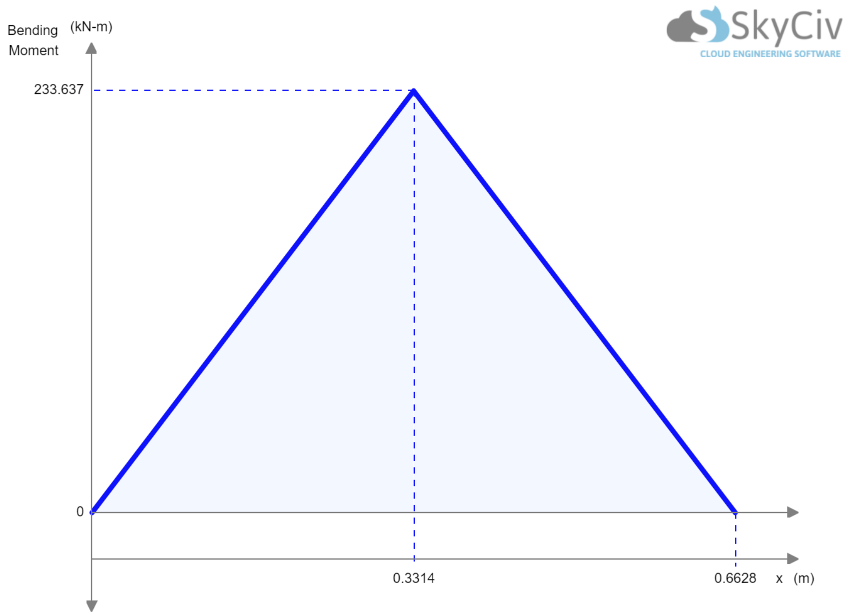

Figure 2.3.2: Moment Diagram

The peak torsion is constant throughout the beam, up to the point where the part is pulled through:

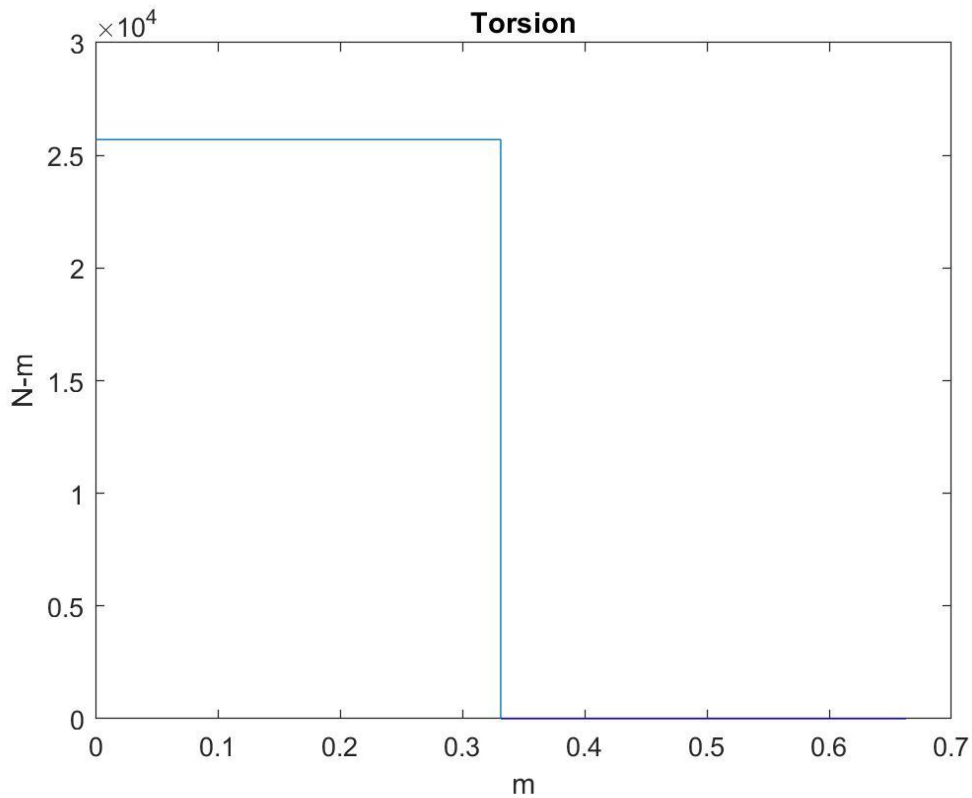

Figure 2.3.3: Torsion Diagram

The shear, moment, and torsion diagrams were only created for one beam. Since the forces exerted will be the same, the diagrams will be identical for the top beam just opposite signs.

3. Failure

3.1 Static Failure

The first thing to examine in the design of the roller is to determine whether or not the roller can bear the desired forces before any motion is considered. This calculation will look at one roller, assuming that the rollers will be the same. The static failure was calculated using the Tresca (MSS) failure criteria and looking at the failure criteria based on yield strength. This was chosen because the material chosen is a ductile material, it was assumed that the tensile yield strength and the compression yield strength are equal and opposite, and a conservative design was desired. A conservative design is appropriate due to the high cost and long term use of the rollers.

3.1.1 Initial Values

This section discusses the initial values that were used and assumptions that were made in order to calculate the factor of safety.

The ultimate strength was found from MATWEB[2].

$S_y = 455\ MPa = 4.55\ x10^8\ Pa$

The moment value was found based on the moment diagram, as seen in section 2.3. The tensile strength, diameter, and length, all came from the given values in the project description. The length of the whole shaft was assumed to be .6628 m and the bearings were simplified to be point forces on the end (rather than distributed forces). The maximum moment was found to be in the middle of the bar, thus the length would be .3314 m.

$M = 233637$ Based on Moment Diagram

$T = 25.7\ kN * m = 25700\ N * m$

$d = 0.26\ m$

$L = 331.4\ mm = .3314\ m$

3.1.2 Tresca Factor of Safety Calculation

To find the factor of safety, first σ_y , σ_x, τ_{xy} and were found based on the given forces and torque. σ_y was assumed to be zero since there are no forces acting in that direction. σ_1 and σ_2 were found from rom σ_x and τ_{xy}.

