1 Nonlinear Pendulum

The Nonlinear Pendulum function:

$$1)\ \ddot{\theta} + \Omega^2 \sin(\theta) = 0$$

where $\Omega^2 = gl^{-1}$.

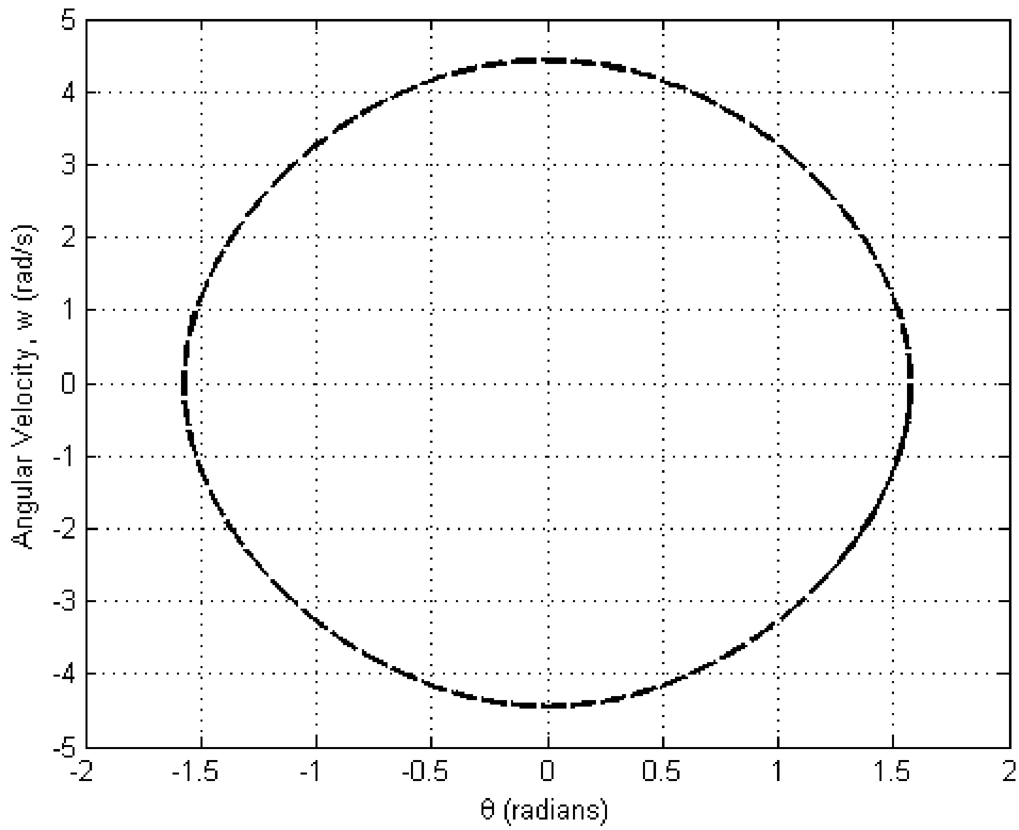


Figure 1: A fourth-order Runge Kutta subroutine computes the motion of the pendulum. The plot represents the phase space trajectory, ω vs. θ. The initial conditions are $x_0 = \frac{\pi}{2}$ and $\omega_0 = 0$.

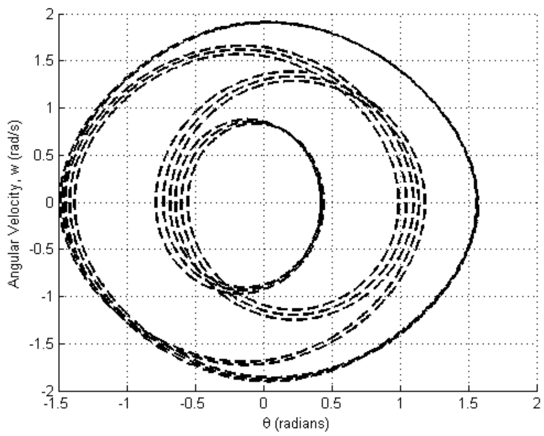
2 Forced Nonlinear Pendulum

The Forced Nonlinear Pendulum function:

