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Stacking sequence optimization of composite laminates for maximum buckling load using permutation search algorithm

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ABSTRACT

Due to the involvement of large number of design variables, it is still one of the key concerns to build an efficient optimization algorithm for the stacking sequence design of composite laminates with various constraints. In this work, the flexural stiffness parameters are expressed in terms of the ply orientations, which helps to formulate the maximum of buckling load factor as a problem of identifying the optimum ply orientation at stacking positions. Afterwards, we suggest a permutation search (PS) algorithm to reduce the evaluations in stacking sequence optimization of composite laminates. In the first stage, permutation operations are sequentially performed for each permutation position, and in the second stage a repair strategy is adopted for overcoming the violation of constraints while maximizing the value of the objective function. A comparison has been performed between the PS and three genetic algorithm (GAs) methods. It has been demonstrated that the number of process analyses for stacking sequence optimization are greatly reduced by the PS algorithm. The novel PS algorithm combined with the modified repair strategy outperforms the studied GA methods for constrained stacking sequence optimization of composite laminate both in computational performance and finding the optimal objective value with high reliability.

1. Introduction

Due to the high specific strength and stiffness, composite materials have become widespread during the last three decades. The variation of thickness, fiber orientation as well as stacking sequence provides a large possibility of tailoring for achieving the required mechanical properties, such as in-plane, flexural and buckling behavior of composite laminates, which also makes the optimization of composite laminates has attracted increasing attention. The common problem is to develop optimization routines to minimize the weight of composite structures subjected to mechanical, blending and manufacturing constraints. In optimization studies, the flexural stiffness may be formulated as a linear function of lamination parameters and material invariants. Moreover, the lamination parameters are often treated as independent design variables and they can be expressed as the trigonometric functions of ply orientation, which are interrelated through the functions of stacking sequence [1–4]. Thus, the goal is often achieved by formulating the design of composite laminates as an optimization problem with stacking sequence and ply orientation considered as design variables. However, extremely high dimensional mixed-integer variables make it a challenging work to build an efficient optimization model to find the global optimum [2].

The problem of optimal design of composite laminates has been investigated by many researches. An extensive review of the topic can be found in a recent paper by Ghiasi et al. [3]. Due to its simple coding, escaping of gradient calculations, suitability for large variety of problems and capable of more likely to find global optima, genetic algorithm (GA) become the most popular heuristic method for stacking sequence optimization of composite laminates [2,3]. While it does not require the gradient or sensitivity coefficient evaluations, the population-based evolutionary algorithm can be computationally time consuming and expensive since large number of generations are usually required before converging to the optimal solution and each generation may consists of a large number of evaluations. Another major concern associated with GA is the premature convergence, which may happen if the initial population is not appropriately selected [3].

To reduce computing cost by decreasing the evaluation time and to increase the convergence rate by reducing the risk of premature convergence, many modifications have been suggested, such

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as the multi-level methods [5], the parallel computing method [5–7], and the Hybrid GA method [8,9], etc. In addition, some problem-dependent operators have been introduced to modify the standard genetic operators [10–15]. In stacking sequence optimization problems, some permutation operators are considered, such as the "gene swap", the "inversion" and the "mutation", etc. [16–20]. It is proved that these permutation operations can handle more effectively the search for an optimal stacking sequence by reducing the number of generations to convergence and the dimensionality of the design space [15,16].

However, in some optimization problems, the decision of permutation or sort itself is the main objective [21–23], such as the permutation optimum design of composite wing structure [4,16,24]. In wing structural optimization, the wing structure is often customarily divided into panels or regions, on which constraints from the overall structural design are imposed. Because of the industrial requirements and practical manufacturing considerations, the layer thickness for each panel or region is usually fixed and the fiber orientation angles are often limited to a discrete set. Thus, the optimization of the overall wing structure often specifies or imposes various constraints, on individual panel, such as the number of 0°, ±45°, 90° plies and in-plane loads. As a result, the optimization design is limited to stacking sequence permutations of given plies with fixed ply numbers of each orientation. Since GA is originally developed for unconstrained optimization problem, whereas the optimal design of stacking sequence is usually constrained by limitations in material's strength, weight, cost or other criteria, that must be incorporated. In order to add the constraints to the objective function of constrained permutation optimal problems, penalty function strategy is the most popular technique which may not be satisfied strictly [8,12], and may result in high cost of dealing with constraints. At the same time, the use of standard crossover operator in the permutation optimization problem may lead to inadmissible design individuals, which may then result in the variation of the ply numbers of each orientation. Therefore, appropriate operation, such as "gene repair" or "fixing-up" operation, is required to handle the constraint of ply numbers. Meanwhile, additional operations are also necessary for permutation if some other design constraints are assigned, such as the contiguity requirement [15,16]. The need for additional operations and the high cost of dealing with constraints via penalty function lead to the development of repair strategy to handle the constraints [16,18]. However, since the efficiency of GA is often sensitive to operations or parameters, unsuitable operations or parameters during the process of permutation and repair may result in heavy computational cost or even failure in finding the optimal solution [15].

In this study, based on the search or sort strategies of sequential search [21] and the selection sort [23] methods, a permutation search (PS) algorithm as well as an improved repair strategy is proposed to improve the numerical efficiency of the constrained sequence optimization of composite structures. The proposed algorithm takes the permutation of the ply angles as design variable, which governs the buckling behavior of composite laminates. Meanwhile, the number-of-ply constraints are handled by the improved repair strategies. The accuracy and computational efficiency of the present PS algorithm are then compared with three GA methods.

2. Optimization formulation

Considering the industrial and manufacturing requirements, symmetric and balanced stacking sequences are usually adopted in design, where the ply angles are pre-assigned and loading conditions of each panel are often specified. In this study, a symmetric laminated plate of rectangular shape composed of 0°, 45°, −45°, and 90° is examined under in-plane loading condition. The plate is simply supported at the edges and subjects to normal loads per unit length fF_x and fF_y in X and Y directions, respectively, and a shear load per unit length fF_{xy}, where f is a scalar amplitude parameter to the reference load F. (Fig. 1a). We consider the N plies laminate plate with its length (X direction) and width (Y direction) dimensions represented by a and b. For symmetric laminates, only $N/2$ plies are necessary to characterize the entire laminate, as shown in Fig. 1b. Again, due to the symmetry and balanced laminate, the extensional–flexural and the shear–extensional couplings can be eliminated. Here, the panel is designed to maximize the buckling load subject to a constraint on the number of plies of each orientation.

To simplify the optimization of the problem, we first take the normal and the shear loading conditions in consideration separately and then focus on the combined effects of different loading conditions. For a palte subject normal load, the governing differential equation for the bending of the panel reads [25]

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}=F_x\frac{\partial^2 w}{\partial x^2}+F_y\frac{\partial^2 w}{\partial y^2} \quad (1)$$

Solution to Eq. (1) can be obtained via direct approach, such as the Navier approach and has the following general form for various boundary conditions

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}\overline{X}_m(x)\overline{Y}_n(x) \quad (2)$$

where the functions $\overline{X}_n(x)$ and $\overline{Y}_n(x)$ are a set of functions that satisfy the boundary conditions, which are uniformly convergent, complete and orthogonal. Practically, because of uniform convergence, a finite number of terms are sufficient to provide any desired accuracy. Note that solutions for different boundary conditions can be obtained by defining corresponding forms of functions $\overline{X}_n(x)$ and $\overline{Y}_n(x)$ and substituting into Eq. (2).

For a plate that is simply supported on all four edges, we have

$$\text{at } x=0 \text{ and } x=a \quad w=M_x=-D_{11}\frac{\partial^2 w}{\partial x^2}-D_{12}\frac{\partial^2 w}{\partial y^2}=0 \quad (3)$$

$$\text{at } y=0 \text{ and } x=b \quad w=M_y=-D_{12}\frac{\partial^2 w}{\partial x^2}-D_{22}\frac{\partial^2 w}{\partial y^2}=0 \quad (4)$$

Then Eq. (2) can be written as [26]

$$w(x,y)=\sum_{n=1}^{N}\sum_{m=1}^{M}A_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \quad (5)$$

where m and n are integers. Under in-plane biaxial loading, the plate may buckle into m and n half waves along the length and width directions, respectively.

