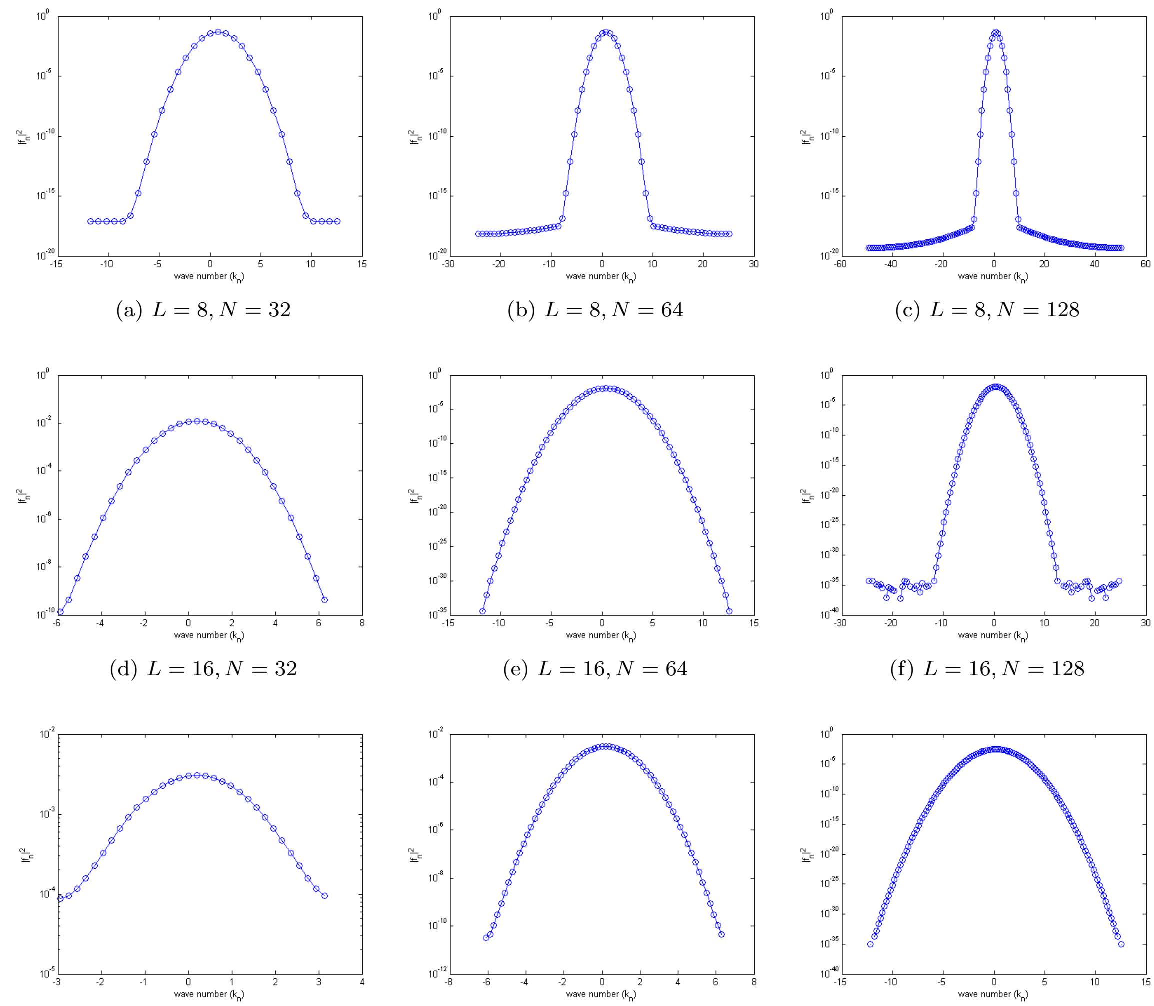
1 Power Spectrum

The function used:

$$f(x) = exp[-(x - x_0)^2]$$



(a) $L = 8, N = 32$ (b) $L = 8, N = 64$ (c) $L = 8, N = 128$

(d) $L = 16, N = 32$ (e) $L = 16, N = 64$ (f) $L = 16, N = 128$

(g) $L = 32, N = 32$ (h) $L = 32, N = 64$ (i) $L = 32, N = 128$

Figure 1: The general shape of the power spectrum is a Gaussian. An increase in N with L constant shows a decreased peak and an increased width of the Gaussian. An increase in L with N constant shows a decreased width of the Gaussian. An increase in L and N shows a constant width of the Gaussian.

