MATH 299Q: Homework 5

Quiver Representations

Let k be an algebraically closed field.

1. Recall Homework 3, which asked you to compute the AR-quiver for a quiver Q with $\Delta_Q = \overrightarrow{\mathbb{A}_3}$ and generalize this pattern to $\Delta_Q = \overrightarrow{\mathbb{A}_n}$.

   Using the solutions to that problem, derive a formula for the number of indecomposable representations in $\operatorname{rep} Q$ for $\Delta_Q = \mathbb{A}_n$ for any arrow orientation. i.e., find the number of vertices in the AR-quiver of Q in terms of $n = |Q_0|$.

   [Hint: Consider the construction of AR-quiver that utilized regular $n+3$-gons.]

2. Let Q be a connected quiver without orientated cycles with n vertices, fix some $\mathbf{d} = (d_i) \in \mathbb{Z}^n_{\geq 0}$ and define the space of all representations $M \in \operatorname{rep} Q$ with dimension vector $E_{\mathbf{d}} := \{M \in \operatorname{rep} Q \mid \underline{\dim} M = \mathbf{d}\}$. It should be clear that $E_{\mathbf{d}}$ is a k-vector space.

   Define the group
$$G_{\mathbf{d}} := \prod_{i \in Q_0} \mathrm{GL}_{d_i}(k).$$
   This group acts naturally on $E_{\mathbf{d}}$ via conjugation; if $g = (g_i) \in G_{\mathbf{d}}$, $M = (M_i, \varphi_\alpha) \in E_{\mathbf{d}}$, and $i \xrightarrow{\alpha} j \in Q_1$, then $(g \cdot \varphi)_\alpha = g_j \varphi_\alpha g_i^{-1}$. We denote the orbit of $M \in E_{\mathbf{d}}$ under $G_{\mathbf{d}}$ by $\mathcal{O}_M := \{g \cdot M \mid g \in G_{\mathbf{d}}$. Show that the orbit $\mathcal{O}_M$ is the isoclass of the representation M, i.e., $\mathcal{O}_M = \{M' \in \operatorname{rep} Q \mid M \cong M'\}$.