Substituting Eq. (5) into the governing partial differential Eq. (1), the load multiplier f can be expressed as [19]:

$$\lambda_{c,n}(m,n)=\pi^2\frac{D_{11}\left(\frac{m}{a}\right)^4+2(D_{12}+2D_{66})\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2+D_{22}\left(\frac{n}{b}\right)^4}{F_x\left(\frac{m}{a}\right)^2+F_y\left(\frac{n}{b}\right)^2} \quad (6)$$

where, the coefficients of the flexural stiffness D_{ij} (i, j = 1, 2, 6) depend on the lamination sequence.

The critical buckling load factor $\lambda_{c,n}$ is given by the minimum of $\lambda_{c,n}$ (m, n) over possible values of m and n, which varies with the total number of plies, the plate geometry, and the loading case. Herein, considering the insignificant contributions of coupling coefficients D_{16} and D_{26} special attention will be paid to D_{11}, D_{12}, D_{22} and D_{66}. For a composite laminate panel made of a single fibrous material, the elements of the flexural stiffness matrix can

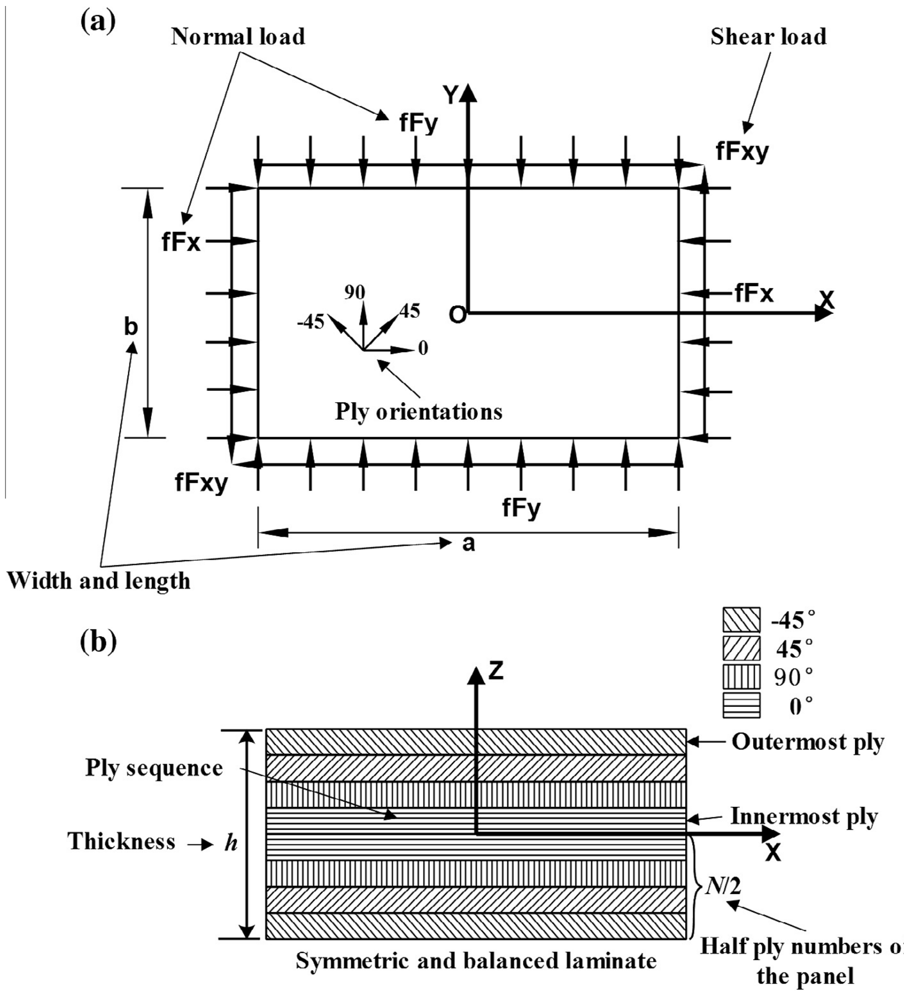

Fig. 1. Laminate composite panels under in-plane loading: (a) geometry and load condition, and (b) ply sequence location.

be expressed in terms of only two lamination parameters and material invariants [1,20]

$$\begin{aligned}
D_{11} &= \frac{h^3}{12}(U_1 + U_2W_1^* + U_3W_2^*),\\
D_{22} &= \frac{h^3}{12}(U_1 - U_2W_1^* + U_3W_2^*),\\
D_{12} &= \frac{h^3}{12}(U_4 - U_3W_2^*),\\
D_{66} &= \frac{h^3}{12}(U_5 - U_3W_2^*),
\end{aligned} \tag{7}$$

where, h is the total panel thickness, U_i ($i = 1,\ldots,5$) are the material invariants, and W_1^* and W_2^* are the lamination parameters in the form of

$$W_1^* = \frac{12}{h^3}\int_{-h/2}^{h/2} z^2\cos 2\theta \mathrm{d}z, \quad W_2^* = \frac{12}{h^3}\int_{-h/2}^{h/2} z^2\cos 4\theta \mathrm{d}z, \tag{8}$$

where, θ represents the ply orientation and z is the vertical position along the thickness coordinate.

Due to the fact that laminates are composed of unidirectional layers ply by ply, lamination parameters are trigonometric functions of the ply orientation, and they are interrelated to the functions of stacking positions [1]. The lamination parameters can be simplified in terms of the stacking position k and ply orientation θ

$$\begin{aligned}
W_1^* &= \frac{8}{N^3}\sum_{k=1}^{N/2}(\cos 2\theta_k)(k^3 - (k-1)^3),\\
W_2^* &= \frac{8}{N^3}\sum_{k=1}^{N/2}(\cos 4\theta_k)(k^3 - (k-1)^3)
\end{aligned} \tag{9}$$

where, the stacking position k denotes the laminate position from the middle plane to the outermost ply (Fig. 2).

We can formulate the relation between the flexural stiffness parameters and the stacking position k and the ply orientation θ by substituting Eq. (9) into Eq. (7),

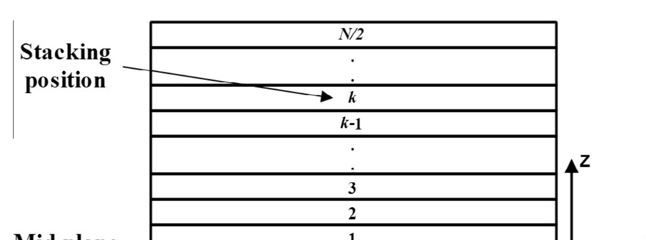

Fig. 2. Illustrations of stacking positions from the middle plane to outermost position.

$$D_{11} = \frac{h^3}{12}\left[U_1 + \frac{8U_2}{N^3}\sum_{k=1}^{N/2}(\cos 2\theta_k)(k^3-(k-1)^3) + \frac{8U_3}{N^3}\sum_{k=1}^{N/2}(\cos 4\theta_k)(k^3-(k-1)^3)\right]$$
$$D_{22} = \frac{h^3}{12}\left[U_1 - \frac{8U_2}{N^3}\sum_{k=1}^{N/2}(\cos 2\theta_k)(k^3-(k-1)^3) + \frac{8U_3}{N^3}\sum_{k=1}^{N/2}(\cos 4\theta_k)(k^3-(k-1)^3)\right]$$
$$D_{12} = \frac{h^3}{12}\left[U_4 - \frac{8U_3}{N^3}\sum_{k=1}^{N/2}(\cos 4\theta_k)(k^3-(k-1)^3)\right]$$
$$D_{66} = \frac{h^3}{12}\left[U_5 - \frac{8U_3}{N^3}\sum_{k=1}^{N/2}(\cos 4\theta_k)(k^3-(k-1)^3)\right] \quad (10)$$