2 Differentiation of Fourier Series

The derivative of the function:

$$f(x) = -2(x - x_0)exp[-(x - x_0)^2]$$

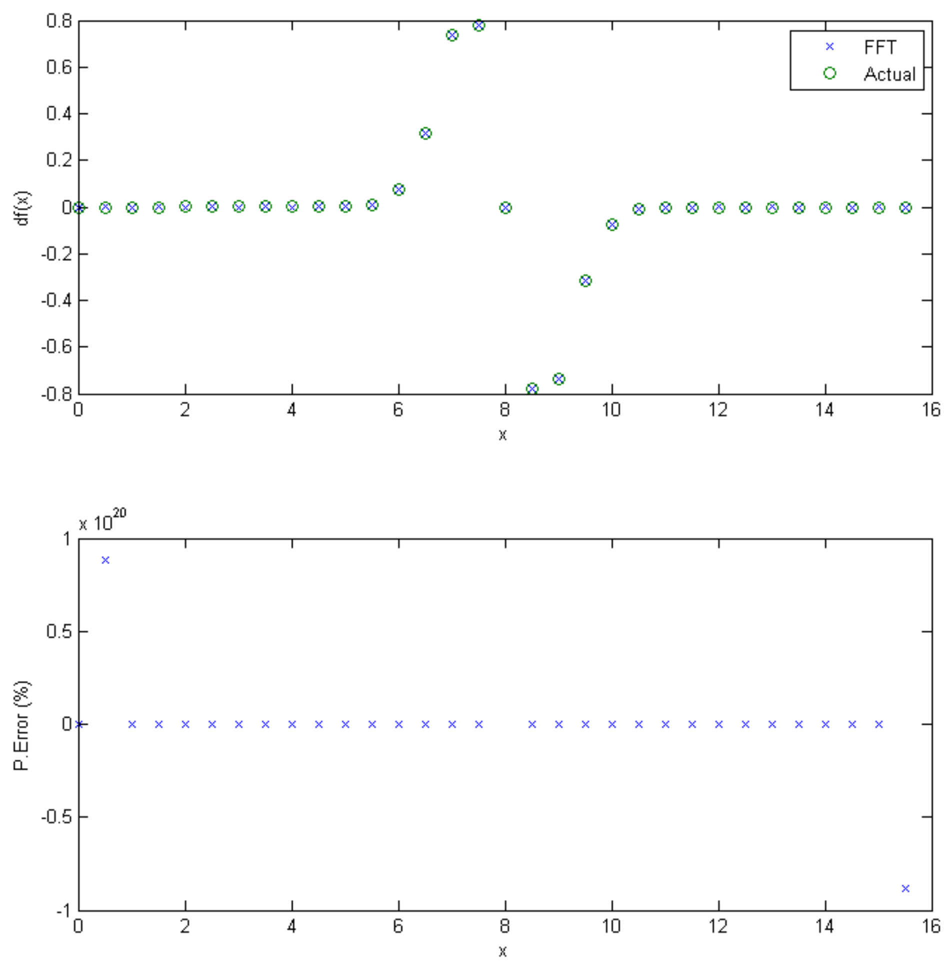


Figure 2: $L = 16, N = 32$

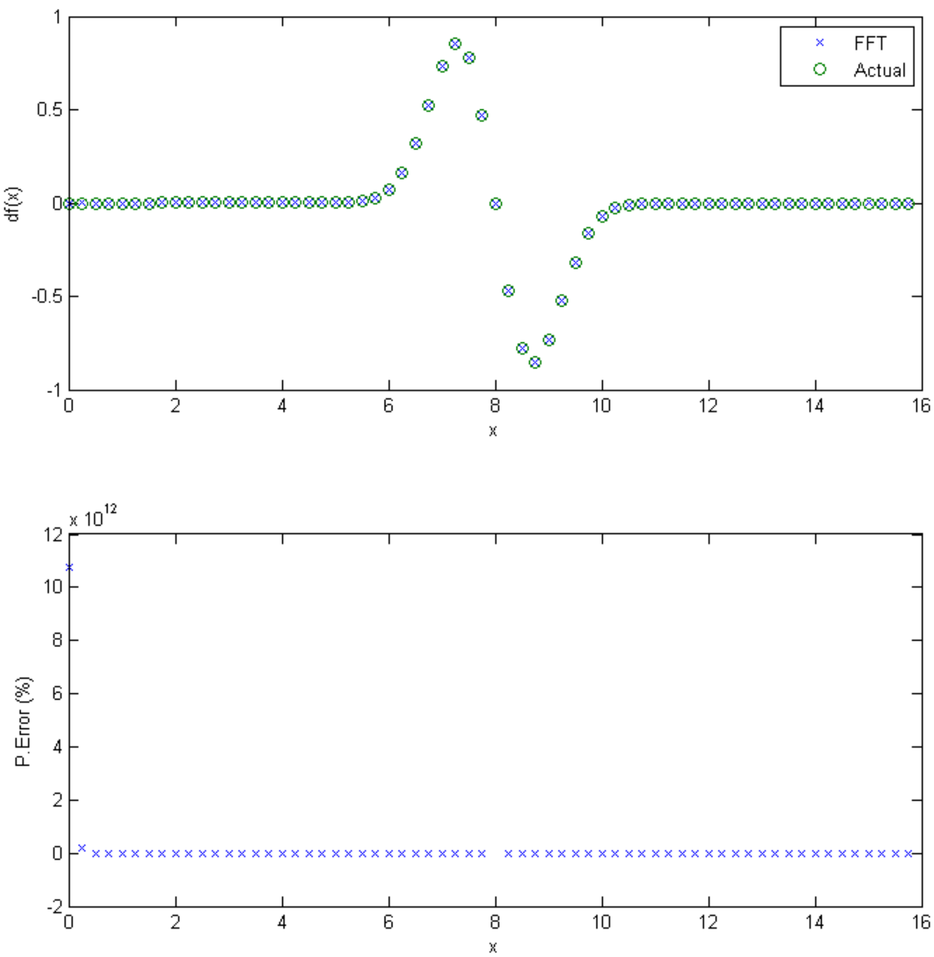


Figure 3: $L = 16, N = 64$

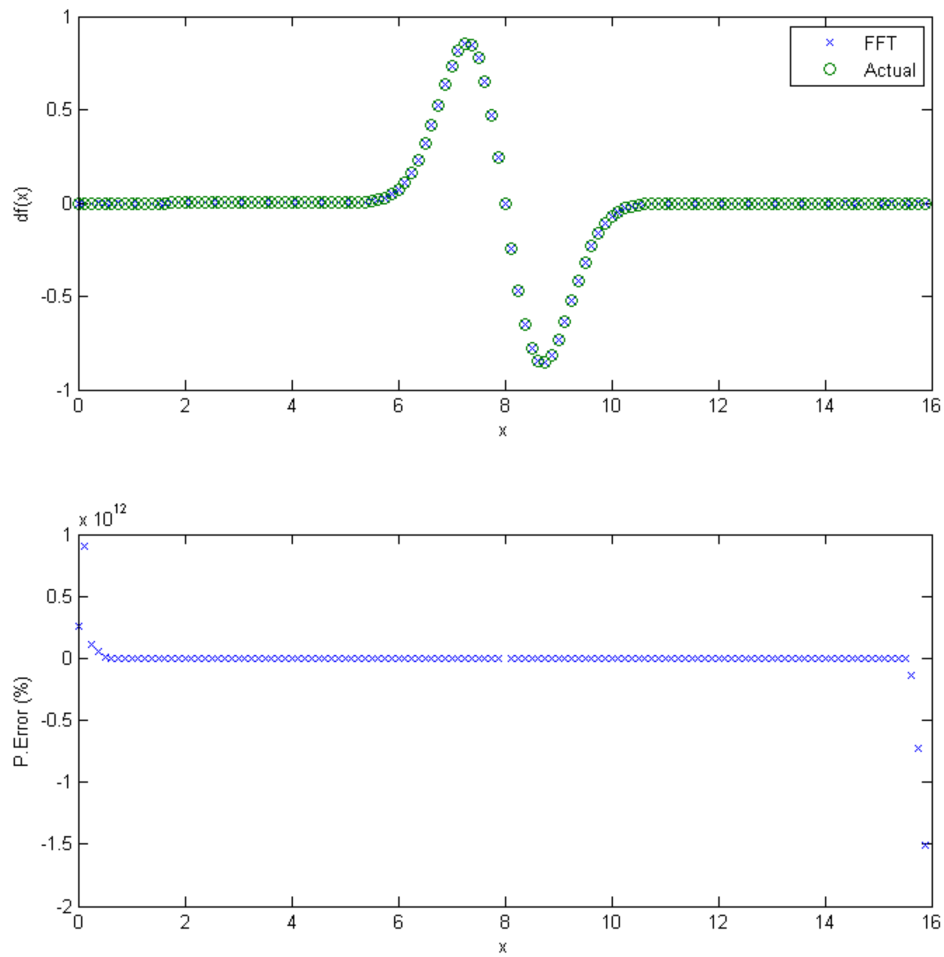