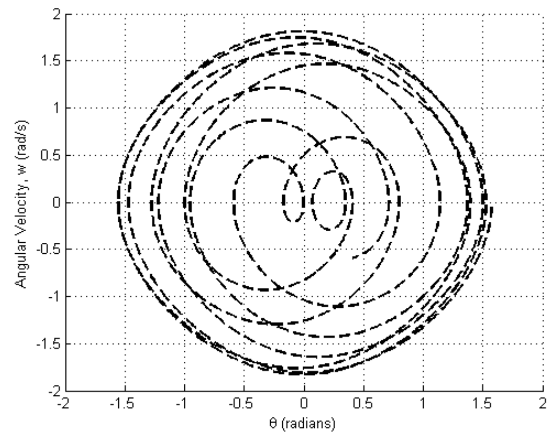
$$1)\ \ddot{\theta} + \Omega^2 \sin(\theta) = \text{f}\sin(\tilde{t})$$

where $\tilde{t} = \text{t}\omega_0^{-1}$, $\Omega^2 = gl^{-1}\omega_0^{-2}$, and $\text{f} = A\omega_0^{-1}$.

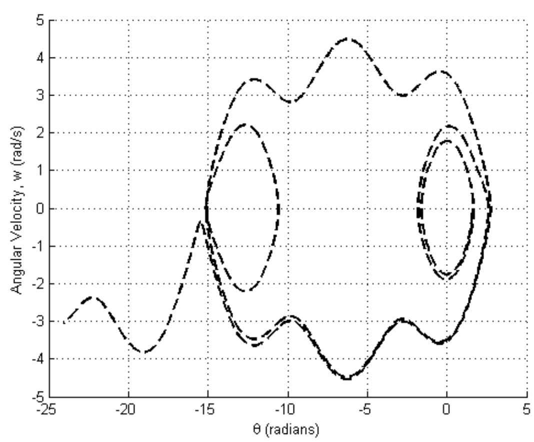
2.1 Chaotic Pendulum



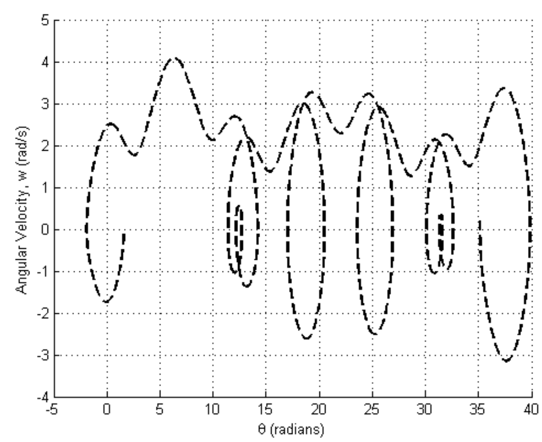
(a) $\Omega^2 = 3/2$, f $= 1/2$



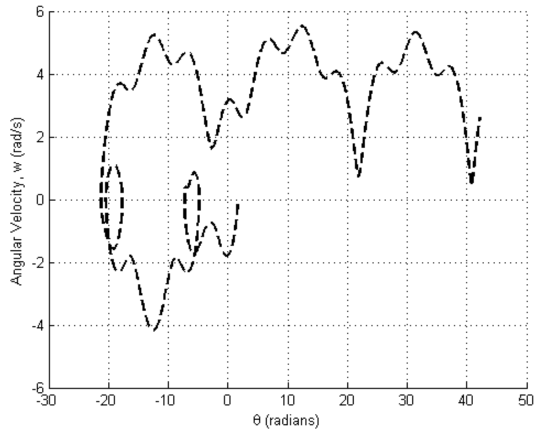
(b) $\Omega^2 = 3/2$, f $= 3/4$



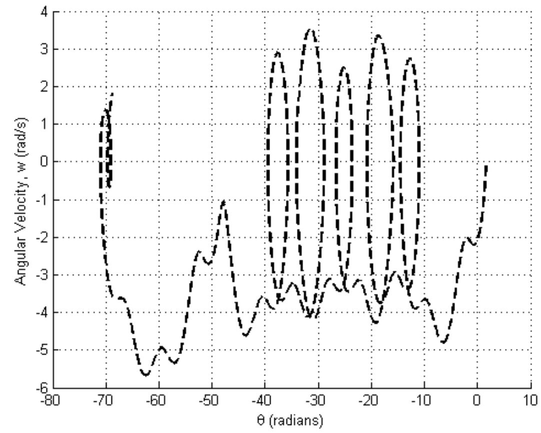
(c) $\Omega^2 = 3/2$, f $= 1$



(d) $\Omega^2 = 3/2$, f $= 5/4$



(e) $\Omega^2 = 3/2$, f = 3/2



(f) $\Omega^2 = 3/2$, f = 2

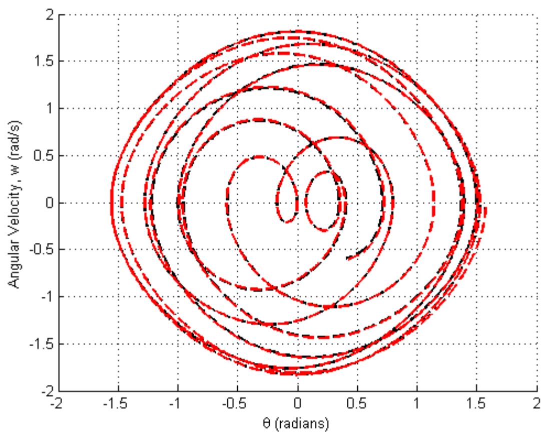
