1 Parabolic PDE: Thermal Diffusion

Using an implicit method, the thermal diffusion equation:

$$\frac{dT}{dt} = \alpha \frac{d^2T}{dz^2}$$

where $\Delta t = \frac{2\pi}{N}$, $\Delta z = \frac{10}{N}$, and $N = 64$.

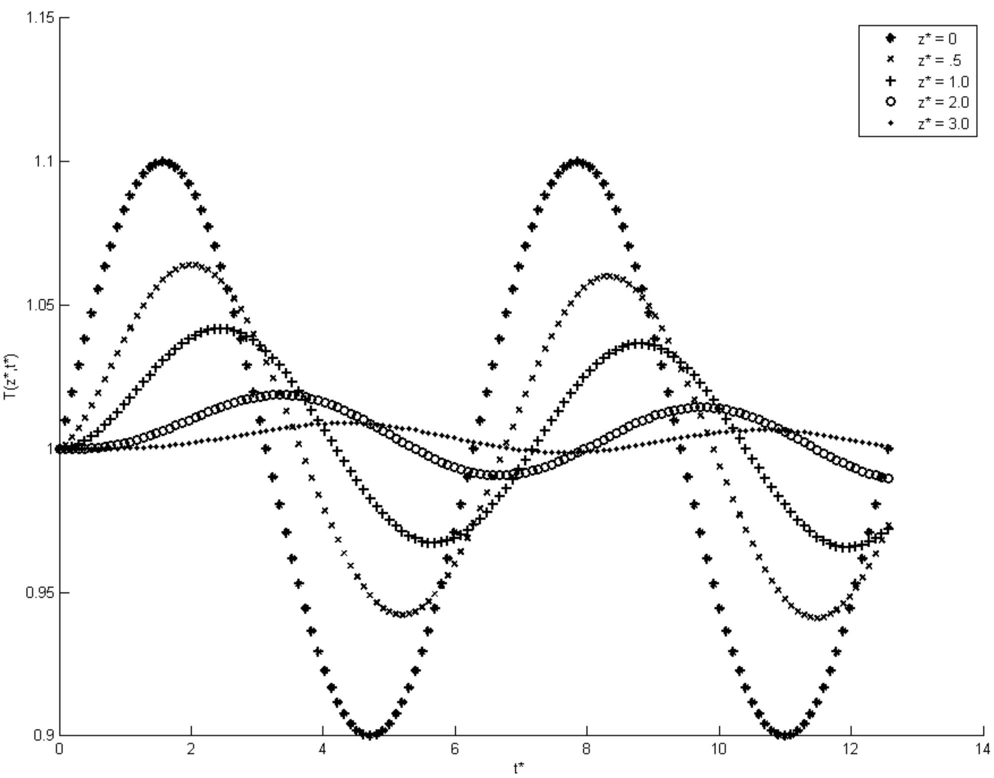

Figure 1: The temperature is simulated over 2 periods of oscillation. The corresponding depths $z^* = 0, .5, 1.0, 2.0, 3.0$ are indicated in the legend. As the skin depth increases the temperature shows phase lags and decreases in amplitude to the surface's temperature. At $z^* \approx 3.0$, the phase is shifted by π and the minimum temperature at the depth occurs at the maximum temperature at the surface and vice versa.

2 1D Hydrodynamic Equations

The explicit method for both the linear and non-linear set of hydrodynamic equation is very sensitive to the N intervals and the perturbation magnitude. The non-linear set needed very small perturbations to plot higher N intervals. The magnitudes of the gas pressure, gas density, and fluid velocity differ depending on the set of equations and N intervals. The plots chosen represent the best resolutions.

2.1 Linear Hydrodynamic Wave Equation

Consider small perturbations:

$$\begin{array}{ll} p = p_0 + \tilde{p} & |\tilde{p}| \ll p_0 \\ \rho = \rho_0 + \tilde{\rho} & |\tilde{\rho}| \ll \rho_0 \\ v = v_0 + \tilde{v} & |\tilde{v}| \ll c_s \end{array}$$

where $v_0 = 0$, $\rho_0 = 5$, $p_0 = 2$. For $N = 32$, I choose $\Delta t = \frac{5}{N}$, $\Delta x = \frac{5}{N}$, and $\gamma = 1.2$ for the linear set of hydrodynamic equations.