We can further simplify Eq. (10) and rewrite it in the form of

$$D_{11} = \sum_{k=1}^{N/2}(D_{11})_k = \sum_{k=1}^{N/2}\left\{\frac{h^3U_1}{6N} + \frac{2h^3}{3N^3}(k^3-(k-1)^3)[U_2(\cos 2\theta_k) + U_3(\cos 4\theta_k)]\right\}$$
$$D_{22} = \sum_{k=1}^{N/2}(D_{22})_k = \sum_{k=1}^{N/2}\left\{\frac{h^3U_1}{6N} + \frac{2h^3}{3N^3}(k^3-(k-1)^3)[-U_2(\cos 2\theta_k) + U_3(\cos 4\theta_k)]\right\}$$
$$D_{12} = \sum_{k=1}^{N/2}(D_{12})_k = \sum_{k=1}^{N/2}\left\{\frac{h^3U_4}{6N} + \frac{2h^3}{3N^3}(k^3-(k-1)^3)[-U_3(\cos 4\theta_k)]\right\}$$
$$D_{66} = \sum_{k=1}^{N/2}(D_{66})_k = \sum_{k=1}^{N/2}\left\{\frac{h^3U_5}{6N} + \frac{2h^3}{3N^3}(k^3-(k-1)^3)[-U_3(\cos 4\theta_k)]\right\} \quad (11)$$

We see from Eq. (11) that the overall flexural stiffness parameters D_{ij} can be expressed in sum of those at each position $(D_{ij})_k$. Moreover, for a specified stacking position k the flexural stiffness parameters D_{ij} only depends on the trigonometric functions of ply orientation θ_k. In other words, for any stacking position k, the ply angle θ_k is the only design variable in the optimization of flexural stiffness parameters.

Substituting Eq. (11) into Eq. (6) yields

$$\lambda_{c,n} = \sum_{k=1}^{N/2}(\lambda_{c,n})_k = \frac{\pi^2}{F_x\left(\frac{m}{a}\right)^2 + F_y\left(\frac{n}{b}\right)^2}\sum_{k=1}^{N/2}\left[(D_{11})_k\left(\frac{m}{a}\right)^4 + 2((D_{12})_k + 2(D_{66})_k)\left(\frac{m}{a}\right)^2\left(\frac{n}{b}\right)^2 + (D_{22})_k\left(\frac{n}{b}\right)^4\right] \quad (12)$$

Then, the buckling load factor can be obtained by solving Eq. (12). Note that the critical buckling load factor $\lambda_{c,n}$ is a function of the flexural stiffness parameters, the plate aspect ratio, and the loading case.

For a specified optimization formulation, the normal buckling load $\lambda_{c,n}$ can be expressed as the sum of $(\lambda_{c,n})_k$ at each stacking position k and hence in terms of the ply angle θ_k at each position. This implies that in this case superposition principles are suitable for the evaluation of buckling load $\lambda_{c,n}$. Therefore, the maximum of the buckling load factor $\lambda_{c,n}$ reduces to a problem of identifying the optimum ply angle θ_k at stacking position k under the constraint of the ply numbers of each orientation angle.

Another issue is the shear load induced buckling. Consider the stability of simply supported rectangular plates subjected to a uniform shear load F_{xy}, the solutions can be obtained by the Galerkin method. For a panel simply supported on all four edges as described by Eqs. (3) and (4), the following solution which satisfy the assumed form of Eq. (5) has been given by Whitney [25]

$$\pi^4[D_{11}m^4 + 2(D_{12}+2D_{66})m^2n^2R^2 + D_{22}n^4R^4]A_{mn} - 32mnR^3b^2F_{xy}\sum_{i=1}^{M}\sum_{j=1}^{N}M_{ij}A_{ij} = 0$$
$$\text{where } M_{ij} = \frac{ij}{(m^2-i^2)(n^2-j^2)}\begin{cases} m\pm i\ odd \\ n\pm j\ odd\end{cases} = 0 \quad \begin{cases} i = m\ m\pm i\ even \\ j = n\ n\pm j\ even\end{cases} \quad (13)$$

where, R is the plate aspect ratio a/b.

It is clear Eq. (13) is a classical eigenvalue problem. The critical buckling load, $F_{c,xy}$, can be obtained by iterations, which usually does not depend on the direction of the shear stress and thus leads to the positive and negative pairs of the eigenvalues. In this case, standard iteration procedures must be modified. In addition, convergence of the solution is slow. As a result, analyzing of shear loading induced buckling mode for a finite plate is computationally expensive. Also, the solution given by Eq. (13) is approximate which may have significant errors, and it cannot conventionally be applied to long plates [25].

In contrast, accurate analytical solutions are available for a plate with infinite length in one direction. In this work, the solutions for a plate of infinite length in the x direction are used as an approximation. Consider an infinite plate simply supported at the edges of $y = \pm b/2$, subjected to a uniformly distributed shear load, the critical shear buckling load factor $\lambda_{c,s}$ can be written as [16]

$$\lambda_{c,s} = \frac{4\beta_1(D_{11}D_{22}^3)^{1/4}}{b^2F_{xy}}, \quad \text{for } 1 \le \Gamma \le \infty,$$
$$= \frac{4\beta_1\sqrt{(D_{22}(D_{12}+2D_{66})}}{b^2F_{xy}}, \quad \text{for } 0 \le \Gamma \le 1, \quad (14)$$

where the variable Γ is defined as [27]

$$\Gamma = \frac{\sqrt{D_{11}D_{22}}}{D_{12}+2D_{66}} \quad (15)$$

and, values of coefficients β_1 can be found in reference [16].

Obviously, Eq. (14) is an accurate solution, which is simpler and more computationally cheap than the solution through Eq. (13). At the same time, the error of this approximation to the solutions of a finite plate is acceptable [26]. Therefore, these solutions provide the limiting cases for a plate of finite length and it is reasonable to adopt the analytical solutions for a plate of infinite length in one direction as approximations.

In case of normal and shear loads being applied to the panel simultaneously, the governing differential equation reads [28]

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} = -F_x\frac{\partial^2 w}{\partial x^2} + 2F_{xy}\frac{\partial^2 w}{\partial x\partial y} \quad (16)$$

where F_x is the uniformly distributed normal load per unit length along x or y direction, and F_{xy} is the shear force uniformly distributed along all edges. Due to the symmetry of variables x and y in Eq. (16), only normal load along x direction is considered here.

We seek an approximate solution satisfying Eq. (5) in the form

$$w = A\sin(\pi y/b)\sin(\pi/s(x - y\tan\varphi)) \tag{17}$$

This expression satisfies the boundary conditions and represents the surface of inclined waves, the length of which in the x direction is s with an angle of inc1ination φ from the y axis. The relation of normal force F_x and shear force F_{xy} can be obtained by substituting Eq. (17) into Eq. (16) and integrate it with respect to y from 0 to b and with respect to x from 0 to s [28]

$$F_x = \frac{\pi^2\sqrt{D_{11}D_{22}}}{b^2}\left[\sqrt{\frac{D_{11}}{D_{22}}}\gamma + \frac{2(D_{12}+2D_{66})}{\sqrt{D_{11}D_{22}}}(\alpha^2\gamma+1)\right. \\ \left. + \sqrt{\frac{D_{22}}{D_{11}}}\left(\alpha^4\gamma + 6\alpha^2 + \frac{1}{\gamma}\right)\right] + 2\alpha F_{xy} \tag{18}$$

where $\gamma = \left(\frac{b}{s}\right)^2, \alpha = \tan\varphi$.

