ANDRII VOITKIV **F2022DATA602**

ProbabilityExerciseSetThree is due on Thursday, September 15, 2022 at 11:59pm.

The number of attempts available for each question is noted beside the question. If you are having trouble figuring out your error, you should consult the textbook, or ask a fellow student, one of the TA's or your professor for help.
There are also other resources at your disposal, such as the Mathematics Continuous Tutorials. Don't spend a lot of time guessing – it's not very efficient or effective.

Make sure to give lots of significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2 \wedge 3$ instead of 8, $\sin(3*\mathrm{pi}/2)$instead of -1, $e \wedge (\ln(2))$ instead of 2, $(2+\tan(3))*(4-\sin(5)) \wedge 6-7/8$ instead of 27620.3413, etc.

Problem 1. **(1 point)**

A recent poll suggests that 66% of Canadians use their mobile device to comparison shop while they are in a store. You are to randomly pick 4 Canadians from a population made up of people who have a mobile device. You ask each the question: Do you use your mobile device to comparison shop while you are in a store? The number who respond "yes" is counted and represented by the random variable X. Its probability distribution is given below.

X	0	1	2	3	4
$P(X=x)$	0.0134	0.1038	0.3021	0.391	0.1897

(a) What can you say about the tendency of X? Pick all that apply.

- A. X is more likely to take on larger rather than smaller values.
- B. The probability distribution of X is right-skewed.
- C. The probability distribution of X is left-skewed.
- D. X is more likely to take on smaller rather than larger values.
- E. The probability distribution of X is perfectly symmetric.

(b) Compute the expected value, or mean, of X.

$E(X) = \mu_X =$ _____ (use two decimals in your answer)

(c) Compute the standard deviation of X.

$SD(X) = \sigma_X =$ _____ (use two decimals in your answer)

Answer(s) submitted:

- AC
- 2.64
- 0.95

submitted: (correct)
recorded: (correct)

Correct Answers:

- AC
- 2.64
- 0.947417542586161

Problem 2. **(1 point)**

You walk into the Rolling Thunder Casino, and are immediately faced with a table that offers the following game: A player pays \$2 to play. The player can win \$ 2, \$ 4, or \$ 5 with probabilities 0.28, 0.08, and 0.03, respectively. A random variable X is to represent the player's profit on a single play of the game.

The partial probability distribution of X is provided.

X	-2	0	2	3
$P(X = x)$	?	0.28	0.08	0.03

(a) $P(X = -2) =$ ____ (Use two decimals in your answer.)

(b) Sketch the distribution of probabilities from (a), with the values of X on the x-axis and $P(X = x)$ on the y-axis.

What can you say about the tendency of the random variable X? The distribution of X is
• ?
• symmetrical
• right skewed
• left skewed
.

(c) Compute the expected value of X and the standard deviation of X.

$E(X) =$ ____ (use two decimals in your answer)

$SD(X) =$ ____ (use two decimals in your answer)

Answer(s) submitted:

- 0.61
- right skewed
- -0.97
- 1.46

submitted: (correct)
recorded: (correct)
Correct Answers:

- 0.61
- right skewed
- -0.97
- 1.44537192445405

Problem 3. **(1 point)**

A hockey player is to take 3 shots on a certain goalie. The probability he will score a goal on his first shot is 0.35. If he scores on his first shot, the chance he will score on his second shot increases by 0.1; if he misses, the chance that he scores on his second shot decreases by 0.1. This pattern continues to on his third shot: If the player scores on his second shot, the probability he will score on his third shot increases by another 0.1; should he not score on his second shot, the probability of scoring on the third shot **decreases** by another 0.1.

A random variable X counts the number of goals this hockey player scores.

(a) Complete the probability distribution of X below. Use four decimals in each of your entries.

X	0	1	2	3
$P(X = x)$	____	____	____	____

(b) How many goals would you expect this hockey player to score? Enter your answer to four decimals.

$E(X) =$ ____

(c) Compute the standard deviation the random variable X. Enter your answer to two decimals.

$SD(X) =$ ____

Answer(s) submitted:

- 0.4144
- 0.3039
- 0.1951
- 0.0866
- 0.954
- 0.98

submitted: (correct)
recorded: (correct)
Correct Answers:

- 0.414375
- 0.303875
- 0.195125
- 0.086625
- 0.954
- 0.97666985209947

Problem 4. **(1 point)**

A person is to walk into a casino and play a certain game. The chance the person will win the game is 0.38. Once they play the first game, win or lose, they are to play the game 3 more times for a total of 4 games.

A random variable X is to count how many of the 4 games the gambler wins.

(a) Finish the probability distribution of X below. Use four decimals in each of your entries.

X	0	1	2	3	4
$P(X=x)$	____	____	____	____	____

(b) From the distribution you found in part (a), what can you say about the distribution of X?

The distribution of X is
- ?
- symmetrical
- skewed to the right
- skewed to the left

, with an mean of ____ games won and a standard deviation of ____ games won.

(Enter your answers to two decimals.)