Figure 4: $L = 16, N = 128$

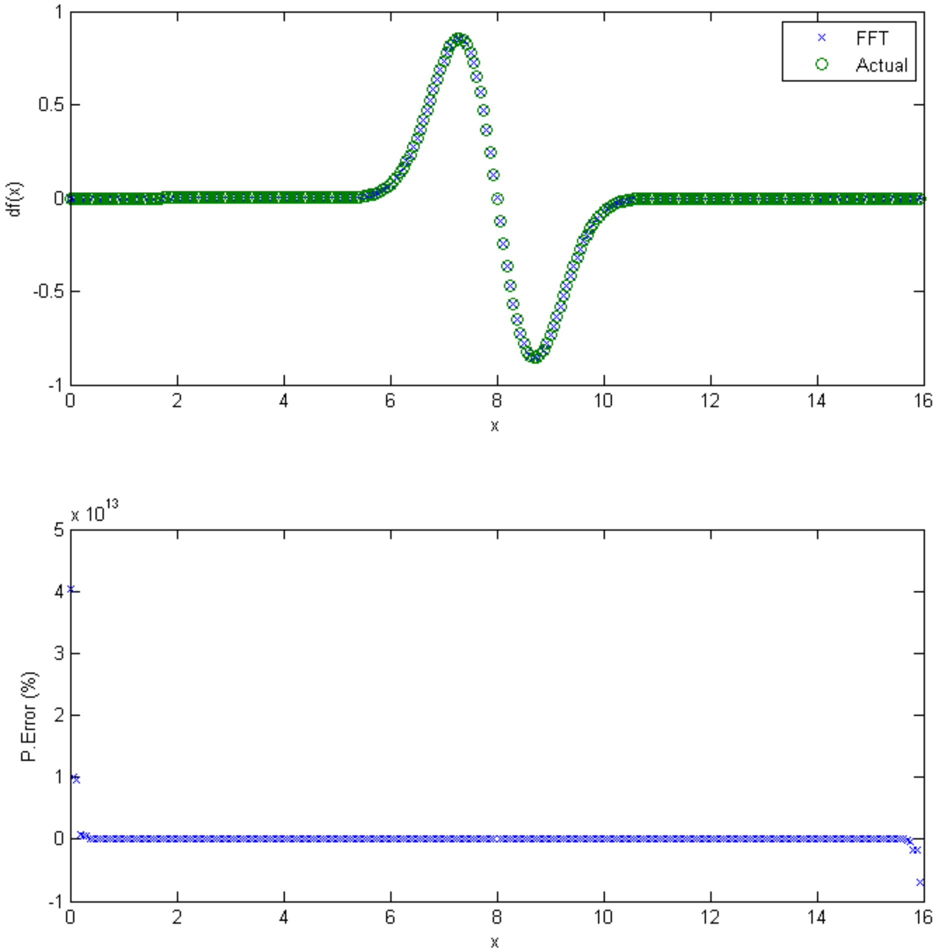


Figure 5: $L = 16, N = 256$

The solutions for the actual and the FFT calculated values are plotted. The error results are very good for non-zero values. However, the error is more sensitive for values approximately equal to zero. The error is exponentially convergent as observed in the plots.

3 Poisson's Equation

Consider a uniformly circular distributed charge:

$$\rho(r) = \begin{cases} \dfrac{1}{\pi r^2} & \text{for r} < 1 \quad (1) \\ 15 & \text{r} = 0 \quad (2) \\ 0 & \text{otherwise} \quad (3) \end{cases}$$

where $\frac{q}{\epsilon_0} = 1$.