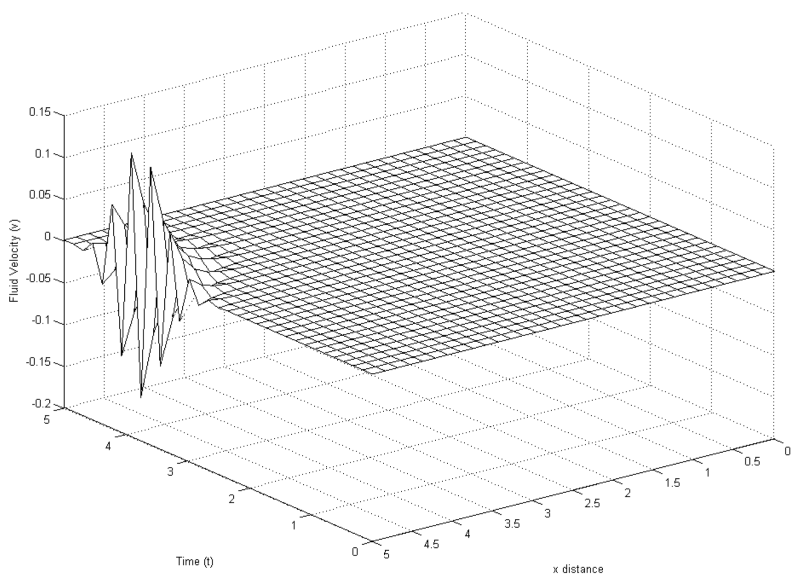

Figure 2: The plot shows velocity as a function of position and time.

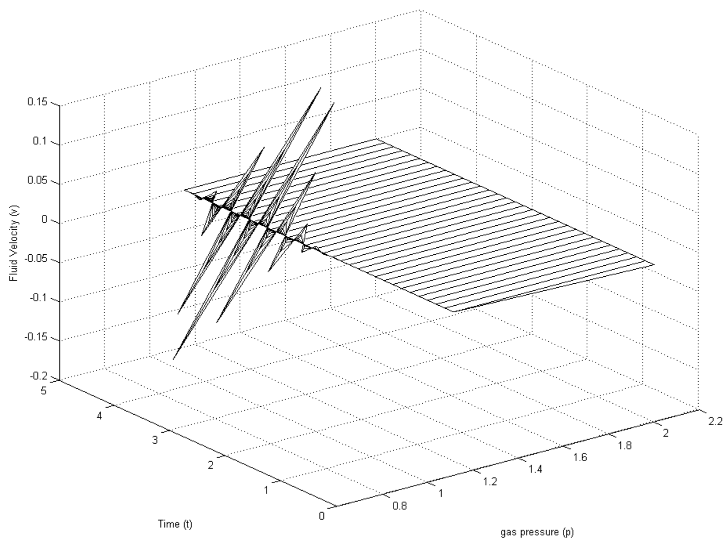

Figure 3: The plot shows the velocity's behavior with time and pressure.

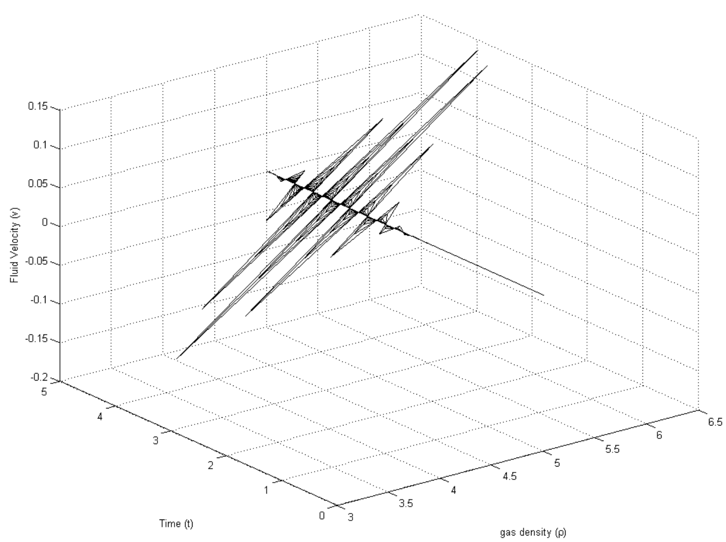

Figure 4: The plot shows the velocity's behavior with time and gas density.

2.2 Non-linear Hydrodynamic Equations

Consider small perturbations proportional to the position and time, where $\sigma \ll 1$:

$$\begin{aligned} p(x,t) &= p_0 + \sigma x t \\ \rho(x,t) &= \rho_0 + \sigma x t \\ v(x,t) &= v_0 + \sigma x t \end{aligned}$$

where $v_0 = 0$, $\rho_0 = 5$, $p_0 = 2$. For $N = 50$, I choose $\Delta t = \frac{5}{N}$, $\Delta x = \frac{5}{N}$, and $\gamma = 1.2$ for the non-linear set of hydrodynamic equations.

