

Modeling Infectious Disease Dynamics: Integrating Contact Tracing-based Stochastic Compartment and Spatio-temporal Risk Models

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Introduction

Infectious diseases have a significant impact on population lives and put enormous pressure on healthcare systems globally.

However, strong interventions imposed to prevent these diseases from spreading may also impact society—making crucial the prioritization of risky areas when applying these protocols.

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In this work, we propose a new **spatio-temporal stochastic model** that allows us **to describe the temporally varying spatial risk**.

We do this by using **(digital) contact-tracing data**, i.e., data collected from the process of identifying individuals' previous infected contacts aiming to test, isolate and trace them.

Introduction



Figure 1: Illustration of Contact-Tracing data with labeled **Susceptible**, **Infected**, and **Recovered** people.

Methodology

Base-SIR modeling (Hernández-Orallo et al., 2020)

Assuming a closed environment, we have 5 compartments and 7 events, namely

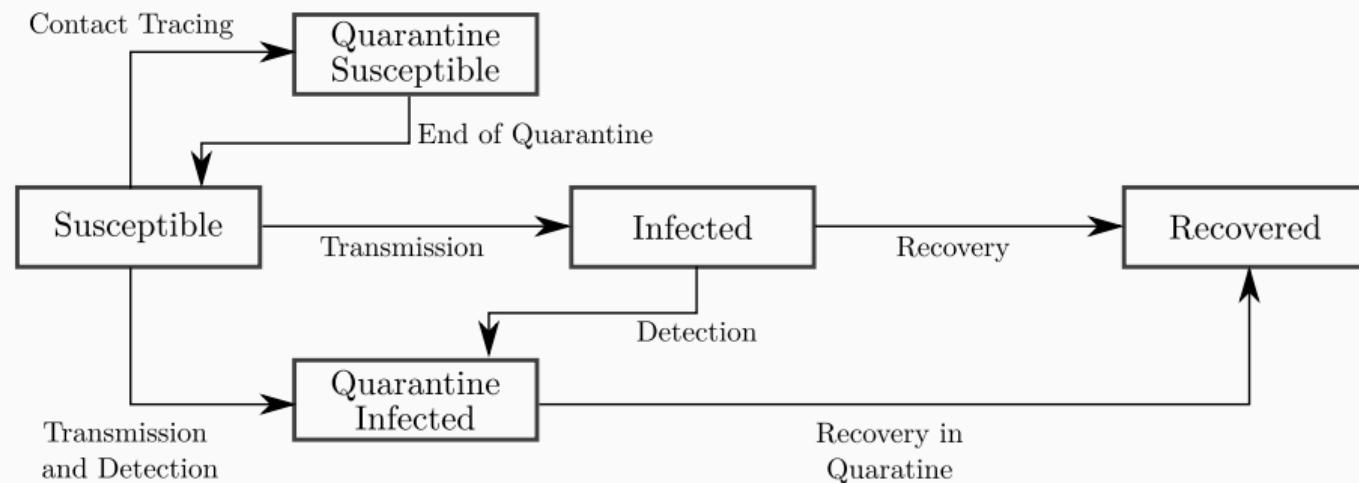
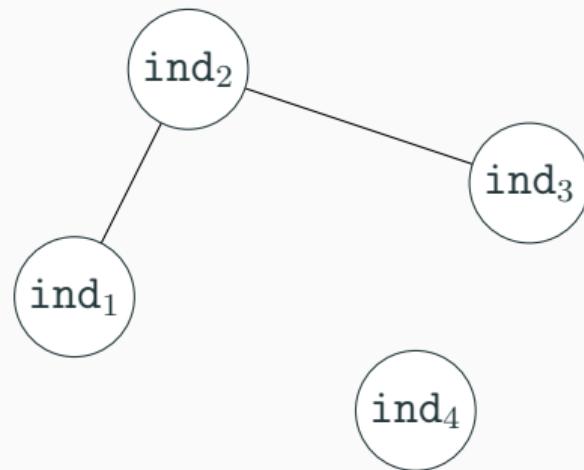


Figure 2: Diagram for the 5 compartments (S, I, R, Q_S, Q_I) and 7 possible transfers.

