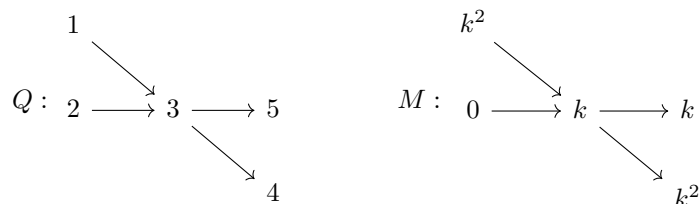


MATH299Q: Midterm Quiver Representations

March 10, 2023

Let k be an algebraically closed field.

1. (20 points) Let Q be the quiver and M a representation of Q :



Is M indecomposable? If it is not, give a decomposition of M such that it is the direct sum of nonzero indecomposable representations.

2. (20 points) Let Q be a finite quiver, $\text{rep } Q$ the category of representations of Q and $L, M, N \in \text{rep } Q$. Suppose $f : L \rightarrow M$ and $g : M \rightarrow N$ are two morphisms of representations.
- (a) (10 points) State carefully the conditions for which f is a **section** and g is a **retraction**.
- (b) (10 points) Define one of the following representations; $\ker f$, $\text{coker } g$.
3. (a) (5 points) What is the “goal” of representation theory?
- (b) (10 points) State the Krull-Schmidt theorem.
4. Let Q be the quiver $1 \longrightarrow 2$, with the three representations

$$S_1 : k \longrightarrow 0 \qquad S_2 : 0 \longrightarrow k \qquad M : k \xrightarrow{1} k .$$

- (a) (10 points) Construct a short exact sequence consisting of **all three** representations.
- (b) (5 points) Construct a short exact sequence consisting of **two** representations.
- (c) (10 points) State explicitly what the morphisms in (a) are.
5. (a) (10 points) Define what a category is.
- (b) (5 points) Define what it means for \mathcal{C} to be additive.
- (c) (5 points) Give an example of an abelian k -category.