MATH 299Q: Homework 5 Solutions

Quiver Representations

Let k be an algebraically closed field.

1. We use a regular $n+3$-gon to construct the AR-quiver for $\mathbb{A}_n$ type quivers. Hence, the number of indecomposable representations of Q is equal to the number of all diagonals in an $n+3$-gon *minus the* n *diagonals we use to construct the associate triangulation.*

   For any vertex a of a polygon the diagonals starting at a can end anywhere besides itself or its two neighbors. So there are n diagonals starting at each vertex, $n+3$ vertices, and we count each diagonal twice so there are $n(n+3)/2$ diagonals total. Subtracting our n diagonals then yields;

$$|\Gamma_{Q_0}| = \frac{n(n+3)}{2} - n = \frac{n^2+3n-2n}{2} = \frac{n^2+n}{2} = \frac{n(n+1)}{2}.$$

   You can verify this works by examining the solutions of Homework 3.

2. *Proof.* Suppose that $M = (M_i, \varphi_\alpha)$ and $M' = (M'_i, \varphi'_\alpha)$ are in the same orbit, then there exists some $g \in G_{\mathbf{d}}$ such that $g \cdot M = M'$. That is, for each arrow $i \xrightarrow{\alpha} j$ in Q the following diagram commutes:

$$\begin{tikzcd} M_i \arrow[r, "\varphi_\alpha"] \arrow[d, "g_i"'] & M_j \arrow[d, "g_j"] \\ M'_i \arrow[r, "\varphi'_\alpha"'] & M'_j \end{tikzcd}$$

   Therefore, g is a morphism of representations, moreover since each g_i is an element of $\mathrm{GL}_{d_i}(k)$ we have that it is invertible and thus an isomorphism. That is, $M \cong M'$.

   It follows immediately that if $M \cong M'$ then there is a $g \in G_{\mathbf{d}}$ such that $g \cdot M = g(M) = M'$. $\square$