NE 255: Homework # 03

Due 18 October 2016

Professor Rachel Slaybaugh

Andrew S Voyles

Problem 1

(20 points) Derive the 1st order form of SP_5 with isotropic source and vacuum boundary conditions.

Solution

XXXXXXXXXXXXXXXXXXX

Problem 2

Consider the integral

$$\int_{4\pi} d\hat{\Omega} \left|\hat{\Omega}\right| \tag{1}$$

The LQ$_N$ quadrature set is given in Figure 1. Recall that $\mu_i = \eta_i = \xi_i$ for a given level, i.

Multidimensional Discrete Ordinates

Table 4-1 Level Symmetric S_N Quadrature Sets LQ_n[a]

Level	n	μ_n	w_n[b]
S_4	1	0.3500212	0.3333333
	2	0.8688903	
S_6	1	0.2666355	0.1761263
	2	0.6815076	0.1572071
	3	0.9261808	
S_8	1	0.2182179	0.1209877
	2	0.5773503	0.0907407
	3	0.7867958	0.0925926
	4	0.9511897	
S_{12}	1	0.1672126	0.0707626
	2	0.4595476	0.0558811
	3	0.6280191	0.0373377
	4	0.7600210	0.0502819
	5	0.8722706	0.0258513
	6	0.9716377	
S_{16}	1	0.1389568	0.0489872
	2	0.3922893	0.0413296
	3	0.5370966	0.0212326
	4	0.6504264	0.0256207
	5	0.7467506	0.0360486
	6	0.8319966	0.0144589
	7	0.9092855	0.0344958
	8	0.9805009	0.0085179

[a]Data from Ref. 5.
[b]See Fig. 4-3 for ordinate directions corresponding to weight

Figure 1: LQ$_n$ quadrature

(a) (5 points) Use the S_4 LQ$_N$ quadrature set to execute this integral.

Solution

XXXXXXXXXXXXXXXXXXXXXXX

(b) (10 points) Repeat it with S_6. What do you observe?

Solution

XXXXXXXXXXXXXXXXXXX

(c) (10 points) Write a short code to execute this integration (and higher orders if you'd like). Try a few different functions. Turn in the code and the evaluation of these functions. Include comments on what you observe.

Solution

XXXXXXXXXXXXXXXXX

Problem 3

(a) (5 points) Briefly compare the diffusion equation, deterministic methods, and monte carlo methods in terms of complexity, accuracy, run time, and range of applicability.

Solution

XXXXXXXXXXXXXXXXXXXXXXXX

(b) (5 points) Given what you've learned about deterministic methods so far, discuss strengths and weaknesses.

Solution

XXXXXXXXXXXXXXXXXXXXXXXXXXXXx

Problem 4

Write a function that generates the associated Legendre Polynomials:

$$P_\ell^m(x) = \frac{(-1)^m}{2^\ell \ell!} \left(1 - x^2\right)^{m/2} \frac{d^{\ell+m}}{dx^{\ell+m}} \left(x^2 - 1\right)^\ell \tag{2}$$

Use this function in a function that generates spherical harmonics:

$$Y_{\ell m}(\theta, \phi) = (-1)^m \sqrt{\frac{2\ell + m}{4\pi} \frac{(\ell - m)!}{(\ell + m)!}} P_{\ell m}(\cos(\theta))\, e^{im\phi} \tag{3}$$

(a) (30 points) Generate and plot the following $\ell = 0, 1, 2$ for $-\ell \le m \le \ell$ (recall we can relate the negate m to positive m values). You will need to discretize θ and ϕ fairly finely (I suggest 30 increments in each to start so you get a real sense of the shape of the harmonics).

Solution

XXXXXXXXXXXXXXXXXX

(b) (20 points) Now, we will approximate the external source. Using the S_4 quadrature to do the integrations and $q_e = 1$ for all angles: use the equations for external source we developed in class (eqns. 19-21), calculate the external source for $\ell = 0, 1, 2$.

Solution

XXXXXXXXXXXXXXXXXXXXXX

Problem 4 continued on next page...

