ANDRII VOITKIV **F2022DATA602**
ProbabilityExerciseSetFour is due on Thursday, September 15, 2022 at 11:59pm.

The number of attempts available for each question is noted beside the question. If you are having trouble figuring out your error, you should consult the textbook, or ask a fellow student, one of the TA's or your professor for help.
There are also other resources at your disposal, such as the Mathematics Continuous Tutorials. Don't spend a lot of time guessing – it's not very efficient or effective.

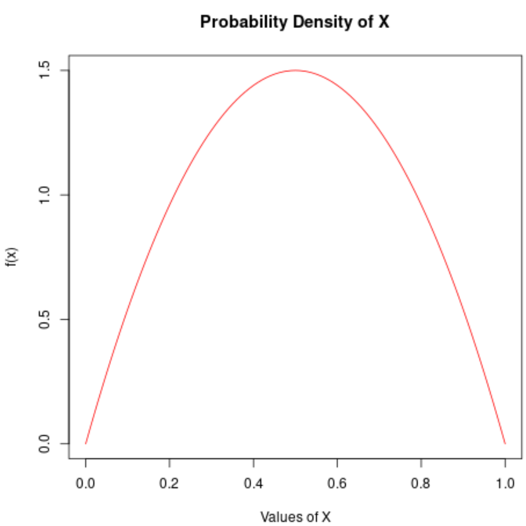
Make sure to give lots of significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2 \wedge 3$ instead of 8, $\sin(3*\mathrm{pi}/2)$instead of -1, $e \wedge (\ln(2))$ instead of 2, $(2+\tan(3))*(4-\sin(5))\wedge 6-7/8$ instead of 27620.3413, etc.

Problem 1. **(1 point)**

A random variable X has the following probability density function

$f(x) = 6x(1-x)$ for $0 \le x \le 1$.

Visually, the distribution of X is demonstrated below.



Use R Studio to create necessary function(s), answer the questions below.

(a) Compute $P(0.26 \le X \le 0.48) =$ _____ (Enter your answer to four decimals)

(b) Compute $P(0.81 \le X \le 1) =$ _____(Enter your answer to four decimals)

(c) Compute the mean/expected value of X, or $\mu_X = E(X)$.

_____ (use two digits after the decimal if rounding...)

(d) Compute the standard deviation of X, or $\sigma_X = SD(X)$.

_____ (use two digits after the decimal if rounding...)

Answer(s) submitted:

- 0.3024
- 0.0946
- 0.5
- 0.22

submitted: (correct)
recorded: (correct)
Correct Answers:

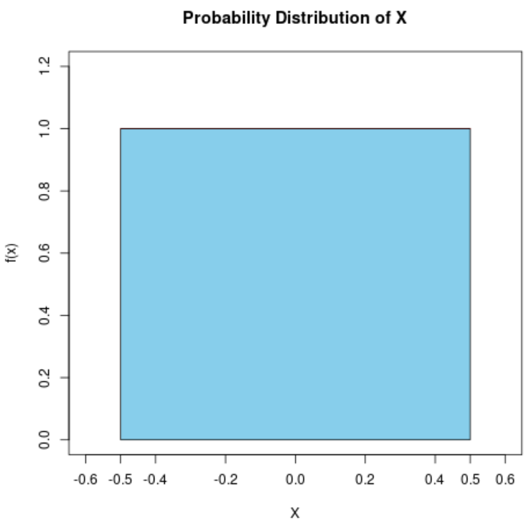
- 0.302368
- 0.094582
- 0.5
- 0.223606797749979

Problem 2. **(1 point)**

A local river feeds into a dam. Upstream from this dam, the daily flucuations in the water level of the river (in metres) X can be modeled by the following probability density function

$f(x) = 1$ for $-0.5 \leq x \leq 0.5$.

Visually, the distribution of X is demonstrated below.



Use R Studio to create necessary function(s), answer the questions below.

(a) Compute the probability that on a certain day, the level of the water has dropped by more than 0.27 meters.

_____ (Enter your answer to four decimals)

(b) Compute the probability that on a certain day, the level of the water has risen above 0.27 meters.

_____ (Enter your answer to four decimals)

(c) By how much do you expect the water to fluctuate on a daily basis? Enter your answer below, to one decimal.

_____ meters

(d) Compute the standard deviation in the daily flucuation of the water, σ_X or $SD(X)$. Use two decimals in your answer.

_____ meters

(e) Compute the probability that the daily fluctuation in the river's water level will fall within one standard deviation of the expected daily water fluctuation.