Methodology

Base-SIR modeling

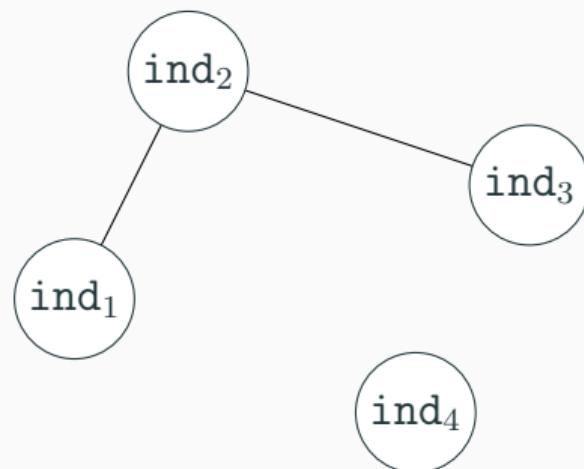
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Methodology

Base-SIR modeling

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$$G(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

such that $G_{ij}(t) = G_{ji}(t)$, for all pairs (i, j) , and $G_{ii}(t) = 0$, $\forall i$.

Methodology

Base-SIR modeling

Regarding the tracing data, the *number of infectious contact* $\mathcal{K}_i(t)$ can be defined as follows

$$\mathcal{K}_i(t) = \sum_{j=1}^N G_{ij}(t) I_j(t),$$

where N is the total number of individuals, and $I_j(t)$ is an indicator function for the event in which the individual j can infect others.

Methodology

Base-SIR modeling

Also, as a way to track back the contacts, we can define, for some t and a time period Δ , a function that verify whether an individual has had contact with at least one traced individual; that is, we want to define $C_i(t, \Delta)$, such that

$$C_i(t, \Delta) = \begin{cases} 1 & \text{if } \sum_{j=1}^N (\max_{\tau \in [t-\Delta, t]} G_{ij}(\tau)) D_j(t) > 0 \\ 0 & \text{otherwise,} \end{cases}$$

such that $D_j(t)$ is an indicator function for the event in which the individual j is infected and traced.

Methodology

Event	Rate Equation	Description
$S \rightarrow I$	$(1 - \mathcal{C}_i(t, \Delta)) \cdot b \cdot \mathcal{K}_i(t)$	Non-traced infected individuals.
$S \rightarrow Q_S$	$\mathcal{C}_i(t, \Delta) \cdot (1 - b \cdot \mathcal{K}_i(t))$	Non-infected but traced individuals.
$S \rightarrow Q_I$	$\mathcal{C}_i(t, \Delta) \cdot b \cdot \mathcal{K}_i(t)$	Infected and traced individuals.
$I \rightarrow Q_I$	δ	Detection of infected individuals.
$Q_S \rightarrow S$	τ_Q	End of quarantine for susceptible individuals.
$I \rightarrow R$	γ	Infected individuals recovered from the disease.
$Q_I \rightarrow R$	τ_Q	End of quarantine for infected individuals.

Such that $\mathcal{K}_i(t)$ is the *number of infectious contacts*, $\mathcal{C}_i(t, \Delta)$ is an indicator function for the event in which an individual has had contact with at least one traced and infected person, b is the *probability of transmission*, δ is the *detection rate*, $1/\tau_Q$ is the *average quarantine time*, and γ is the *recovery rate*.

Methodology

New spatio-temporal-SIR modeling

As in Mahmood et al. (2021), we can incorporate the idea of *temporally varying spatial risk*; meaning that, for a time windows $(t - 1)$, we will collect information on the individuals that walked within a particular cell (determined by a superimposed grid over the studied region), and use such data to update their rates for the next step of an iterative process.