Considering force F_{xy} as temporarily constant, we can get the following relations by seeking the minimum value of F_x as a function of α and γ

$$F_{xy} = -\frac{2\pi^2\sqrt{D_{11}D_{22}}}{b^2}\cdot\alpha\left(\frac{D_{12}+2D_{66}}{\sqrt{D_{11}D_{22}}}\gamma + \sqrt{\frac{D_{22}}{D_{11}}}(3+\alpha^2\gamma)\right), \tag{19}$$

$$\gamma = \sqrt{\frac{D_{22}}{D_{11} + 2(D_{12}+2D_{66})\alpha^2 + D_{22}\alpha^4}}. \tag{20}$$

Substituting Eq. (20) into Eq. (18) and into Eq. (19) and assume that $D_1 = D_{11}, D_2 = D_{22}, D_3 = D_{12} + 2D_{66}$, then we get the relations

$$F_x = \frac{\pi^2\sqrt{D_1D_2}}{b^2}\bullet 2\left(\frac{D_3}{\sqrt{D_1D_2}} + 3\alpha^2\sqrt{\frac{D_2}{D_1}}P + \sqrt{1+\frac{2D_3}{D_1}\alpha^2+\frac{D_2}{D_1}\alpha^4}\right) + 2\alpha F_{xy}, \tag{21}$$

$$F_{xy} = -\frac{\pi^2\sqrt{D_1D_2}}{b^2}\bullet 2\alpha\left(3\sqrt{\frac{D_2}{D_1}} + \frac{D_3+\alpha^2D_2}{\sqrt{1+\frac{2D_3}{D_1}\alpha^2+\frac{D_2}{D_1}\alpha^4}}\right)$$

where $\alpha = \tan\varphi$.If we vary parameter α from zero to infinity and if values of F_x are plotted along the abscissa and corresponding values of F_{xy} as ordinates, a quadratic parabola form of the relation between F_x and F_{xy} is

$$\frac{F_x}{F'_{c,x}} + \frac{F_{xy}{}^2}{F'^2_{c,xy}} = 1 \tag{22}$$

where the critical compressive force $F'_{c,x}$ is the intercept on the abscissa when shear forces are absent, while the critical shear force $F'_{c,xy}$ is the intercept on the ordinates when normal forces are absent.

After determining $F'_{c,x}$ and $F'_{c,xy}$, it is always possible to find the critical normal load for a given shear force, or vice versa, from Eq. (22), or a critical value of λ_{cr} if there is a given ratio between $F'_{c,x}$ and $F'_{c,xy}$. Eq. (22) can be rewritten by defining the critical load amplitudes of normal and shear load as $\lambda_{c,n} = F'_{c,x}/F_x$ and $\lambda_{c,s} = F'_{c,xy}/F_{xy}$, respectively

$$\frac{1}{\lambda_{c,n}} + \frac{1}{\lambda^2_{c,s}} = 1 \tag{23}$$

Assume that $\lambda_c^{(m,n)}$ is the critical load amplitudes of combined normal and shear loads, then the following relation can be introduced to approximate the interaction between normal and shear forces

$$\frac{1}{\lambda_c^{(m,n)}} = \frac{1}{\lambda_{c,n}^{(m,n)}} + \frac{1}{\lambda^2_{c,s}} \tag{24}$$

where $\lambda_{c,n}^{(m,n)}$ and $\lambda_{c,s}$ are the critical load amplitudes under normal and shear loads, respectively. Shear buckling occurs independent of the sign of the shear load. To avoid buckling, $\lambda_{c,n}^{(m,n)}$ and $\lambda_{c,s}$ have to be greater than one. Therefore, the combined buckling load factor $\lambda_c^{(m,n)}$ is always more critical than the normal buckling load factor $\lambda_{c,n}^{(m,n)}$.

The buckling load λ_c can be taken to be the minimum of the load factors

$$\lambda_c = \min\left\{|\lambda_{c,s}|, \lambda_c^{(m,n)}\right\} \tag{25}$$

3. Permutation optimization scheme

3.1. Permutation search (PS) algorithm

It has been demonstrated that the stacking sequence optimization of composite laminates can be formulated as a permutation problem [4]. In the field of computer programming, several methods have been developed to solve the permutation problems, such as the sequential search [21] and the sort methods [22,23], etc. In this section, based on the theoretical derivation of Section 2, we suggest a two-step strategy to identify the optimal stacking sequence for maximum buckling load factor. In the first step, a ply orientation at stacking position k is selected through swap strategy. The ply orientation will not change after it has been identified. In the second step, we vary the stacking position k from the last to the first position, sequentially. Thus, a search scheme named PS algorithm is developed and employed to reduce the number of evaluations in constrained stacking sequence optimization of composite laminates while maintaining accuracy.

Given the set of all permutations of sequence $T_N = (\theta_1, \theta_2, \ldots, \theta_N)$, where θ_k ($k = 1, 2, \ldots, N$) is the ply orientation, any permutation $\boldsymbol{\pi} \in T_N$ can be viewed as an ordered arrangement of the elements in T_N, where $\pi(i)$ is the element at position i. Then, new permutations $\boldsymbol{\pi} \in T_N$ can be obtained by using different operators [22], such as "swap", "interchange", "insert" and so on. As a result, the optimal problem can be summarized as maximizing the buckling load factor λ_c via different operations on the permutation $\boldsymbol{\pi}$. Here in our work, three operators are used, in which the "swap" operator is mainly used to obtain the global optimal sequence of the laminate while the "delete" and "insert" operators are used to deal with continuity constraints.

Swap: $\sigma < i, j >$, where $1 \leqslant i \leqslant j \leqslant N$, is the generator

$$(\theta_1, \theta_2, \ldots, \theta_{i-1}, \underline{\theta_i}, \theta_{i+1}, \ldots, \theta_{j-1}, \underline{\theta_j}, \theta_{j+1}, \ldots, \theta_N) \\ \downarrow \\ (\theta_1, \theta_2, \ldots, \theta_{i-1}, \underline{\theta_j}, \theta_{i+1}, \ldots, \theta_{j-1}, \underline{\theta_i}, \theta_{j+1}, \ldots, \theta_N)$$

The swap $\sigma\,\langle i, j\rangle$ as the effect of switching elements $\boldsymbol{\pi}(i)$ and $\boldsymbol{\pi}(j)$ in the permutation $\boldsymbol{\pi}$. The distance between two elements is defined as $d\ (i, j) = |i - j|$. Theoretically, any permutations can be obtained by a series of swap operations as long as the constraints on the ply numbers have been satisfied.

For the constrained composites optimization problem, another two operators "delete" and "insert" are defined.

Delete: $\alpha\,\langle k\rangle$, where $1 \leqslant k \leqslant N$, is the generator

$$(\theta_1, \theta_2, \ldots, \theta_{k-1}, \underline{\theta_k}, \theta_{k+1}, \ldots, \theta_N) \\ \downarrow \\ (\theta_1, \theta_2, \ldots, \theta_{k-1}, \theta_{k+1}, \ldots, \theta_N)$$

where the element θ_k is deleted from the original permutation $\boldsymbol{\pi}$, and the total ply numbers N changes from N to $N - 1$ with $\boldsymbol{\pi} = (\theta_1, \theta_2, \ldots, \theta_{k-1}, \underline{\theta_k}, \ldots, \theta_{N-1})$.

Insert: $\beta\langle k\rangle$, where $1 \leqslant k \leqslant N$, is the generator

$$(\theta_1, \theta_2, \ldots, \underline{\theta_k}, \theta_{k+1}, \ldots, \theta_N)$$
$$\downarrow$$
$$(\theta_1, \theta_2, \ldots, \underline{\theta_k}, \underline{\underline{\theta_i}}, \theta_{k+1}, \ldots, \theta_N)$$

where, a new element θ_i is inserted to the original permutation $\boldsymbol{\pi}$, and the total ply numbers N changes from N to $N+1$ with $\boldsymbol{\pi} = (\theta_1, \theta_2, \ldots, \theta_{k-1}, \underline{\theta_k}, \ldots, \theta_{N+1})$.

The PS algorithm is presented in Appendix A, and the process is also summarized as follows.

Step 1 *(Initialization):* Randomly generate M initial populations with constraints of the ply numbers of each ply orientation, and then evaluate the objective function value λ_c. The initial values of the generation C and the searching position of the current layer P are set to be 1 and $N/2$ (here, $P = N/2 - k + 1$ in the sequence, where k is the stacking position in Fig.2), respectively. Execute the following steps from the first initial population $u = 1$.

Step 2 *(Swap):* Sequentially compare the ply angle θ at the searching position P with other positions of $P-1, P-2, \ldots, 2, 1$. If same ply orientation is detected, jump to the next stacking position; otherwise, if a different ply angle appears, swap them and evaluate the objective function λ_c.