Figure 2: The initial conditions are $x_0 = \frac{\pi}{2}$ and $\omega_0 = 0$. The chaotic pendulum is plotted over course of 50 seconds. The various constants Ω^2 and f are noted in the figures. With f > 1, the pendulum deviates largely from its initial position. With f < 1, it deviates closely to its initial position.

2.2 Lyopunov Exponent

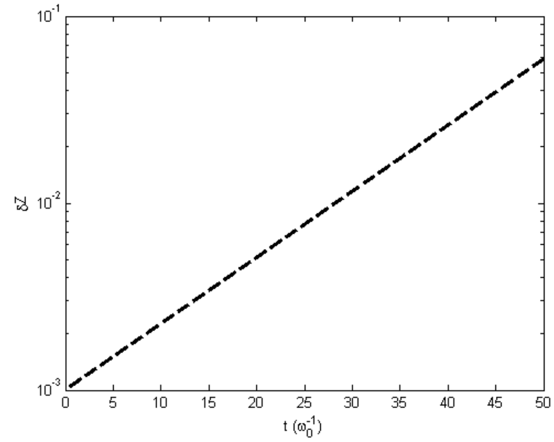
The Lyoponuv Exponential function:

$$1)\ \delta\mathrm{Z} = \delta\mathrm{Z}_0 \mathrm{e}^{\lambda_L \tilde{t}}$$

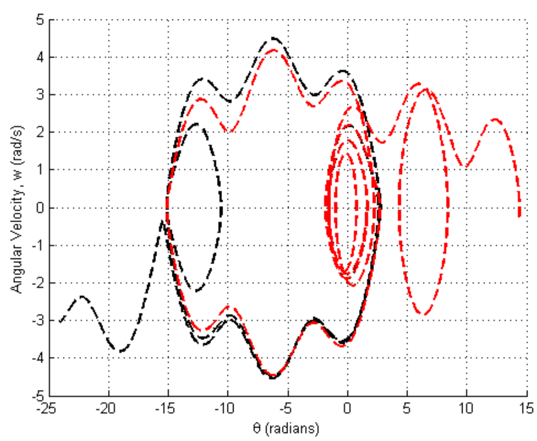
where $\tilde{t} = \mathrm{t}\omega_0^{-1}$, $\Omega^2 = gl^{-1}\omega_0^{-2}$, and $\mathrm{f} = A\omega_0^{-1}$.



