MATH 299Q: Homework 4

Quiver Representations

Let k be an algebraically closed field.

1. Fix Q to be the Kroenecker 2-quiver;

$$1 \underset{\beta}{\overset{\alpha}{\rightrightarrows}} 2 \; .$$

Associate to Q, the *path algebra* kQ (recall this is a k-algebra).

(a) What is $\dim kQ$?

(b) An *endomorphism* is a morphism from M to itself, that is for some $M \in \operatorname{rep} Q$ the morphism $f : M \to M$. The collection of all endomorphisms, $\operatorname{End} M$, of a fixed representation M, much like $\operatorname{Hom}(-,-)$, forms a natural k-vector space under composition of morphisms. Let M be the following representation of Q;

$$k^2 \underset{\begin{bmatrix} 00 \\ 10 \\ 01 \end{bmatrix}}{\overset{\begin{bmatrix} 10 \\ 01 \\ 00 \end{bmatrix}}{\rightrightarrows}} k^3 \; .$$

What is $\dim(\operatorname{End} kQ)$?

2. Fix Q to be as follows;

$$\begin{tikzcd}
 & & & 2 \arrow[dl, "\beta"'] & \\
6 & 5 \arrow[l, "\zeta"'] & 4 \arrow[l, "\epsilon"'] & & 1 \arrow[ul, "\alpha"'] \arrow[dl, "\gamma"] \\
 & & & 3 \arrow[ul, "\delta"] &
\end{tikzcd}$$

With the set of relations $R = \{\alpha\beta - \gamma\delta, \beta\epsilon, \delta\epsilon\zeta\}$ generating the admissible ideal $I = \langle R \rangle$. Compute the indecomposable projective representations $\mathcal{P}(1)$, $\mathcal{P}(3)$, and $\mathcal{P}(4)$ of (Q, I).