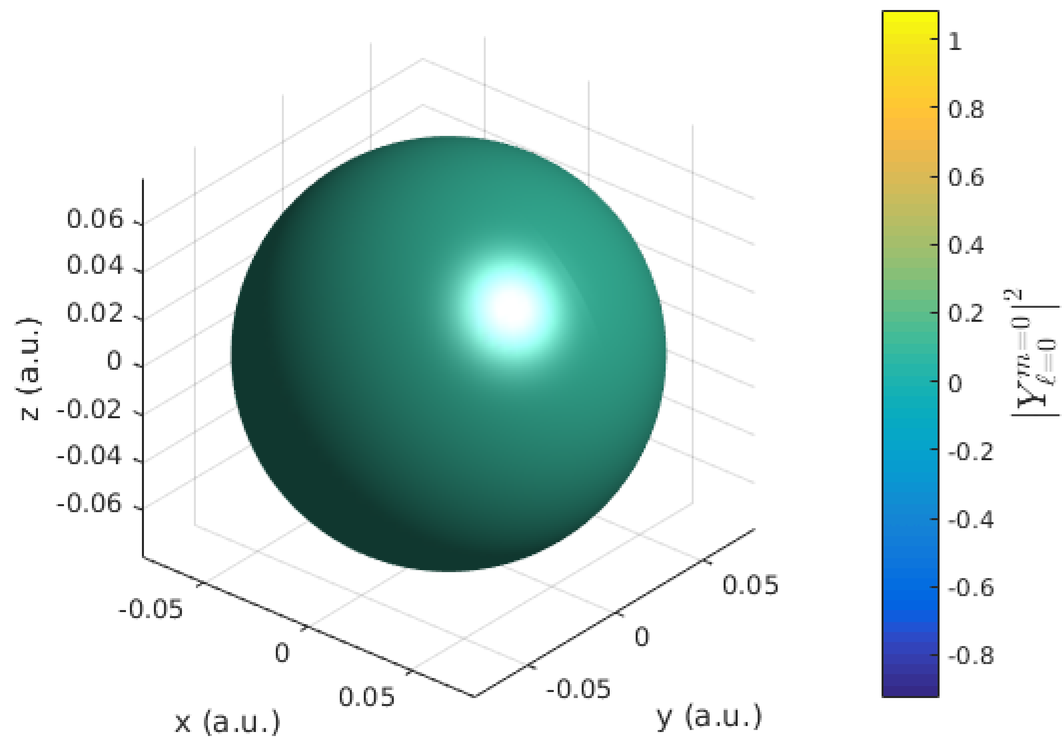

Figure 2: Y00

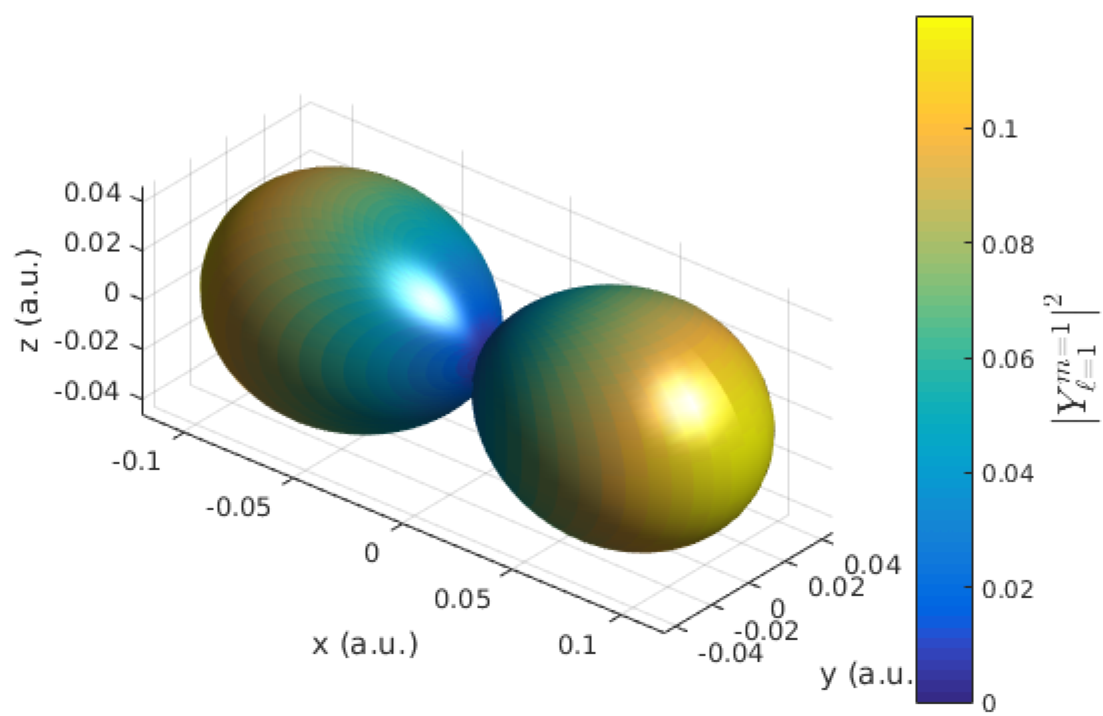

Figure 3: Y11

Problem 4 continued on next page. . .

