MATH 299Q: Homework 2 Solutions

Quiver Representations

1. *Proof.* It suffices to find a morphism between the two representations.

   Suppose $M = (M_i, \varphi_\alpha)$, using the constructions discussed in class we have $\operatorname{im} f = (f(M_i), \psi_\alpha)$ with $\psi_\alpha(f_i(m_i)) = f_j\varphi_\alpha(m_i)$ for all arrows $i \xrightarrow{\alpha} j \in Q_1$. On the other hand, $M/\ker f = (M_i/\ker f_i, \chi_\alpha)$ (recall coker f, $\chi_\alpha : (m_i + \ker f_i) = \varphi_\alpha(m_i) + \ker f_j)$. Since f_i is linear we have the induced isomorphism of vector spaces

   $$g_i : M_i/\ker f_i \to f_i(M_i) \qquad g_i : m_i + \ker f_i \mapsto f_i(m_i).$$

   For each arrow $i \xrightarrow{\alpha} j \in Q_1$ we have $\psi_\alpha g_i = g_j \varphi_\alpha$, which shows g is a morphism of representations. $\square$

2. We have the following representations;

   $$L:\ 0 \longrightarrow k \qquad M:\ k \longrightarrow k \qquad N:\ k \longrightarrow 0\ .$$

   Then we have the short exact sequence

   $$0 \longrightarrow L \xrightarrow{f} M \xrightarrow{g} N \longrightarrow 0\ .$$

   With maps $f = (f_1, f_2) = (0, 1)$ and $g = (g_1, g_2) = (1, 0)$.