_____ (use four decimals in your answer)

Answer(s) submitted:

- 0.23
- 0.23
- 0
- 0.29
- 0.58

submitted: (correct)
recorded: (correct)
Correct Answers:

- 0.23
- 0.23
- 0
- 0.288675134594813
- 0.57735

Problem 3. **(1 point)**

In a certain geographical region in the southwestern United States, the number of earthquakes receives various from one month to the next with a mean of three earthquakes per month (assume 30 days in a one-month period). When an earthquake does occur, the magnitude of the earthquake on the Richter scale A can be modeled by the probability density function

$f(a) = \frac{1}{3.8} e^{\left(\frac{-a}{3.8}\right)}$ for $a > 0$

Use R Studio to create the necessary function(s) and visualization(s) of the probability density function of A.

Use R Studio to create necessary function(s), answer the questions below.

(a) What can you say about the distribution of the magnitude of earthquakes in the particular region of the SW United States? Complete the statement below.

The distribution of time passing between earthquakes is
- ?
- is left-skewed
- is right-skewed
- is symmetrical

with a mean of ____ and a standard deviation of ____.
(Enter numerical answers to two decimals.)

(b) Compute the probability that the magnitude the next earthquake will be between 2 and 7 on the Richter scale.

____ (Enter your answer to four decimals)

(c) An earthquake has just occurred in this region. Compute the probability that the magnitude of this earthquake will exceed 7.

____ (Enter your answer to four decimals)

(d) The cumulative distribution function of A is given below as

$F(a) = 1 - e^{\left(\frac{-a}{3.8}\right)}$ for $a > 0$

50-percent of the time, the magntitude of an earthquake on the Richter scale will be at most how much? ____ Richter scale units (Enter your answer to two decimals)

(e) If the magntitude of an earthquake is at least 6 on the Richter scale, compute the probability that it will be at least 7.

____ (Enter your answer to four decimals)

Answer(s) submitted:
- is right-skewed
- 3.8
- 3.8
- 0.4323
- 0.1585
- 2.63
- 0.7686

submitted: (correct)
recorded: (correct)
Correct Answers:
- is right-skewed
- 3.8
- 3.8
- 0.432294088567203
- 0.158483425334028
- 2.634
- 0.768620526593736

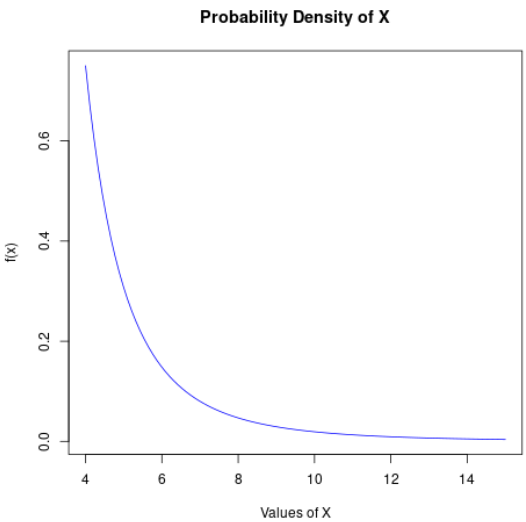
Problem 4. **(1 point)**

The Pareto probability model is often used in Economics to describe the distribution of incomes. The probability density function of the Pareto distribution is provided below.

$f(x) = \left(\frac{a*b^a}{x^{a+1}}\right)\ x \geq b$

Suppose a random variable X follows a Pareto distribution with $a = 3$ and $b = 4$. The probability density function of X is then

$f(x) = \left(\frac{3*4^3}{x^{3+1}}\right) = \left(\frac{192}{x^4}\right)\ x \geq 4$



Use R Studio to create necessary function(s), answer the questions below.

(a) Compute $P(5.5 \leq X \leq 8) =$ ____ (Enter your answer to four decimals)

(b) Compute the mean/expected value of X.

$\mu_X = E(X) =$ ____ (Enter your answer to two decimals)

(c) Compute the standard deviation of X.

$\sigma_X = SD(X) =$ ____ (Enter your answer to two decimals)

(d) Compute the probability below:

$P(X \geq 10 | X \geq 5) =$ ____ (Enter your answer to two decimals)

Answer(s) submitted:

- 0.2597
- 6
- 3.46
- 0.125

submitted: (correct)
recorded: (correct)
Correct Answers:

- 0.259673178061608
- 6
- 3.464102
- 0.125000000000905

Problem 5. (1 point)

After 8:00pm on any Thursday, the amount of time a person spends waiting in line to get into a well-known pub is a random variable represented by X. Suppose we can model the behavior of X with the Exponential probability distribution with a mean of waiting time of 42 minutes.

(a) Provide the value of the standard deviation of this distribution. Enter your answer to two decimals.