Step 3 *(Optimal sequence selection):* Repeat the swap process until the ply angle at current searching position P has been compared with all other stacking positions of $P-1, P-2, \ldots, 2, 1$. Compare the objective function values for all new generated permutations. For the studied searching position P, define the stacking sequence with maximum λ_c as the initial stacking sequence for the next searching position $P-1$.

Step 4 *(Convergence judgment):* Searching continues until two generations' permutations are the same. It indicates that the stacking sequence converges to the optimal solution.

Step 5 *(Search for all M initial populations):* After reaching convergence for the initial population u, assume $u = u+1$, $C = 1$ and $P = N/2$, and turn to Step 2, find the optimal stacking sequence for the next initial population $u = u+1$. Repeat until all M initial populations have been searched and optimized.

Step 6 *(Global optimum selection):* Comparing all obtained optimal stacking sequences for M initial populations, choose the one maximizing the objective function as the optimum stacking sequence and save it into an array PS(k), where k is the stacking position.

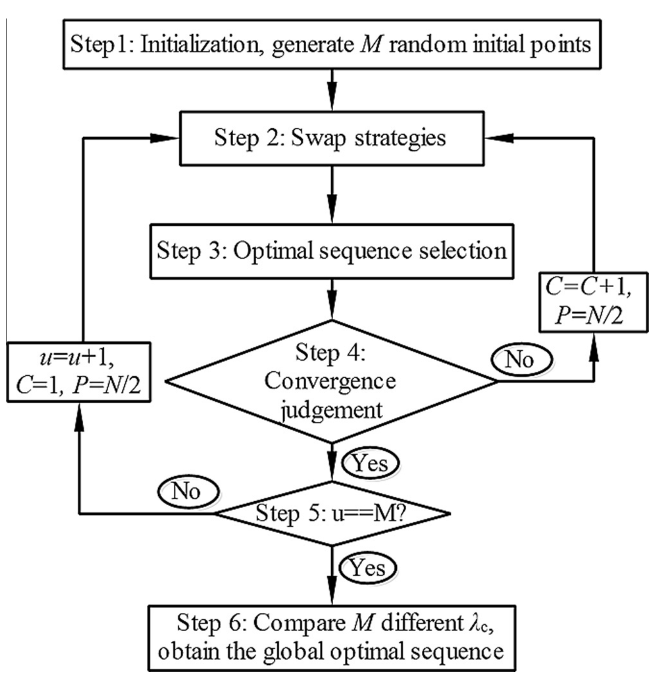

Fig. 3. Flowchart of the permutation search (PS) algorithm.

It should be pointed out that for any initial population u with P stacking positions, the maximum number of evaluations in one generation C appears if all stacking positions have different ply orientations and the maximum evaluation number can be calculated by

$$E_c = P(P-1)/2 \tag{26}$$

where, $P = N_0 + N_{45} + N_{90}$ is the total stacking positions.

Theoretically, considering the ply numbers of all orientations, the design space of the problem is

$$E_t = P!/(N_0!N_{45}!N_{90}!) \tag{27}$$

According to Eqs. (26) and (27), the number of evaluation E_c depends only on the total stacking positions. Once the number of stacking positions has been given, the number of evaluations needed at each position can be determined, which is independent of the number of orientation angles. Due to the only dependence of evaluation times on the stacking position, we believe that the more the ply orientation angles are, the more efficient of the proposed algorithm. Also, from Eq. (27), it is clearly that the searching space of PS algorithm is far less than the design variable space, which will be discussed later. Intrinsically, the "swap" and "selection" strategies (step 2 and step 3 in PS algorithm) designed for the proposed algorithm are based on the theoretical Eqs. (11) and (12), which makes the performance of the algorithm efficient in stacking sequence design. From the point of computer science, due to its rapid convergence the proposed permutation search algorithm can be far more effective in solving stacking sequence optimization problems.

The flowchart of the PS algorithm is shown in Fig. 3 and an example to illustrate the PS searching process is given in Appendix B.

3.2. Repair strategy

Previous works [8,18] indicated that dealing with continuous constraints by a penalty approach was significantly less efficient than deal with the solutions directly. In Section 3.1, the global optimal stacking sequence has been obtained from the PS analysis, however, in which all related constraints have been ignored. In this section, constraints are introduced and imposed directly to the global optimal solution passed from the PS analysis. At this stage, to minimize the change to the obtained maximum buckling load factor, it is preferred to perform as fewer operations as possible. In other words, significant variation of the optimum results from PS analysis should be avoided. In this work, the "delete" and "insert" operators are mainly adopted.

The following steps illustrate the repair strategy.

Step 1 Check the optimal permutation passed from the PS analysis (PS(k), k is the stacking position) for their conformance to the specified constraints, and save all the plies orientations, positions and the total position numbers of these permutations that violate constraints into an array named by "Pos_violate".

Step 2 Judge whether there are any non-zero elements in the array "Pos_violate". If all elements are zero, end the repair process and the global result is the optimum stacking sequence. Otherwise, start from the first non-zero element in the array "Pos_violate", which corresponds to the left-most position of contiguity violation; find the closest (both

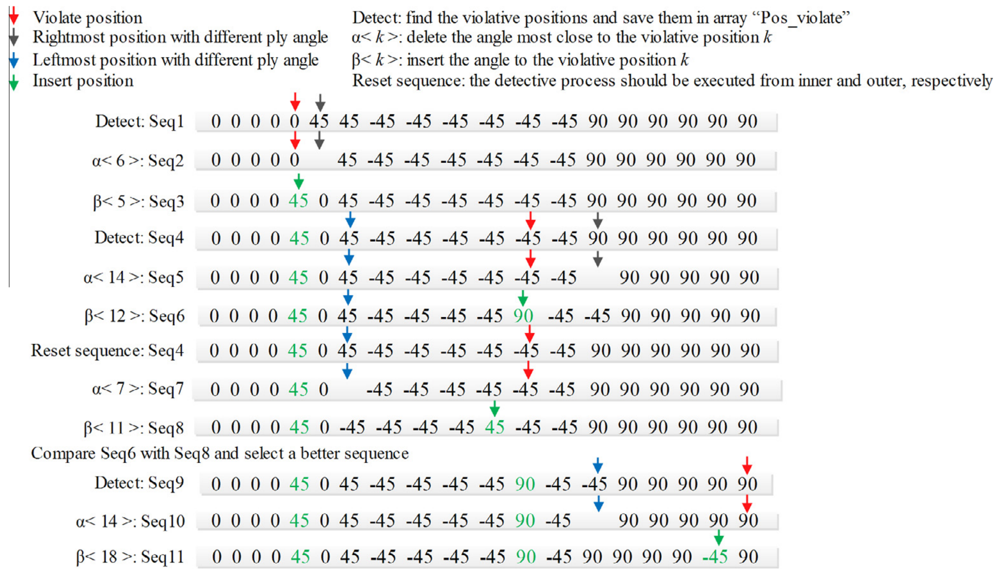

Fig. 4. Illustrations of the repair strategy for dealing with continuity constraints.

Table 1
Physical properties of the graphite-epoxy plate.

Material properties	
Young's modulus (longitudinal)	$E_1 = 18.5 \times 10^6$ psi (127.59 GPa)
Young's modulus (transverse)	$E_2 = 1.89 \times 10^6$ psi (13.03 GPa)
Shear modulus	$G_{12} = 0.93 \times 10^6$ psi (6.4 GPa)
Poisson's ratio	$v_{12} = 0.3$
Ply thickness	$T = 0.005$ in (0.0127 cm)

Table 2
Optimized ply numbers of composite laminate panels with various load cases.

Load case	Loading (lb/in)			(N_0, N_{45}, N_{90})	Total
	F_x	F_y	F_{xy}		
1	−20,000	−2000	1000	(9, 18, 9)	36
2	−15,000	−2000	1000	(8, 17, 8)	33
3	−10,000	−2000	1000	(7, 15, 7)	29
4	−5000	−2000	1000	(6, 12, 6)	24
5	0	−2000	1000	(4, 8, 4)	16
6	0	−16,000	8000	(8, 16, 8)	32
7	15,980	−14,764	10,160	(9, 8, 13)	30
8	−16,657	1963	828	(13, 7, 15)	35

Table 4
Computational efficiency of PS.