Methodology

New spatio-temporal-SIR modeling

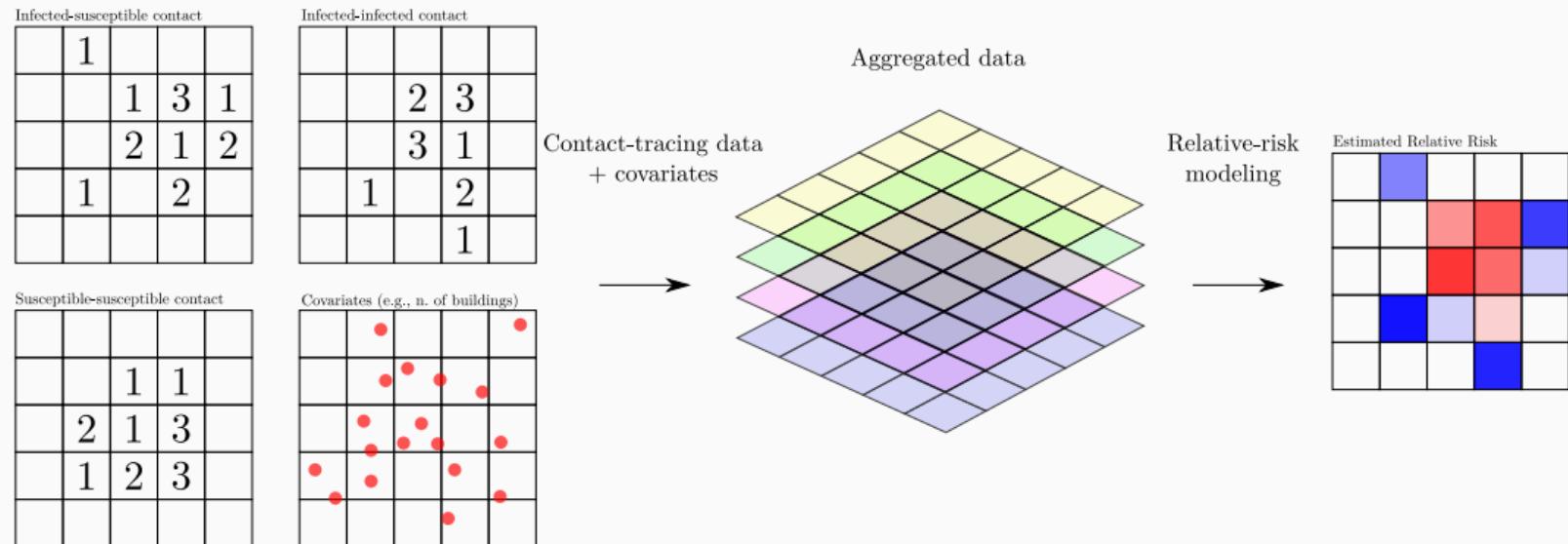


Figure 3: Computed Relative Risk (RR) from contact-tracing data and covariates.

Methodology

New spatio-temporal-SIR modeling

For Y_j representing the number of contacts between one infected and one susceptible individual in a cell j (as before), we will set

$$Y_j \sim \text{Poisson}(E_j \cdot \theta_j), \text{ for all } j, \quad (1)$$

where $\log(\theta_j) = \beta_0 + \beta_1 \cdot x_{1j} + \cdots + \beta_p \cdot x_{pj} + u_j$, such that x_{1j}, \dots, x_{pj} are covariates that affect the risk (e.g., # of buildings) in j , and $u_j \stackrel{\text{i.i.d.}}{\sim} \text{Normal}(0, \sigma_u^2)$.

Also, $E_j = (\text{cont}_j \cdot \theta_{\text{all}})$, such that

$$\theta_{\text{all}} = \frac{\sum_{\text{all}j} Y_j}{\sum_{\text{all}j} \text{cont}_j},$$

where $\text{cont}_j = (c_j^{is} + c_j^{ii} + c_j^{ss})$.

Methodology

New spatio-temporal-SIR modeling

In Model (1), the Relative Risk (RR) θ_j quantifies whether area j has higher ($\theta_j > 1$) or lower ($\theta_j < 1$) risk than the average risk in the population.

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Also, as we may not observe contacts in some cells, we cannot say much about whether these regions are more or less risky than the overall considered area.

In this case, if $\text{cont}_j = 0$ for some j , we chose to set RR_j to 1.

Simulation

Using the Simulation of Urban Mobility (SUMO) library, we simulated the movement of 1,000 pedestrians in part of Valencia, Spain, for 150 days.