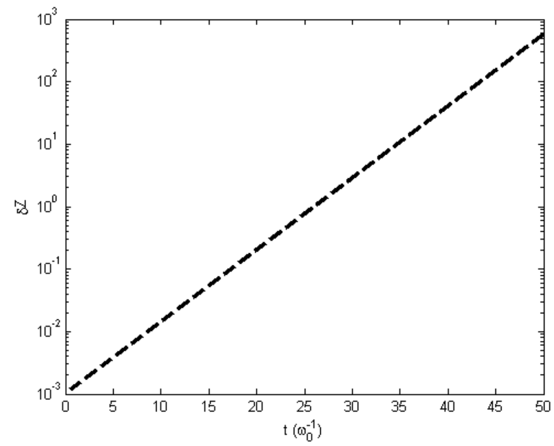
(a) $\Omega^2 = 9/4$, f = 3/4



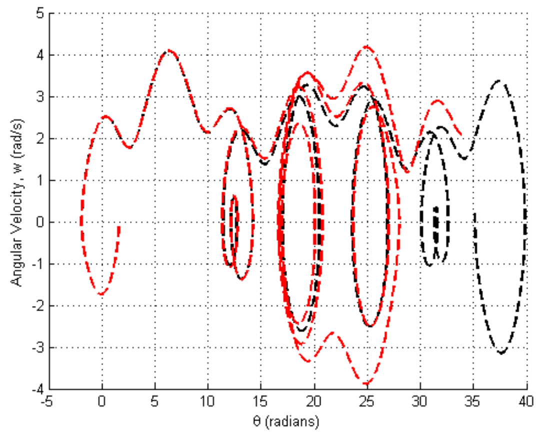
(b) $\lambda_L = 0.0814$



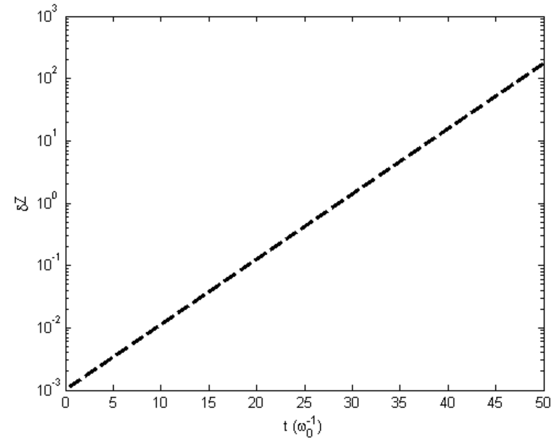
(c) $\Omega^2 = 9/4$, f = 1



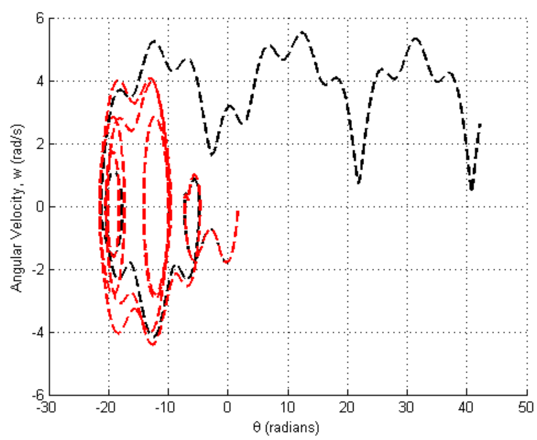
(d) $\lambda_L = 0.2648$



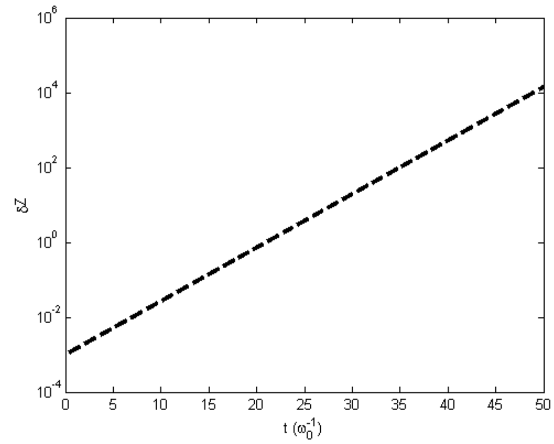
(e) $\Omega^2 = 9/4$, f $= 5/4$



(f) $\lambda_L = 0.2409$



(g) $\Omega^2 = 9/4$, f $= 3/2$



(h) $\lambda_L = 0.3293$

Figure 3: The plots: (a),(c),(e),(g) represent the phase trajectory of two chaotic pendulums with initial conditions differing by 1E-03. The mean Lyopunov Exponent is noted and plotted alongside its corresponding phase trajectory.