Answer(s) submitted:

- 0.1478
- 0.3623
- 0.333
- 0.1361
- 0.0209
- skewed to the right
- 1.52
- 0.97

submitted: (correct)
recorded: (correct)

Correct Answers:

- 0.14776336
- 0.36225856
- 0.33304416
- 0.13608256
- 0.02085136
- skewed to the right
- 1.52
- 0.970772887960928

Problem 5. **(1 point)**

A recent poll has suggested that 68 % of Canadians will be spending money - decorations, halloween treats, etc. - to celebrate Halloween this year.

24 Canadians are randomly chosen, and the number that will be spending money to celebrate Halloween is to be counted. This count is represented by the random variable X.

Part (a) Compute the probability that 14 of these Canadians indicate they will be spending money to celebrate Halloween.

$P(X=14)=$ ____ (use four decimals in your answer)

Part (b) Compute the probability that between 8 and 14 of these Canadians, inclusive, indicate they will be spending money to celebrate Halloween.

$P(8 \leq X \leq 14)=$____ (use four decimals in your answer)

Part (c) How many of the 24-Canadians randomly chosen would you expect to indicate they will be spending money to celebrate Halloween? Compute the standard deviation as well.

$E(X)=\mu_X=$ ____ (use two decimals in your answer)

$SD(X)=\sigma_X=$ ____ (use two decimals in your answer)

Part (d) Compute the probability that the 11-th Canadian random chosen is the 7-th to say they will be spending money to celebrate Halloween.

____ (use four decimals in your answer)

Answer(s) submitted:

- 0.0998
- 0.2101
- 16.32
- 2.29
- 0.148

submitted: (correct)
recorded: (correct)

Correct Answers:

- 0.0998065046481655
- 0.210132865800201
- 16.32
- 2.28525709713371
- 0.148040859188799

Problem 6. **(1 point)**

A certain airline has 170 seats available for a flight from YYC (Calgary International Airport) to LAX (Los Angeles International Airport). Because people with reservations **do not show up** for their flight 10% of the time, the airline always overbooks this flight. That is, there are more passengers that have tickets on the flight than there are seats.

Suppose the airline has 182 passengers booked for 170 seats. Assume one person showing up for the flight does not affect others who may, or may not, show up for this flight.

(a) How many people (with tickets) does the airline expect to **show up** for this flight? Provide the standard deviation as well. Enter your answers to two decimals.

The expectation of the number of people showing up for the flight = ____

The standard deviation of the number of people who show up for the flight = ____

(b) When the flight takes off from YYC, what is the probability that there will be 5 seats empty? Enter your answer to four decimals.

$P(5\ seats\ empty) =$ ____

(c) What is the chance that a passenger with a flight reservation will not make it to LAX due to overbooking? Use four decimals in your answer.

____

Answer(s) submitted:

- 163.8
- 4.0472
- 0.0967
- 0.042

submitted: (correct)
recorded: (correct)

Correct Answers:

- 163.8
- 4.04722126896961
- 0.0966839537755408
- 0.0420320401974392

Problem 7. **(1 point)**

Your statistics professor hands you a fair die, with each side having the same chance of being the uppermost face. You are asked to toss the die 7 times and count the number of times the die shows a topside of six.

This count is represented by the random variable X.

(a) Complete the probability distribution of X below. Use four decimals in each of your entries.

X	0	1	2	3	4	5	6	7
$P(X = x)$	____	____	____	____	____	____	____	0.00000357

(b) From the distribution you found in part (a), what can you say about the distribution of X?

The distribution of X is
- ?
- symmetrical
- skewed to the right
- skewed to the left

, with an mean of ____ sixes and a standard deviation of ____ sixes.

(Enter your answers to two decimals.)

(c) As requested, you have tossed the die 7-times and observed $X = 6$ sixes. If you were to repeat the 7-tosses of this die, what is the probability you will observe at least the same number of sixes as this? Enter your answer to four decimal places.

____

(d) In part a) you just gave the distribution for how many times a 'fair' die tossed 7 times should show a six. Let's say you tried this experiment, and you happened to observe 6 out of the 7 times rolling a 6 (aka $P(X = 6)$). Feeling suspicious you try another 7 times...and again got either 6 or all 7 (out of 7) to be a six(aka $P(X \geq 6)$). Compare with what should likely have happened (refer to part a). What would you say this information means? Select the **most appropriate** answer.

- A. The die appears to be fair. Since, $X = 6$ suggests this is not an unusual event.
- B. The distribution of the random variable X appears to be skewed to the right.
- C. The die appears to favor an outcome of a six. Since, $X = 6$ should be an unusual event yet I continue to observe it.
- D. The die appears not to be fair. $X = 6$ is an usual event which explains why I'm seeing it so often.
- E. The distribution of the random variable X appears to be roughly symmetrical.

Answer(s) submitted:
- 0.2791
- 0.3907
- 0.2344
- 0.0781
- 0.0156
- 0.0019
- 0.0001
- skewed to the right
- 1.17
- 0.99
- 0.0001
- C

submitted: (correct)
recorded: (correct)

Correct Answers:
- 0.279081647233653
- 0.390714306127115
- 0.234428583676269
- 0.0781428612254233
- 0.0156285722450847
- 0.00187542866941017
- 0.000125028577960678
- skewed to the right
- 1.16666666666667
- 0.986013297183269
- 0.000128600823045222
- C

Problem 8. **(1 point)**

A random variable X is to count the number of customers entering a 24-hour convenience store every 10-minutes. The mean of X is found to be $\lambda = 4.8$ customers per 10-minute interval.