Case	N_{total}	(N_0, N_{45}, N_{90})	E_{des}	E_c	C	E_{obj}	E_{obj}/E_{des}
1	36	(9, 18, 9)	4.412×10^{14}	730	3	256	5.80×10^{-13}
2	33	(8, 17, 8)	1.5017×10^{13}	528	3	226	1.50×10^{-11}
3	29	(7, 15, 7)	2.6618×10^{11}	406	3	196	7.36×10^{-10}
4	24	(6, 12, 6)	2.4986×10^{9}	276	3	139	5.56×10^{-8}
5	16	(4, 8, 4)	9.0090×10^{5}	120	2	63	6.99×10^{-5}
6	32	(8, 16, 8)	7.7359×10^{12}	496	3	207	2.68×10^{-11}
7	30	(9, 8, 13)	2.9114×10^{12}	435	3	570	1.96×10^{-10}
8	35	(13, 7, 15)	2.5178×10^{12}	595	3	316	1.26×10^{-12}

rightmost and leftmost) positions with different ply orientations. Then, replace the ply angle of rightmost position with different ply angle by that of the first violation position, which corresponding to the first non-zero element in "Pos_violate", and save the obtained permutation stacking sequence into the permutation array R $(k+1)$. At the same time, replace the leftmost ply with different orientation angles by the same violation ply at array "Pos_violate" and save the stacking sequence into the permutation matrix L $(k+1)$. Calculate the corresponding objective

Table 3
Optimum stacking sequence from the permutation search (PS) for eight loading cases.

Case	Stacking sequence without and with continuity constraints	Number of evaluations	Failure load factor λ_c
1	$[(\pm45)_{18}/(90_2)_9/(0_2)_9]_s$	256	0.9482
	$[(\pm45)_{18}/(90_2)_2/0_2/(90_2)_2/0_2/(90_2)_2/(0_2)_2/90_2/(0_2)_2/90_2/(0_2)_2/90_2/0_2]_s$	262	0.9481
2	$[(\pm45)_{17}/(90_2)_8/(0_2)_8]_s$	226	0.9484
	$[(\pm45)_{17}/(90_2)_2/0_2/(90_2)_2/0_2/90_2/0_2/90_2/(0_2)_2/90_2/(0_2)_2/90_2/0_2]_s$	232	0.9483
3	$[(\pm45)_{15}/(90_2)_7/(0_2)_7]_s$	196	0.9100
	$[(\pm45)_{15}/(90_2)_2/0_2/90_2/0_2/90_2/(0_2)_2/90_2/0_2/90_2/0_2/90_2/0_2]_s$	203	0.9097
4	$[(\pm45)_{12}/(90_2)_6/(0_2)_6]_s$	139	0.8713
	$[(\pm45)_{12}/(90_2)_2/0_2/90_2/0_2/90_2/(0_2)_2/90_2/0_2/90_2/0_2]_s$	143	0.8705
5	$[(\pm45)_8/(90_2)_4/(0_2)_4]_s$	63	0.7810
	$[(\pm45)_8/(90_2)_2/0_2/90_2/(0_2)_2/90_2/0_2]_s$	65	0.7756
6	$[(\pm45)_{16}/(90_2)_8/(0_2)_8]_s$	207	0.7810
	$[(\pm45)_{16}/(90_2)_2/(0_2)_2/90_2/(0_2)_2/90_2/(0_2)_2/90_2/0_2/90_2/0_2]_s$	211	0.7740
7	$[90_2/(\pm45)_2/(90_2)_8/\pm45/90_2/\pm45/90_2/(\pm45)_3/(90_2)_2/\pm45\ /(0_2)_9]_s$	570	1.6246
	$[90_2/\pm45/(90_2)_2/\pm45/(90_2)_2/\pm45/(90_2)_2/\pm45/(90_2)_2/\pm45/(90_2)_2/\pm45/(0_2)_2/\pm45/(0_2)_2/90_2/(0_2)_2/90_2/(0_2)_2/\pm45/0_2]_s$	579	1.6153
8	$[(\pm45)_7/(90_2)_{15}/(0_2)_{13}]_s$	316	1.1024
	$[(\pm45)_7/(90_2)_2/0_2/(90_2)_2/0_2/(90_2)_2/0_2/(90_2)_2/0_2/(90_2)_2/(0_2)_2/90_2/(0_2)_2/90_2/0_2/90_2/(0_2)_2/90_2/0_2/90_2/0_2]_s$	333	1.1022

function values of these permutations in matrixes $R(k+1)$ and $L(k+1)$, compare and save the one gives larger objective value into PS $(k+1)$, which will be used as the initial permutation for next operation. Note that the ply angle of the violation position is replaced by that of the leftmost or rightmost position with different ply orientations, which is desirable since it minimizes the effect on the buckling load passed from the PS analysis.

Step 3 Repeat **Step 2** until all ply angles of the violated positions are replaced by those of the closest positions with different ply orientations. During this process, it should be ensured that the buckling load factor from PS process is not significantly changed.

An example to demonstrate the repair strategy to deal with constraints is shown in Fig. 4.

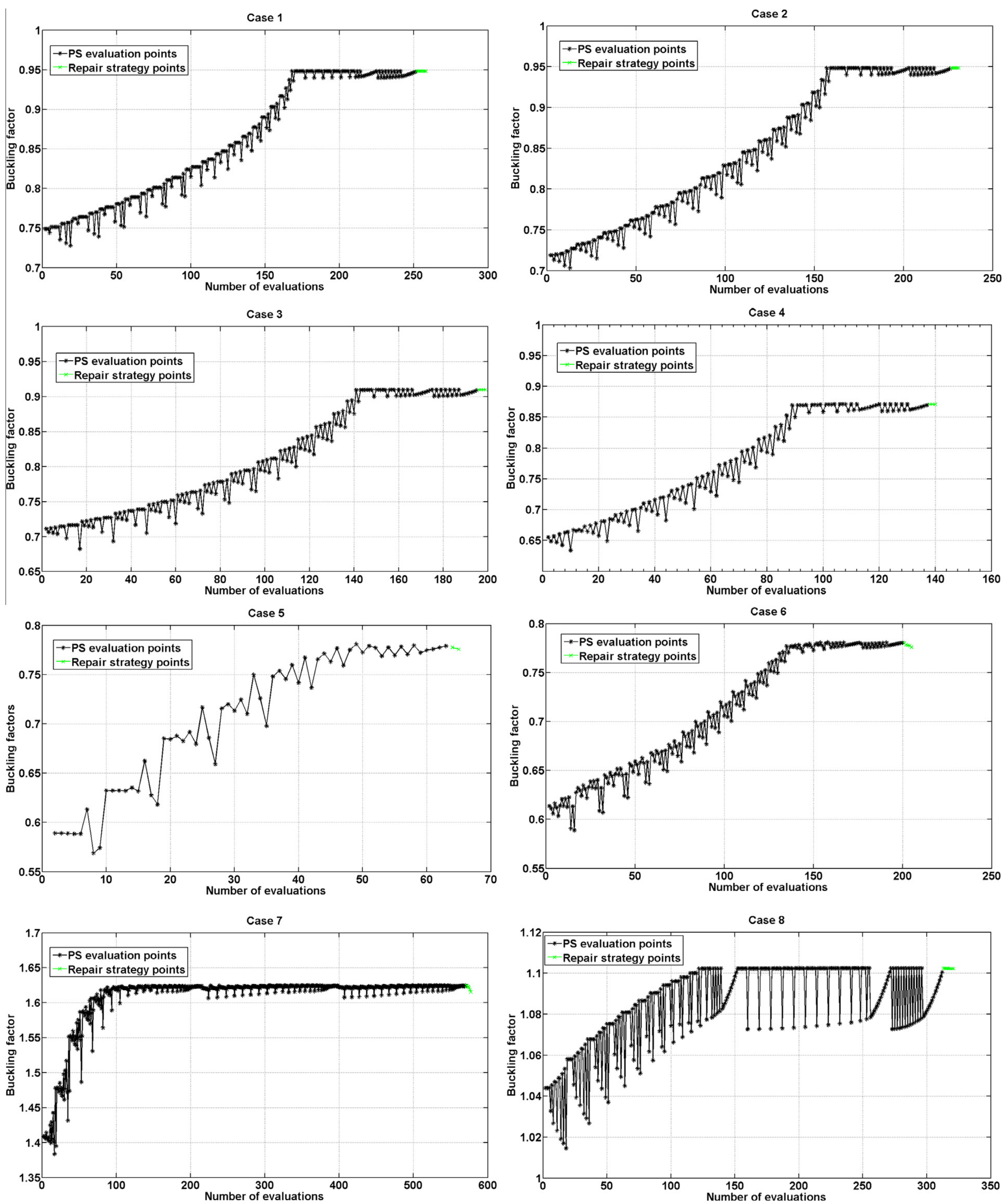

Fig. 5. Illustrations of the search process under various load cases. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 5
Comparison of computational efficiency of the PS and three genetic algorithms (GAs) without continuity constraints.

