

MATH 299Q: Homework 4

Quiver Representations

Let k be an algebraically closed field.

- Fix Q to be the Kroenecker 2-quiver;

$$1 \begin{array}{c} \xrightarrow{\alpha} \\ \xrightarrow{\beta} \end{array} 2 .$$

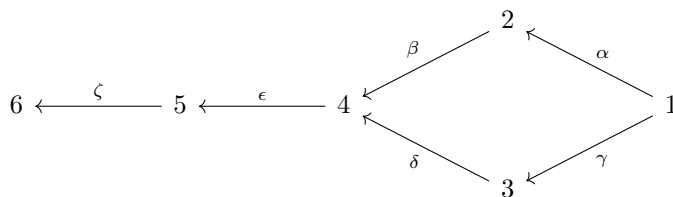
Associate to Q , the *path algebra* kQ (recall this is a k -algebra).

- What is $\dim kQ$?
- An *endomorphism* is a morphism from M to itself, that is for some $M \in \text{rep } Q$ the morphism $f : M \rightarrow M$. The collection of all endomorphisms, $\text{End } M$, of a fixed representation M , much like $\text{Hom}(-, -)$, forms a natural k -vector space under composition of morphisms. Let M be the following representation of Q ;

$$k^2 \begin{array}{c} \xrightarrow{\begin{bmatrix} 10 \\ 01 \\ 00 \end{bmatrix}} \\ \xrightarrow{\begin{bmatrix} 00 \\ 10 \\ 01 \end{bmatrix}} \end{array} k^3 .$$

What is $\dim(\text{End } kQ)$?

- Fix Q to be as follows;



With the set of relations $R = \{\alpha\beta - \gamma\delta, \beta\epsilon, \delta\epsilon\zeta\}$ generating the admissible ideal $I = \langle R \rangle$. Compute the indecomposable projective representations $\mathcal{P}(1)$, $\mathcal{P}(3)$, and $\mathcal{P}(4)$ of (Q, I) .