$\sigma_y = 0$

$\sigma_x = \frac{Mc}{I} = \frac{32\ M}{\pi d^3} = \frac{32*233637}{\pi*0.26^3} = 1.35x10^8 Pa$

[2] *AISI 1095 Hot Rolled Steel*. MatWeb. (n.d.). Retrieved December 6, 2021, from http://www.matweb.com/search/DataSheet.aspx?MatGUID=21bc72229925455db41e3cea6bb7625a&ckck=1.

$$\tau_{xy} = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(25700)}{\pi(0.26)^3} = 7.45x10^6 Pa$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{(\frac{\sigma_x + \sigma_y}{2})^2 + \tau_{\tau_{xy}}^{\ 2}}$$

$$\sigma_{1,2} = \frac{1.35x10^8 + 0}{2} \pm \sqrt{(\frac{1.35x10^8 + 0}{2})^2 + (7.45x10^6)^2} = 1.35x10^8 Pa, \ -4.09x10^5 \ Pa$$

Based on the above calculations, σ_2 is actually σ_3 and σ_2 is zero. This is determined via the Mohr's circle. Here one positive value and one negative value was found for σ . σ_2 is always the mid-value. Furthermore, we are assuming the third principal stress is zero. Since σ_1 is always the highest principal stress and σ_3 is always the lowest, the found positive value has to be σ_1 and the found negative value has to be σ_3. Then the factor of safety was calculated based on the tresca criteria. It was found to be 3.34.

$$\sigma_1 = 1.35x10^8 Pa$$

$$\sigma_2 = 0 \ Pa$$

$$\sigma_3 = -4.09x10^5 \ Pa$$

$$\eta = \frac{S_y}{\sigma_1 - \sigma_3} = \frac{4.55x10^8}{1.35x10^8 + 4.09x10^5}$$

$$\eta = 3.34$$

This factor of safety is much higher than one, so the roller is unlikely to fail due to static failure. This assessment will be revisited later in comparison to the dynamic failure.

3.2 Dynamic Failure

The rollers that shape the part must be able to repeatedly apply the same force, cyclically, without failing. Thus, it is critical to examine the factor of safety of the rollers and their predicted life spans.

3.2.1 Calculation of Stresses

To determine failure due to fatigue, one has to examine the loading conditions and the resulting stresses. This will refer back to the shear-moment-torque diagrams in section two.

From these diagrams, one can calculate the maximum bending shear and maximum torsional shear. Bending stress is given by:

$$\sigma_{max} = \frac{32M}{\pi d^3} K_f$$

Because there are no filets or other geometry in the cylinder that we are assuming to be the geometry of the roller, $K_f = 1$. Therefore, plugging in the known values yields:

$$\sigma_{max} = \frac{32M}{\pi d^3} = \frac{32(233637)}{\pi (0.26)^3} = 135.401 MPa$$

Torsional shear is given by the equation:

$$\tau_{max} = \frac{16T}{\pi d^3} K_{fs}$$

The same as for bending shear, there are no stress concentrations so $K_{fs} = 1$. Therefore, plugging in the known values yields:

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(25700)}{\pi (0.26)^3} = 7.45 MPa.$$

The bending stress that the rollers experience is repeated because the amplitude fluctuates between 0 and σ_{max}. The torsional stress is constant. The next step is to find the midrange stresses and amplitudes so we can apply the modified von Mises to find the stress resulting from the combined loading conditions. The equations to calculate midrange and amplitude stresses are as follows:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{135.401 + 0}{2} = 67.70\ MPa$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{135.401 + 0}{2} = 67.70\ MPa$$

$$\tau_m = \frac{\tau_{max} + \tau_{min}}{2} = \frac{7.45 + 7.45}{2} = 7.45\ MPa$$

$$\tau_a = \frac{\tau_{max} - \tau_{min}}{2} = \frac{7.45 - 7.45}{2} = 0\ MPa$$

Modified von Mises (simplified to remove stress concentration factors and axial loading since neither occur in this case):

$$\sigma'_a = [\sigma_a^2 + 3\tau_a^2]^{\frac{1}{2}} = [67.70^2 + 3(0)^2]^{\frac{1}{2}} = 67.70\ MPa$$

$$\sigma'_m = [\sigma_m^{\,2} + 3\tau_m^{\,2}]^{\frac{1}{2}} = [67.70^2 + 3(7.45)^2]^{\frac{1}{2}} = 68.912\ MPa$$

3.2.2 Endurance Limit

Fatigue analysis is dependent on calculating the limit at which a material fails, given a certain number of cycles of stress. The process to find this value (which corresponds to "unlimited life" or 10^6 cycles) involves calculating an initial S_e' based on laboratory data (Mischke correlation from RR Moore testing) for the material of interest then correcting that limit based on environmental factors with a set of coefficients called Marin factors.

For materials with an ultimate tensile strength less than or equal to 1400 MPa, the Mischke correlation for S'_e is:

$$S_e' = 0.5S_{ut} = 0.5(830) = 415\ MPa.$$

Next, Marin factors must be applied to correct for the differences between laboratory RR Moore testing and application in the real world. The first factor, k_a, accounts for different surface finishes. We assume that the roller (which is simplified as a cylinder here but is not in reality) would have shapes machined into it to form the connecting rods. Therefore, for a machined finish k_a:

$$k_a = 4.51(S_{ut})^{-0.265} = 0.7597$$

The next Marin factor, k_b, corrects for the size of the shaft. The given correlations only go up to diameters of 254mm, which is just shy of the shaft we are tasked with analyzing (260mm). Because there is only a 6mm difference between the upper bound bound and the roller, use:

$$k_b = 1.51d^{-0.157} = 0.6307$$

Loading factor, k_c, accounts for the different stresses that affect the bar. This is relevant because the endurance limit based on RR Moore testing is based on only bending stress and does not account for torsion or axial stress. However, because this is a combined loading condition and we calculated the combined stresses using modified von Mises, the correction factor k_c is incorporated in that calculation and is set to one here.