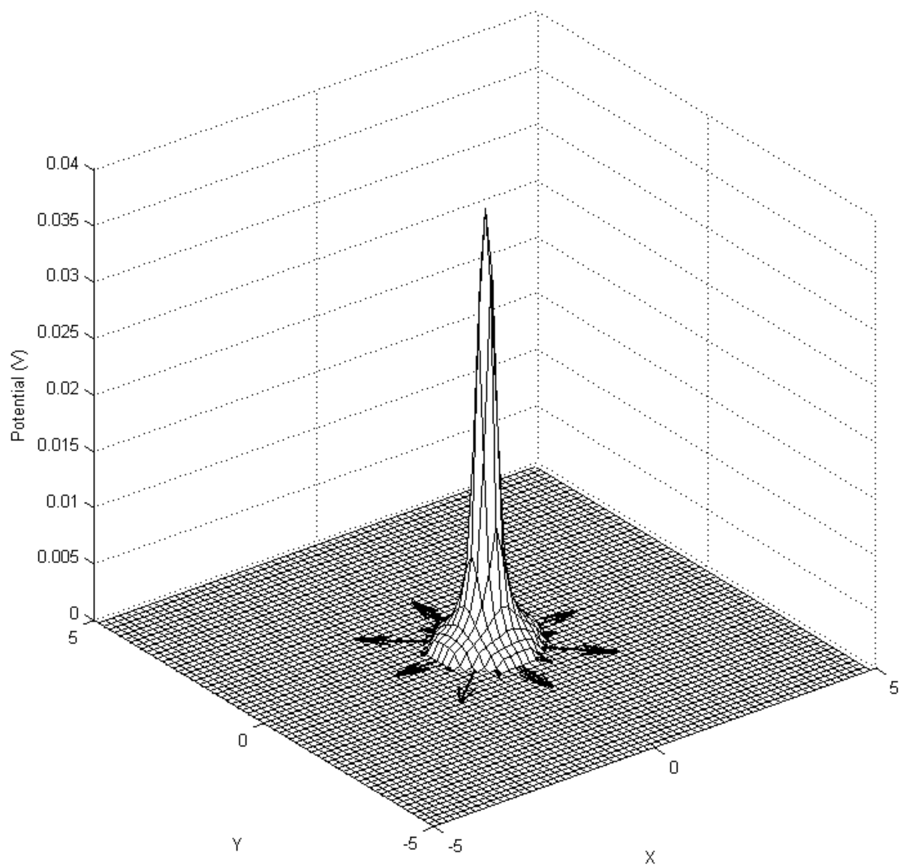


Figure 6: Set $N = 64$ and overlay a scaled vector plot of the electric field over the contour plot of the potential.

Consider an exponential charge distribution:

$$\rho(r) = \begin{cases} e^{r^2} & \text{for r} < 1 \qquad (4) \\ 0 & \text{otherwise} \qquad (5) \end{cases}$$

where $\frac{q}{\epsilon_0} = 1$.

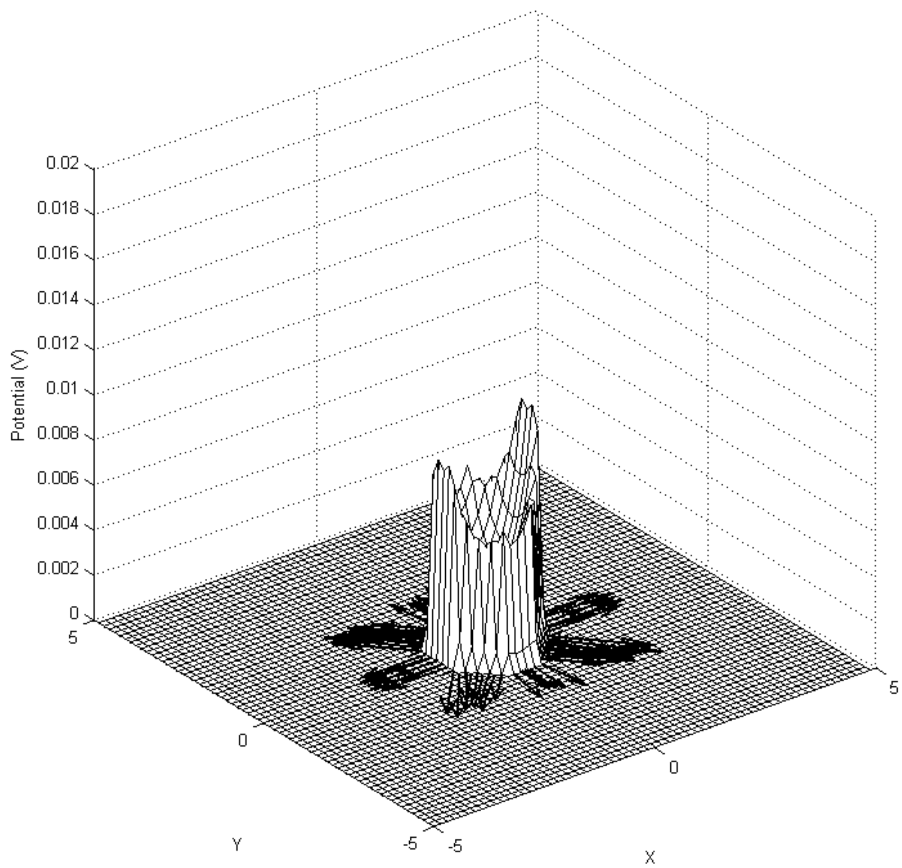


Figure 7: Set $N = 64$ and overlay a scaled vector plot of the electric field over the contour plot of the potential.