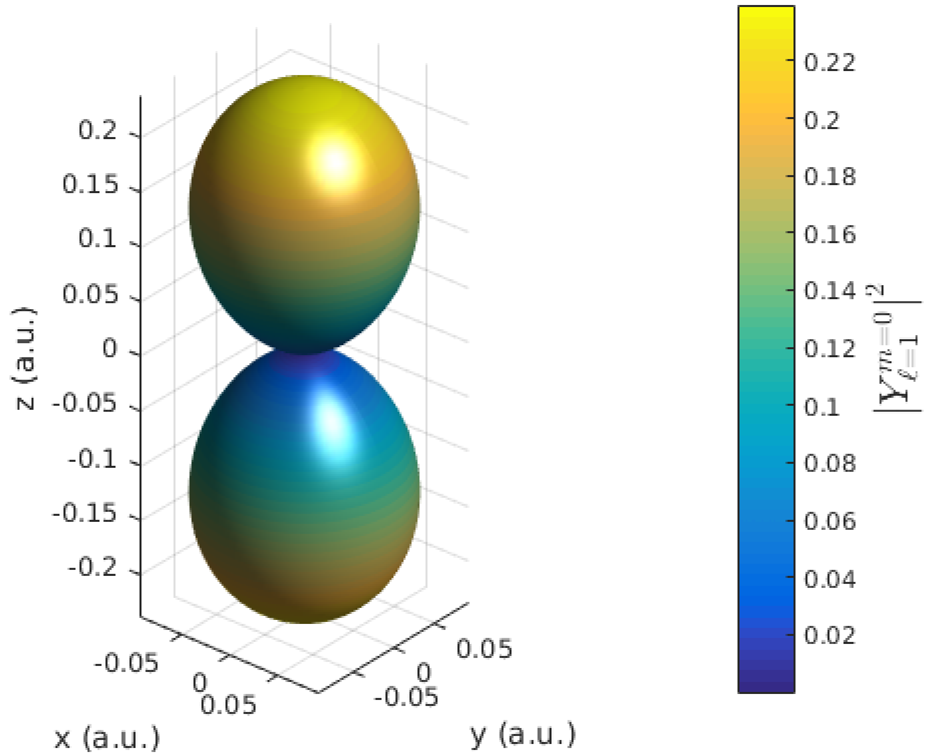

Figure 4: Y10

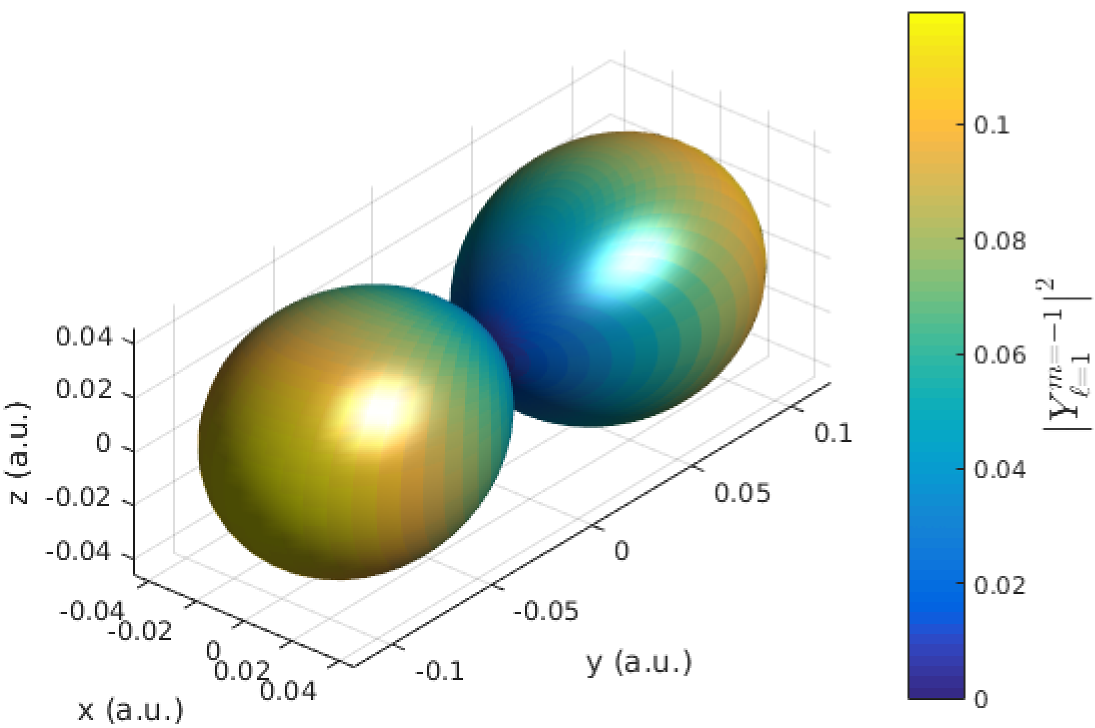