The next Marin factor, k_d, accounts for different temperatures in which the shaft will operate. This is fairly intuitive given that colder temperatures increase the brittleness of metals making them more prone to fractures and higher temperatures lower the yield strength, increasing plasticity. While the rollers come in contact with hot pieces (1100 C), it is reasonable to assume that the rollers will never get that hot because of their relative size to the connecting rods. Additionally, some form of cooling is assumed to be used, whether that be natural or forced convection. This will aid in keeping the temperature of the rollers lower than the connecting rods. To be conservative, we will still account for the temperature and will assume that the rollers get quite hot and heat up to 400 celsius during operation. This corresponds to k_d=0.900 (Table 6-4, Shigley).

The last Marin factor we will use is the reliability factor, k_e. This factor accounts for desired performance of the shaft being analyzed or designed. Because we want to be sure that the roller will not fail due to reliability issues, k_e=0.753 which corresponds with 99.9% reliability.

The endurance limit for actual application is then:

$$S_e = k_a k_b k_c k_d k_e S'_e = (0.7597)(0.6307)(1)(0.900)(0.753)(415) = 134.76\ MPa$$

3.2.3 Factor of Safety for Fatigue

Now that we found the endurance limit and combined stresses, we can calculate the factor of safety corresponding to our desired life. AISI 1095 HR steel is brittle so the Gerber equation should be used.

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right]$$

Therefore, the factor of safety guarding against failure by fatigue is:

$$n_f = \frac{1}{2}\left(\frac{830}{68.912}\right)^2 \frac{67.70}{134.76}\left[-1 + \sqrt{1 + \left(\frac{2*68.912*134.76}{830*67.70}\right)^2}\right] = 1.94.$$

The conclusion of this process is that the rollers will be able to produce the desired number of components (nearly three million passes total) without failing due to fatigue. An easy way to improve this would be to increase the endurance limit by ensuring that the rollers are adequately cooled. A tool steel could also be chosen but that comes with the cost tradeoff of significantly more expensive materials. Furthermore, this factor of safety is lower than that of the static failure (which was 3.34). Thus, it is predicted that the roller will fail dynamically.

4. Bearings

When looking at the role of bearings in the roll forge, the deflection and slope of the roller at the bearings is critical. The rigidity of the setup ensures controlled and consistent forging for the preforms. The bearing function can be compromised by excess slope of supported shafts, in this case the roller. This section will be looking at the peak deflection of the roller, slope of the deflection at the bearings, catalog data for bearings in the design, and the entire assembly of bearings.

4.1 Peak Deflection of a Roller

The deflection method for Castigliano's approach was used to calculate the deflection of the structure. This method was chosen because it only requires one integration. The values for the dM/dF terms were calculated from the Shear-Moment diagrams for each section. Shear forces act on the roller; however, these forces contribute to deflection very minimally so they are not included in the calculations. The roller in the forge was treated as a shaft with a semicircular cross section to represent a simplified die.

$$I = \frac{\pi}{8} r^4$$

The grooves and indentations of the dye will not affect the calculations much like the grooves and keyways will have a minimal effect on a shaft’s deflection. For this reason, they are ignored. This calculation was also computed from the beginning of one bearing to the beginning of the other. This is to model the bearings as simple supports in Castigliano’s Method. This assumption is valid because while the bearings do restrict movement, their restriction is not equivalent to a fixed wall. The bearings are still able to move and give with the shaft. The material of the roller is AISI 1095 with a modulus of elasticity of 200 GPa. The length is taken from the start of one bearing to the end of the other. This is because there is still a reaction force through the entirety of the bearing, even to the end.

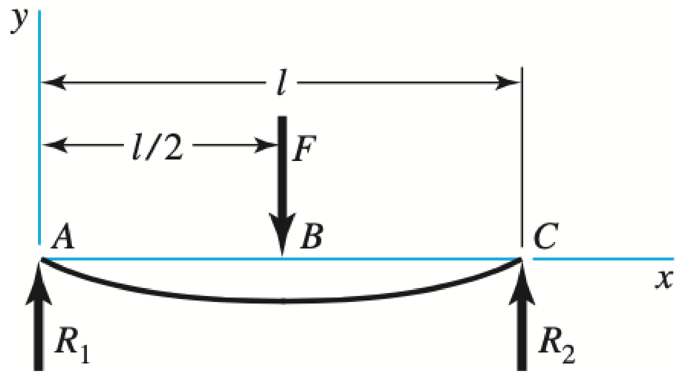

Figure 4.1.1: Simple supports with a center load taken[3].

4.1.1 Calculations:

F	1.41 MN
l	0.6628 m
E	200 GPa
I	$\frac{\pi}{8}r^4 = \frac{\pi}{8}0.13^4 = 1.122 \times 10^{-4} m^4$

$$y_{max} = -\frac{Fl^3}{48EI}$$

$$y_{max} = -\frac{(1.41\times10^6 N)(0.6628m)^3}{48(200\times10^9 Pa)(1.122\times10^{-4} m^4)} = -0.000868m = -0.868mm$$

There is a very minimal deflection predicted in the roller. This is due to the high modulus of elasticity and the geometry of the semi cylindrical die. The peak deflection will occur at the center of the roller, but is not representative of the impact occurring at the bearings.

To confirm the deflection on the shaft, a SolidWorks FEA was run on a semicircular shaft with ball bearing connections and material AISI 1095. The force was applied at the center of the shaft as it rotated.

[3] Budynas, R. G., Nisbett, J. K., Tangcchaichit, K., & Shigley, J. E. (2021). *Shigley's Mechanical Engineering Design*. McGraw Hill Education.

