

MATH 299Q: Homework 1 Solutions

Quiver Representations

Let k denote an algebraically-closed field.

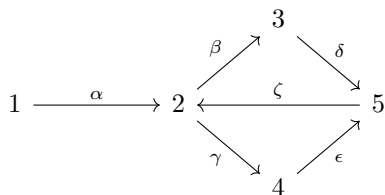
1. (a) We have $\text{im } T := \{w \in W \mid w = T(v), v \in V\}$ and $\ker T := \{v \in V \mid T(v) = 0\}$.
- (b) There are various ways of describing these. For instance, T is injective if and only if $\ker T = \{0\}$, T is surjective if $\text{im } T = W$. If T is an isomorphism then $\dim V = n = m = \dim W$.
- (c) *Proof.* Let $f, g \in \text{Hom}(V, W)$. Define k -vector space structure of $\text{Hom}(V, W)$ to be

$$(f + g)(v) = f(v) + g(v) \quad (\lambda f)(v) = \lambda(f(v))$$

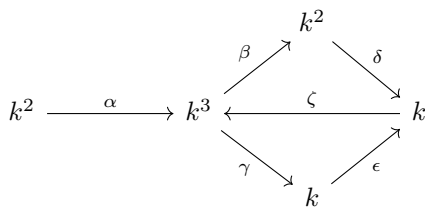
where $v \in V$ and $\lambda \in k$. It follows that since f and g are linear operators, the two operations preserve linearity and thus $\text{Hom}(V, W)$ is a k -vector space. \square

2. For example,

(a)

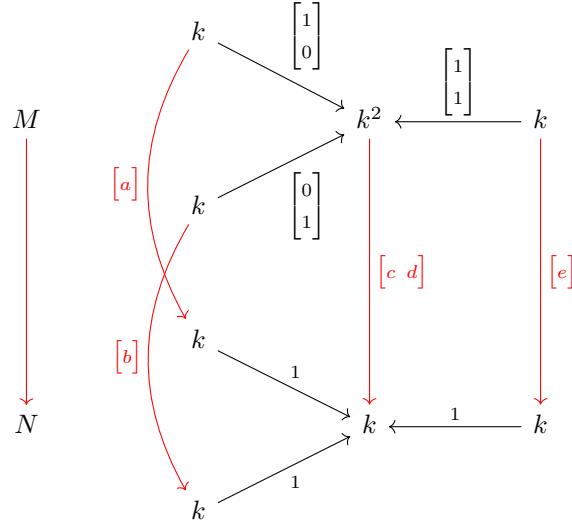


(b)



With appropriate linear maps $\alpha, \beta, \gamma, \delta, \epsilon$.

3. *Proof.* It follows that a morphism $f : M \rightarrow N$ is a choice of scalars $a, b, c, d, e \in k$ such that the following diagram commutes:



We then get the following relations

$$a = c, \quad b = d, \quad c + d = e.$$

Therefore, $a, b \in k$ completely determines the morphism. Since every choice of a and b yields a unique morphism we have that $\text{Hom}(M, N) \cong k^2$. \square