Figure 5: Y1-1

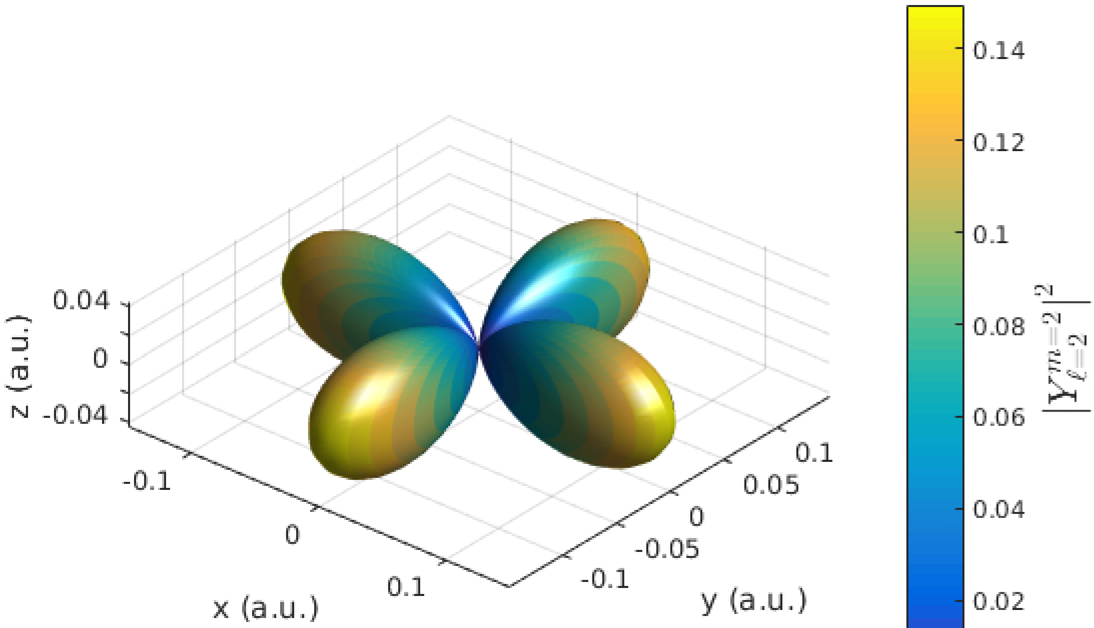

Figure 6: Y22