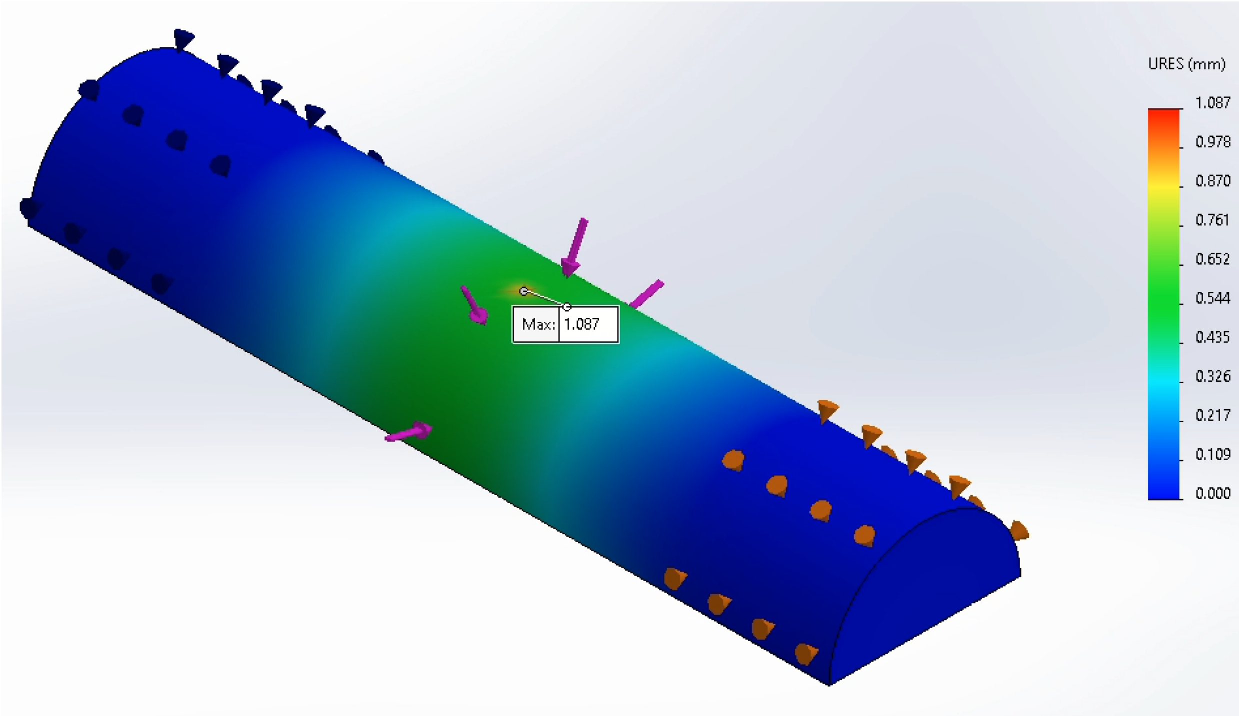

Figure 4.1.2: Image of peak deflection on SolidWorks shaft.

There appears to be a minor discrepancy with the deflection models. The FEA predicted approximately 1 mm of deflection rather than a little over 0.8 mm. This is a very small discrepancy considering that the difference is only 1/1000th of the diameter of the shaft. There is likely no reason to be concerned with the deflection discrepancies between the FEA and Castigliano's. The values are very close and are of the same magnitude and thus confirm each other despite not matching perfectly.

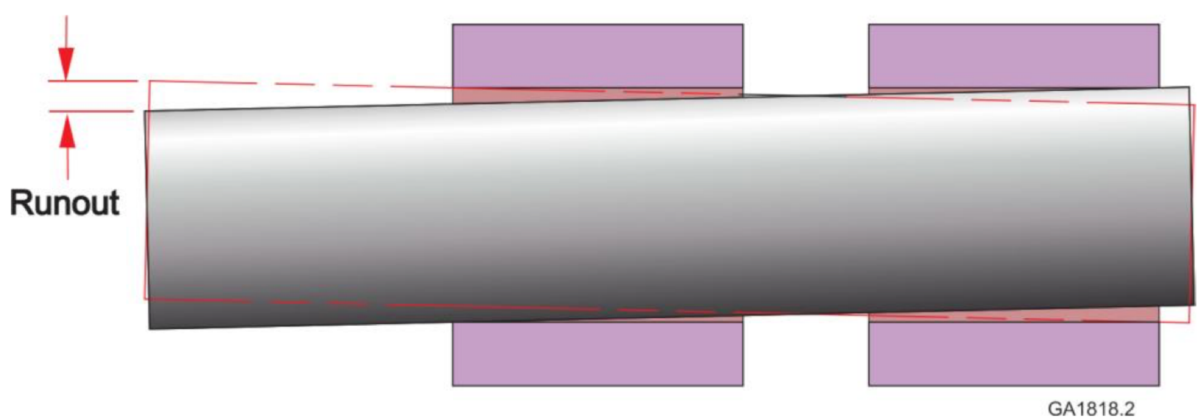

Figure 4.1.3: Explanation of dynamic runout.

Dynamic runout is a concern in this model. Dynamic runout is eccentric rotation in a shaft that can cause it to be off center[4]. Excessive runout can be caused by high deflections and different types of bearings. The way to mitigate runout is with minimal deflection and bearing types that reduce clearance (the red height in Figure 5.1.3). The shaft deflection of the roller is larger than the requirements of no runout (0.05 mm deflection). Different bearing types need to be considered to reduce runout and ensure quality preforms. The obvious bearing type to reduce clearance is a journal bearing because there is almost no space between the bearing and the shaft. However, that is not relevant for the application of the roll forge and was not in the scope of this class. Further research into journal bearings may be warranted. For the remainder of this section, there will be ball bearing and roller bearing comparisons.