Case	Average number of evaluations			
	Required for 80% reliability [16]			Required for 99% reliability
	SGA	PMX	GR	PS
1	10,432	1328	1184	256
2	8600	1224	856	226
3	5216	1024	776	196
4	3304	824	608	139
5	1672	560	408	63
6	5112	848	480	207
7	2176	336	360	570
8	5024	840	296	316

See reference [16]: SGA: standard GA; PMX: partially mapped crossover; GR: gene-rank crossover.

Table 6
Comparison of computational efficiency of the PS and three GAs with continuity constraints.

Case	Number of evaluations			
	Required for 80% reliability [16]			Required for 99% reliability
	SGA	PMX	GR	PS
1	672	792	456	262
2	536	792	400	232
3	368	658	352	203
4	224	496	304	143
5	80	272	184	65
6	400	552	416	211
7	3512	336	288	579
8	48	696	352	333

See reference [16]: SGA: standard GA; PMX: partially mapped crossover; GR: gene-rank crossover.

4. Results and discussion

While complex composite structures could have been used to validate the efficiency of the proposed algorithm, the work in this paper focuses on the optimization of a composite panel for specified ply numbers and in-plane loadings studied by Liu et al. [16] as a benchmark problem. The symmetric and balanced composite laminate panel is simply supported with in-plane normal and shear loadings fF_x, fF_y, and fF_{xy}, applied. Both the length and width of the plate are assumed to be 24 inches. It should be pointed out that the proposed PS algorithm can be applied to a plate of various length to width ratio since the expressions of the critical buckling load factors $\lambda_{c,n}$ and $\lambda_{c,s}$ (Eqs. (12) and (14)) are applicable for composite laminates with various aspect ratios. The ply thickness of 0.005 inches is used. In addition, the total number of 0°, ±45°, and 90° plies is fixed. The material properties of the composite laminate panel are presented in Table 1.

As described above, the constraints for manufacturing requirements and composite lay-up rules are usually applied to the composite laminates for optimizing their stacking sequences [10,16,24,29,30]. In this problem, the buckling load of the composite panel for given total ply number of 0°, ±45, and 90° is maximized considering the following constraints: (1) The maximum continuous number of layers with same fiber orientation angle must be no more than four; and (2) The sum of the ply numbers of different orientations must be equal to the given total number of 0°, ±45°, and 90° plies [16,31].

In this work, eight optimization cases with different in-plane normal and shear loadings are solved (Table 2). The optimized results of ply numbers and the corresponding loading conditions are also presented in Table 2. Note that the ply numbers from the overall optimization process have been rounded into integers, which are used as one of the constraints in the following permutation process.

In this work, by considering the symmetry and balance requirements and the reduce of design space, the constraints of ply number and contiguity are incorporated into gene coding, where 0° and 90° plies are assumed in pair while ±45° plies stack together. Thus, continuous constraint is applied only to 0° and 90° plies and only a quarter of total plies in the laminate are considered. The number of evaluation is calculated by performing 100 optimization runs, and averaging the numbers of evaluations of these reached the optimum at each initial point. Before the execution of PS, the initial parameters are set as: initial permutation $M = 1$ and the maximum generation is set as MAX_C = 10. The detailed optimum stacking sequences as well as the numbers of evaluations and failure load factors are given in Table 3 for both without and with continuous constraints.

As shown in Table 3, without considering continuous constraints the optimal sequence obtained by PS algorithm often violates the continuous constraints of layers with same fiber orientation angle should be no more than four. In contrast, by performing the repair strategy, the continuous constraints can be met within a few number of repair evaluations. Under the principle of minimal changing of the global optimal sequence, the decrease in the objective value λ_c due to the consideration of continuous constraints is nonsignificant. Also, since the repair strategy is designed to minimize the number of permutation operations, it is seen from Table 3 that the difference of the maximum buckling load factors is neglectable between these with and without repair operations.

From Eqs. (26) and (27), both the design space and the search space can be evaluated. For the unconstrained situations, the results are presented in Table 4, in which N_0, N_{45}, N_{90}, are the ply numbers of 0_2, ±45 and 90_2 layers, respectively, N_{total} is the total ply number, E_c is the theoretical value of design space in one generation C of PS algorithm, which is calculated by Eq. (26), E_{des} is the scale of design space which is evaluated by Eq. (27), E_{obj} is the evaluation time in finding optimum. The numerical efficiencies of the proposed algorithm can be clearly demonstrated by comparing the analysis of objective function E_{obj} with the design space E_{des}. From Table 4, it is clearly that the numbers of analyses are reduced exponentially in all studied cases. In fact, in PS algorithm, the buckling loading factor is formulated as a function of ply angles at each stacking position, which provides a direct search way for predicting the optimum ply orientation at each stacking position.

The search process and the variations of buckling load factors are shown with respect to the number of evaluations in Fig. 5, where black star and green cross lines represent the PS results without and with repair strategy, respectively. It is clear in Fig. 5 that only a few generations are performed to reach an optimal solution, implying the fast convergence and high accuracy of PS algorithm.

In this work, the efficiencies of four different methods are discussed in terms of the number of evaluations to reach convergence. Also, the average of evaluation numbers to obtain the global optimum at a given level of reliability are listed in Tables 5 and 6 for problems with and without constraints, respectively, where "PS" represents the proposed PS algorithm, "SGA" represents "standard GA" methods, while "PMX" and "GR" represent "partially mapped crossover GA" and "gene-rank crossover GA" [16], respectively.

First, 100 optimization runs are performed for each random generated initial population. Then check how many runs reach the optimum buckling load factor. At the same time, the number of evaluation for each run is obtained, which is defined as the number of evaluations satisfying given reliability requirement during one complete run. Herein, the reliability is defined by the percentage of solutions with the buckling load factor greater than or equal to the optimal solution. Moreover, the average numbers of evaluations presented in Tables 5 and 6 are obtained by running 100 searching for each load case and the numbers of evaluations are then determined by satisfying the specified reliability requirement. Note that, based on our numerical experiments it is hard for "SGA", "PMX" and "GR" to obtain reliability higher than 80%. In contrast, there is no difficulty for PS to reach 99% or higher reliability. Therefore, 99% reliability is used as a threshold value for the proposed PS method, while 80% reliability is adopted for the three GA algorithms. Comparison of reliability validates the robust of the PS method and also indicates the sensitivity of other three GA methods in the selection of initial population or other parameters.

From Tables 5 and 6 it is clear that PS performs much better than the studied three GA algorithms. Generally speaking, the PS method has faster convergence rate (Fig. 5) and higher reliability except for case 7. Although the GR performs better than PS in case 7, however, PS can lead to higher buckling loading factor. Moreover, the evaluations are calculated for 99% reliability for PS, which is also much higher than that of GAs.