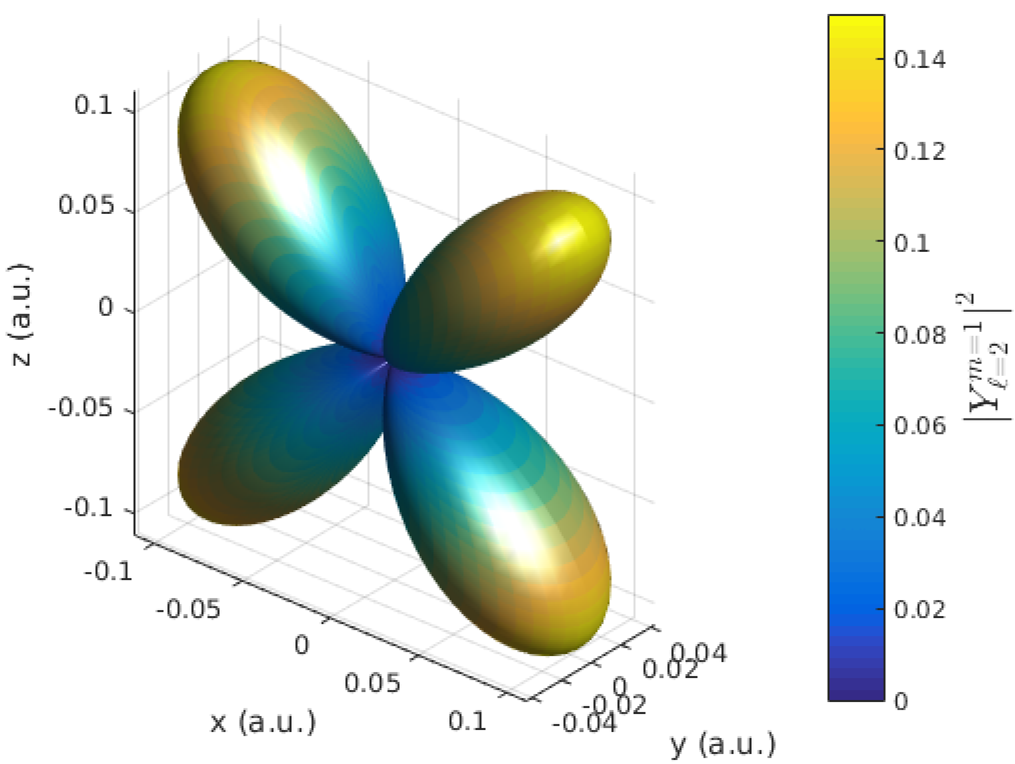

Figure 7: Y21

Problem 4 continued on next page. . .