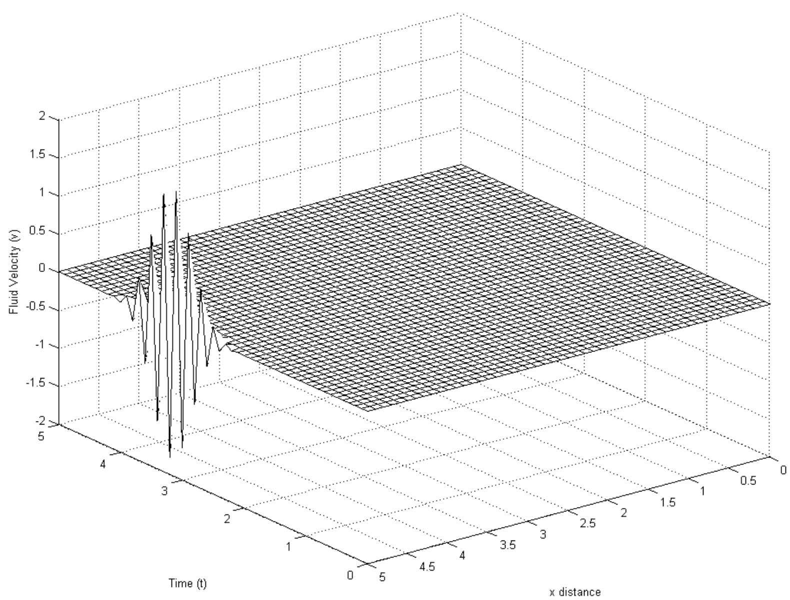

Figure 5: The plot shows velocity as a function of position and time.

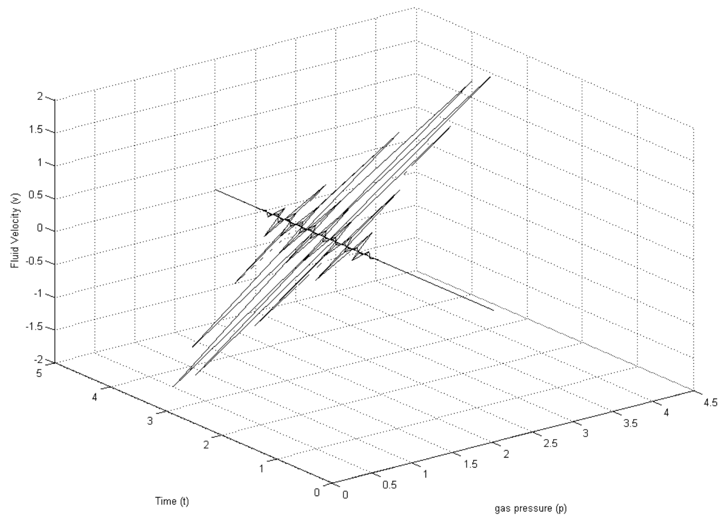

Figure 6: The plot shows the velocity's behavior with time and pressure.

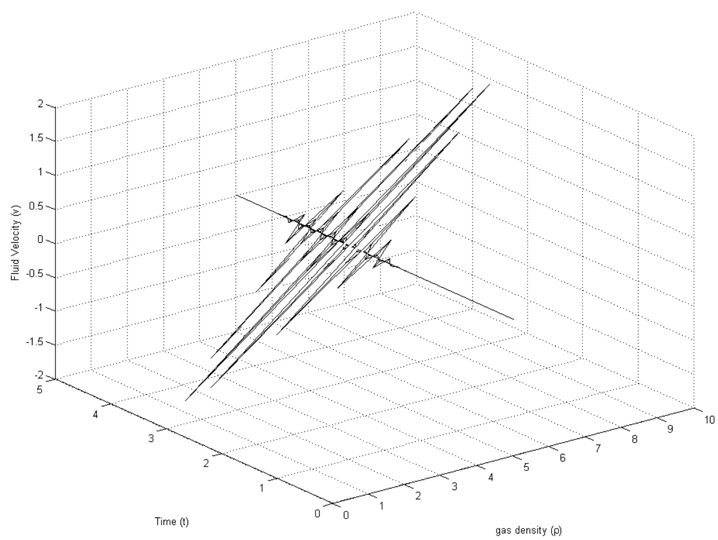

Figure 7: The plot shows the velocity's behavior with time and gas density.