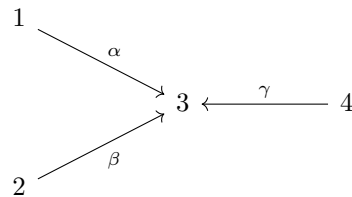


# MATH 299Q: Homework 1

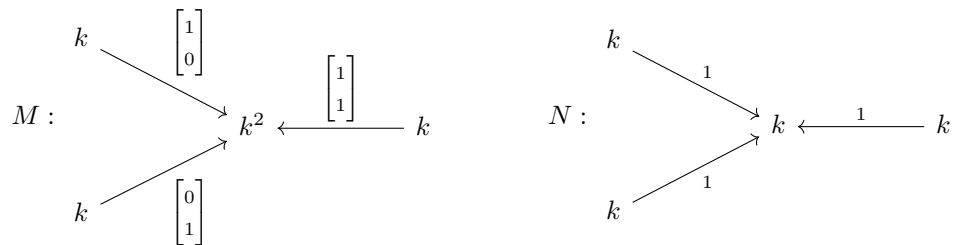
## Quiver Representations

Let  $k$  denote an algebraically-closed field.

1. Suppose  $V$  and  $W$  are two finite-dimensional  $k$ -vector spaces, with  $\dim V = n$  and  $\dim W = m$ . Let  $T \in \text{Hom}(V, W)$ , that is  $T : V \rightarrow W$ .
  - (a) Define  $\text{im } T$  and  $\ker T$ .
  - (b) What does it mean for  $T$  to be *injective*? What about *surjective*? What conditions on  $n$  and  $m$  must we have for  $T$  to be an *isomorphism*?
  - (c) Show that  $\text{Hom}(V, W)$  has a natural  $k$ -vector space structure.  
 [Hint: For  $f, g \in \text{Hom}(V, W)$  show that  $f + \lambda g \in \text{Hom}(V, W)$  for  $\lambda \in k$ .]
2.
  - (a) Draw your favorite (finite) quiver!
  - (b) Give a non-trivial representation of the quiver you drew in (a).
3. Let  $Q$  be the quiver



and consider the representations  $M$  and  $N$ :



Prove that  $\text{Hom}(M, N) \cong k^2$ . [Hint: Draw a morphism between  $M$  and  $N$ .]