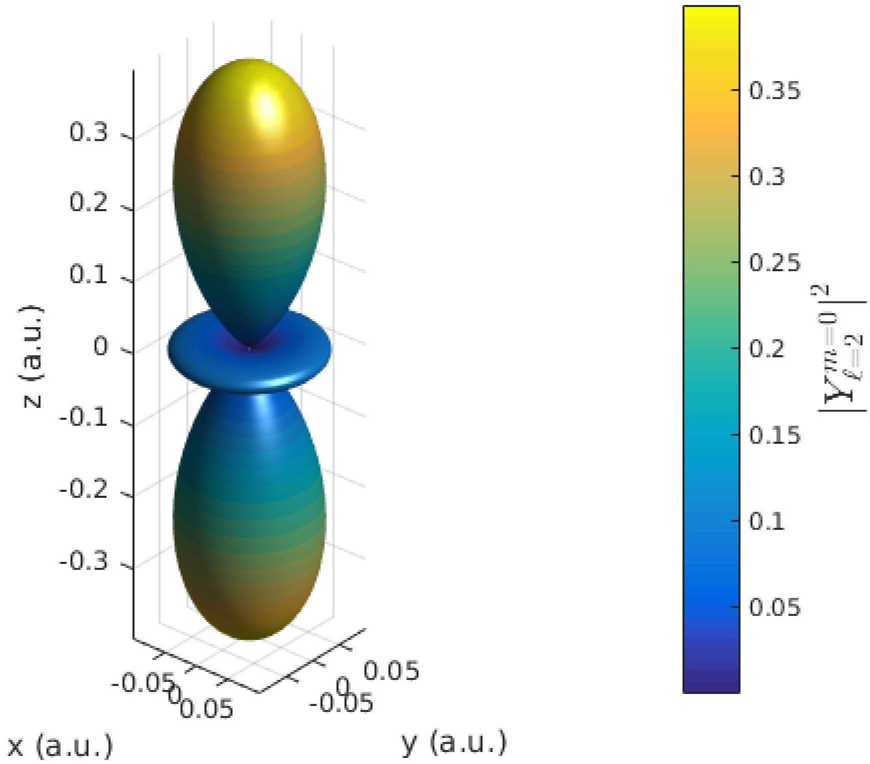

Figure 8: Y20

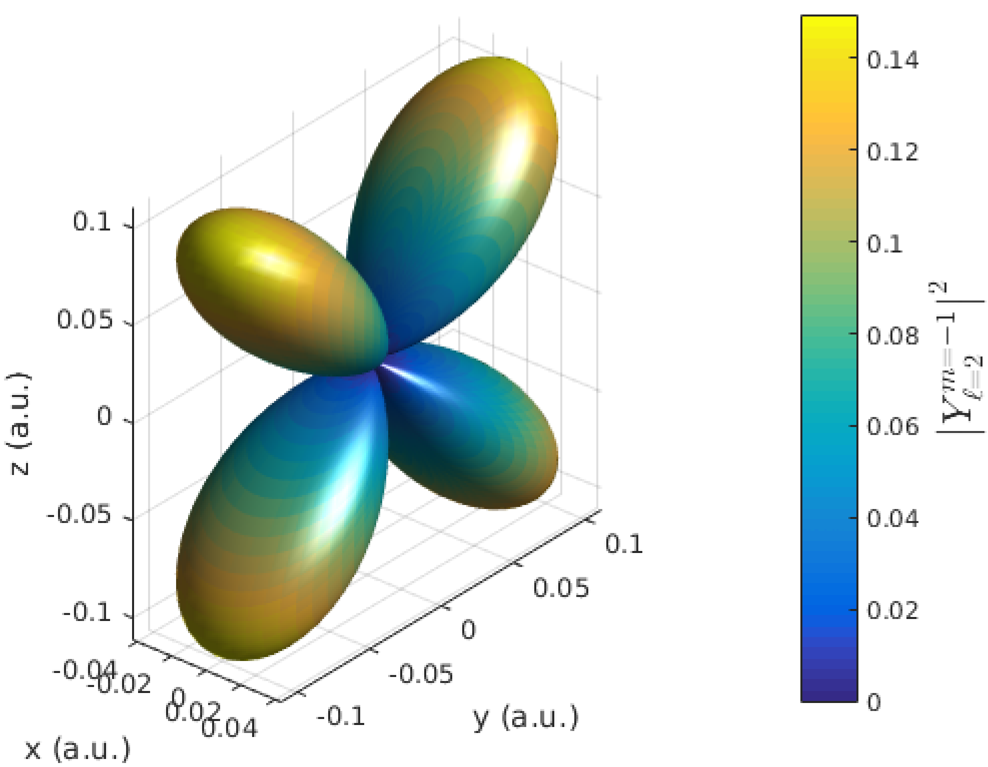

Figure 9: Y2-1

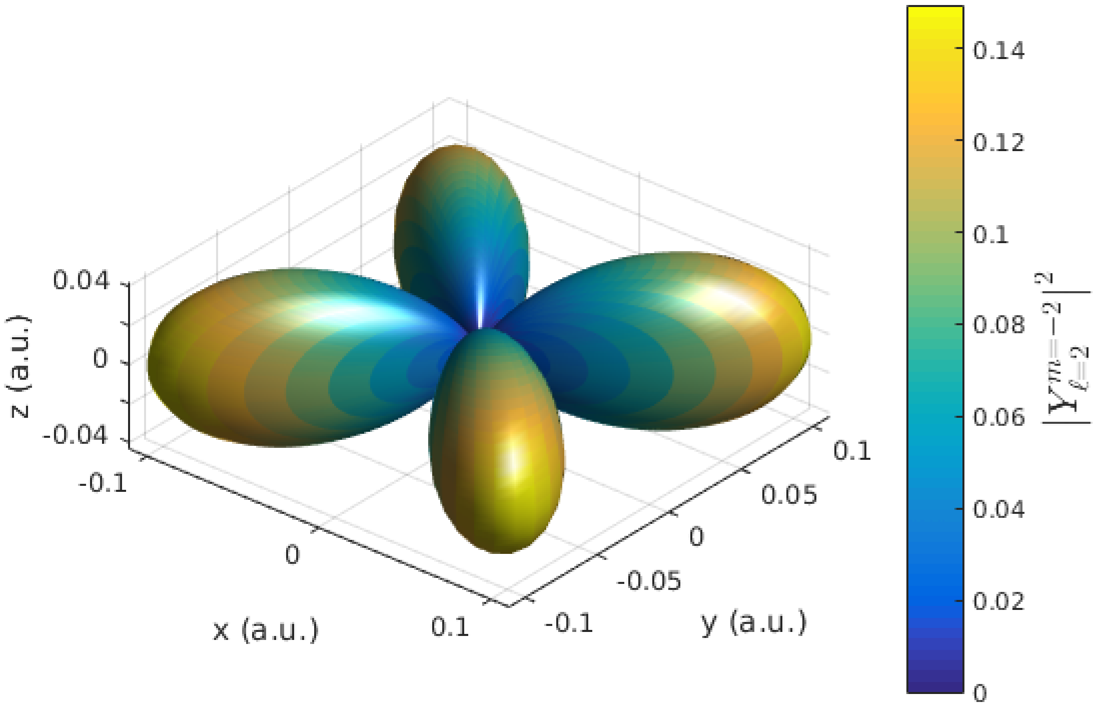

Figure 10: Y2-2

Problem 5

(5 points) What are the major nuclear data libraries and which countries manage them?

(a) Briefly describe what each term in the Transport Equation physically represents.

Solution

A) Time rate of change of the neutron angular flux, the change in the neutron angular flux for an energy group over the entire volume, as a function of time..

B) Streaming losses, the rate at which neutrons exit the control volume.

C) Total interaction losses, the rate at which neutrons are absorbed or outscattered from an energy and solid angle group.

D) External source gains, the rate at which neutrons enter the system, or from a generic point/line/distributed/etc source (other than fission) in the system.

E) Inscattering source gains, the rate at which neutrons are scattered into an energy and solid angle group.

F) Fission source gain, the rate at which neutrons are born through fission, into a particular energy group.

(b) Rewrite the time independent form of the equation to include azimuthal symmetry. Show the steps needed to get there.