4.2 Slope of the Roller at the Bearings

The slope of the shaft at the bearings is indicative of the rigidity of the roll forge. If the slope is much less than a radian, the roll forge is very rigid and it is accurate to use a bearing. If the slope at the bearing is much greater than a radian, a ball or roller bearing may not be the correct choice of fixture because of the axial forces that now come into play.

4.2.1 Calculations:

F	1.41 MN
l	0.6628 m
E	200 GPa
I	$\frac{\pi}{8}r^4 = \frac{\pi}{8}0.13^4 = 1.122 \times 10^{-4} m^4$
x	0 (bearing is at the start of the shaft)

$$\theta_{bearing} = \frac{F}{16EI}(4x^2 - l^2)$$

$$\theta_{bearing} = \frac{(1.41\times10^6 N)}{16(200\times10^9 Pa)(1.122\times10^{-4} m^4)}(4(0)^2 - (0.6628)^2) = -0.0017\, rad$$

[4] *Handling shaft deflection, runout, Vibration, & axial motion*. Kalsi Seals Handbook. (n.d.). Retrieved December 6, 2021, from https://www.kalsi.com/handbook/D04_Shaft_deflection_runout_vibration_and_axial_motion.pdf.

There is a very small slope at the bearings. A ball or roller bearing fixture is an adequate choice in this design and will likely not experience axial loading conditions.

4.3 Catalog Data

Knowing that the bearing is the correct fixture for the rollers, the C_{10} can be calculated to select the proper size bearing for the load. The $C_{10}$is based on a 90% reliability per bearing. For the initial calculations of one bearing, a 90% reliability is standard. The desired life was taken from the initial data given in the problem and the life of five years. Number of revolutions was also calculated based upon the given data.

4.3.1 Calculations:

$\mathcal{L}_D$	$\frac{8\ hours}{1\ day} \times \frac{260\ days}{1\ year} \times 5\ years\ =\ 10400\ hours$
n_d	$\frac{0.5\ revolutions}{1\ second} \times \frac{60\ seconds}{1\ minute}\ =\ 30\ rev/min$
L_R	$\mathcal{L}_R n_R 60\ =\ 10^6\ revolutions$
F_D	0.705 MN (radially on the bearing)
a	3 (ball bearing)

$$C_{10}\ = F_D(\frac{\mathcal{L}_D n_d 60}{L_R})^{1/a}$$

$$C_{10}\ =\ (0.705\ \times\ 10^6 N)(\frac{(10400\ hours)(30\ rev/min)60}{10^6})^{1/3}\ =\ 1.872\ MN$$

$$x_D\ =\ \frac{L_D}{L_R}\ =\ 18.72$$

For one bearing with 90% reliability, the catalog data indicates a bearing with 1.872 MN of force. That is nearly 19 times that of the reference life - marginally larger. The catalog load rating is also much higher than the force applied on the bearing to achieve this goal.

4.4 Bearings in Assembly

There is not only one bearing in this assembly though. In fact, there are four which need to be considered. If all of the bearings were at a 90% reliability, the assembly's reliability would be much lower at 65%.

$$R_D = \Pi_{all,i} R_i = (0.9)^4 = 0.6561$$

The goal is for the entire assembly to be 90% reliable. This means that each bearing must have a much larger reliability.

$$R = 0.9^{1/4} = 0.975\, per\ bearing$$

The catalog life rating with the larger reliability is achievable through the Weibull distribution. For this calculation, the application factor value was assumed to be 3.0 given Table 11-5[5] where the roll forge is considered machinery with moderate impact. The largest value in this category was chosen to be conservative with the catalog rating estimate. The values for x_0, $(\theta - x_0)$, and β were also assumed based upon a similar problem in the textbook (example 11-3).

4.4.1 Calculations (Ball Bearing):

a_f	3.0
F_D	0.705 MN (radially on the bearing)
x_D	18.72
x_0	0.02
$(\theta - x_0)$	4.439
R_D	0.975
β	1.483
a	3 (ball bearing)

[5] Budynas, R. G., Nisbett, J. K., Tangcchaichit, K., & Shigley, J. E. (2021). *Shigley's Mechanical Engineering Design*. McGraw Hill Education.

$$C_{10} = a_f F_D [\frac{x_D}{x_0 + (\theta - x_0)(ln\frac{1}{R_D})^{1/\beta}}]^{1/a}$$

$$C_{10} = 3.0(0.705 \times 10^6 N)[\frac{18.72}{0.02 + (4.439)(ln\frac{1}{0.975})^{1/1.483}}]^{1/3} = 7.67\ MN$$

Because of the increased reliability and addition of the application factor, the catalog life rating had a 309.72% increase for ball bearings. This was to be expected since there are now higher metrics for the bearing.

4.4.2 Calculations (Roller Bearing):

a_f	3.0
F_D	0.705 MN (radially on the bearing)
x_D	18.72
x_0	0.02
$(\theta - x_0)$	4.439
R_D	0.975
β	1.483
a	10/3 (roller bearing)

$$C_{10} = a_f F_D [\frac{x_D}{x_0 + (\theta - x_0)(ln\frac{1}{R_D})^{1/\beta}}]^{1/a}$$

$$C_{10} = 3.0(0.705 \times 10^6 N)[\frac{18.72}{0.02 + (4.439)(ln\frac{1}{0.975})^{1/1.483}}]^{3/10} = 6.74\ MN$$

The current recommendation of a bearing for this roll forge is the SKF Spherical Roller Bearing: Double Row[6]. The bore diameter is the correct size, the dynamic load factor is slightly

[6] *Spherical roller bearing: Double row, 10.2362 in bore Dia. (in.), 260 mm bore dia. (MM)*. Grainger. (n.d.). Retrieved December 6, 2021, from https://www.grainger.com/product/SKF-Spherical-Roller-Bearing-Double-36MF58.

lower than the one calculated for the roller bearing at this high of a reliability, but is still much higher than that of the single bearing. This also decreases the clearance to reduce runout over a ball bearing. In the roller bearing it would be harder to bend the shaft downwards where the inner and outer ring of the bearing would touch since it is a cylinder. This is not the case in the ball bearing though. The only drawback to this solution is the cost: $18,313.05 per bearing. Because of this high cost, other bearing solutions should be explored. It should be noted that the cost of changing a bearing would be much higher. Between the labor, stopping of the production line, and cost of the new bearing, it is likely more effective to only have one bearing for the lifetime of the roll forge. Ultimately, the bearing could be a source of failure within the roll forge because of the low reliability compounding due to the assembly. If there is an adequate bearing in place and regular maintenance with lubricant, the point of failure for the roll forge will be elsewhere. Other bearing options would require lowering the catalog rating. This can be done by having bearings in assembly on the same shaft, changing the reliability, or even changing the bearing type.