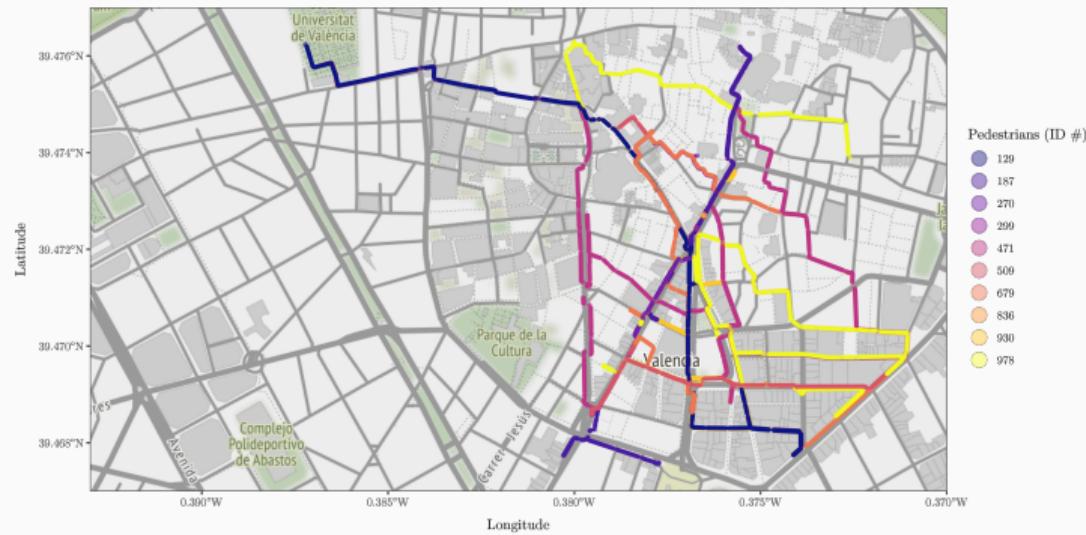


Figure 4: SUMO simulated trajectories for 10 randomly selected pedestrians in the “day 1” in Valencia. “ID #” refers to the pedestrian code number in the data set.

Implementation

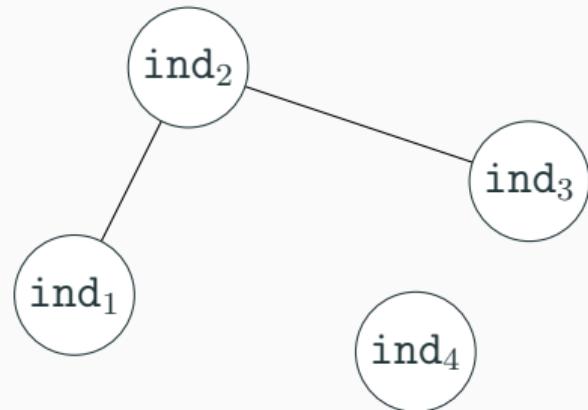
Based on the simulated data set, we define a model for the Relative Risks (RRs) for all $j = 1, 2, \dots, 230$, and for all time windows. In particular,

$$Y_j \sim \text{Poisson}(E_j \cdot \theta_j), \text{ for } j = 1, \dots, 230, \text{ such that } \text{cont}_j \neq 0,$$

where $\log(\theta_j) = \beta_0 + \beta_1 \cdot x_{1j} + u_j$, such that x_{1j} represents the number of buildings in cell j and $u_j \stackrel{\text{i.i.d.}}{\sim} \text{Normal}(0, \sigma_u^2)$.

Implementation

Now, to incorporate this temporally varying spatial risk into our compartment model, we will update the $G(t)$ matrix, such that $G^*(t)$ at the position (k, ℓ) will be given by RR_j , if individuals k and ℓ have been in contact at cell j for the time-window $(t - 1)$; and 0, if there was no contact (as before).



$$G^*(t) = \begin{bmatrix} 0 & RR_j & 0 & 0 \\ RR_j & 0 & RR_j & 0 \\ 0 & RR_j & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