Solution

The first step is to make the time-independent assumption:

$$\frac{\partial \psi}{\partial t} = 0 \tag{4}$$

This also removes all time dependence terms:

$$\begin{aligned} \hat{\Omega} \cdot \nabla \psi(\mathbf{r}, E, \hat{\Omega}) + \Sigma_t(\mathbf{r}, E)\psi(\mathbf{r}, E, \hat{\Omega}) = S(\mathbf{r}, E, \hat{\Omega}) \\ + \int_0^\infty \int_{4\pi} \Sigma_s(\mathbf{r}, E' \rightarrow E, \hat{\Omega}' \rightarrow \hat{\Omega})\psi(\mathbf{r}, E', \hat{\Omega}')d\hat{\Omega}'dE' \\ + \frac{\chi_p(E)}{4\pi} \int_0^\infty \int_{4\pi} \nu(E')\Sigma_f(\mathbf{r}, E')\psi(\mathbf{r}, E', \hat{\Omega}')d\hat{\Omega}'dE' \end{aligned} \tag{5}$$

For the azimuthal symmetry assumption, scattering is only a function of $\mu = \hat{\Omega}' \cdot \hat{\Omega}$, the cosine of the scattering angle. This gives us the simplifications:

$d\hat{\Omega} = \sin(\theta)d\theta d\varphi = d\mu d\varphi$; $\mu = \cos(\theta)$; $d\mu = \sin(\theta)d\theta$.

$$\int_{4\pi} d\hat{\Omega} = \int_0^{2\pi} d\varphi \int_{-\pi/2}^{\pi/2} \sin(\theta)d\theta = \int_0^{2\pi} d\varphi \int_{-1}^{1} d\mu = 4\pi \tag{6}$$

$$\psi(\mathbf{r}, \hat{\Omega}, E)d\hat{\Omega} = \psi(\mathbf{r}, \varphi, \mu, E)d\varphi d\mu = \psi(\mathbf{r}, \mu, E)d\varphi d\mu \tag{7}$$

The scattering cross section is no longer dependent on φ:

Problem 5 continued on next page. . .

$$\Sigma_s(\mathbf{r}, \hat{\Omega}' \to \hat{\Omega}) \to \Sigma_s(\mathbf{r}, \hat{\Omega}' \cdot \hat{\Omega}) = \Sigma_s(\mathbf{r}, \mu) \tag{8}$$

Thus,

$$\int_{4\pi} d\hat{\Omega}\, \psi(\mathbf{r}, \hat{\Omega}, E) = \int_0^{2\pi} d\varphi \int_{-1}^{1} d\mu\, \psi(\mathbf{r}, \hat{\Omega}, E) = 2\pi \int_{-1}^{1} d\mu\, \psi(\mathbf{r}, \mu, E) \tag{9}$$

Finally, the fission term can now be written in terms of the scalar flux:

$$\begin{aligned}\frac{\chi_p(E)}{4\pi} \int_0^\infty \int_{4\pi} \nu(E')\Sigma_f(\mathbf{r}, E')\psi(\mathbf{r}, E', \hat{\Omega}') d\hat{\Omega}' dE' &= \frac{\chi_p(E)}{4\pi} \int_0^\infty \nu(E')\Sigma_f(\mathbf{r}, E') dE' \int_{4\pi} \psi(\mathbf{r}, E', \hat{\Omega}') d\hat{\Omega}' \\ &= \frac{\chi(E)}{2} \int_0^\infty dE'\, \nu(E')\Sigma_f(\mathbf{r}, E')\phi(\mathbf{r}, E')\end{aligned} \tag{10}$$

Combining these all together:

$$\begin{aligned}\mu \cdot \nabla\psi(\mathbf{r}, E, \mu) + \Sigma_t(\mathbf{r}, E)\psi(\mathbf{r}, E, \mu) &= S(\mathbf{r}, E, \mu) \\ &+ 2\pi \int_0^\infty dE' \int_{-1}^{1} \Sigma_s(\mathbf{r}, E' \to E, \mu')\psi(\mathbf{r}, E', \mu') d\mu' \\ &+ \frac{\chi(E)}{2} \int_0^\infty dE'\, \nu(E')\Sigma_f(\mathbf{r}, E')\phi(\mathbf{r}, E')\end{aligned} \tag{11}$$