5. Conclusions

An effective permutation search (PS) algorithm as well as its corresponding repair strategy to improve the numerical efficiency of the constrained permutation optimization of composite laminates has been developed. Based on the formulation of flexural stiffness parameters in terms of the ply orientation, the optimization has been reduced to identify the optimum ply angle at each stacking position. In the PS algorithm, the ply orientation at each stacking position, which governs the buckling properties of composite laminate structures, is taken as the design variable. Meanwhile, a modified repair strategy is developed to handle the continuous constraints passed from the overall structure design stage. To validate the proposed searching algorithm, the stacking sequence optimization of a symmetric composite panel with specified ply numbers and loading conditions is investigated. The accuracy and computational efficiency of the present PS algorithm are then compared with the standard genetic algorithm (GA), and two improved permutation GA methods. From results of the number of evaluation and the optimal buckling load factor, it is observed that the PS algorithm accompany with an appropriate repair strategy can greatly decrease the number of processes to reach convergence while keeping comparable accuracy. Hence, the proposed method is numerically efficient in the stacking sequence optimization of composite laminates with continuous constraints. The algorithm can be applied to composite panels with various aspect ratios and a large number of ply orientations, which makes it can readily be extended to cases with complex structures. In addition, considering the only dependence of evaluations on the stacking position, the high efficiency characteristic of the algorithm can be more apparent for complex composite structures.

Acknowledgments

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Appendix A. Permutation search (PS) algorithm

The strategies of the present PS algorithm are summarized in Algorithm 1, where N is the total number of plies, C is the generations, P is the stacking position of the current layer, CP is the stacking position to be compared in the current layer, M and Max_C are the number of initial random sequences and the initial iteration times predefined, respectively.

Algorithm 1: Permutation search algorithm

Initialization: Generate M initial populations according to the ply numbers of each orientation angle, and set the maximum iteration times be MAX_C
For u = 1: M
C = 1
While C <= MAX_C
  Initial_sequence = Random_Seq (u)
  Swap strategies:
  For $P = N/2: -1: 2$
For $CP = P - 1: -1: 1$
If $\theta_p \neq \theta_{cp}$ **Then**
    swap the ply angles of positions: $\sigma < P, CP >$, evaluate the objective value λ_c and save.
End if
End for
  Select the sequence with best objective value λ_c as the initial stacking sequence for the comparison of next stacking position P.
End for
Select the optimal sequence for the Cth iteration
Best_sequence (C) = Optimal_sequence_C
Convergence judgement:
If Best_sequence (C) = = Best_sequence (C − 1) **Then**
  The algorithm converges to an optimal solution.
  Optimal_sequence (u) = Best_sequence (C)
Break
Else
  C = C + 1
Endif
End While
End For
Select the global optimal sequence of the M Optimal_sequences.

Appendix B. Example of optimize stacking sequence design of laminate panel using the PS algorithm

In this example, considering the symmetry of the studied panel, the ply numbers are assumed to be $(N_0, N_{45}, N_{90}) = (3, 4, 2)$ in half laminate, that means the total ply numbers of the panel N_{total} is 18. However, it does not require that pairs of the same orientation layers stack together. The coefficient D_{11} of the flexural stiffness is adopted as the objective function. It is assumed that the initial population is randomly generated and has the form of $[(90)_2/-45/(0)_2/(45)_2/0/-45]_s$. Although, a panel with ply orientation angles of 0°, ±45°,and 90° is here adopted to validate the algorithm, however, combination of any ply orientation angles can be analyzed with the proposed algorithm.

To obtain the optimum stacking sequence of the example panel, the following processes are performed using the PS algorithm:

Search for the last stacking position P9 (the underlined red characters represent the identified positions where ply orientations will be interchanged in the next permutation operation, the blue characters represent those have been interchanged in the last permutation operation, and the green characters represent the optimal ply orientations and they will not be considered in further process)

	P1	P2	P3	P4	P5	P6	P7	P8	P9
Initial stacking sequence- P9:	90	90	-45	0	0	45	45	0	-45
Permutation 1 $(\sigma < P8, P9 >)$:	90	90	-45	0	0	45	45	-45	0
Permutation 2 $(\sigma < P7, P9 >)$:	90	90	-45	0	0	45	0	-45	45
Permutation 3 $(\sigma < P5, P9 >)$:	90	90	-45	0	45	45	0	-45	0
Permutation 4 $(\sigma < P3, P9 >)$:	90	90	0	0	45	45	0	-45	-45
Permutation 5 $(\sigma < P2, P9 >)$:	90	-45	0	0	45	45	0	-45	90

Based on the calculations of the objective function for permutations 1–5, it is found that the stacking sequence of permutation 5 leads to greatest value of flexural stiffness coefficient D_{11}, which is thus used as the initial stacking sequence for the next search process.

Search for the second last stacking position P8:

	P1	P2	P3	P4	P5	P6	P7	P8	P9
Initial stacking sequence-P8:	90	-45	0	0	45	45	0	-45	
Permutation 6 $(\sigma < P7, P8 >)$:	90	-45	0	0	45	45	-45	0	
Permutation 7 $(\sigma < P6, P8 >)$:	90	-45	0	0	45	0	-45	45	90
Permutation 8 $(\sigma < P4, P8 >)$:	90	-45	0	45	45	0	-45	0	
Permutation 9 $(\sigma < P2, P8 >)$:	90	0	0	45	45	0	-45	-45	
Permutation 10 $(\sigma < P1, P8 >)$:	-45	0	0	45	45	0	-45	90	

The stacking sequence of permutation 10 in this search process gives the greatest value of D_{11}, and thus is used as the initial stacking sequence for the next search process.

Search for the stacking position P7:

	P1	P2	P3	P4	P5	P6	P7	P8	P9
Initial stacking sequence:-P7	-45	0	0	45	45	0	-45		
Permutation 11 $(\sigma < P6, P7 >)$:	-45	0	0	45	45	-45	0		
Permutation 12 $(\sigma < P5, P7 >)$:	-45	0	0	45	0	-45	45	90	90
Permutation 13 $(\sigma < P3, P7 >)$:	-45	0	45	45	0	-45	0		
Permutation 14 $(\sigma < P1, P7 >)$:	0	0	45	45	0	-45	-45		

The stacking sequence of permutation 14 in this search process gives the greatest value of D_{11}, and thus is used as the initial stacking sequence for the next search process.

Search for the stacking position P6:

	P1	P2	P3	P4	P5	P6	P7	P8	P9
Initial stacking sequence-P6:	0	0	45	45	0	-45			
Permutation 15 $(\sigma < P5, P6 >)$:	0	0	45	45	-45	0	-45	90	90
Permutation 16 $(\sigma < P4, P6 >)$:	0	0	45	0	-45	45			
Permutation 17 $(\sigma < P2, P6 >)$:	0	45	45	0	-45	0			

The stacking sequence of permutation 16 is used as the initial stacking sequence for the next search process.

Search for the stacking position P5:

	P1	P2	P3	P4	P5	P6	P7	P8	P9
Initial stacking sequence-P5:	0	0	45	0	-45				
Permutation 18 $(\sigma < P4, P5 >)$:	0	0	45	-45	0	45	-45	90	90
Permutation 19 $(\sigma < P3, P5 >)$:	0	0	0	-45	45				
Permutation 20 $(\sigma < P2, P5 >)$:	0	45	0	-45	0				

The stacking sequence of permutation 19 is used as the initial stacking sequence for the next search process.

Search for the stacking position P4:

	P1	P2	P3	P4	P5	P6	P7	P8	P9
Initial stacking sequence-P4:	0	0	0	-45	45	45	-45	90	90
Permutation 21 ($\sigma < P3, P4 >$):	0	0	-45	0					

It is found that the value of objective function produced by the initial stacking sequence for position P4, that is the stacking sequence of permutation 19, is greater than permutation 21. Therefore, the stacking sequence of permutation 19 is still the preferred population and is used for stacking position P3. According to the PS strategy, there is no need to operate the results of permutation 19. In other words, since the largest target value is obtained by the stacking sequence passed from the top step (permutation 19), the sequence will be kept. However, this is the first generation of PS, another generation is performed to verify the optimal solution and it will find the same solution, indicating that the PS convergent to the optimum in two generations. Therefore, the optimum stacking sequence for the studied problem is $[(0)_3/-45/(45)_2/-45/(90)_2]_s$ and the explore space of PS is 37. Note that in this example the continuous constraint has not been considered.

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