Part (a) What is the probability that 3 customers will enter this convenience store within the next 10-minutes?

$P(X = 3) =$ _____ (use four decimals in your answer)

Part (b) Compute the probability that between 2 and 9 customers will enter the convenience store within the next 10-minutes?

$P(2 \leq X \leq 9) =$_____ (use four decimals in your answer)

Part (c) Compute the probability that more than 6 customers will walk into the convenience store in the next 10-minutes.

$P(more\ than\ 6\ customers\ enter) =$ _____ (use four decimals in your answer)

Part (d) A random variable Y is to count the number of customers that walk into this convenience store in hour. How many customers do you expect in an hour? Compute the standard deviation of this count as well.

$E(Y) = \mu_Y =$ _____ (use two decimals in your answer)

$SD(Y) = \sigma_Y =$ _____ (use two decimals in your answer)

Answer(s) submitted:

- 0.1517
- 0.9271
- 0.2092
- 28.8
- 5.37

submitted: (correct)
recorded: (correct)

Correct Answers:

- 0.151690697607537
- 0.927126297295941
- 0.209195415815402
- 28.8
- 5.3665631459995

Problem 9. **(1 point)**

On a typical day at a local popular mall, the number of shoplifters caught by mall security fluctuates from day to day, with an average of 8.6 caught per day.

Suppose you are to count the number of shoplifters apprehended at this mall today. (Assume today is a typical day at the mall.)

Part (a) Compute the probability that mall security apprehended 7 shoplifters.

_____ (use four decimals in your answer)

Part (b) Compute the probability that mall security will apprehend at least 7 shoplifters. Enter your answer to four decimals.

_____

Part (c) Compute the probability that between 7 and 11 (inclusive) shoplifters will be apprehended. Enter your answer to four decimals.

_____

Part (d) Think about the distribution of the number of shoplifters mall security apprehends on a typical day. What can you say about this distribution? Select the most appropriate reason below.

- A. The distribution of values is roughly symmetrical, with a mean of 8.6 shoplifters and a standard deviation of 2.93257565972304 shoplifters.
- B. The distribution of values is skewed to the right, with a mean of 8.6 shoplifters and a variance of 2.93257565972304 shoplifters.
- C. The distribution of values is roughly symmetrical, with a mean of 8.6 shoplifters and a variance of 2.93257565972304 shoplifters^2.
- D. The distribution of values is skewed to the right, with a mean of 8.6 shoplifters and a standard deviation of 2.93257565972304 shoplifters.
- E. The number of shoplifters apprehended each day is 8.6.

Part (e) On a typical day, the mall is open from 10:00am to 9:00pm, a total of 11 hours.You are interested in the number of shoplifters apprehended in any given hour.

How many shoplifters would you expect mall security to catch in any given hour? Provide the variance of this count as well.

$\mu =$ _____ (use three decimals in your answer)

$\sigma^2 =$ _____ (use three decimals in your answer)

Answer(s) submitted:

- 0.1271
- 0.7543
- 0.5943
- D
- 0.782
- 0.782

submitted: (correct)
recorded: (correct)

Correct Answers:

- 0.127094301286281
- 0.754323548832576
- 0.594331106780654
- D
- 0.781818181818182
- 0.781818181818182

Problem 10. **(1 point)**

The number of people entering a security check-in lineup in a 15-minute interval at a medium sized airport can be modeled by the following probability model:

$$P(X = x) = \frac{e^{-20.8}(20.8)^x}{x!} \qquad x = 0, 1, 2, \cdots$$

Part (a) What does 20.8 represent in the probability model? Select the **most appropriate** explanation below.

- A. 20.8 is the rate at which people enter the security check-in lineup every 15 minutes.
- B. 20.8 represents the average number of people who enter the a security check-in lineup every 15-minutes.
- C. 20.8 represents a weighted-average of the number of people who enter the a security check-in lineup every 15-minutes.
- D. 20.8 represents how much skewed the distribution of values is.
- E. 20.8 is the standard deviation of the distribution of people entering the security check-in lineup every 15-minutes.

Part (b) Compute the probability that 18 people enter the security check-in lineup in a 15-minute interval. Use four decimals in your answer.

$P(X = 18) =$ _____

Part (c) Compute the probability that at least 5 people will enter the security check-in lineup in a **5-minute** interval. Enter answer to four decimals.

_____

Part (d) In the past 15-minutes, you have been told that somewhere between 13 and 18 people, inclusive, have entered the security lineup. Compute the probability that this uncertain number is 17.

_____ (use four decimals in your answer)

Answer(s) submitted:

- C
- 0.0768
- 0.8208
- 0.2294

submitted: (correct)
recorded: (correct)
Correct Answers:

- C
- 0.0768199084891687
- 0.820839308036198
- 0.229351763907862