Results

Base-SIR model

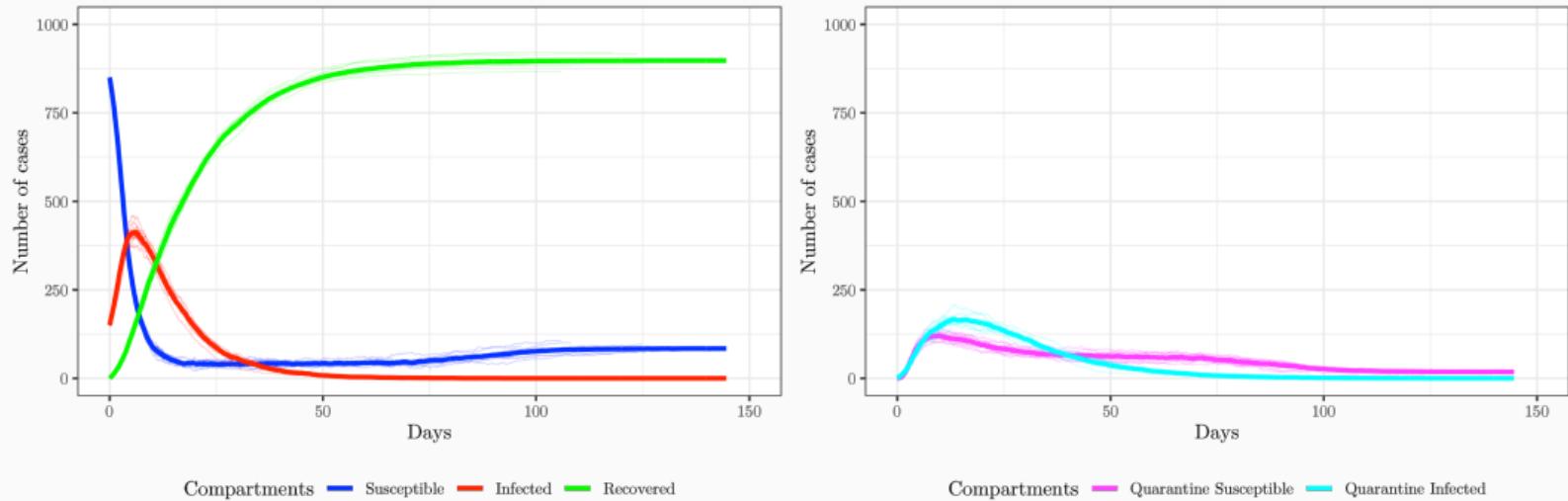


Figure 5: Number of individuals in each compartment (S , I , R , Q_S , Q_I) over the days using the base-SIR model. Light lines represent the 10 realizations of the simulated epidemic, and the bold line represents their average.