5. Gears

Within the roll forge, the gear train plays the vital role of power transmission. Through it, torque is increased by a factor of 5 and rotational speed is decreased by a factor of approximately 0.2 from the motor to the rollers. The gear train also ensures that the rollers are both rotating in the direction necessary to feed the billet through the semi-cylindrical dies as it is preformed. As the gears transmit this power over the rated life of the machine, they experience wear and fatigue from bending and contact stresses. It is vital to analyze these stresses, as they limit the amount of power the gears are able to reliably transmit over the lifespan of the machine. For this analysis, the maximum bending and contact stresses are first determined, then the transmitted load via each mechanism, and finally the acceptable power transmission limit. The smallest of these values is then the limiting factor for the entire gear train.

5.1 Gear Train Analysis

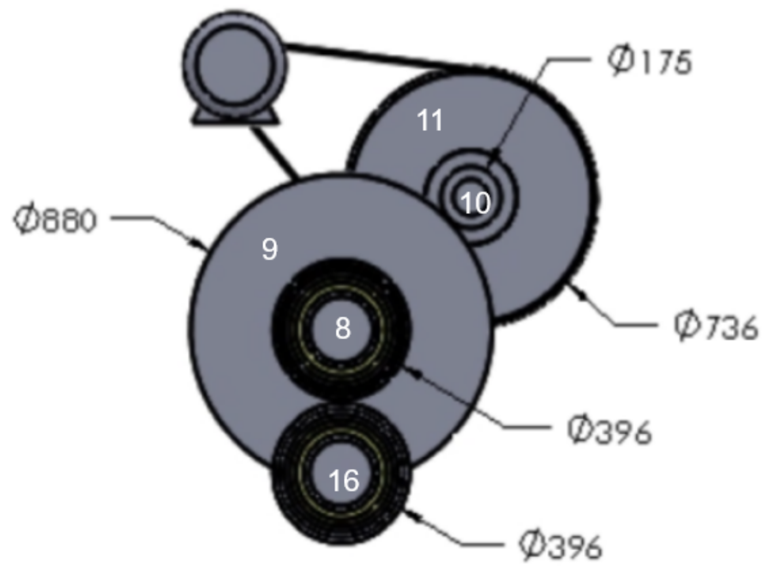

Figure 5.1.1. Gear train with provided pitch diameters.

$$e = \frac{\Pi N_{drive}}{\Pi N_{driven}} \qquad d_p = \frac{N}{p} \qquad e = \frac{\Pi d_{p,\,drive}}{\Pi d_{p,\,driven}} = \frac{d_{10} d_8}{d_9 d_{16}} = \frac{d_{10}}{d_9} = 0.19886$$

The gear train value (e) is equal to the product of the number of teeth on the driving gears divided by the product of the number of teeth of the driven gears in the train. This is a useful value, as many other properties are also proportional to this value. For example, the torque of the output value in a gear train is equal to the input torque divided by e. As the pitch diameter is equal to the number of teeth divided by the diametral pitch, the gear train value is also equal to the product of the driving pitch diameters divided by those of the driven gears. Concentric gears are neither driven nor driving, so the only driving gears are gears 10 and 8. The only driven gears in this train are gears 9 and 16. In this case, as gears 8 and 16 have the same diameters, they cancel out in the gear train value calculation. This gear train value assumes that gear 16 is the output gear, and gear 10 is the input gear, although the motion of gear 10 comes from the flywheel (gear 11). The gear train value is positive because gears 10 and 16 rotate in the same direction; there is a reversing of direction between gears 10 and 9, then again between gears 8 and 16.

5.2 Tabulated Values

Gear #	d_p [mm]	Speed (n) [RPM]	Speed (v) [m/s]	Maximum Torque [kNm]
10	175	150.86	1.382	5.11
11	736	150.86	5.814	5.11
9	880	30	1.382	25.7
8	396	30	0.622	25.7
16	396	30	0.622	25.7

The pitch diameter of each gear was obtained from figure 7. The maximum torque experienced by the rollers, powered by gears 8 and 16, was given. The output speed n was given, as the rollers complete a full rotation every 2 seconds. The tangential velocity of gear 16 could then be calculated using the following formula:

$$v = \frac{\pi d n}{60}$$

It is known that a driven and driving gear pair must have the same tangential velocity, and as gears 8 and 16 have the same diameter, they must have the same rate of rotation as well. This means they must also have the same torque. Next, gear 9 is on the same axis as gear 8, so they must have the same torque and rate of rotation. Due to their different diameters, however, gear 9 has a different tangential velocity. Since gears 9 and 10 mesh, they must have the same tangential velocity; however, due to their differing diameters, they must have different rates of rotation, which can be calculated using the same relation above. Gears 11 and 10 are coaxial as well, and so must have the same rate of rotation and torque. The relationship between the torque of gear 10 (driver) and that of the output gear can be obtained by multiplying the output torque by the gear train value, obtaining the final tabulated values.