$\sigma_X =$ _____ minutes

(b) Suppose you are in line to get into the pub. Compute the probability that you will have to wait between 29 and 37 minutes to get in. Answer with four decimals.

$P(29 \leq X \leq 37) =$_____

(c) It has been 30 minutes since you entered the lineup to get into the pub, and you are still waiting. What is the chance that you will have waited at most 58 minutes, in total? Use four decimals in your answer.

$P(wait\ in\ total\ at\ most\ 58\ minutes) =$ _____

(d) 60% of the time, you will wait at most how many minutes to get into this pub? Enter your answer to two-decimals.

_____ minutes

Answer(s) submitted:

- 42
- 0.0869
- 0.4866
- 38.48

submitted: (correct)
recorded: (correct)

Correct Answers:

- 42
- 0.0869492117903072
- 0.486582880967408
- 38.4842107387145

Problem 6. (1 point)

The number of customers entering a 24-hour convenience store every 10-minutes can be modeled by the Poisson distribution with a a mean of $\lambda = 4.8$ customers. You are to look at the amount of time passing between successive customers entering the convenience store, represented by X.

(a) How many **minutes** can you expect to pass between successive customers entering the convenience store?

_____ (use at least two decimals in your answer)

(b) Compute the probability at most 2.5 minutes will pass between the arrival of one customer and the **next** customer. In answering this question, be sure to use your answer in **(a)** to two decimals. Enter your answer using four decimals, and, if necessary, use your answer in part (a).

$P(X \leq 2.5) =$_____

(c) Find the probability that at least 2 to at most 2.75 minutes pass between the entry of two customers in the store. Use four decimals in your answer.

$P(2 \leq X \leq 2.75) =$ _____

(d) At least 1 minute has passed since the last customer entered the store. What is the probability that in total, at least 4 minutes will pass until the next customer enters this store? Use four decimals in your answer.

_____ (use four decimals in your answer)

Answer(s) submitted:

- 2.08
- 0.6988
- 0.1158
- 0.2369

submitted: (correct)
recorded: (correct)

Correct Answers:

- 2.083333
- 0.69880584591709
- 0.115757581615888
- 0.236927704093964

Problem 7. **(1 point)**

The exam scores on a certain Society of Actuaries (SOA) professional examination are Normally distributed with a mean score of $\mu = 68\%$ and a standard deviation of $\sigma = 5\%$.

(a) Compute the probability that a random chosen person who is writing this SOA exam will score at most 70%.

_____ (use four decimals in your answer)

(b) What proportion of all persons writing this SOA Exam will score between 73% and 83% on the exam? Use four decimals in your answer.

_____

(c) 28% of all persons writing this SOA Examination will not pass. What is the minimum mark needed to pass this exam? Enter your answer to two decimal places, and enter as a percentage.

_____%

Answer(s) submitted:

- 0.6554
- 0.1573
- 65.09

submitted: (correct)
recorded: (correct)
Correct Answers:

- 0.655422
- 0.157305
- 65.085

Problem 8. **(1 point)**

The song-length of tunes in the Big Hair playlist of a Statistics professor's mp3-player varies from song to song. This variation can be modeled by the Normal distribution, with a mean song-length of $\mu = 4.1$ minutes and a standard deviation of $\sigma = 0.55$ minutes. Note that a song that has a length of 4.5 minutes is a song that lasts for 4 minutes and 30 seconds.

While listening to a song, the professor decides to shuffle the playlist, which means the mp3-player is to randomly pick a song within this particular playlist, and play this next.

If using/working with z-values, use three decimals.

(a) What is the probability that the next song to be played is between 3.6 and 4.7 minutes long? Answer to four decimals. _____

(b) What proportion of all the songs in this playlist are longer than 5 minutes? Use four decimals in your answer. _____

(c) 10% of all the songs in this playlist are at most how long, in minutes? Enter your answer to two decimals, and keep your answer consistent with how the song length has been expressed in this problem.

_____ minutes

(d) There are 213 songs in the Big Hair playlist. How many of these would you expect to be longer than 5 minutes in length? Use two decimals in your answer.

_____ songs

(e) From the time he set his mp3-player to shuffle, there has been 17 songs randomly chosen and played in succession. What is the chance that the 17-th song played is the 8-th to be longer than 4.1 minutes? Enter your answer to four decimals (hint: use Binomial distribution).

_____

Answer(s) submitted:

- 0.6807
- 0.0509
- 3.4
- 10.84
- 0.0873

submitted: (correct)
recorded: (correct)
Correct Answers:

- 0.680692
- 0.050882
- 3.395
- 10.837866