Results

New spatio-temporal-SIR modeling

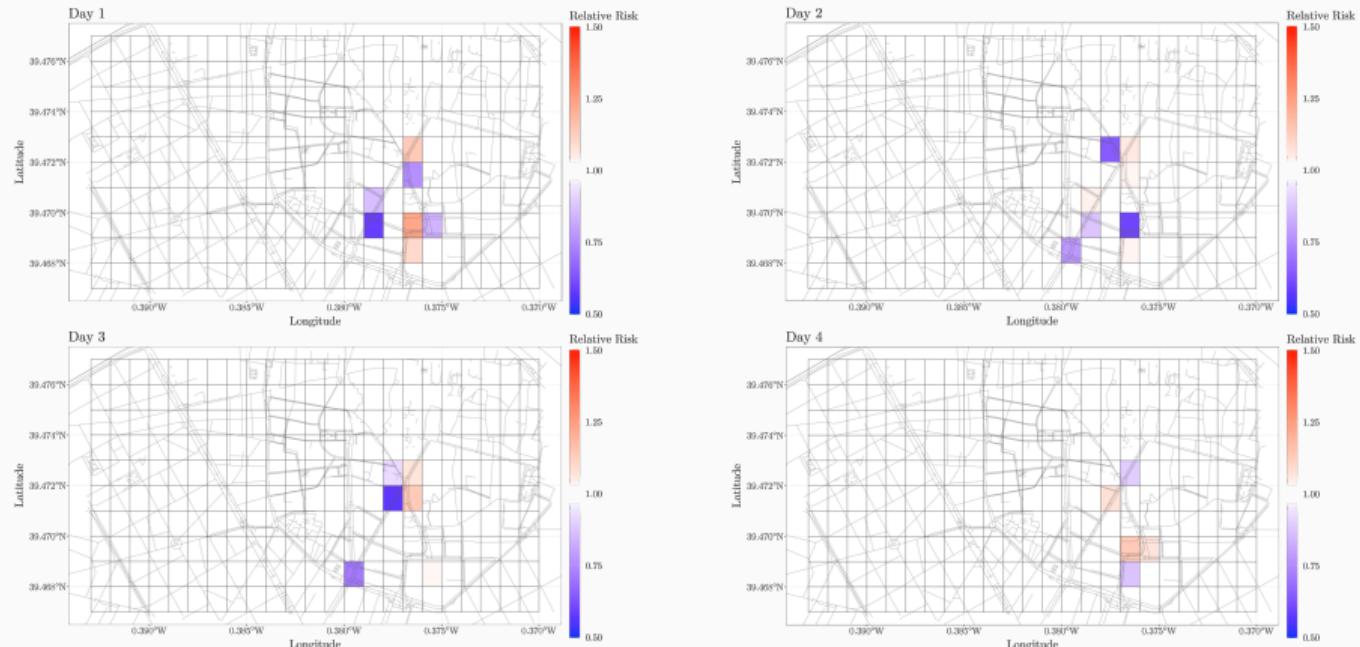
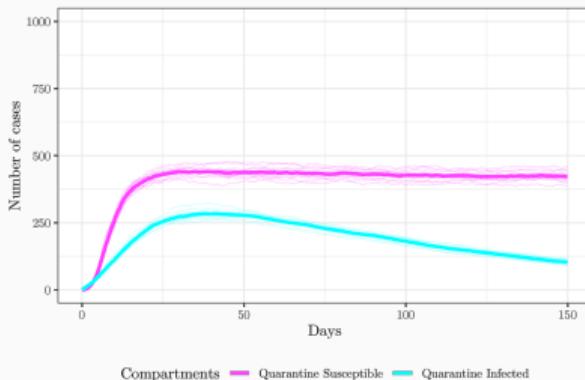
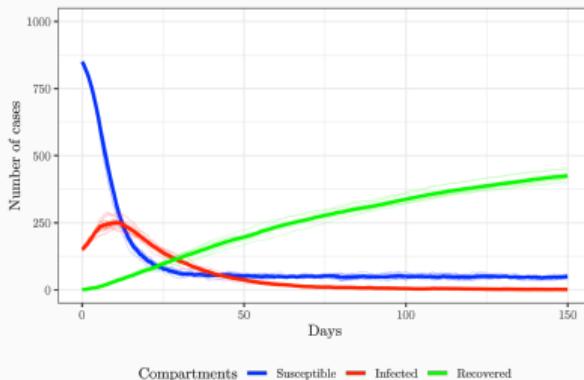
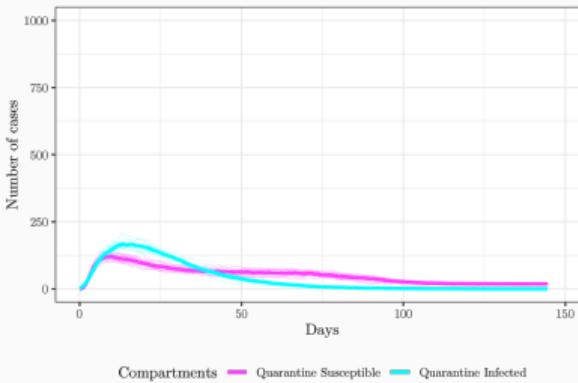
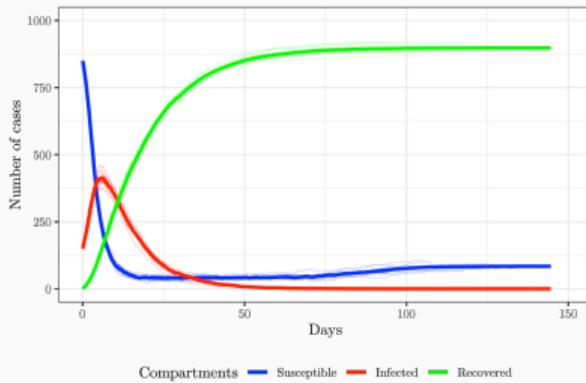


Figure 6: Estimated RRs for all cells in the second window of days 1, 2, 3, and 4.

“Base-SIR” (T) and “new spatio-temporal-SIR model” (B)



Discussion

1. With the “new spatio-temporal-SIR” model, since riskier areas affect more the disease transmission, people who visit these cells are more likely to be quarantined and the peak of the simulated epidemic was reduced.

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3. One could argue that not allowing people to go out would prevent any new cases to be observed. However, as these restrictions also impact other areas of the citizens’ lives, having frequently updated information may help politics to focus on areas that matter.
4. Contact-tracing data is difficult to obtain due to privacy issues.

References

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