5.3 Bending Failure

In the mode of failure analysis, we analyzed the relationship between gears 10 and 9 as a pinion driving a larger gear. We made this decision because as the smallest gear, the pinion was the most likely to fail, a prediction that would be reinforced through our calculations. We will assume that $N_{10} = 16$ and $N_9 = 81$. This is obtained via the calculation for the minimum number of teeth on a pinion,

$$N_P = \frac{2k}{(1+2m)sin^2(\phi)}\,(m + \sqrt{m^2 + (1 + 2m)sin^2(\phi)}$$

and

$$m = \frac{N_9}{N_{10}} = \frac{D_9}{D_{10}} = 5.028$$

Where k=1 for full-depth teeth, and ϕ=20 °, such that $N_P = N_{10}$= 15.747 = 16 teeth. Then by the same gear train number, $N_9 = 81$. The gears were assumed to be made from AISI 4340 steel, chosen for its strength. This is also a fairly common material for gear manufacturing, as noted on both its MatWeb page[7] and in the textbook[8].

First, we calculated the allowable bending stress using the AGMA metric standard.

$$\sigma_{all}^{bend} = \left(\frac{S_t Y_N}{S_F Y_\theta Y_Z}\right)$$

The factors were evaluated as follows.

S_t is the allowable bending stress, such that $S_t = 0.533\ H_B + 88.3$ MPa (fig 14-2, textbook) for grade 1, through-hardened steel. The Brinell hardness, H_B, for AISI 4340 steel is 321[9].

$$S_t = 0.533(321) + 88.3 = 259.393 \text{ MPa}$$

Y_N is the stress-cycle factor, such that $Y_N = 6.1514N^{-0.1192}$ (fig. 14-14, textbook). The roll forge executes 360 cycles per hour, 8 hours a day, for 260 days. Gear 10 has a rotational speed of 150.86 RPM, and gear 9 has a rotational speed of 30 RPM.

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[7] *AISI 4340 Steel, normalized.* MatWeb (n.d.). Retrieved December 10, 2021, from http://www.matweb.com/search/DataSheet.aspx?MatGUID=4a3cfc1e1cfd451091e67d3f3b66bb80.

[8] "Textbook" refers to *Shigley's Mechanical Engineering Design*, as cited.

[9] *AISI 4340 Steel, normalized.* MatWeb (n.d.). Retrieved December 10, 2021, from http://www.matweb.com/search/DataSheet.aspx?MatGUID=4a3cfc1e1cfd451091e67d3f3b66bb80.

$$N_{10} = 150.86*60*8*260 = 1.89*10^7 \text{ cycles}$$

$$Y_{N,10} = 6.1514N_{10}^{-0.1192} = 0.8353$$

$$N_9 = 30*60*8*260 = 3.74*10^6 \text{ cycles}$$

$$Y_{N,9} = 6.1514N_9^{-0.1192} = 1.0126$$

S_F is the safety factor. For design purposes at this time, we will be setting $S_F = 1$, to determine the upper limit of the power transmissible by this gear system. To calculate more conservative values, we would utilize a safety factor of around 3 or higher.

Y_θ is the temperature correction factor. This is definitely worth taking into account for hot rolling steel, but without objective correlations, it is better to assume $Y_\theta = 1$.

Y_Z is the reliability factor. Assuming reliability of 0.99, $Y_Z = 1$.

Thus,

$$\sigma_{all,10}^{bend} = \left(\frac{259.393*0.853}{1*1*1}\right) = 216.66 \text{ MPa}$$

$$\sigma_{all,9}^{bend} = \left(\frac{259.393*1.0126}{1*1*1}\right) = 262.662 \text{ MPa}$$

To determine the acceptable power transmission, W^t, we must next calculate the transmitted load via bending using the AGMA metric standard. For this portion, we will analyze gear 10, as it is shown to have a lower allowable bending stress.

$$\sigma_b = W^t K_O K_V K_S \frac{K_H K_B}{b m_t Y_J}$$

We can then rearrange this equation to get

$$W^t = \frac{\sigma_b b m_t Y_J}{K_O K_V K_S K_H K_B}$$

With the factors calculated as follows.

K_O is the overload factor; as we are not planning to overload our motor, $K_O = 1$.

K_V is a precision factor. For these purposes, we will assume moderate commercial precision of $Q_V = 5$, such that

$$K_V = \left(\frac{A+\sqrt{200V}}{A}\right)^B, B = 0.25(12 - Q_V)^{2/3}, \text{ and } A = 50 + 56(1 - B).$$

As previously calculated, V for gear 10 is 1.382 m/s, making B = 0.9148, A = 54.7697, and K_V = 1.274.

Factor b is the face width of the gears; from the provided schematics, it appears that gears 9 and 10 have the same face width. They appear to be about half the width of the bearings' diameter, making b = 130 mm.

Factor m_t is the transverse metric module, $m_t = \frac{D}{N cos\phi}$. For gear 10, D = 175 mm. We assumed that the pinion, gear 9, had 16 teeth, with a pitch angle of 20°. This makes m_t = 11.639.

Factor K_H is the load-distribution factor, such that

$$K_H = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$$

Where $C_{mc} = 1$ since we will assume the gears are uncrowned, C_e = 1 for gears not especially adjusted, and

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F \text{ (eqn 14-32)} = 0.2505$$

$$C_{ma} = 0.2 \text{ (fig 14-11)}$$

Such that K_H = 1.4505.

For the rim-thickness factor K_B, we will assume that the gears are manufactured to the accepted standard of tooth height such that K_B = 1.

Factor Y_J is evaluated graphically using fig. 12-6 for a 20°, full-depth tooth where N_{10}=16 to get Y_J = 0.28.

Plugging into the previously found equation,

$$W^t = \frac{(216.66\ MPa)(130\ mm)(11.639)(1.4505)(1)}{(1)(1.274)(1)(0.28)(1)} = 1.333\ MN$$

Finally, we can evaluate for the maximum power transmission as

$$W^t = \frac{60{,}000H}{\pi Dn} \Rightarrow H = \frac{\pi Dn}{60{,}000W^t}$$

$$H = \frac{\pi(175\ mm)(150.857\ rpm)}{60{,}000(1.333\ MN)} = 1.039 \text{ kW}$$

This value is low, to be sure. Pinions are frequently made of a stronger material than the other gears in a gear train for exactly this reason. This is also without taking into account a factor of safety, or a correction factor for the high operating temperatures.

5.4. Wear and Fatigue

Next, we calculated the allowable contact stress between gears 10 and 9 using the AGMA metric standard.

$$\sigma_{all}^{contact} = \left(\frac{S_C Z_N Z_W}{S_H Y_\theta Y_Z}\right)$$

The terms were evaluated as follows. Factors Y_θ and Y_Z are the same as for the allowable bending stress calculations. The factor S_H serves the same purpose as S_F in the allowable bending stress formula, and will also be equal to 1 for the purpose of determining the maximum allowable contact stress.

S_t is the allowable contact stress, such that $S_C = 2.22\ H_B + 200$ MPa (fig 14-5, textbook) for grade 1, through-hardened steel. The Brinell hardness, H_B, for AISI 4340 steel is 321[10].

$$S_C = 2.22(321) + 200 = 912.62 \text{ MPa}$$

Z_N is the stress-cycle factor, depending on the number of cycles undergone by the gear. (fig. 14-15, textbook). For the previously calculated number of cycles, the correlations are as follows:

$$Z_{N,10} = 1.448N_{10}^{-0.023} = 0.9849$$

$$Z_{N,9} = 2.466N_9^{-0.056} = 1.0566$$

Thus,

$$\sigma_{all,10}^{contact} = \left(\frac{912.62*0.9849}{1*1*1}\right) = 898.831 \text{ MPa}$$

$$\sigma_{all,9}^{contact} = \left(\frac{912.62*1.0566}{1*1*1}\right) = 964.284 \text{ MPa}$$

These are the allowable stresses due to wear and fatigue, notably higher than the allowable bending stresses.

To determine the acceptable power transmission, W^t, we must next calculate the transmitted load via contact stress using the AGMA metric standard. For this portion, we will analyze gear 10, as it is shown to have a lower allowable bending stress.

$$\sigma_c = Z_E\sqrt{W^t K_O K_V K_S \frac{K_H Z_R}{d_w b Z_I}}$$

We can then rearrange this equation to get

[10] http://www.matweb.com/search/DataSheet.aspx?MatGUID=4a3cfc1e1cfd451091e67d3f3b66bb80

$$W^t = \frac{\sigma_c^2 d_w b Z_I}{Z_E^2 K_O K_V K_S K_H Z_R}$$

K_O, K_V, b, K_H, and K_S are the same as previously calculated for bending stress, with the remaining factors calculated as follows.

Z_E is an elastic coefficient, taken as 191 $\sqrt{MPa}$ as given in table 14-8.

d_{w1} is the pinion pitch diameter of 175 mm.

Factor Z_I is a geometry factor accounting for pitting resistance. As gears 10 and 9 are taken to be spur gears made from the same material, we can calculate this factor as

$$Z_I = \frac{\cos\phi \sin\phi m_G}{2(m_G - 1)} \quad \text{where } m_G = \frac{N_G}{N_P} = \frac{1}{e} = 5.029$$

Such that $Z_I = 0.2006$.

Z_R is a surface condition factor; we will assume that there are no surface abnormalities on these gears.

Plugging into the previously found equation, we can find the load as

$$W^t = \frac{(898.831 MPa)^2 (175mm)(130\ mm)(0.2006)}{(191\sqrt{MPa})^2 (1.274)(1.4505)} = 54.690 \text{ kN}$$

Which is significantly lower than the load calculated for bending stress.

Finally, we can evaluate for the maximum power transmission as

$$W^t = \frac{60{,}000H}{\pi D n} \Rightarrow H = \frac{\pi D n}{60{,}000 W^t}$$

$$H = \frac{\pi(175\ mm)(150.857\ rpm)}{60{,}000(0.05469\ MN)} = 25.27 \text{ kW}$$

This value, maximum power transmission taking fatigue as the limiting factor, is significantly higher than that found using bending stress as the limiting factor. That suggests that when this gear train does fail, it will be due to the failure of gear 10 due to bending stresses. The relative values may change if different correction factors are used, but with as stark a contrast as we have between these two values, it is likely that this mechanism of failure would not change.

6. Conclusion

Overall, the roll forge as designed, and with the assumptions we made, should be functional for the desired task desired task. Our analysis showed that the deflection of the roller would be minimal so the roller will be able to deliver the desired force to the billet and shape the part correctly. This predicted slope was also not significant enough to warrant concerns about shaft runout with ball or roller bearings; however, it should still be kept in mind when selecting a bearing. Calculation of static failure likewise yielded results suggesting that part would survive. The factor of safety guarding against static failure could be improved by using a thicker shaft which would decrease the bending stress in the roller. This change would also improve the fatigue factor of safety for the same reasons, in addition to improving the roller's ability to handle the hot temperatures of the billet being fed between the rollers.

The bearings we found that would be suitable for use in the roll forge are very expensive; but if one is to consider maintenance that must be performed on the machine, spending more money upfront to prevent complicated work later (i.e., disassembling and reassembling the machine to replace bearings) is an adequate offset to prevent difficult and costly maintenance.

The gear train is limited by the bending stress that gear ten can handle. Compared to wear, bending stress is the obvious limiting factor to the power that the gears can withstand. Because of the large difference between maximum power for bending and wear, it is unlikely that changes we make to the gears would shift that balance (although both values would increase).

This project provided a useful exercise in the analysis and selection of machine components. Seeing other groups present their findings demonstrated the wide array of approaches that can be taken with an open-ended problem and illustrated the diversity of ideas that come from such tasks.

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